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# A Minimal Thermodynamic Framework for Economic Dynamics

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# A Minimal Thermodynamic Framework for Economic Dynamics

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**Abstract**

In the tradition of Georgescu-Roegen, we introduce a framework for economic modeling in which all inputs and outputs are aggregated into a single construct: energy. The objective of the model is to present a minimal foundational framework that describes the physical limits of economic activity, accounting for the transition out of the Malthusian equilibrium into the modern era of economic growth and a contemporary phase of degrowth. The model conceives the economic system as energy flows traversing three main reservoirs: the human system, the environment, and a finite resource. The economic module is intentionally simple in order to isolate the structural role of energy transformation in shaping long-run macroeconomic dynamics. Under simple parametric assumptions, the economy behaves like a damped harmonic oscillator, suggesting that sustained exponential growth in material throughput is incompatible with thermodynamic constraints. We also present an estimate of the parameters in the economic tradition of "calibration" and an illustrative simulation under constant-parameter assumptions. This framework presents several avenues for further research, including historical estimates, the impact of recycling technology on degrowth, and the expansion of economic and demographic modules. The model does not claim to exhaust the mechanisms governing economic growth, but aims to clarify the biophysical constraints within which such mechanisms must operate.

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# 1 Introduction

Economic science has posited theoretical alternatives to the growth paradigm, most notably by introducing energy as a factor of production into an aggregate production function, maintaining most of the analytical apparatus of neoclassical economics (Keen et al., 2019; Kümmel et al., 1985). This approach assumes limits to growth motivated by the finitude of natural resources and the limited capacity of ecosystems to withstand the perturbations caused by human economic activity (Ayres and Warr, 2005; Kümmel, 2011; Stern, 2011). In contrast to that approach, Georgescu-Roegen inspired a literature that explicitly considers economic activity's finitude based on the first law of thermodynamics: energy cannot be created nor destroyed, only transformed (Daly, 1997; Couix, 2020; Georgescu-Roegen, 1971). Models in this school view the economy as a set of material flows that traverse the system, modified by services or funds (such as labor) to produce stocks (of capital or finished products). While these frameworks offer rich descriptive detail, their complexity can make it difficult to isolate the structural mechanisms governing long-run growth dynamics.

In contrast to those approaches, the present article proposes a minimal physical model of the economy, which can also serve as a framework for economic modeling. The framework aims to ground economic growth or degrowth within the bounds of the physical laws of energy transformation. The main advantages of the current proposal are that the limits to growth are not an add-on to the theory but are derived from within; the model is also useful for describing both growth and degrowth regimes; and it is simple enough to accommodate further sophistication for different modeling objectives. The model is deliberately minimal. It does not incorporate price dynamics, technological choice, or institutional feedback. Its purpose is to isolate the thermodynamic structure within which such mechanisms operate. Subsequent research can embed richer economic modules within this framework. The objective is not to replace existing economic models, but to clarify the biophysical constraints within which they operate. The present article is inscribed within the naturalistic approaches of evolutionary economics, taking the stance of ontological monism, which assumes that the economy and nature are interdependent realities, and a heuristic strategy that applies a generic concept of evolution (Witt, 2008).

The proposed framework is a compartment model[1]: a system of differential equations, deterministically governing the flows between the compartments by physical and economic relations. The economic system is described as flows of energy traversing three main compartments: the biomass reservoir, the human system and a finite resource. All compartments are regarded as receptacles or reservoirs where energy is stored. Therefore, living beings and capital are both considered only in their energetic equivalents. This reduction is methodological and intended to isolate thermodynamic constraints, not to deny the institutional or qualitative dimensions of capital. Following the second law, energy flows are unidirectional from lower entropy to higher entropy (Annila and Salthe, 2009; Garrett, 2012; Kåberger and Månsson, 2001). We model the exploitation of natural resources as an interface between the environment and civilization, dependent on human technology and activity (economic equilibrium); this interface is a parsimonious model for the concept of a fund. All energy dissipation (also called *anergy*) is collected into an anergy reservoir; in this way, we account for energy conservation.

Results show that the economy behaves like a forced, damped harmonic oscillator, with a forcing function that decays exponentially under simple parametric estimates based on current empirical

[1]For general reference: Bischoff and Dedrick (1970); Calvetti and Somersalo (2012); Metzler (1971)

research on global energy utilization. Analogously to the impossibility of a perpetual motion machine, this model obtains the impossibility of sustained exponential growth in material and energetic throughput. In other words, degrowth is not imposed as an assumption but obtained as a result. The model's qualitative predictions are a regime of growth followed by a regime of degrowth. The forecasts must be interpreted cautiously, as parameter estimates are based on data collected from different sources and numerous simplifications. The model also suggests some counterintuitive results about sustainable technology. An increase in solar energy utilization, *caeteris paribus*, enables the economy to accumulate more energy, which in turn promotes resource exploitation, ultimately leading to further exergy depletion in the environment. Higher energy efficiency generates a similar effect.

If the term "Energetic-Malthusian equilibrium" is employed to denote a regime in which economic output is limited by subsistence-level energetic availability, the primary aim of this article can be more appropriately articulated as providing framework for posing the question: "if a minimal model suggests a return to Energetic-Malthusian equilibrium, what additional elements must be incorporated into our understanding of the economy or its interaction with the physical system to demonstrate a pathway toward a non-Malthusian steady state?" Subsequent economic modeling can then concentrate on this inquiry. Furthermore, this framework aims to characterize the physical limitations of conventional energy policies, such as wind, solar, and recycling, and their interactions with energy markets, with the ultimate objective of identifying policy approaches that foster prosperity within a finite world.

The rest of the article is structured as follows. In section 2, we describe the literature concerning economic models that emphasize energy transformation, mainly the neoclassical school and its aggregate production function, and the literature stemming from Georgescu-Roegen. We also show how these models relate to the present exposition. In section 3, we introduce the analytical description of the model and the theoretical results. Numerical simulations are reserved for section 4, where we also discuss how to estimate the parameters. The article closes with concluding remarks in section 5, in which we discuss extensions such as modeling anergic inefficiencies, incorporating recycling technology, and considering energy markets.

# 2 Energy Modeling Paradigms

Models that incorporate energy as a fundamental building block can be grouped into two main categories: (i) models derived from the Georgescu-Roegen flow-fund theory and (ii) models inscribed in the neoclassical theory of production, which rests on the concept of aggregate production function. The latter family is broader and spans a larger literature. We will begin by describing the similarities and discrepancies between the current proposal and the Georgescu-Roegen flow-fund theory, as they are conceptually closer to the one presented in this article.

Georgescu-Roegen's Work represents one of the most influential critiques of the Solow-Stiglitz paradigm of macroeconomic modeling. These criticisms were summarized by Daly (1997): i) the Solow model ignores the role of natural resources; in fact, the continuous exertion of labor and the maintenance and increase of the capital stock are possible because of the depletion of natural resources. ii) Even when natural resources are included in the Solow paradigm, production functions commonly employed assume substitutability between capital and natural resources. The

neoclassical paradigm is incomplete since it considers "a circular, self-feeding and self-renewing process" (Heilbroner et al., 1990). Because the Solow framework abstracts from explicit material and energetic accounting, it does not incorporate the thermodynamic constraints governing energy conservation and entropy generation. In Georgescu-Roegen's paradigm, dependence on the physical limits of energy transformation is explicitly incorporated.

According to Georgescu-Roegen (1971), the basic unit of analysis is a "process" through which energy and materials flow; hence, it is delimited by a boundary. Elements that traverse the process are of two distinct categories: *flows* and *funds*. Flow elements are the objects of transformation; they enter the process as inputs and exit as outputs. Fund elements are the agents of transformation of the flows, and they are not transformed by the process. Georgescu-Roegen describes the processes that make up the economy: the production of goods, the production of capital goods, consumption itself, and nature. The main flows described by the model are commodities, natural resources, and waste. The law of conservation of matter is at the core of this model, governing the transformation of resources into intermediate goods, consumption goods, and, ultimately, waste. This transformation is supported by funds whose actions are termed a *service*. Funds are mainly land and labor. The funds themselves are degraded in the process, but their volume is maintained through additional material flows, which are termed the process fund.

