

Global research trends and emerging directions in algebraic thinking in education from 1991 to 2024

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Abstract

Algebraic thinking is a foundational aspect of mathematics education, enabling learners to recognize patterns, generalize, reason abstractly, and solve complex problems. This study presents a global bibliometric analysis of 293 Scopus-indexed publications (1991–2024), using VOSviewer and Biblioshiny to examine publication trends, influential sources, collaboration networks, and thematic developments. Results reveal an annual growth rate of nearly 9%, with three phases of development: early exploration (1991–2004), expansion and technology integration (2005–2018), and interdisciplinary connections (2019–2024). The United States, Canada, and Australia dominate in scholarly impact, while Indonesia and China show rising output but limited citation influence. International collaboration remains modest, with only 16% of studies involving cross-national authorship. Keyword analysis highlights five major themes: early algebra, problem-solving pedagogy, mathematical modeling, interdisciplinary integration, and teacher education. This review identifies key trends and gaps, offering directions for strengthening collaboration and integrating technology into algebra education.

Introduction

Algebra occupies a central position in the school mathematics curriculum because it develops logical thinking and deductive reasoning, thereby enhancing students' overall mathematical competence (Thorpe, 2018). Through learning, algebraic thinking naturally emerges as a cognitive process closely connected to algebra itself and contributes to a coherent and sustainable understanding of mathematics. It is commonly defined as the capacity to generalise (Mason, 1996; Fred et al., 2022), reason, represent (Kieran, 1992), and justify abstract mathematical relationships (Mason, 1996). Among these, generalisation plays a pivotal role by enabling learners to extend patterns into general rules, while reasoning, representation, and proof complement this process to form a holistic framework of algebraic understanding. Hence, algebraic thinking extends far beyond symbolic manipulation to include interpreting and applying mathematical ideas, recognising multiple forms of representation, and constructing logical inferences for problem-solving, prediction, justification, and modelling (Chimoni et al., 2023a; Coles & Ahn, 2022; Hodgen et al., 2018). Despite its recognised importance, many students continue to face difficulties in developing algebraic thinking (Asquith et al., 2007; Herscovics & Linchevski, 1994; MacGregor & Stacey, 1995). Prior research has often focused on isolated aspects-such as task design (Afonso, 2019; Guler & Celik, 2021) or the role of number sense and symbolic understanding (Somasundram, 2021; Walkoe, 2014)-without providing a comprehensive or systematic view of the field.

Recent reviews have offered valuable insights but remain limited in scope and depth. For example, A Systematic Review on Algebraic Thinking in Education (2013–2021) concentrated mainly on elementary and lower-secondary levels, providing descriptive analyses with limited implications for classroom practice or policy (Sibgatullin et al., 2022). Similarly, Research Trends on Early Algebra in the Middle School (Fardian et al., 2024) examined 234 Scopus-indexed journal articles (2013–2024) but restricted its scope to journal-based studies. The citation impact of these reviews remains modest, and their

analyses are largely confined to publication trends and keyword mapping rather than deeper pedagogical perspectives.

These limitations reveal several gaps in the existing literature: (i) a predominance of descriptive rather than explanatory analyses of how algebraic thinking develops; (ii) fragmented exploration of influencing factors, with few large-scale or integrative studies; (iii) limited cross-national comparisons; (iv) bibliometric reviews with narrow temporal or database coverage; and (v) insufficient attention to practical implications-particularly the role of teachers in nurturing algebraic reasoning.

To address these issues, the present study conducts a comprehensive global bibliometric review of algebraic thinking research published between 1991 and 2024. The analysis is guided by four key research questions:

RQ1: How has research on algebraic thinking evolved over time and across regions?

RQ2: Which sources (journals, authors, and articles) have contributed most significantly, and how is their scholarly impact reflected in citation and ranking patterns?

RQ3: What collaboration networks among authors, institutions, and countries are shaping the current research landscape?

RQ4: What dominant and emerging themes can be identified, and what implications do they hold for mathematics education?

By addressing these questions, this paper aims to provide a holistic overview of the global research landscape on algebraic thinking, generate evidence based insights to guide future investigations, and inform pedagogical innovation within mathematics education.

Methodology

This study follows a standard scientific mapping approach consisting of five key stages: research design, data collection, data analysis, visualization of results, and interpretation of findings, as suggested in previous bibliometric studies (Börner et al., 2005; Zupic & Čater, 2015).

The research begins by establishing the main question: how recent publications on algebraic thinking in mathematics education, indexed in the Scopus database, can be quantitatively examined using bibliographic data.

The data collection process involves three main steps: gathering, screening, and refining the dataset.

In the first step, data were retrieved from Scopus, one of the most widely used databases for bibliometric research due to its extensive coverage compared with other sources such as Web of Science, Google Scholar, Microsoft Academic, and Dimensions (Moral-Muñoz et al., 2020; Mongeon & Paul-Hus, 2016). The search was conducted using the advanced search function on the Scopus platform. Keywords

related to algebraic thinking, including “thinking algebra,” “algebraic thinking,” and “algebraic reasoning,” were applied to titles, abstracts, and keywords. To ensure relevance to education, additional terms such as “teach,” “learn,” and “educate” were included. The search was limited to articles and conference papers written in English. Publications up to the year 2023 were considered, as records for 2024 may not yet be fully indexed due to database update delays. The search was performed on October 21, 2024, yielding an initial dataset of 388 documents.

In the second step, the dataset was screened by reviewing titles, abstracts, and keywords to remove studies that were not directly related to the research focus. After this filtering process, 293 documents remained for analysis.

In the final step, data cleaning was conducted to correct inconsistencies in the dataset, including variations in author names and institutional affiliations.

For data analysis and visualization, a combination of bibliometric techniques was applied. Descriptive statistics were used to summarize the overall characteristics of the dataset, while publication trends over time were examined to identify the development of the research field. Contributions from different countries were assessed based on publication output and citation impact. The analysis was supported by tools such as Scopus analytics, VOSviewer, and Biblioshiny.

Results

General Publication Characteristics

Table 1
General information about data

Description	Results
MAIN INFORMATION ABOUT DATA	
Timespan	1991:2024
Sources (Journals, Books, etc)	125
Docs	293
Annual Growth Rate %	8.96
Document Average Age	8.04
Average citations per doc	17.73
References	10552
DOCUMENT CONTENTS	
Keywords Plus (EN)	352
Author's Keywords (DE)	588
AUTHORS	
Authors	661
Authors of single-authored docs	50
AUTHORS COLLABORATION	
Single-authored docs	56
Co-Authors per Doc	2.86
International co-authorships %	16.04
DOCUMENT TYPES	
Article	211
Book chapter	23
Conference paper	59

The primary information about the dataset discussed in the article is presented in Table 1. The total number of publications in the collection is 293, which includes 211 articles, 23 book chapters, and 59 conference papers published across 125 different sources from 1991 to 2024. The total number of citations recorded during the study was 5195, averaging 17,73 per document. The h-index of this collection is 38, indicating that out of the 293 publications under study, 38 have been cited at least 38 times. A total of 661 authors have contributed to publications in this field, averaging 2.26 authors per

publication. Fifty authors published research independently, resulting in 56 scientific publications authored by a single individual, which accounts for 19.11% of the total publications.

Publication Trends Over Time

The number of publications and cumulative citations on algebraic thinking in mathematics education is shown in Fig. 1. This study presents a global bibliometric analysis of 293 publications indexed in the Scopus database between 1991 and early 2024. The results show that the field has expanded significantly over the past three decades, though with some noticeable fluctuations. Three phases of development can be identified. The initial phase (1991–2004) produced only a few scattered studies, mainly attempting to distinguish arithmetic reasoning from algebraic reasoning. During the growth phase (2005–2018), publication output increased steadily, reflecting growing attention to early algebra and the integration of technology into teaching and learning. The recent phase (2019–2024) reached its highest output in 2018 and 2021, followed by a temporary decline in 2023 and a modest recovery in early 2024. Cumulative citations have continued to increase but appear to have stabilised since 2021, suggesting that while research activity remains vibrant, the field is likely moving into a stage of consolidation and maturity rather than rapid expansion.

(Source: data collection results of the authors' group) (dup: abstract ?)

Geographical Distribution of Publications and Citations

According to data from Scopus, authors from 184 countries and territories participated in the studies on this topic. The number of publications from each country/territory is recorded when at least one author or co-author is associated with the agency's name and address in that location country/territory.

Table 2

Top nine countries/territories with the most significant number of publications on the topic of algebraic thinking

Rank	Country/ Territory	TP	%	TC	Citation Ranking	%	TC/TP
1	United States	119	40.60%	2121	1	40.80%	17.8
2	Indonesia	31	10.60%	26	8	0.50%	0.8
3	Spain	27	9.20%	59	5	1.10%	2.2
4	Australia	17	5.80%	200	3	3.80%	11.8
5	Canada	14	4.80%	529	2	10.20%	37.8
6	Turkey	12	4.10%	32	6	0.60%	2.7
7	United Kingdom	10	3.40%	66	4	1.30%	6.6
8	RBrazil	7	2.40%	17	9	0.30%	2.4
9	China	7	2.40%	29	7	0.60%	4.1

TP: Total publications TC: Total citations CR: Citation Ranking

(Source: data collection results of the authors' group)

(Source: data collection results of the authors' group)

The top nine countries and territories with the highest number of publications on algebraic thinking are shown in Table 2 and Fig. 2. The United States leads with 119 publications (10.6%) and 2,121 citations (40.8%), nearly four times higher than Indonesia, the second country in output, with an average of 17.8 citations per document. This strong citation performance highlights the global influence and quality of U.S. research. Canada ranks second in citation impact with 14 publications (fifth in volume) but an average of 37.8 citations per article, the highest in the table. Australia is fourth in output with 17 publications (5.8%) and third in total citations (200), averaging 11.8 citations per paper. Together, the U.S., Canada, and Australia have played a leading role in shaping research directions.

Europe also contributes significantly, with Spain publishing 27 articles (9.2%) and receiving 59 citations (2.2 per paper), the UK contributing 10 publications (3.4%) with 66 citations (6.6 per paper), and Turkey producing 12 publications (4.1%) with 32 citations (2.7 per paper). Among emerging contributors, Indonesia stands out with 31 publications (10.6%), though with modest citation impact (0.8 per paper). By contrast, China and Brazil, each with seven publications (2.4%), show stronger averages of 29 and 17 citations per paper, respectively. These patterns illustrate both the dominance of established research powers and the growing participation of emerging countries in the field of algebraic thinking. Building on these national patterns, the next section examines international collaboration to assess how countries interact and form research networks.

International Collaboration

Figure 3 illustrates the international cooperation network on algebraic thinking in mathematics education. This network comprises 24 countries/territories collaborating with others in at least three published works.

(Source: data collection results of the authors' group)

Each color cluster in VOSviewer illustrates the degree of scientific cooperation among countries/territories. Countries/territories that share the same color often have a high frequency of co-authorship, forming distinct research clusters. For instance, the purple cluster includes the United States, Turkey, Israel, Taiwan, and others; the green cluster comprises Australia, Brazil, New Zealand, and others; while the red cluster features Indonesia, Sweden, Malaysia, and others. The diversity of color clusters indicates that the cooperative network is somewhat scattered, but this does not imply division. The connections between the clusters, depicted through the seams, reveal ongoing interaction and exchange among countries/territories in the research field. The size of the buttons represents how frequently each country appears in the publications. The United States, represented by the most prominent round button, affirms its leading role in the number of studies and the level of connectivity. Indonesia and Spain also have relatively large button sizes, reflecting their frequency of notable appearances. Countries/Territories with smaller nodes, such as Chile, Sweden, and Israel, demonstrate lower participation rates, yet they maintain diverse connections with numerous partners. The links from the United States to most other clusters are quite thick, signifying the country/territory's extensive multilateral cooperation. Meanwhile, countries on the periphery, such as Taiwan, Mexico, and Israel, often have fewer partners or collaborate intensively with specific groups. Despite its large node, Indonesia predominantly connects with countries/territories within the same cluster (such as Sweden and Malaysia) or with others outside the cluster. This suggests that Indonesia has significant potential to expand its cooperation beyond the region, reaching out to other prospective partners.

Most Frequent Publication Sources

The authors on the topic of algebraic thinking published their research based on 125 different sources, including 211 articles in scientific journals, 59 publications in conference proceedings, and 23 publications in a book series. Table 3 shows the list of the top nine most published sources, with more than two-thirds of the most published sources consisting of eight journals, one conference proceeding, and one book series.

Table 3
Top nine most published sources of algebraic thinking

Rank	Source	Type	TP	TC	CiteScore 2021	SJR 2021	SNIP 2021
1	Educational Studies In Mathematics	JN	23	878	Q1	5.2	1.476
2	Journal Of Physics Conference Series	CP	20	36	Q4	0.5	0.183
3	ZDM - International Journal on Mathematics Education	JN	18	482	Q1	4.7	1.403
4	Journal Of Mathematical Behavior	JN	11	183	Q1	3.8	1.081
5	Algebra In The Early Grades	BS	10	459			
6	Mathematics Education Research Journal	JN	8	143	Q1	2.9	0.957
7	Infancia Y Aprendizaje	JN	7	32	Q2	1.7	0.682
8	International Journal Of Science And Mathematics Education	JN	7	80	Q1	3.5	1.005
9	Journal For Research In Mathematics Education	JN	7	688	Q1	4.1	1.337
CP: Conference Proceeding, JN: Journal, BS: Book series, TP: Total Publication, TC: Total Citation							

(Source: data collection results of the authors' group)

Seven journals in the Q1 category (based on CiteScore or SJR) within the Top nine indicate that researchers in algebraic thinking often prefer to publish in prestigious journals with a rigorous review process, receiving high regard from international academics. Notably, most of these journals focus on the broader field of mathematics education (e.g., Educational Studies in Mathematics, ZDM - International Journal on Mathematics Education, Journal of Mathematical Behavior), not solely on algebra. Thus, algebraic thinking is closely linked to the larger issues of mathematics education. Among them, Educational Studies in Mathematics not only leads in the number of published papers (with 23 publications) but also has a very high citation rate (878 citations), demonstrating this journal's significant impact. Despite publishing only seven articles, the Journal for Research in Mathematics Education boasts nearly 100 citations per article, reinforcing this journal's prominent status, primarily since the US National Council of Teachers of Mathematics (NCTM) supports it. While many journals are associated with the US or Europe, most function internationally, featuring editorial boards composed of scholars from various countries/territories, which illustrates the widespread recognition of the importance of algebraic thinking in mathematics education.

Most Influential Authors

According to data from the Scopus database, 661 authors have contributed to research on algebraic thinking. The top seven authors with the most publications in this field are displayed in Table 4. These authors hail from Indonesia (2), Spain (2), Australia (1), Cyprus (1), and the United States (1). One author, Suryadi, D. (Universitas Pendidikan Indonesia), has eight publications, while Cañadas, M.C. (Universidad de Granada) has seven. Blanton, M. (North Carolina State) and Godino, J.D. (Universidad de Granada) follow closely, with six publications each, but they have a higher number of citations than the other authors in the top seven list, with 198 and 58 citations, respectively. The remaining three authors each have five publications.

Table 4
Top seven most published authors on algebraic thinking

Rank	Author	Institution	Country	TP	TC	TP/TC
1	Suryadi, D.	University of Education Indonesia,	Indonesia	8	10	1.3
2	Cañadas, M.C.	Universidad de Granada	Spain	7	22	3.1
3	Blanton, M.	North Carolina State	United States	6	198	33
4	Godino, J.D.	Universidad de Granada	Spain	6	58	9.7
5	Herman, T.	Universitas Pendidikan Indonesia	Indonesia	5	30	6
6	Pitta-Pantazi, D.	University of Cyprus	Cyprus	5	25	5
7	Stephens, M.	University of Melbourne	Australia	5	36	7.2
TP: Total publications TC: Total citations						

(Source: data collection results of the authors' group)

Figure 4 presents the annual publication and citation numbers for the top seven authors, also listed in Table 4 above. These leading authors have primarily published within the last six years. All three authors Suryadi, Cañadas and Blanton, have a substantial publication history, with 18 publications on this topic in the past five years. Notably, Blanton. has received 198 citations. Among the other authors, Stephens has one publication from 2012, while the others have primarily been published from 2013 onwards.

(Source: data collection results of the authors' group)

Figure 5 illustrates the collaboration between authors, with each node representing an author. The size of the nodes corresponds to the number of publications, the lines between the nodes indicate the cooperation between the respective authors, and the thickness of the lines represents the number of collaborative publications published.

It can be observed that the collaboration among authors researching algebraic thinking occurs within small groups. This collaboration is relatively diverse, involving contributors from many countries/territories. Some typical groups illustrated in Fig. 5 include a group of eight authors, namely Blanton (Spain), Brizuela (Taiwan), Gardiner and Stephens (United States), Knuth (Italy), Sawrey (Portugal), Strachota (South Korea), and Stroud (Argentina). This group has co-authored six studies published between 2005 and 2022. Another group consists of nine authors from various parts of the world, including Ayala-altamirano (Brazil), Cañadas and Polo-Blanco (Indonesia), Goñi-Cervera and Torres (Spain), Molina (Malaysia), Moreno and Vergel (United States) and Pinto (Germany). These authors collaborated on seven studies on this topic, published between 2017 and 2023.