It is important to note that mainstream economic modeling rests on the idea of a *stock* since production depends on the sizes of labor and capital stocks. On the contrary, in Georgescu-Roegen's model, production depends on how much of the stock of capital and labor is utilized in a given time interval, *i.e.,* the *service* it provides, and what is the quantity of intermediate commodities that are required to keep these services running. All the flows and funds are organized according to the processes they feed into or are fed by. This multi-process model is strictly disaggregated. Georgescu-Roegen criticized the use of aggregated variables since the flows and funds involved in the different processes are not "qualitatively identical and, hence, have no common measure" (Georgescu-Roegen, 1971, p. 244). Aggregated variables, while analytically convenient, may obscure the qualitative heterogeneity of matter and energy traversing the processes. (Couix, 2020).

The model crystallizes as a dynamic system of equations called the Open Leontief system, which, in essence, is a careful dynamic accounting of the material inputs and outputs in the different processes that constitute the economy. Despite the successful description of the economy as a whole, especially in terms of its maintenance or subsistence, the model faces conceptual and technical challenges in accommodating growth. Growth presupposes an increase in the product flow, requiring the creation of additional processes. These new processes must be "primed" for production; their process fund or maintenance fund must be completed before production begins. In simple terms, this requires an accumulation of output before growth is even possible. To accommodate growth, the model needs lags or delays, a development formalized by Mayumi (2005) who also shows the parametric restrictions required for a solution to exist.

The flow-fund conceptualization has given rise to a relatively focused but substantive literature in economic modeling, such as the Multi-scale Integrated Analysis of Society and Ecosystem Metabolism (MuSIASEM) (Giampietro et al., 2009) and the Ecological Microeconomic post-Keynesianism (Dafermos et al., 2017). MuSIASEM has been applied to study the metabolic patterns of selected national economies (Giampietro et al., 2010) and local economies (Acevedo-De-los Ríos et al., 2024; Pérez-Sánchez et al., 2019; Wang et al., 2017).

The model described in this article rests on the physical dimension of the economy in the spirit of Georgescu-Roegen. Accounting for the physical foundation requires us to abide by the first and second laws of thermodynamics: energy conservation and the loss of exergy, as described by the law of entropy. Nevertheless, instead of following the disaggregation path, this model takes the opposite approach and aggregates all stocks and funds into a single construct: energy. Aggregation in the proposed model discussed is based on the biophysical school of thought, which considers energy as the only fundamental non-substitutable biophysical input (Cleveland et al., 1984; Cleveland, 1992; Gilliland, 1975). A high level of aggregation aims to isolate the relationship between the environment and the economy.

The model is driven by mechanical time. We acknowledge the limitations of dynamic models raised by Mayumi (2005), who claims that it is impossible to represent genuine qualitative changes in the production process using a dynamic system since qualitative change requires the emergence of a new production process. Therefore, it is impossible to reach the new equilibrium using the old processes by the addition of infinitesimal steps. Moreover, Rosen's (1985) critique that social systems cannot be represented by reactive models (like systems of differential equations) applies. According to Rosen, the behavior of a social system, like the economy, depends on predicted future states and inputs; the economy is an "anticipatory system". Despite these critiques, differential equation systems remain analytically useful on the grounds of their ability to describe both a regime of growth and the growth process, as well as a simpler description of a fund.

Having sufficiently explored Georgescu-Roegen's paradigm, our discussion now focuses on the alternative approach advanced by the aggregate production function (APF). The APF extends the neoclassical production function by incorporating energy as an additional factor. Either in the form $f(K, L, E)$ or as the underlying input of both factors, *i.e.*, $f(K(E), L(E))$ (Keen et al., 2019). This approach hinges on the total derivative of $f$ to obtain the marginal productivities, as is usual in the neoclassical tradition. Energy in these models is considered to be *exergy*, the energy that can produce work understood in the physical sense. Building on the ideas of Georgescu-Roegen, these models hold that it is not the volume (or magnitude) of the stocks that matters, but the services they provide. In addition, they consider functional forms in which capital and exergy are imperfect substitutes. One of the more prolific strands of literature stems from the linear-exponential (LINEX) production function developed by Kümmel et al. (1985), obtained by assuming flexible mathematical forms of factor productivities and then proceeding by partial integration. This literature has offered an explanation for the Solow residual (Ayres and Warr, 2005) and incorporated the effects of information and communication technologies (ICT) (Warr and Ayres, 2012). Other studies have verified that energy plays a crucial role in explaining total factor productivity (Santos et al., 2018).

The model presented in this article is amenable to the traditional stocks of labor and capital and shares the same conceptual focus on their services rather than their magnitudes. However, the present approach differs from the APF on many fundamental levels. First, the current proposal does not use a production function, since energy is the only input and economic activity is an energy flow along a gradient. In other words, the present model describes energy flow and its contribution to growth. Also, the models with aggregate production functions differ in how they incorporate the physical limits of the productive process. The literature stemming from Ayres and Warr (2005) acknowledges the importance of natural resource inputs and waste outflows, but

the latter play a minor role because it focuses on explaining growth. Waste outflows are used to compute energy efficiency, rather than as explicit binding constraints. Other approaches, such as Kümmel (2011), introduce pollution functions that slow growth once critical thresholds are reached; recycling functions are incorporated analogously. This approach results in a growth equation with a limit-to-growth multiplier. A more conservative approach is that of Stern (2011), who also introduces energy as a factor of production in a CES production technology with low substitutability between capital and labor. By adding additional parameters to the CES function, energy could be a constraint on growth or an enabler of growth.

In this essay, the law of entropy is a foundational building block and not embedded on top of the model. Hence, the limits to growth Emerged endogenously from the thermodynamic structure of the model. Waste-energy flows are explicitly considered, more closely resembling Georgescu-Roegen's approach. The effects of waste flows can be modeled as impacting the parameters that govern energy transformation rather than the rate of growth itself, as done in the APF literature.

The current article relates to other approaches that explicitly consider limits to growth or model degrowth. This article suggests that sustained exponential growth in material throughput is incompatible with the physical constraints on energy and matter transformations (Hickel and Kallis, 2020; Ward et al., 2016), and instead derives a steady-state economy (Daly, 2012). These approaches are not exclusive to ecological economics, as neoclassical economists have explored how degrowth can be achieved (Bilancini and D'Alessandro, 2012; Heikkinen, 2015; Lange, 2018). Given the high level of aggregation in our approach, it is not immediately clear how the degrowth trajectories derived in this article relate to policy, prices, or labor markets, but they could be studied in subsequent research. Our approach offers a significant complement to disaggregated and market-specific neoclassical approaches.

A review would not be complete without discussing the contributions in Econophysics, from which the most resemblance comes from the work of Herbert et al. (2023) and Nitzbon et al. (2017). Both analyze the interdependence of the Earth system and human society with the aim of describing the evolution of natural and socioeconomic subsystems. The model presented in this article simplifies the analysis of these authors, with inherent advantages and disadvantages. Both authors establish a parametric dependence between sustainable and collapse regimes; sustainability is possible, albeit under strict parametric assumptions. We defend the need for an alternative thermodynamic framework because the aforementioned models assign a limited role to human agency and the impact of markets. In Nitzbon et al. (2017), the economic module is made of derived quantities that depend mostly on efficiency parameters and one policy parameter: investment. In Herbert et al. (2023), there is an explicit supply and demand for energy that depend on two policy parameters: intensity and friction (which models recycling technologies). It is unclear how to incorporate price effects in both models. We advocate for a richer parameter set to describe different energy policies and to provide a framework that enables an economic module to ground a theory of production and market dynamics.

# 3 A Thermodynamic Framework of the Economy

The main premise of this model is to aggregate all economic inputs and outputs into their energy equivalents. This extreme aggregation is possible due to the equivalence of energy and mass in

Einstein's special theory of relativity (Kümmel, 2011, pp. 33-34). The model conceives of the economy as an energy flow from one stage to the next; each stage is represented by a compartment or reservoir. The main influx of energy into the system comes from the sun, which inputs energy at a rate denoted by $\eta$. The main reservoir of this energy is denoted by $B$, the biomass reservoir. For the purposes of this article, we assume that the biomass reservoir excludes all humans. Historically, the human system has extracted energy from a finite resource reservoir denoted by $F$.

Several nuances are worth expounding. First, the only energy inflow into the biomass reservoir comes from the sun. Hence, the system is not closed. Second, the term $\eta$ refers to the rate at which potential "useful" energy from the sun is captured for further transformation. Note that this rate differs from the solar energy inflow and even from solar energy absorption. The rate $\eta$ not only considers the difference between the solar influx and the terrestrial outflux (Kleidon and Lorenz, 2004), but how much of this difference is captured by living subsystems (biomass on earth). The proliferation of lifeforms is considered an accumulation of energy in the reservoir $B$. After millions of years and additional energy inflows, the demise of these lifeforms gives rise to the present reserves of fossil fuels, which are considered part of the stock of energy in $F$. Ultimately, all mass in the Earth's crust, including minerals such as coal, diamonds, and iron ore, can be considered part of humanity's energy endowment, accounted for in $F$. We do not consider geothermal energy for simplicity, nor the absorption of energy by other terrestrial subsystems, such as energy storage in the oceans or atmospheric heating. Third, the model must abide by the first law of thermodynamics: "energy can neither be created nor destroyed". It follows that once energy is captured within the system, it will be either stored, transformed, or escape the system; the model should account for all three.

After considering energy inputs, we proceed to a brief discussion about energy transformation. Not all available energy can produce useful work; for that reason, it is important to provide a measure of "useful" energy; that concept is called *exergy*. It is defined as the maximum amount of *work* that can be performed as a subsystem approaches thermodynamic equilibrium with its reference environment. Energy cannot be "spent" or "used up" since it is *conserved*. Only a portion (exergy) can be converted from any energy quantity to produce work. The remainder is called *anergy*. Therefore, the first law of thermodynamics is stated as:

$$\text{energy} \equiv \text{exergy} + \text{anergy} = \text{constant} \tag{1}$$

Usage of the term "anergy" was proposed by Kümmel (2011), who borrowed it from medicine and psychology. Its usage is not as widely accepted as "exergy". We also adopt its use because it allows us to track energy transformation in ways that resemble accounting.

All energy transformation processes are governed by the second law of thermodynamics, also known as the law of entropy. Most readers are familiar with the Clausius statement of the second law: "the entropy of the universe increases in the course of any spontaneous change". In other words, the entropy of any system, together with its surroundings, always increases. There is no prohibition that the system or the surroundings individually undergo decreasing entropy, provided there is a compensating change elsewhere. For the present discussion, dwelling on the intricacies of the physics definition of work, temperature, or heat engines is unnecessary. It will be enough to recall that the change in entropy, denoted by $\Delta S$, is given by

$$\Delta S = \Delta Q/T$$

where $\Delta Q$ denotes the heat provided into the system reversibly, and $T$ denotes the temperature at which the transfer occurred (Atkins, 2010; Zemansky and Dittman, 1997). Therefore, thermodynamic entropy has the dimension of energy divided by temperature, and its units are Joules per Kelvin (J/K). The second law of thermodynamics in the present context means that in all energy-conversion processes, which produce entropy, *anergy* increases at the expense of useful exergy; and that no energy-conversion process can be performed at 100% efficiency, *i.e.,* anergy will always be produced. We will also use the term "energy dispersion" as a synonym for anergy generation. Statistical thermodynamics interpretations of entropy as chaos or disorder will not be employed. Since we will rely on entropy estimation for different subsystems, we will also define anergy generation as entropy generated times the environmental temperature at which this energy transformation takes place, $\Delta A_{an} \approx \Delta S_{gen} \times T_0$; bearing in mind that $\Delta S = \Delta S_{\text{transfer}} + \Delta S_{gen}$.

The principles' first application occurs at the environmental level, disregarding human activity. From the energy of the sun, the light reactions of photosynthesis convert carbon dioxide ($CO_2$) to sugars and other organic molecules that make up living organisms (biomass); this considers not only the biomass of vegetation, but also of herbivore fauna that feeds on it, and carnivore fauna. We consider this the "growth" phase, the energy accumulation in $B$. We will assume that energy accumulation in the biomass reservoir is proportional to its volume. In other words, disregarding energy losses, the energy reservoir will grow according to $B'(t) = \eta B(t)$, we are assuming that the accumulation rate is fixed. However, burning these organic molecules by respiration (controlled oxidation within animal bodies) and dispersing energy as body heat generates anergy. We will denote the anergy production rate by $\delta$, and its production will be assumed to be proportional to the size of the reservoir $B$. Dead biomass is also considered part of the reservoir; although its energetic processes differ from living biomass, we can still regard it as exergy-consuming and anergy-producing. Hence, disregarding human activity, we have to adjust the equation of growth for the biomass reservoir to $B'(t) = (\eta - \delta)B(t)$. Note that the biomass reservoir feeds the finite resource reservoir ($F$) at rate $\theta$. However, in historical time, $\theta = 0$, which we will assume from now on.

Human activity requires extracting energy from $B$ and $F$. We will discuss first the extraction from the biosphere. Extraction rate is denoted by $\alpha$. We use $\beta \in [0, 1]$, a dimensionless coefficient, to model energy extraction inefficiencies. Therefore, $\alpha\beta$ is the net extraction rate. We will use analogous terms for the extraction from the finite resource reservoir. Extraction rate will be denoted by $\alpha_F$, extraction efficiency will be denoted by $\beta_F$, and the term $\alpha_F\beta_F$ will be the net extraction rate. We will also assume the presence of technology that allows for direct solar energy extraction. It will be denoted by $\eta_k$, a rate of absorption. All energy sourced by human activity will be accumulated in the reservoir $K$, termed "the economy" but a more proper name could be "the human reservoir" or "the human system". It is essential to observe that the usual units of analysis of "the economy" (capital and labor) are not explicitly described but aggregated into one. The objective is to achieve a plain description of the economy from an energy transformation perspective.

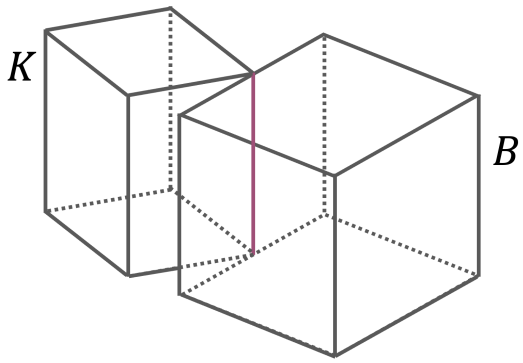
Aggregating capital and labor obviates three other important economic processes: production, consumption, and investment. Although they are important for economic analysis, they are cumbersome to account for from an energy perspective since everything that is produced is either consumed, hence employed in the metabolism of humans to sustain their existence and to grow in numbers, or saved, increasing the stock of capital, which in this perspective is just accumulated energy. Therefore, consumption and investment are aggregated and expressed as the growth of the human system ($K$).

We will assume that direct solar energy absorption ($\eta_k$) is directly proportional to the size of the human system. However, energy extraction from other sources is assumed to be proportional to the size of the energy reservoir but less than unitary and proportional to the size of the economy. The reason for such an assumption is the idea of a "fund". A cornerstone of Georgescu-Roegen's approach (Georgescu-Roegen, 1971, pp. 224-228), funds are the agents of the economic process that act upon the flow elements to control their transformation. Fund elements provide a service, but the process does not transform them. Traditionally, fund elements are capital, human labor, and land (Couix, 2020, p. 3). The absence of transformation within the funds is a fiction since they suffer from wear and tear and need a steady flow of matter to keep them running. Fund elements are also a "stock", but a stock of services. The number of workers or tools can be increased or decreased, but what matters is the total services their "stock" can provide. Furthermore, those services cannot be provided at the same time. A worker can provide 8 hours of labor everyday for a month, but it is unable to provide $30 \times 8 = 240$ hours all at once.