(Source: data collection results of the authors' group)

Keyword Trend Over Time

Figure 6 illustrates the research trends in algebraic thinking over time. The trend analysis identifies three main clusters in the evolution of algebraic thinking research: conceptual foundations, pedagogical strategies, and professional/technological support. In the early 2010s, studies emphasized core concepts such as early algebra, generalization, patterns, and functions, which laid the groundwork for algebraic thinking. These ideas, including functional thinking, have remained central, reflecting sustained interest in how students generalize and structure mathematical relations from an early age (Blanton & Kaput, 2011). Alongside this, research on pedagogical strategies has been consistent, with problem solving emerging as a recurring and central theme, showing the importance of how students engage with algebraic ideas through reasoning, heuristics, and task design (Kieran, 2018). A third cluster focuses on professional and technological support. Earlier work highlighted teacher knowledge and professional development, while later studies pointed to educational computing and technology, marking a shift toward digital integration. By the late 2010s, computational thinking became a prominent theme, indicating a broader move in mathematics education toward technology-enhanced learning and algorithmic reasoning (Wing, 2006; Radford, 2014). Overall, the trajectory reflects a shift from consolidating basic concepts and teacher expertise to integrative approaches that connect pedagogy, technology, and practical competence.

(Source: data collection results of the authors' group)

The most influential studies

The ten most cited articles on algebraic thinking, listed in Table 5, account for 1,588 citations, or 30.57% of the total in this dataset. These works have shaped the field by addressing central issues such as the transition from arithmetic to algebra and the challenges students face in this process (Sfard & Linchevski, 1994; Radford, 2000, 2008). They also examine how learners grasp and apply concepts like variables and equivalence (Knuth et al., 2005), while analyzing the abstract nature of algebra and the ways in which meaning is assigned to symbols and structures (Sfard & Linchevski, 1994; Vukovic &

Lesaux, 2013; Radford, 2000, 2008). Broader definitions are also considered, with Kaput (2017) advocating for the early introduction of algebra at the elementary level. Teacher-focused research is a recurring theme, highlighting instructional strategies, professional development, and questioning techniques that foster algebraic reasoning (Jacobs et al., 2007; Franke et al., 2009; Blanton & Kaput, 2005; Nathan & Koedinger, 2000).

The two most cited papers were published in *Educational Studies in Mathematics* (Q1, Springer). Sfard and Linchevski (1994) lead with 233 citations (7.77 per year), emphasizing the need for flexibility in the use of algebraic symbols. Jacobs et al. (2007) follow with 206 citations (12.12 per year), demonstrating how teacher professional development enhances students' reasoning. Franke et al. (2009) rank third with 200 citations, focusing on the role of teacher questioning. Kaput's (2017) chapter is fourth with 186 citations (26.57 per year), outlining a roadmap for algebra teaching. Vukovic and Lesaux (2013) are fifth with 154 citations, emphasizing language in mathematical development. Other influential works include Radford (2000) with 143 citations, Blanton and Kaput (2005) with 139, Knuth et al. (2005) with 115, Nathan and Koedinger (2000) with 108, and Radford (2008) with 104.

Citation data from Semantic Scholar show that these studies are cited 234 times as background, 116 in methods, and 13 in results, confirming their role as theoretical and methodological foundations for later work in algebraic thinking.

Table 5
Top ten most cited studies in algebraic thinking

	Document Title	Source Title	Author(s)	PY	TC	TC/ Year	BC	MC	RC
1	The gains and the pitfalls of reification - The case of algebra	Educational Studies in Mathematics	Sfard A., Linchevski L.	1994	551	18.36	234	28	11
2	Professional development focused on children's algebraic reasoning in elementary school	Journal for Research in Mathematics Education	Jacobs V.R., Franke M.L., Carpenter T.P., Levi L., Battey D.	2007	206	12.12	111	13	6
3	Teacher questioning to elicit students' mathematical thinking in elementary school classrooms	Journal of Teacher Education	Franke M.L., Webb N.M., Chan A.G., Ing M., Freund D., Battey D.	2009	312	13.33	145	20	13
4	What is algebra? What is algebraic reasoning?	Algebra in the Early Grades	Kaput J.J.	2017	236	26.57	156	21	8
5	The language of mathematics: Investigating the ways language counts for children's mathematical development	Journal of Experimental Child Psychology	Vukovic R.K., Lesaux N.K.	2013	195	14	64	6	12
6	Signs and meanings in students' emergent algebraic thinking: A semiotic analysis	Educational Studies in Mathematics	Radford L.	2000	286	5.96	139	19	4
7	Characterizing a classroom practice that promotes	Journal for Research in Mathematics Education	Blanton M.L., Kaput J.J.	2005	352	7.32	153	116	8

BC: Background Citations, MC: Methods Citations, RC: Results Citations, PY: Publication Year, TC: Total Citations

	Document Title	Source Title	Author(s)	PY	TC	TC/ Year	BC	MC	RC
	algebraic reasoning								
8	Middle school students' understanding of core algebraic concepts: equivalence and variable	ZDM - International Journal on Mathematics Education	Knuth E.J., Alibali M.W., McNeil N.M., Weinberg A., Stephens A.C.	2005	115	6.05	5		
9	An investigation of teachers' beliefs of students' algebra development	Cognition and Instruction	Nathan M.J., Koedinger K.R.	2000	211	4.5	96	10	11
10	Iconicity and contraction: A semiotic investigation of forms of algebraic generalizations of patterns in different contexts	ZDM - International Journal on Mathematics Education	Radford L.	2008	229	6.5	114	18	8
BC: Background Citations, MC: Methods Citations, RC: Results Citations, PY: Publication Year, TC: Total Citations									

(Source: data collection results of the authors' group)

Keyword Co-Occurrence Thematic Clustering

Among the 293 publications analyzed, authors employed a total of 881 keywords. The VOSviewer mapping reveals four major thematic clusters in algebraic thinking research. The red cluster includes terms such as algebraic thinking, early algebraic thinking, generalization, patterns, and representations, focusing on how students - particularly at the primary level - develop algebraic thinking through pattern recognition and generalization. The blue cluster, with keywords like algebraic reasoning, teacher education, professional development, and teacher knowledge, highlights research on strengthening teachers' professional capacity to improve algebra instruction. The green cluster centers on problem solving, thinking skills, and mathematical tasks, emphasizing the role of algebraic thinking in problem-solving and cognitive development. The yellow cluster comprises mathematical modeling, functions, and computational thinking, reflecting the integration of modeling approaches and digital competencies into algebra learning.

A comparison of the clusters reveals distinct orientations: the red cluster is learner-centered, while the blue cluster is teacher-centered. The other two clusters serve as bridges and extensions. Despite their different emphases, the clusters complement one another: learner-focused research provides a basis for teacher education, while improving teachers' expertise enhances student learning outcomes. At the same time, the modeling-oriented yellow cluster extends the scope by connecting algebraic thinking with higher-level mathematical skills and technology integration, illustrating the diversity and cross-fertilization that characterize this research domain.

Figure 7 presents four thematic clusters, highlighting complementary perspectives on learner-centered concepts, teacher development, problem-solving, and the integration of technology.

(Source: data collection results of the authors' group)

Summary of Major Research Themes

The analysis of the keyword co-occurrence network reveals five major global research trends in the field of algebraic thinking: (1) early algebra development, (2) problem-solving instruction, (3) mathematical modeling, (4) technology integration and interdisciplinary approaches, and (5) teacher training and professional development.

These five trends are not isolated; rather, they are closely interconnected, reflecting the field's developmental progression from foundational theories to modern, integrated approaches.

1. Early algebra development serves as the foundational layer of the field. This research trend focuses on how students particularly at the elementary and middle school levels develop the capacity to recognize patterns, generalize, and represent mathematical relationships (Mason, 1996; Blanton & Kaput, 2011). Studies by Warren & Cooper (2008) and Acosta, Alsina, & Pincheira (2023) highlight the effectiveness of 'pattern and structure' activities. They found that these interventions help students build early algebraic representation and generalization skills, which lay a strong foundation for later functional thinking and abstract reasoning.

2. Problem-solving instruction views algebraic thinking as a core cognitive process in mathematical reasoning. Studies in this cluster examine how students use algebraic representations and equations to analyze, model, and solve mathematical or real-world problems (Fuchs et al., 2010; Mustaffa et al., 2018). Hiebert and Grouws (2007) argue that problem-solving activities not only deepen students' conceptual understanding but also foster their ability to reason and justify core components of algebraic thinking.

3. Mathematical models are a bridge that helps students connect abstract algebraic concepts to real-world situations. Rivera (2010) points out that the use of models, diagrams, and visual representations allows students to make natural connections between models, functional relationships, and algebraic expressions. When working with these representations, students develop a more structured view and

deeper algebraic thinking. In addition, many other studies have shown that modeling instruction contributes to reducing reasoning errors and improving problem-solving quality in word problems in students (Blum & Leiss, 2007).

4. Technology integration and interdisciplinary approaches represent a rapidly expanding research trend in recent years. Studies by Bråting and Kilhamn (2021), Agatolio, Albanese, & Moro (2018), and Huang, Brizuela, & Wong (2008) indicate that the use of digital tools, programming, computational thinking, and STEM-based environments has a significant impact on students' algebraic thinking. Technology-enhanced learning environments enable students to visualize functional relationships, test patterns, and experiment with multiple representations, thereby broadening their capacity for modeling and generalization-core skills for learning mathematics in the 21st century.

5. Teacher training and professional development constitute a critical and enduring theme. This research direction highlights the importance of teachers' deep content knowledge, pedagogical content knowledge (PCK), and metacognitive skills in nurturing algebraic thinking among students (Kieran, 2018; Jacobs et al., 2007; Franke et al., 2009; Kramarski, 2008). Numerous studies have demonstrated that teachers' questioning strategies, feedback practices, and analysis of students' written work strongly influence the development of students' algebraic reasoning and overall learning outcomes.

While these five research trends have distinct emphases, they share some core elements: generalization, representation, reasoning, and modeling. Algebraic thinking lays the foundation for later problem-solving and modeling processes; problem-solving allows teachers to observe and support student reasoning; and technology and interdisciplinary teaching create opportunities to extend these competencies to new, dynamic learning environments.

In summary, research on algebraic thinking has shifted from focusing separately on students or teachers to adopting an integrated model that encompasses the learner, the teacher, and technology. This integrated paradigm promotes a stronger alignment between theory, pedagogical innovation, and technological advancement, ultimately supporting the development of students' mathematical competencies in the digital age (Radford, 2014).

Discussion

The bibliometric results reveal a consistent and accelerating expansion of algebraic thinking research from 1991 to early 2024, evolving through three developmental phases: a foundational stage focused on the transition from arithmetic to algebraic reasoning (1991–2004); a phase of rapid growth and technological integration (2005–2018); and a recent period (2019–2023) characterised by interdisciplinary perspectives such as computational thinking. These patterns align with Sibgatullin et al. (2022), who reported a surge in global research peaking around 2019, followed by a levelling trend toward 2024.

Geographically, research activity remains concentrated in the United States, Canada, Australia, and the United Kingdom, with growing participation from Indonesia and China. However, the citation impact

remains uneven, echoing the regional disparities identified by Fardian et al. (2024). RQ1 thus underscores the globalisation of the field but also the unequal distribution of scholarly influence across regions.

Publication and citation patterns are dominated by a small cluster of Q1 journals particularly Educational Studies in Mathematics, IJMEST, and The Journal of Mathematical Behavior. This concentration reinforces the status of international venues as key arenas for theoretical exchange but also highlights a potential lack of diversity in publication outlets. Citation peaks linked to seminal figures such as Kaput and Kieran confirm the continuing centrality of foundational frameworks. Although emerging countries like Indonesia and Spain have increased their contributions, their global visibility remains modest, indicating structural barriers related to language, access, and academic networks.

Collaboration patterns show gradual improvement but limited depth. Only 16% of studies involve international co-authorship, with most networks confined to regional clusters led by the United States. This lack of cross-national collaboration restricts knowledge transfer and limits the dissemination of pedagogical innovations across contexts. Stronger, long term partnerships especially between developed and developing regions could enhance methodological diversity and the global relevance of research on algebraic thinking.

Thematic analyses refine this picture by identifying five dominant orientations: early algebraic thinking, problem-solving, mathematical modelling, technology integration, and teacher education. Over time, attention has shifted from teacher-focused research toward technology-enhanced and learner-centred paradigms. This reflects broader digital transformations in education but also raises concerns about the diminishing visibility of teacher knowledge and professional development (Sibgatullin et al., 2022; Pincheira & Alsina, 2022). Meanwhile, computational thinking and technology-mediated learning have expanded rapidly, supporting trends noted by Fardian et al. (2024) and Veith et al. (2023).

Despite these transformations, core constructs early algebra and algebraic thinking remain central, reflecting both theoretical maturity and the need for renewed conceptual work on generalisation, relational reasoning, and functional thinking. The field is thus transitioning from symbolic instruction toward integrated, technology-driven frameworks that connect learners, teachers, and tools within dynamic ecosystems of learning. Sustaining this evolution requires balancing technological innovation with pedagogical depth and human interaction.

This study, while comprehensive, is not without limitations. The dataset includes only English-language publications from Scopus, excluding other databases and non-English sources. The timeframe ends in early 2024, leaving the impact of artificial intelligence on mathematics education yet to be captured. Citation patterns are also skewed toward developed contexts. Future research should expand data coverage (e.g., Web of Science, ERIC, Dimensions), promote equitable collaboration, and revisit teacher centred inquiry through scalable professional development models. Recent studies point to several promising directions for future research, including the development of AI driven feedback systems, the design of pattern generalisation tasks, and the integration of algebra with coding or robotics. For

example, Wu et al. (2022) introduced an intelligent tutoring system that provides step-by-step support for solving algebraic word problems, Li (2025) discussed how tools like Ernie Bot and Khan Academy offer personalised learning pathways in elementary mathematics. Together, these studies illustrate the potential of combining AI technologies with algebra instruction to support adaptive reasoning and differentiated learning.

Ultimately, the field of algebraic thinking research has entered a stage of consolidation-mature, data-rich, and increasingly interdisciplinary. To sustain its growth, it must remain inclusive, theoretically grounded, and responsive to technological change while preserving the human core of teaching and learning.

Conclusion

Based on 293 Scopus-indexed publications from 1991 to early 2024, this study presents a comprehensive bibliometric overview of research on algebraic thinking. The results show a gradual shift from foundational investigations toward more interdisciplinary and technology-supported directions, forming three identifiable phases of development. Contributions from emerging economies have increased in recent years, yet their citation impact remains limited, and influential work continues to be concentrated within a small set of countries and journals.

Thematic analysis indicates five persistent research orientations: early algebra, problem solving, mathematical modelling, technology integration, and teacher education. A noticeable movement from teacher-centered studies to technology-enhanced learning reflects ongoing digital transformation in mathematics education. Nevertheless, weak international collaboration continues to constrain the global spread of knowledge and methodological diversity, pointing to an urgent need for stronger cross-regional research partnerships.

Looking ahead, the field's progress will depend on its ability to demonstrate clear benefits for classroom practice, particularly in diverse and under-resourced contexts. Technological advances-such as digital tools and AI-offer potential to support deeper forms of algebraic reasoning, but their effectiveness relies on sustained teacher professional agency and active student participation in constructing mathematical ideas. Ensuring a stronger alignment between innovation and pedagogy is therefore essential if algebraic thinking research is to contribute meaningfully to educational equity and the development of students' mathematical thinking worldwide.

Declarations

Ethics, Consent to Participate, and Consent to Publish declarations: not applicable

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Informed consent: All participants were fully informed about the purpose of the study, assured of confidentiality, and provided verbal informed consent prior to participation. Verbal consent was deemed appropriate because the survey involved minimal risk and did not affect participants' privacy.

Identifying information: All author and institutional details have been removed to ensure anonymity for peer review.