An interface parameter is used to parsimoniously model the idea of a "fund". Since only a fraction of the total capital stock (or human labor) can be allocated to the biomass reservoir for energy extraction, an interface parameter is required to regulate their interaction. The interface parameter is $1/\phi$ where $\phi > 1$, therefore energetic output is given by $\alpha\beta K(t)^{1/\phi}B(t)$. The economist reader should be warned that this familiar algebraic form has a very different meaning from a Cobb-Douglas production function, where we use exponents to denote the relative importance of each factor of production. It is also tempting to interpret the exponents as a normalization of a Cobb-Douglas, but this is not the case. The exponent is to reduce the dimensionality of the capital stock to consider that only a fraction (the fund) interacts with the environment to extract energy for any given unit of time. If extraction is unitary proportional to both system sizes, the energetic flow to the capital stock would be represented as $\alpha\beta K(t)B(t)$. However, such a modeling choice is valid only when $K$ and $B$ are perfectly well-mixed, as in epidemic models where susceptibles can contact infectives or other susceptibles with equal probability; this is also the case in simple predator-prey interactions.

To further the interpretation of the dimensional analogy, consider that the embodied energy in the human system is much smaller than the energetic content of the biomass reservoir; furthermore, not the entire volume of the human system interacts with the environment or the finite resource reservoir to extract energy from them. Only the fund (an interface) is in contact with the energy reservoir, hence exergy flows unidirectionally from $B$ to $K$ through the interface. Therefore, the parameter $\alpha$ is a diffusivity parameter, which considers units of inverse time, representing the rate at which energy flows from $B$ to $K$, and it is an expression of extractive capacity, limited by technology and market considerations. If we assume that both $K$ and $B$ are volumes, and the fund is length density (or capacitance density), energetic flows to the economy would be represented by $\alpha\beta K^{1/\phi}B$, where $\phi = 3$. In simple words, the interface is one-dimensional (a length) with respect to the reservoirs, which are three-dimensional (a volume). Extraction is behaving like a flow from a volume through a slit (a line). Hence, if the time units are years ($yr$), the units of $\alpha$ would be $\frac{1}{yr \cdot J^{1/3}}$. The parameter $\alpha$ indicates how many Joules of energy flow to $K$ per every cubic root of Joules in $E$ per year. Refer to figure 1a for a visualization.

For the economy in the aggregate, extracting from the biomass reservoir, it might be reasonable to assume $\phi \approx 3$. The fund (or interface) represents the linear contact between two volumes over




(a) Conceptual diagram.



(b) Artistic representation.

Figure 1: Interpretation of the interface between the human system and the biomass reservoir; both are considered volumes, while the interface is one-dimensional. Artistic representation was generated using ChatGPT.

the surface of the earth. See figure 1b for an artistic rendering. For conventional oil extractive industries, it seems reasonable to assume $\phi > 3$. The reasoning is that although we can consider the human system and the finite resource as volumes, the "dimension" of the human system "in contact" with the finite resource is less than linear, since most of the finite resource is underground. In the absence of a method to accurately estimate the parameter, we will assume that $\phi \approx 4$. In the reminder, we will use the letter $\psi$ to denote the interface parameter for the finite resource.

Anergy production from all energy conversion processes in the economy is denoted by $\delta_k$, also a rate. This coefficient is significant since the so-called "externalities" from economic activity that negatively impact the environment are all anergy. From the material waste of human metabolism to $CO_2$ and other greenhouse gases, they are the by-products of energy conversion processes. Note that $\delta_k$ also includes capital depreciation, another form of anergy production. The final assumption is that the anergy production of the human system is proportional to its size: the larger the economy, the more we consume, and the more waste is produced. The bigger the capital stock, the more of it is depreciated.

Hence, the rate of growth of the human system is $K'(t) = \alpha\beta K^{1/\phi}B + \alpha_F\beta_F K^{1/\psi}F + (\eta_k - \delta_k)K$. Time dependencies on the right-hand side are omitted for readability. The derivative represents the inflow to the human system, which amounts to the *net* inflow from the biomass reservoir plus the net inflow from the finite reserves. The third term represents the gains from direct solar energy usage $\eta_k K$, and the losses from anergy production $\delta_k K$. The rate of growth of the ecosystem, including human-induced flows, is updated to $B'(t) = (\eta - \delta)B - \alpha K^{1/\phi}B$. The first term represents the net inflow of energy: the growth enabled by the energy inflow from the sun $\eta B$, minus anergy production $\delta B$. The second term represents the energy loss due to human extraction.

Finally, it is necessary to account for anergy production throughout all processes and introduce an anergy reservoir $W$. The evolution of the energy reservoir is given by $W'(t) = \delta B + \delta_k K + \alpha(1-\beta)K^{1/\phi}B$. The first term represents anergy produced by the environment, the second term represents anergy produced by the economy, and the third term represents anergy originating from extraction inefficiencies. Given that we assumed that $\eta$ represents energy cap-

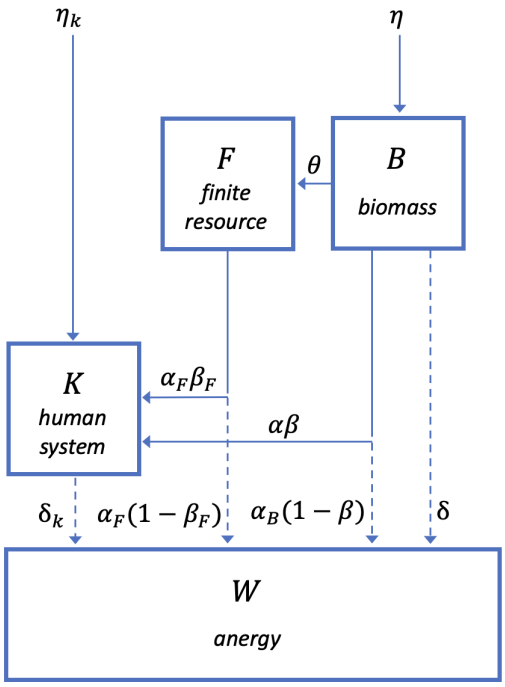



Figure 2: Conceptual diagram of the energetic flows.

tured by the terrestrial systems, it is a convenient simplification to assume that no energy is lost; in other words, the system is open to receive energy from outside but does not lose it. Therefore $B'(t) + K'(t) + W'(t) = \eta B + \eta_k K$, verifying the law of conservation. A more descriptive model would account for the fact that some of this anergy is lost to outer space, but, following a simple accounting rationale, we would treat it as becoming trapped in the system. If the researcher would like to study the inefficiencies arising from the accumulation of anergy (see section 5), careful modeling of anergy flows is needed.

A schematic description of the system is shown in Figure 2. Note that there is no flow from the anergy reservoir $W$ to the biomass reservoir $B$. This flow could be used to misrepresent recycling technologies that incorporate material waste into the energy stock. This flow violates the second law of thermodynamics. By the second law of thermodynamics, both the economy and the environment are increasing in entropy, represented by the flows $\delta$ and $\delta_k$. According to the statistical physics of open systems, increasing entropy means energy dispersion. Therefore, economic activities can be viewed as energy flowing along an energy gradient, and this flow is always "downhill": the economy flows from one state to another by decreasing free energy (exergy) (Annila and Salthe, 2009). Discussion on the possible ways of modeling recycling technology will be left to section 5.