Author Contributions (CRediT taxonomy)

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- Formal analysis: Trinh Thi Phuong Thao
- Investigation: Duong Huu Tong, Hoang Thi Nga
- Data curation: Hoang Thi Nga
- Software: Tang Minh Dung
- Visualization: Tang Minh Dung
- Supervision: Duong Quoc Thai
- Project administration: Duong Quoc Thai
- Writing – original draft: Duong Quoc Thai, Trinh Thi Phuong Thao
- Writing – review & editing: All authors

References

1. Thorpe, J. A. (2018). Algebra: What should we teach and how should we teach it?. In Research issues in the learning and teaching of algebra (pp. 11-24). Routledge.
2. Mason, J. (1996). Expressing generality and roots of algebra. In Approaches to algebra: Perspectives for research and teaching (pp. 65-86). Dordrecht: Springer Netherlands.
3. Fred, J., Valero, P., & Van Steenbrugge, H. (2022). Powerful algebraic ideas: Early algebraic thinking and active citizenship. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.) Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12). (pp. 489–496). Free University of Bozen-Bolzano and ERME. <https://hal.science/hal-03744605v1>
4. Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp. 390–419). New York: Macmillan.
5. Chimoni, M., Pitta-Pantazi, D., & Christou, C. (2023a). Unfolding algebraic thinking from a cognitive perspective. *Educational Studies in Mathematics*, 114(1), 89-108. <https://doi.org/10.1007/s10649-023-10218-z>
6. Coles, A., & Ahn, A. (2022). Developing algebraic activity through conjecturing about relationships. *ZDM–Mathematics Education*, 54(6), 1229-1241. <https://doi.org/10.1007/s11858-023-01515-1>
7. Hodgen, J., Oldenburg, R., & Strømskag, H. (2018). Algebraic thinking. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 32-45). Routledge. <https://doi.org/10.4324/9781315113562-4>
8. Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning*, 9(3), 249-272.
9. Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational studies in mathematics*, 27(1), 59-78.
10. MacGregor, M., & Stacey, K. (1995). The effect of different approaches to algebra on students' perceptions of functional relationships. *Mathematics Education Research Journal*, 7(1), 69-85. <https://doi.org/10.1007/BF03217276>
11. Afonso, D. G. (2019). The development of algebraic thinking in the foundation phase: A comparative study of two different curricula. Cape Peninsula University of Technology.
12. Guler, M., & Celik, D. (2021). The effect of an elective algebra teaching course on prospective mathematics teachers' pedagogical content knowledge. *International Electronic Journal of Mathematics Education*, 16(2), em0636. <https://doi.org/10.29333/iejme/10902>
13. Somasundram, P. (2021). The role of cognitive factors in year five pupils' algebraic thinking: A structural equation modelling analysis. *Eurasia Journal of Mathematics, Science and Technology Education*, 17(1), 1-12. <https://doi.org/10.29333/ejmste/9612>
14. Walkoe, J. (2014). Exploring teacher noticing of student algebraic thinking in a video club. *Journal of Mathematics Teacher Education*, 18(6), 523-550. <https://doi.org/10.1007/s10857-014-9289-0>

15. Sibgatullin, I. R., Korzhuev, A. V., Khairullina, E. R., Sadykova, A. R., Baturina, R. V., & Chauzova, V. (2022). A systematic review on algebraic thinking in education. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(1), 1-15. <https://doi.org/10.29333/EJMSTE/11486>
16. Fardian, D., Suryadi, D., Prabawanto, S., & Hayuningrat, S. (2024). Research trends on early algebra in the middle school: A combined bibliometric and meta-analysis review. *Journal of Elements*, 10(2), 410-440. <https://doi.org/10.29408/jel.v10i2.25539>
17. Börner, K., Chen, C., & Boyack, K. W. (2005). Visualizing knowledge domains. *Annual Review of Information Science and Technology*, 37(1), 179-255. <https://doi.org/10.1002/aris.1440370106>
18. Zupic, I., & Čater, T. (2015). Bibliometric methods in management and organization. *Organizational Research Methods*, 18(3), 429-472. <https://doi.org/10.1177/1094428114562629>
19. Moral-Muñoz, J. A., Herrera-Viedma, E., Santisteban-Espejo, A., & Cobo, M. J. (2020). Software tools for conducting bibliometric analysis in science: An up-to-date review. *Profesional de la Información*, 29(1), 1-20. <https://doi.org/10.3145/epi.2020.ene.03>
20. Ha, C. T., Thao, T. T. P., Trung, N. T., Huong, L. T. T., Dinh Van, N., & Trung, T. (2020). A bibliometric review of research on STEM education in ASEAN: Science mapping the literature in Scopus database, 2000 to 2019. *Eurasia Journal of Mathematics, Science and Technology Education*, 16(10), em1889. <https://doi.org/10.29333/ejmste/8500>
21. Mongeon, P., & Paul-Hus, A. (2016). The journal coverage of Web of Science and Scopus: a comparative analysis. *Scientometrics*, 106(1), 213-228. <https://doi.org/10.1007/s11192-015-1765-5>
22. Pham-Duc, B., Tran, T., Trinh, T.-P.-T., Nguyen, T.-T., Nguyen, N.-T., & Le, H.-T.-T. (2020). A spike in the scientific output on social sciences in Vietnam for recent three years: Evidence from bibliometric analysis in Scopus database (2000-2019). *Journal of Information Science*, 48(5), 623-639. <https://doi.org/10.1177/0165551520977447> <http://www.scopus.com>
23. Blanton, M. L., & Kaput, J. J. (2011). Functional thinking as a route into algebra in the elementary grades. In *Early algebraization: A global dialogue from multiple perspectives* (pp. 5-23). Berlin, Heidelberg: Springer Berlin Heidelberg.
24. Kieran, C. (2018). The early learning of algebra: A structural perspective. In *Research issues in the learning and teaching of algebra* (pp. 33-56). Routledge.
25. Wing, J. M. (2006). Computational thinking. *Communications of the ACM*, 49(3), 33-35.
26. Radford, L. (2014). The progressive development of early embodied algebraic thinking. *Mathematics Education Research Journal*, 26(2), 257-277.
27. Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls of reification - The case of algebra. *Educational Studies in Mathematics*, 26(2-3), 191-228. <https://doi.org/10.1007/bf01273663>
28. Radford, L. (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational studies in mathematics*, 42, 237-268.
29. Radford, L. (2008). Connecting theories in mathematics education: Challenges and possibilities. *ZDM Mathematics Education*, 40(2), 317-327. <https://doi.org/10.1007/s11858-008-0090-3>

30. Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence & Variable1. *Zentralblatt für Didaktik der Mathematik*, 37(1), 68-76.
31. Vukovic, R. K., & Lesaux, N. K. (2013). The language of mathematics: Investigating the ways language counts for children's mathematical development. *Journal of Experimental Child Psychology*, 115(2), 227-244.
32. Kaput, J. J. (2017). 1 What Is Algebra? What Is Algebraic Reasoning?. In *Algebra in the early grades* (pp. 5-18). Routledge.
33. Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for research in mathematics education*, 38(3), 258-288.
34. Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, 60(4), 380-392.
35. Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for research in mathematics education*, 36(5), 412-446.
36. Nathan, M. J., & Koedinger, K. R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and instruction*, 18(2), 209-237.
37. Warren, E., & Cooper, T. (2008). Generalising the pattern rule for visual growth patterns: Actions that support 8-year-olds' thinking. *Educational Studies in Mathematics*, 67(2), 171-185.
<https://doi.org/10.1007/s10649-007-9092-2>
38. Fuchs, L. S., Zumeta, R. O., Schumacher, R. F., Powell, S. R., Seethaler, P. M., Hamlett, C. L., & Fuchs, D. (2010). The effects of schema-broadening instruction on second graders' word-problem performance and their ability to represent word problems with algebraic equations: A randomized control study. *The Elementary School Journal*, 110(4), 440-463. <https://doi.org/10.1086/651191>
39. Mustaffa, N., Said, M. N. H. M., Ismail, Z., & Tasir, Z. (2018). Framework of integrating algebraic thinking in problem-based learning via online environment for school students. In M. J. W. Lee, S. Nikolic, M. Ros, J. Shen, L. C. U. Lei, G. K. W. Wong, & N. Venkatarayalu (Eds.), 2018 IEEE International Conference on Teaching, Assessment, and Learning for Engineering (TALE) (pp. 372-378). IEEE. <https://doi.org/10.1109/TALE.2018.8615390>
40. Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 371–404). Information Age Publishing.
41. Rivera, F. D. (2010). Visual templates in pattern generalization activity. *Educational Studies in Mathematics*, 73(3), 297-328. <https://doi.org/10.1007/s10649-009-9222-0>
42. Blum, W., & Leiss, D. (2007). How do students and teachers deal with modelling problems? In C. Haines et al. (Eds.), *Mathematical Modelling: Education, Engineering and Economics* (pp. 222–231). Horwood.