The model can be summarized as the following system of differential equations:

$$B'(t) = (\eta - \delta)B - \alpha K^{1/\phi} B \tag{2}$$

$$F'(t) = -\alpha_F K^{1/\psi} F \tag{3}$$

$$K'(t) = \alpha\beta K^{1/\phi} B + \alpha_F \beta_F K^{1/\psi} F + (\eta_k - \delta_k)K \tag{4}$$

$$W'(t) = \delta E + \delta_k K + \alpha(1-\beta)K^{1/\phi} B + \alpha_F(1-\beta_F)K^{1/\psi} F \tag{5}$$

Note that human technology determines $\alpha$, $\beta$, $\alpha_F$, $\beta_F$, $\eta_k$ and $\delta_k$. This detailed description allows us to study the economy's evolution as various technological parameters vary. The parameters $\eta$ and $\delta$ are not determined by technology. In the present article, we will assume all parameters to be constant across time.

Finally, we introduce an assumption that ensures local stability and argue for its plausibility through back-of-the-envelope calculations. First, we assume:

**Assumption 1.** $\bar{\eta}_k \equiv \eta_k - \delta_k < 0$.

This assumption means that anergy production exceeds the energetic inflow from solar extraction. The assumption is verified by empirical data, considering that solar capacity in the year 2022 amounted to 1 TW ($\eta_k K \approx 1$ TW) (IEA, 2023); meanwhile, total entropy production by commercial use of energy in human societies is approximately 11 TW ($\delta_k K \approx 11$ TW) [2] (Kåberger and Månsson, 2001). It remains an open question if solar technology can extract more energy than the total entropy production of the economy; it seems far-fetched since it would require extremely high energy efficiency.

## 3.1 Solving the Model

The first observation is that equation 3 describing the dynamics for the finite reservoir is separable, which allows us to find a solution. As this result will be useful later, we summarize it in the following lemma.

**Lemma 1.** *The dynamics for the finite resource reservoir are given by*

$$F(t) = c \exp\left\{-\alpha_F \int_0^t [K(s)]^{1/\psi} ds\right\};$$

*since $F(0) = F_0$, we get $c = F_0$.*

The proof follows the standard method for separable differential equations.

Since the finite reservoir decreases exponentially its steady state value would be $F = 0$. Also, observe that the evolution of the biosphere and the human system do not depend on the anergy reservoir; this allows the system to be solved exclusively for $B$ and $K$; the solution for $W$ can be found afterward. A steady state can be found by setting $B' = K' = 0$, which leads to

$$B^* = \frac{\delta_k - \eta_k}{\alpha\beta}\left(\frac{\eta - \delta}{\alpha}\right)^{\phi-1} \tag{6}$$

$$K^* = \left(\frac{\eta - \delta}{\alpha}\right)^{\phi} \tag{7}$$

Interpreting the effect of the parameters on the equilibrium solution is warranted, especially given some counterintuitive effects. First, note that the steady-state value of the human system does not depend on any technological parameters; it depends only on parameters from the natural systems.

[2] Assuming a surface temperature of 15°C or equivalently 288.15 K.

On the contrary, the steady-state value of the biomass compartment depends on technological parameters. Hence, a higher extraction rate ($\alpha$) or higher efficiency rate ($\beta$) decreases the equilibrium value of the energy reservoir ($B^*$). In the same way, a higher solar absorption ($\eta$) is associated with a higher equilibrium value of $B^*$, and the opposite is true of the anergy outflows ($\delta$). However, counterintuitively, a higher solar absorption rate from the economy ($\eta_k$) decreases the equilibrium value of the biomass reservoir. Also, a higher anergy outflow from the human system ($\delta_k$) results in a higher value of $B^*$. To understand these results, remember that this is the equilibrium of a dynamic system. If there is widespread use of solar extraction technology, *caeteris paribus*, this would allow for higher economic growth, leading to greater exploitation of the biomass reservoir. A higher energy efficiency in the economy has a similar effect.

It is also interesting to observe that the equilibrium value of the human system ($K^*$) does not depend on its inflow and outflow rates ($\eta_k$ and $\delta_k$), nor on the efficiency of extraction parameter ($\beta$). A higher solar absorption rate from the biomass reservoir ($\eta$) supports a larger economy, and the opposite is true for the anergy outflow rate ($\delta$). The counterintuitive effect is that a higher extraction rate ($\alpha$) decreases the energy content of the human system in equilibrium. Once again, this is the feedback effect from the dynamic system: a higher extraction rate depletes the energy reservoir faster, hindering the economy's long-run growth despite a higher initial growth rate. The absolute energetic size of the economy also depends directly on the extraction parameter $\phi$, which represents extraction technology.

The following proposition characterizes the global behavior of the solution for the human system.

**Proposition 1.** *Under assumption 1, the dynamics of the economy $K(t)$ are given by a forced and damped oscillator if $\phi = 3$ and $\psi = 4$.*

The proof is reserved for appendix A.

# 4 Numerical Results

Following the calibration tradition in macroeconomics, the next step is to assign values to the parameters using environmental science estimations. To accomplish this objective, we rely on a few simplifications. First, instead of providing a full description of the human system comprising capital and labor, we will focus exclusively on the capital stock. The reason for that is to set aside the discussion related to how to consider humanity as an energy stock (population) and as an energy output (labor). This discussion is left for a separate investigation. Considering only the human system as the capital stock provides some minimal economic dynamics. Second, instead of considering the energetic content of the biomass reservoir, we will restrict ourselves to the energetic content in plants and animals. Lastly, instead of using all finite resources, we will restrict ourselves to fossil fuel reserves.

For the most part, we use estimations provided by Rosen (2022) since he provides a comprehensive list of estimates for exergy consumption and entropy generation for all of Earth's primary systems. Parameter estimates should be interpreted cautiously, as they depend on various assumptions and originate from studies with diverse research aims. The objective of this section is to investigate the order of magnitude of the parameters as well as to provide a description of the behavior of the dynamic system in the near term. For all practical purposes, this exercise should

| Parameter/Initial Value | Units | Reference |
|---|---|---|
| $\alpha = 0.0013$ | $EJ^{-1/3} \cdot 1/\text{yr}$ | (Rosen, 2022) |
| $\alpha_F = 0.0057$ | $EJ^{-1/4} \cdot 1/\text{yr}$ | (Rosen, 2022) |
| $\beta = \beta_F = 0.3$ | no units | (Gilli, 1996) |
| $\delta = 0.0611$ | 1/yr | (Rosen, 2022) |
| $\delta_k = 0.3024$ | 1/yr | (Rosen, 2022) |
| $\eta = 0.0629$ | 1/yr | (Rosen, 2022) |
| $\eta_k = 0.00215$ | 1/yr | (Rosen, 2022) |
| $\phi = 3$ | no units | Conjectured |
| $\psi = 4$ | no units | Conjectured |
| $K(0) = 1,962$ | $EJ$ | (Chen et al., 2022) |
| $B(0) = 21,725$ | $EJ$ | (Bar-On et al., 2018) |
| $F(0) = 43,454$ | $EJ$ | (Rosen, 2022; BP, 2014) |
| $W(0) = 0$ | $EJ$ | Conjectured |

Table 1: Summary of parameters and initial values

be considered as preliminary and subject to further perfecting. Parameters and initial values are summarized in table 1.

First, the initial values are needed to calculate all the parameters. Global embodied energy in the capital stock was estimated by Chen et al. (2022) at 1,962 $EJ$ in 2014, his latest estimate. The total stock of biomass on Earth is estimated at 550 $G$ tons of carbon (Bar-On et al., 2018), which is converted to chemical energy using the conversion factor of 39.5 $KJ$ chemical energy per gram of carbon. The calculation yields a total of 21,725 $EJ$. For the finite resource reservoir, we consider only fossil fuel reserves as estimated by British Petroleum at the end of 2013 to be 43,454 $EJ$ (British Petroleum, 2014); we are excluding all other mineral resources that could be considered finite. For practical purposes, we are also considering that the anergy reservoir starts at zero, *i.e.,* $W(0) = 0$.