43. Bråting, K., & Kilhamn, C. (2021). Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning*, 23(2), 170-185.
<https://doi.org/10.1080/10986065.2020.1779012>
44. Agatolio, F., Albanese, F., & Moro, M. (2018). How an ambitious informatics curriculum can influence algebraic thinking of primary school children. In S. Pozdniakov & V. Dagienė (Eds.), *Informatics in Schools. Fundamentals of Computer Science and Software Engineering: ISSEP 2018 (Lecture Notes in Computer Science, Vol. 11169, pp. 354-365)*. Cham: Springer International Publishing.
https://doi.org/10.1007/978-3-030-02750-6_27
45. Huang, W., Brizuela, B., & Wong, P. (2008). Integrating algebra and engineering in the middle school classroom. In ASEE (Ed.), *Proceedings of the 2008 Annual Conference & Exposition* (pp. 13.759.1-13.759.18). Pittsburgh, PA, USA: American Society for Engineering Education.
https://docs.rwu.edu/seccm_fp/124
46. Kramarski, B. (2008). Promoting teachers' algebraic reasoning and self-regulation with metacognitive guidance. *Metacognition and Learning*, 3(2), 83-99. <https://doi.org/10.1007/s11409-008-9020-6>
47. Alsina, Á., & Pincheira, N. (2022). El cambio: Un conocimiento esencial del álgebra temprana: Change: Essential knowledge of early algebra. *Revista Científica ECOCIENCIA*, 9(6), 49-76.
48. Veith, J. M., Beste, M. L., Kindervater, M., Krause, M., Straulino, M., Greinert, F., & Bitzenbauer, P. (2023, August). Mathematics education research on algebra over the last two decades: quo vadis?. In *Frontiers in Education* (Vol. 8, p. 1211920). Frontiers Media SA. Wu, S., Li, L., Huang, Y., & Wang, Q. (2022).
49. An Intelligent Tutoring System for Math Word Problem Solving with Tutorial Solution Generation.
50. In *Proceedings of the 2022 IEEE International Conference on Educational Informatization and Reformation (IEIR)*, pp. 34–38. DOI: 10.1109/IEIR56323.2022.10050083
51. Li, M. (2025). Integrating artificial intelligence in primary mathematics education: Investigating internal and external influences on teacher adoption. *International Journal of Science and Mathematics Education*, 23(5), 1283-1308.

Figures

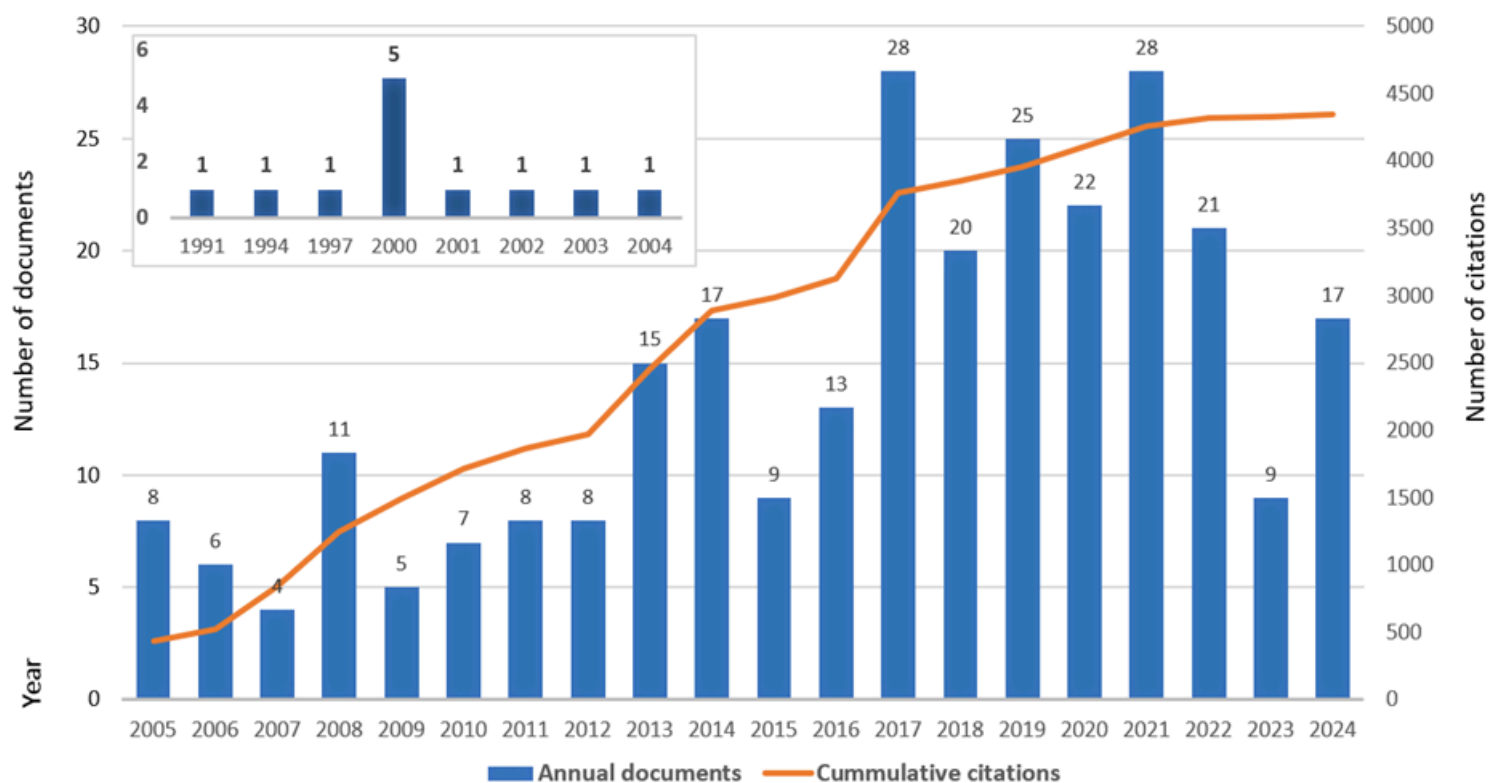


Figure 1

The annual publication count and cumulative citations about algebraic thinking

(Source: data collection results of the authors' group)

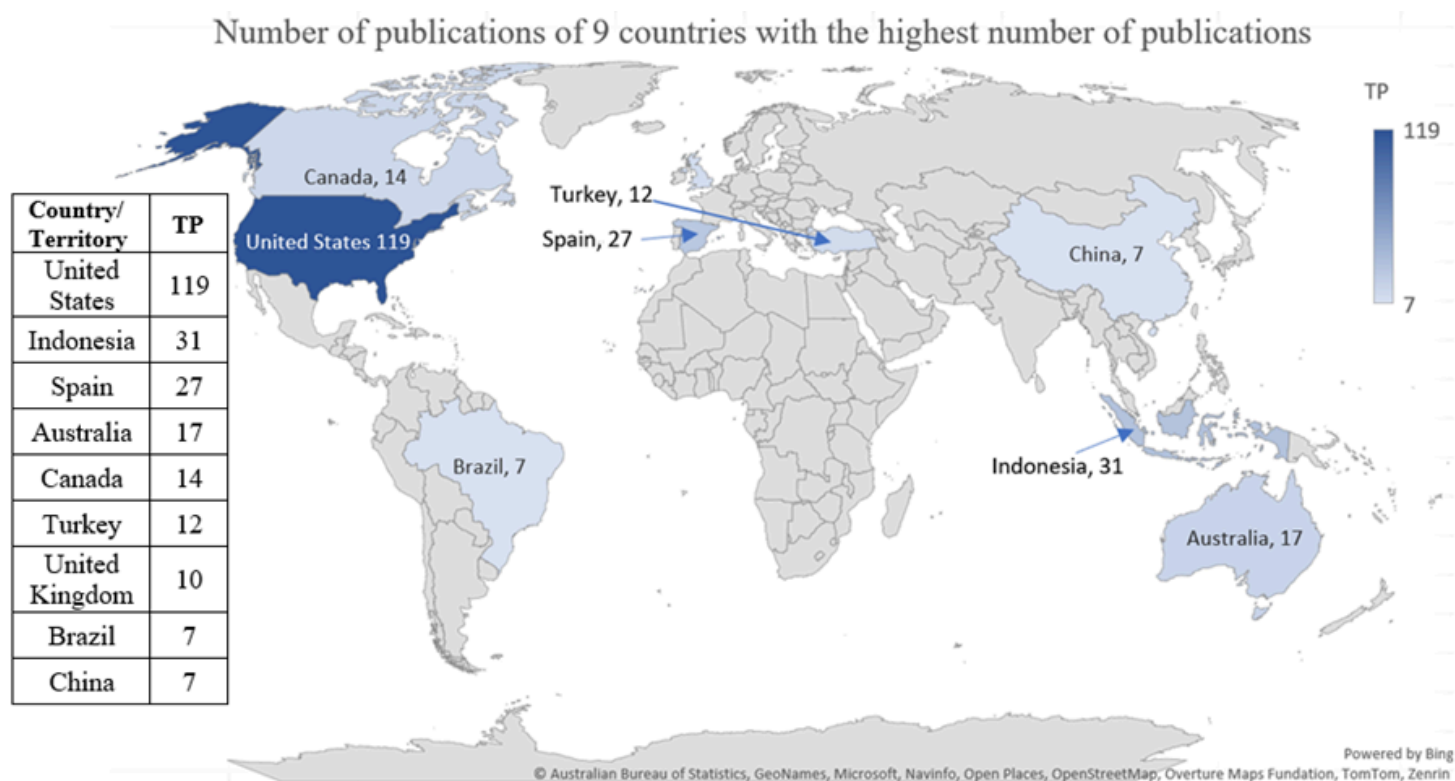


Figure 2

Number of publications of nine countries/territories with the highest number of publications