The exercise in this section is to estimate the parametric flow rates using the change from 2013 to 2014. Hence $t_0 = 2013$, $t_0 + \delta = 2014$, and $\delta = 1$ year. We used the change from 2013 to 2014 because the latest estimate of energy embodied in capital is from 2014. For reference, $K(t_0) = 1962$ $EJ$ and $K(t + \delta) = 2041$ $EJ$. The rationale behind the calculation of the extraction rate is as follows. Start with $B'(t) = (\eta - \delta)B - \alpha K'^{/\phi} B$, then integrate both sides from $t_0$ to $t_0 + \delta$:

$$\begin{aligned} B\left(t_0 + \delta\right) - B\left(t_0\right) &= \int_{t,}^{t_0+\delta} B'(t)dt \\ &= (\eta - \delta) \int_{t_0}^{t_0+\delta} B(t)dt + \alpha \int_{t_0}^{t_0+\delta} K(t)^{1/\phi} B(t)dt. \end{aligned}$$

The left-hand side is disaggregated as $\Delta B = \Delta B_{ex} + \Delta B_{an} + \Delta B_K$ where they are equivalent to

the right-hand side terms

$$\Delta B_{ex} = \eta \int_{t_0}^{t_0+\delta} B(t)dt \tag{8}$$

$$\Delta B_{an} = -\delta \int_{t_0}^{t_0+\delta} B(t)dt. \tag{9}$$

Parameters $\eta$ and $\delta$ are obtained from equations 8 and 9 respectively. The left-hand-side terms are obtained from Rosen (2022), since they are net of human activity. The right-hand-side integrals are calculated numerically from initial and final measurements of $B(t)$, assuming a linear interpretation. To obtain $B(t+\delta)$, we use the net primary production of carbon as estimated by Field et al. (1998) and converted to its energy equivalent. The net primary production of carbon (NPP) is the net amount of carbon taken up by plants and algae for photosynthesis (Gross primary production) minus the carbon lost through respiration. It represents the remaining chemical energy used to create new biomass. We will use this number as the biomass that could have been generated in one year, it represents a biomass generation potential. We finally obtain $\eta = 0.0629$ and $\delta = 0.611$.

The method is analogous for capital. Integrating both sides of equation 4 we obtain

$$\begin{aligned} K\left(t_0+\delta\right) - K\left(t_0\right) = \alpha\beta &\int_{t_0}^{t_0+\delta} K^{1/\phi}(t)B(t)dt \\ &+ \alpha_F\beta_F \int_{t_0+\delta}^{t_0+\delta} K^{1/\psi}(t)B(t)dt \\ &+ \eta_K \int_{t_0}^{t_0+\delta} K(t)dt \\ &- \delta_K \int_{t_0}^{t_0+\delta} K(t)dt. \end{aligned}$$

The previous equation can be written in the shorthand $\Delta K = \Delta K_{bios} + \Delta K_{fin} + \Delta K_{sol} + \Delta K_{an}$.

Solar energy rates are the simplest to compute since $\eta_K = \frac{\Delta K_{sol}}{\int_{t_0}^{t_0+\delta} K(t)dt}$. For the denominator we use a numerical integral assuming a linear interpolation, were initial and final values come from (Chen et al., 2022). For the numerator we use the installed solar capacity in 2013 and assume that all capacity is translated into output which is susceptible to be accumulated as capital, then $\Delta K_{an} = 58.8\ EJ$. This yields $\eta_K = 0.00215$. Energy production from the capital stock is similar, since $\delta_K = \frac{\Delta K_{an}}{\int_{t_0}^{t_0+\delta} K(t)dt}$. From Rosen (2022) we obtain $\Delta K_{an} = 605.3\ EJ$, which allows us to estimate $\delta_K = 0.3024$.

Before proceeding with the estimation for the extraction rates a discussion on the estimation of the efficiency parameters ($\beta$ and $\beta_F$) is required. Note that both parameters refer to first-law efficiency, which relates to the conversion of energy abiding by the conservation of the overall quantity of energy, without direct consideration for the quality of energy (*i.e.,* exergy or anergy) (Smith and Parmenter, 2016). Both parameters refer to the total amount of energy that enters the economic system as inputs in contrast to the energy spent or lost during extraction. Second law efficiency in this context would refer to how much of this energy input is converted into exergy (in this model, amounts to growth) and how much is degraded into anergy. Therefore, we need estimates of first

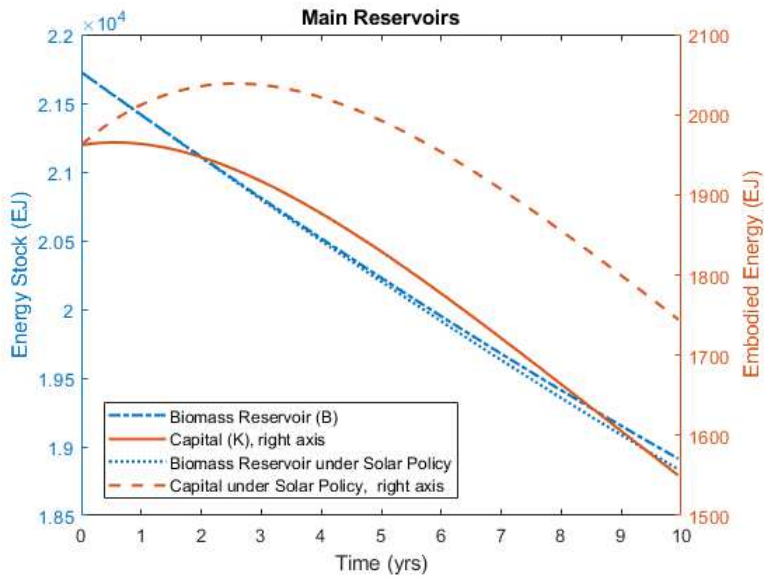



Figure 3: Trajectories under parametric assumptions in Table 1, trajectories under Solar Policy assume solar capacity of 2024 ($\eta_k = 0.0294$).

law efficiency for the global economy; estimates are provided by Gilli et al. (1995), who estimates it just below 30%. For reference, a similar study estimates the first-law efficiency of ferrous metallurgy in the Soviet Union at 31.2% and the overall first-law efficiency of the Soviet economy at 10.6% (Stepanov, 1995). For simplicity, we assume that the first-law efficiency of fossil fuel extraction equals the economy's first-law efficiency ($\beta = \beta_F = 0.3$). This represents an extremely rough estimate.

Estimates for the extraction rates from the biosphere and the finite resource reservoir need a slightly more complicated integral. For the extraction from the finite reservoir,

$$\Delta K_{\text{fin}} = \alpha_F \beta_F \int_{t_0}^{t_0+d} K(t)^{1/\psi} F(t) dt.$$

From Rosen (2022), $\Delta K_{\text{fin}} = -492.3\ EJ$ which corresponds to the total exergy consumption from fossil fuels. To perform the integral in the right hand side we assume that the function $K(t)^{1/\psi}F(t)$ is linear, and $R(t_0) = 43,454\ EJ$, $R(t_0+\delta) = 42,961\ EJ$,$K(t_0) = 1,962\ EJ$, and $K(t_0+\delta) = 2,041$ $EJ$. Finally $\alpha_F = 0.0057$. Estimation for the extraction rate from the biosphere starts with

$$\Delta K_{\text{bios}} = \alpha\beta \int_{t_0}^{t_0+\delta} K(t)^{1/\phi} B(t) dt;$$

$\Delta K_{\text{bios}} = 106.6\ EJ$ from Rosen (2022), and for the numerical integral $B(t_0) = 21,725\ EJ$ and $B(t_0+\delta) = B(t_0) + \Delta B_{ex} = 23,223\ EJ$. Note that we are using the actual Energy consumed by plants and animals rather than the potential energetic generation estimated from NPP. The values for the capital stock remain the same as in the calculation for extraction from finance resources. Our estimate is $\alpha = 0.0013$.