(Source: data collection results of the authors' group)

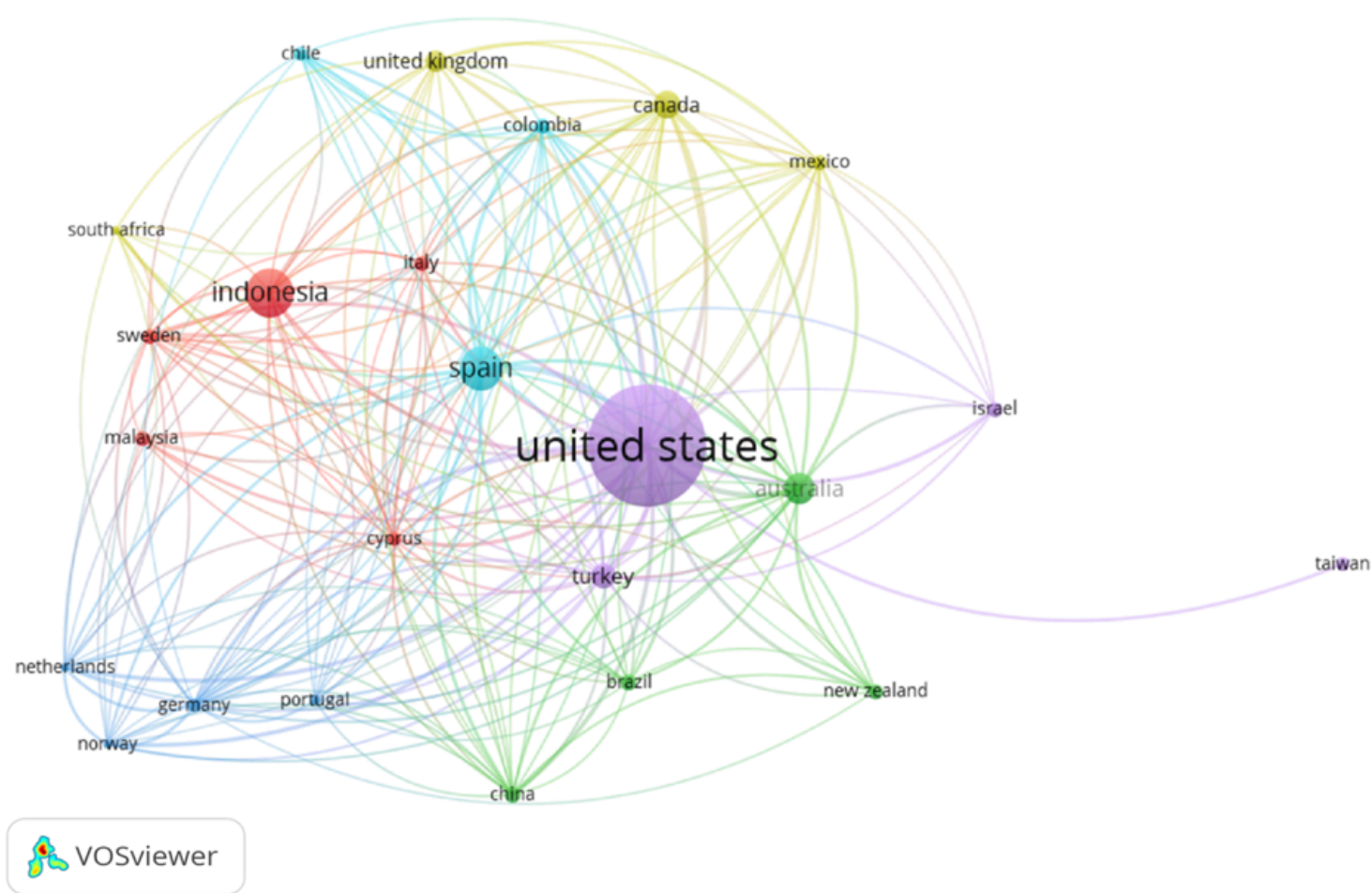


Figure 3

International cooperation network among countries/territories in research on algebraic thinking in mathematics education

(Source: data collection results of the authors' group)

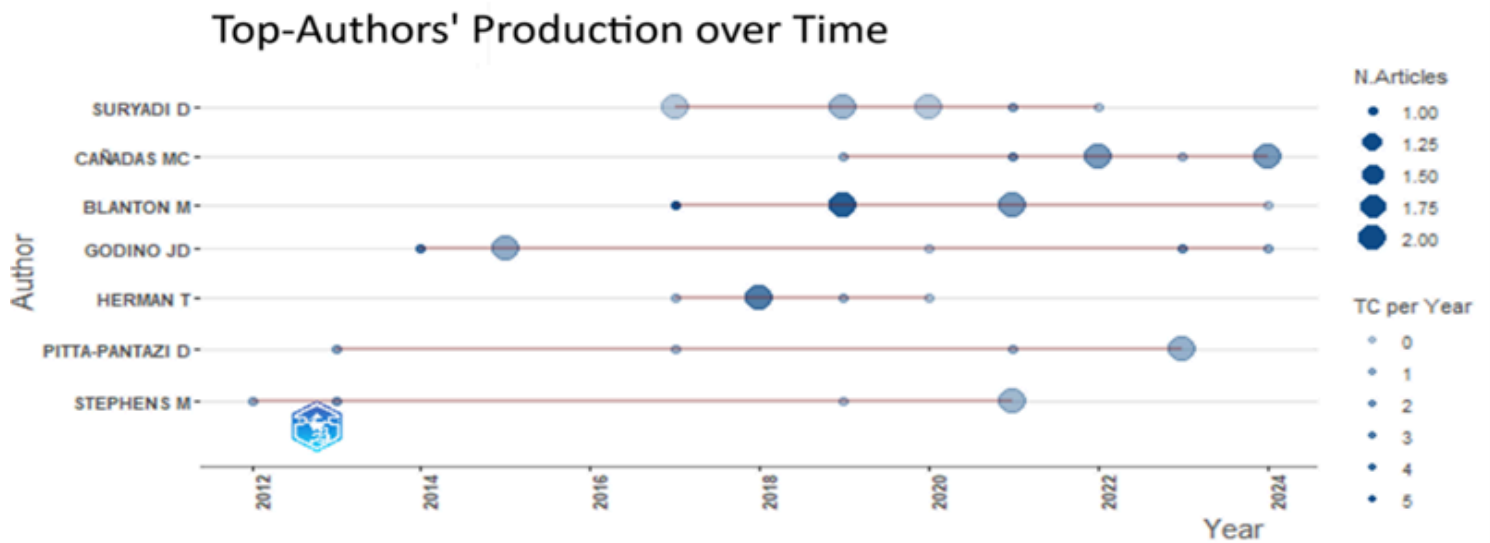


Figure 4

Top seven authors' production over time

(Source: data collection results of the authors' group)