Trajectories for the economy and the finite resource reservoir are shown in Figure 3. The curves

are constructed using the finite difference method, with time increments of 1/20 of a year, spanning 10 years. The model exhibits the expected qualitative trends, such as a short period of overall growth of the capital stock followed by deceleration, a peak, and a trajectory of degrowth. The peak is predicted to occur within the next 10 years. This result agrees with dated forecasts of peak oil (Murphy and Hall, 2011) and even with sectoral estimates of Kroy (2007), who estimate the peak extraction of energy from all sources around 2015 and 2025. The constant depletion of the biomass reservoir accompanies the economic trajectory. Figure 3 also shows the trajectories in an alternative scenario in which the solar extraction rate is switched for an estimation in the year 2024. This scenario portrays what could have happened if, in 2013, the solar installed capacity had been the same as in 2024. This result yields an estimate of $\eta_k = 0.0294$, which is an order of magnitude larger. The qualitative result is that with a stronger solar policy, the capital stock can accumulate further before entering the degrowth regime. The biosphere shows a minor deterioration. After 10 years, a strong solar policy implies a capital stock that is 12% larger; meanwhile, the biosphere only diminishes by 4%.

Results in this section suggest that modern growth is an exceptional epoch between two Malthusian eras of zero growth. If we assume that prior to 1800, extraction from finite sources was minimal ($\alpha_F \approx 0$), Equations 6 and 7 describe the steady state despite $F(t) > 0$. Therefore, the Malthusian steady-state and the steady-state after the depletion of the finite resource reserves are identical. Using the parameterization in this section, we obtain $K^* = 3.12$ EJ. Thus, the Malthusian equilibrium obtains an absolute size of the economy in which its embodied energy is approximately 0.15% that of today's. An estimate of the capital stock in the United States in the year 1800 is approximately $9,000$ million dollars of 2017 (Williamson, 2025), which corresponds to 0.04% of the capital stock in the year 2010. This is a rough validation of the appropriate scale for the capital stock under a Malthusian regime. Although the discrepancy is one order of magnitude, the estimates are not equivalent, since one is in value (dollars) and the other in embodied energy. The damped oscillation trajectory, which under this parameterization exhibits a short burst of growth followed by decay, suggests that as finite reserves deplete, the economy would revert to its pre-industrial equilibrium in the absence of any compensating increase in extraction from alternative sources. This finding is significant for policymakers, as it indicates that the most effective degrowth trend involves more intensive use of renewable sources, along with a deceleration of extraction from finite reserves, so that $F$ remains larger than zero for as long as possible, thereby delaying the reversion to the Malthusian trap.

# 5 Concluding Remarks

In the preceding sections, this article suggests that modern growth is an exceptional phase succeeding a zero-growth era. It also clarifies that modern growth relies on the exploitation of finite resource reserves, as we approach the exhaustion of those reserves, this model shows that humanity is potentially approaching a new steady-state era, imposed by the natural restraints of energy transformation processes.

The reader might suggest that the present framework does not describe an economic model but rather a physical model in which economic activity is embedded. That is a valid objection, but to address the issue would merit a lengthy discussion, which is worth considering separately. Note that the parameters $\alpha$ and $\alpha_F$ correspond to economic equilibrium quantities. A capacity for

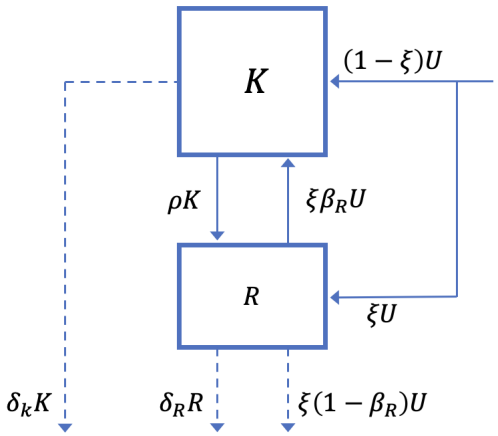



Figure 4: Schematic diagram of the recycling subsystem; the rest of the compartments and flows remain unchanged. Anergy flows are denoted in dashed lines.

energy extraction as a function of the current capital stock represents a potential energy supply rate. Incorporating the effect of prices into energy capacity yields a supply function. An energetic demand function considers the relation between energy consumption rate and price; the demand function would not only be dependent on energetic needs but also on energetic wants, considering the effect of energy uses that go beyond the sustenance of life. In this way, the parameters $\alpha$ and $\alpha_F$ represent the market equilibrium energy rate for energy extraction from biomass and the finite reserves, respectively. Different microfoundations are possible, and they are considered an important goal for further investigation.

Throughout this article, we employed the simplest assumption: that parameters are fixed over time. A better modeling choice is to assume that the extraction rate parameters are not constant and depend on the volumes of energy in the reservoirs. Once extraction rates are explicitly modeled as an economic equilibrium, different dynamics would emerge. For example, as the finite resource reserve depletes, energy prices would necessarily go up, which in turn would ration demand. The net effect would be a decrease in the extraction rate $\alpha_F$. This effect would then delay the Energetic-Malthusian steady state. Another direction for future research is to explore the dynamics of the system once the effect of energy markets is incorporated into the extraction rates.

Since the model presented is just a framework, it is open to further improvements. The current model can be useful to describe the dynamics of the economy in the presence of recycling technology. Under this framework, recycling would require an intermediate waste reservoir between the capital stock, which is matter with a substantial content of high-quality energy (*exergy*), and the anergy reservoir, where the high-quality energy has been depleted. An additional energy input ($\eta_R$) is required to convert waste and reincorporate back into the capital stock. The model must highlight that exergy spent on recycling cannot be spent to increase the human or the capital stock. To simplify the description, we first group all energy inputs as $U \equiv \alpha\beta K^{1/\phi} B + \alpha_F \beta_F K^{1/\psi} F$. The parameter $\xi$ denotes which proportion of energy input $U$ is devoted to recycling, a dimensionless parameter. A diagram of the recycling subsystem is shown in figure 4.

Only a portion of the energy required for recycling is recovered. The parameter $\beta_R$ is another

dimensionless parameter denoting recycling efficiency. The remainder of this energy goes into $W$. The parameter $\rho$ determines the rate at which part of the economy is depreciated but susceptible to recycling; the parameter is in EJ/yr. Another portion of the capital stock is irretrievable and depreciates directly into the anergy stock $W$. The parameter $\delta_k$, which we used earlier, governs this flow. In further refinements, $\xi$ and $\rho$ must be considered as emerging from a market equilibrium.

Another natural extension of the model is to consider the inefficiencies that may arise as the anergy reservoir increases. The model presented in this article only considered a very simple anergy reservoir for purposes of energy conservation. A more proper modeling would consider the thermodynamic equilibrium of the Earth as well as the effects of accumulated anergy, waste, and pollutants on the efficiency of the economy. This would require a more complex elaboration of $W$. One simple extension is to model the damage to the productivity of service funds (capital and labor) as a function of $W(t)$, since the ill effects of civilization on the environment are a function of accumulated anergy. Climate change is associated with the destruction of infrastructure from storms or inundation and the abandonment of capital in coastal areas as sea levels rise (Dietz and Stern, 2015; Naqvi, 2015; Taylor et al., 2016). Climate change can also create a hostile environment that reduces firms' ability to use capital effectively (Dietz and Stern, 2015; Stern, 2013). Labor productivity can be reduced because climate change can affect morbidity, mortality, air quality, the proliferation of infectious diseases, and mood alterations, all of which hinder the ability to perform work tasks (Dell et al., 2014; Kjellstrom et al., 2009; Lan et al., 2022; Taylor et al., 2016). Natural disasters can also cause losses of human lives (Tol, 2002). A simple way to model this scenario is to make the extraction efficiency dependent on the anergy reservoir. Suppose $\beta(t) = \frac{\beta_0 \sigma}{W(t)+\sigma}$ where the parameter $\sigma > 0$. Extraction efficiency would decrease at a decreasing rate, and if $W(t) = 0$, then $\beta(t) = \beta_0$ is the maximum attainable efficiency. Also, as $\sigma$ is larger compared to $W(t)$, the effect of the inefficiency is smaller.