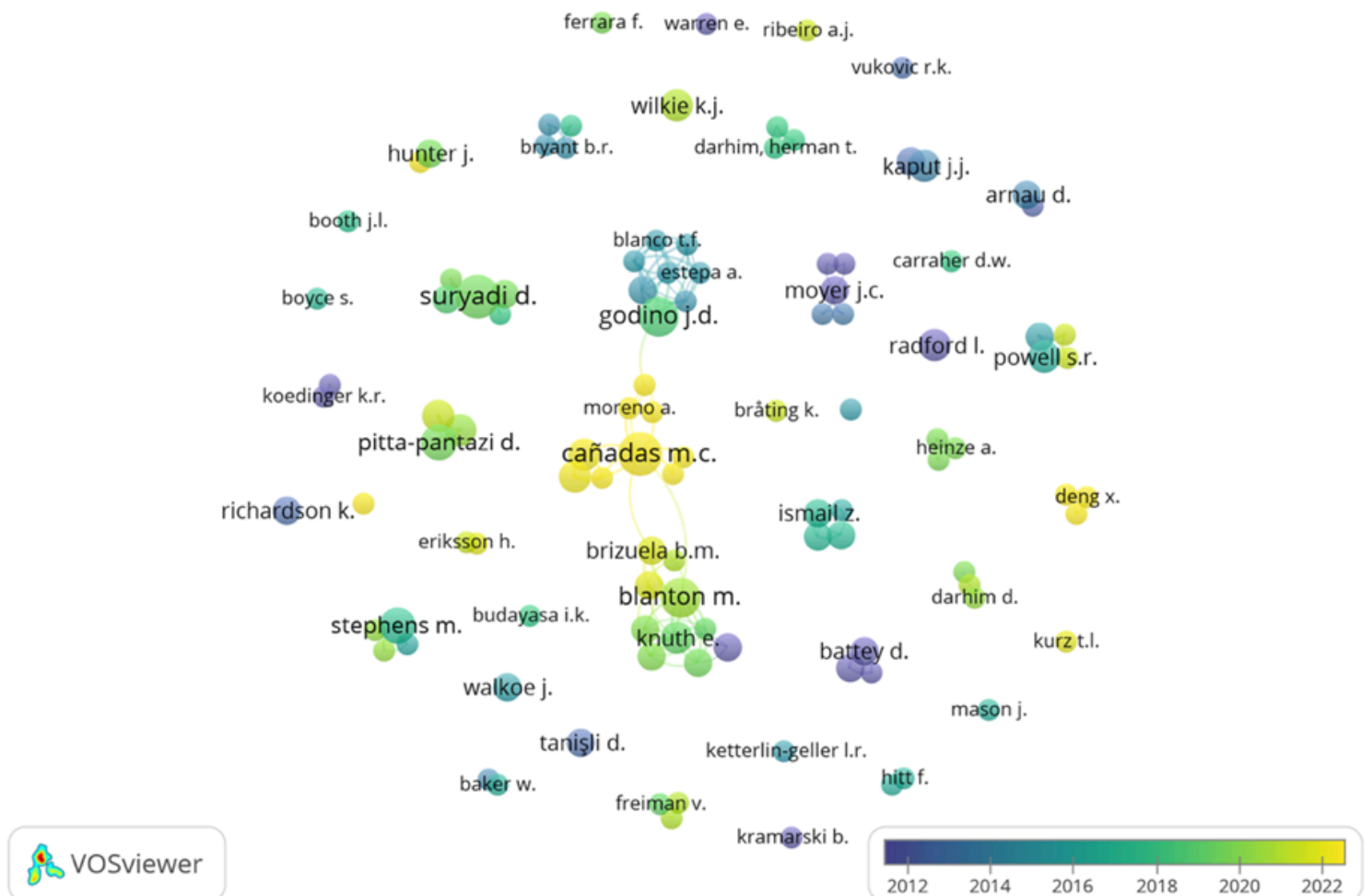


Figure 5

The collaborative network between authors

(Source: data collection results of the authors' group)

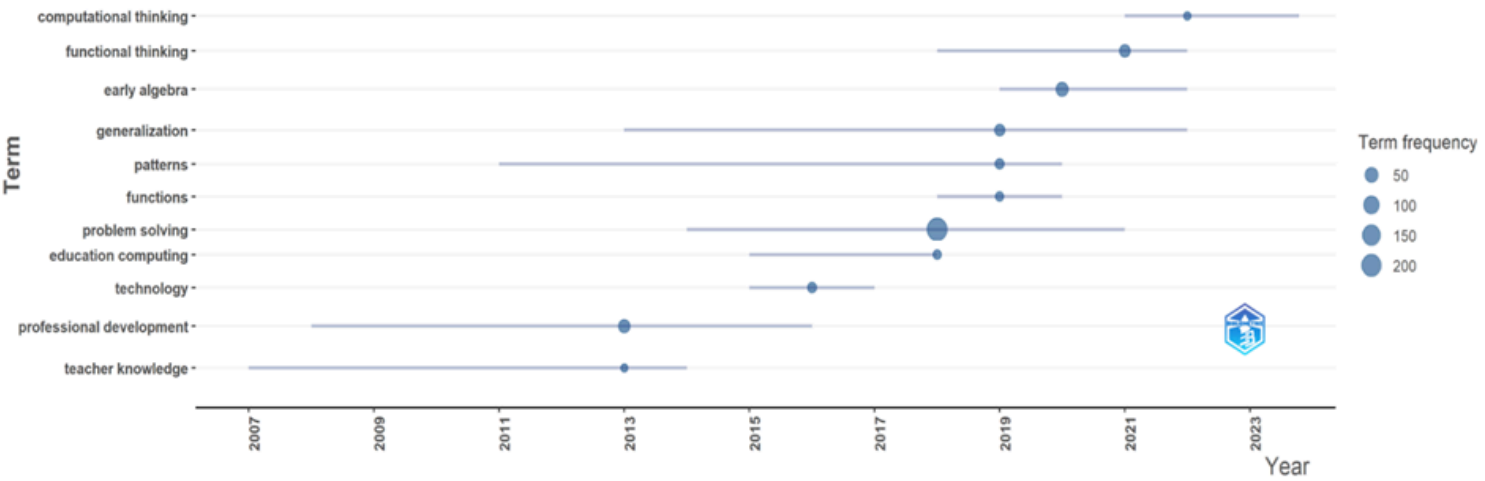


Figure 6

Research trends on Algebraic thinking

(Source: data collection results of the authors' group)

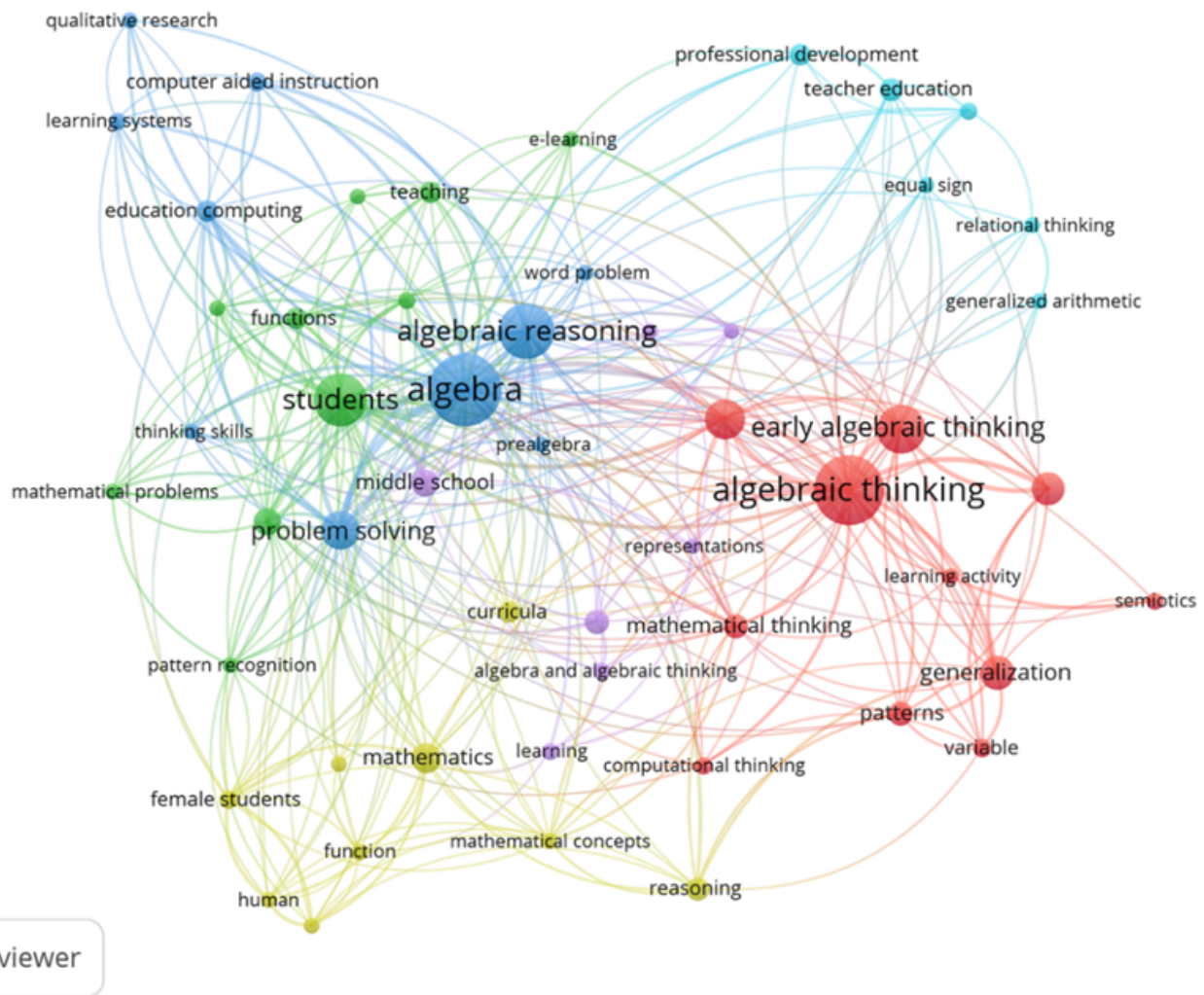


Figure 7

Keyword co-occurrence network of algebraic thinking research.

(Source: data collection results of the authors' group)