But the most relevant contribution and the pressing question posed by this article are the possibilities for a sustainable economy. One objective of the framework was to plainly illustrate the relationship between resource exploitation and civilization. The model proposed highlights the predator-prey interactions between the human system and the environment and suggests where energy markets can be embedded. The model is purposely built "bare bones" to highlight these dependencies. Some models in econophysics (Herbert et al., 2023; Nitzbon et al., 2017) exhibit sustainable regimes, either oscillatory dynamics or steady states. The question posed by this article proceeds in the same minimalist fashion: what is the minimal feature that needs to be added to the description of energy extraction or its interaction with civilization to obtain a non-Malthusian, sustainable, steady state?

The possibility of returning resource-constrained dynamics is an unsettling message, often dismissed by arguing that human ingenuity will eventually find a solution, as it has in the past. Some studies suggest that the escape from the Malthusian trap just before the Industrial Revolution was made possible by exploiting finite resource reserves, which untapped high exergy reserves, thereby increasing the energetic efficiency of civilization (Kümmel, 2011; Wrigley, 2013). The unstated assumption in equations 2 to 5 is that all energy conversion processes are connected to either the biosphere or the capital stock. If we take some speculative liberty and suppose energy conversion processes could be (at least partially) decoupled from the biosphere (since it seems impossible to detach them from the capital stock), the predator-prey relationship between civilization and the biosphere might break down, allowing the biosphere to thrive. Furthermore, any anergy gener-

ated would no longer be trapped in the biosphere, so it would not harm civilization. This leads to speculative energy-based civilization scaling, suggesting that escaping energy constraints could involve developing the technology of a civilization with a Kardashev Type greater than 1 (Ćirković, 2015; Sagan, 1973), where humanity harnesses energy on a solar scale rather than relying solely on incoming or stored solar energy on Earth. Alternatively, humanity could adopt self-adaptation to the environment, enabling civilization to thrive under energy constraints (Ivanov et al., 2020).

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# A Global Solution of the System

**Proposition 1.** *Under assumption 1, the dynamics of the economy $K(t)$ are given by a forced and damped oscillator if $\phi = 3$ and $\psi = 4$.*

*Proof.* Write the system again using the shorthand notation and recalling that the finite reservoir will be called $F$, and its extraction coefficients $\alpha_F$ and $\beta_F$, the extraction coefficients from the biosphere will be left without subscript.

$$\begin{aligned} B' &= \bar{\eta}B - \alpha K^{1/\psi} B \\ K' &= \bar{\eta}_k K + \alpha\beta K^{1/\phi} B + \alpha_F \beta_F K^{1/\psi} F \\ F' &= \alpha_F K^{1/\psi F} \end{aligned}$$

Use a change of variable: $x = B, y = K^{1/\phi}, z = F$. Observe that $\phi y^{\phi-1}\frac{dy}{dt} = \frac{dK}{dt}$ and $y^{\phi/\psi} = K^{1/\psi}$. The system then becomes

$$x' = \bar{\eta}x - \alpha y x \tag{A1}$$

$$\phi y^{\phi-1} y' = \bar{\eta}_k y^{\phi} + \alpha\beta x y + \alpha_F \beta_F y^{\phi/\psi} z \tag{A2}$$

$$z' = -\alpha_F y^{\phi/\psi} z \tag{A3}$$

Now we find the equilibrium solutions, first note that from equation A3 we get $z^* = 0$. This allows to recover the equilibrium solutions for the decoupled system:

$$\begin{aligned} x^* &= \frac{-\bar{\eta}_k}{\alpha\beta}\left(\frac{\bar{\eta}}{\alpha}\right)^{\phi-1} \\ y^* &= \frac{\bar{\eta}}{\alpha} \end{aligned}$$

Now we suppose a perturbation solution $(u, v, w)$ :

$$\begin{aligned} x &= \frac{-\bar{\eta}}{\alpha_\beta}\left(\frac{\bar{\eta}}{\alpha}\right)^{\phi-1} + u \\ y &= \frac{\bar{\eta}}{\alpha} + v \\ z &= 0 + w \end{aligned}$$

Note that $\frac{dx}{dt} = \frac{du}{dt}$, $\frac{dy}{dt} = \frac{dv}{dt}$ and $\frac{dz}{dt} = \frac{dw}{dt}$.

Then, the dynamics of the perturbation solution are given by:

$$\frac{du}{dt} = \frac{\bar{\eta}_k}{\beta}\left(\frac{\bar{\eta}}{\alpha}\right)^{\phi-1} v$$

To move forward, we need to set the values for $\phi$ and $\psi$. Since $\psi = 4$ and $\phi = 3$, equation A2 can be divided by $y$ to obtain

$$3yy' = \eta_k y^2 + \alpha\beta x + \alpha_F \beta_F y^{-1/4} z.$$

After plugging-in the perturbation solution, simplifying while ignoring second order terms obtains

$$3\left(\frac{\bar{\eta}}{\alpha}\right)\frac{dv}{dt} = \frac{2\bar{\eta}_k\bar{\eta}}{\alpha}v + \alpha\beta\frac{du}{dt} + \alpha_F\beta_F\left[\frac{\bar{\eta}}{\alpha}+v\right]^{-1/4}w.$$

Taking the second derivative while ignoring second order terms yields

$$3\left(\frac{\bar{\eta}}{\alpha}\right)\frac{d^2v}{dt^2} = \frac{2\bar{\eta}_k\bar{\eta}}{\alpha}\frac{dv}{dt} + \alpha\beta\frac{du}{dt} + \alpha_F\beta_F\left[\frac{\bar{\eta}}{\alpha}+v\right]^{-1/4}\frac{dw}{dt}.$$

Next, plug in $\frac{du}{dt}$ and $\frac{dw}{dt}$, bearing in mind that $\frac{dw}{dt} = \alpha_r\left[\frac{\bar{\eta}}{\alpha}+v\right]^{3/4}w$,

$$3\left(\frac{\bar{\eta}}{\alpha}\right)\frac{d^2v}{dt^2} == \frac{2\bar{\eta}_k\bar{\eta}}{\alpha}\frac{dv}{dt} + \frac{\bar{\eta}_k\bar{\eta}^2}{\alpha}v + \alpha_F^2\beta_F\left[\frac{\bar{\eta}}{\alpha}+v\right]^{1/2}w;$$

which we can rearrange as the following second order differential equation

$$3\left(\frac{\bar{\eta}}{\alpha}\right)\frac{d^2v}{dt^2} - \frac{2\bar{\eta}_k\bar{\eta}}{\alpha}\frac{dv}{dt} - \frac{\bar{\eta}_k\bar{\eta}^2}{\alpha}v = \alpha_F^2\beta_F\left[\frac{\bar{\eta}}{\alpha}+v\right]^{1/2}w. \tag{A4}$$

To establish the behavior of the right-hand-side term, we will use a first order approximation. Since $v$ is a perturbation term, it stays small relative to $\frac{\bar{\eta}}{\alpha}$; *i.e.*, $\left|\frac{\alpha}{\bar{\eta}}v\right| \ll 1$. We can rewrite $\frac{\bar{\eta}}{\alpha}+v = \frac{\bar{\eta}}{\alpha}\left(1+\frac{\alpha}{\bar{\eta}}v\right)$, and use the Taylor expansion $(1+x)^{-\frac{1}{2}} = 1-\frac{1}{2}x+\mathcal{O}\left(x^2\right)$ to write

$$\left(\frac{\bar{\eta}}{\alpha}+v\right)^{-1/2} = \left(\frac{\bar{\eta}}{\alpha}\right)^{-1/2}\left[1-\frac{1}{2}\frac{\alpha}{\bar{\eta}}v+\mathcal{O}\left(v^2\right)\right].$$

Hence,

$$\alpha_F^2\beta_F\left[\frac{\eta}{\alpha}+v\right]^{1/2}w \approx \alpha_F^2\beta_F\left(\frac{\bar{\eta}}{\alpha}\right)^{-1/2}w \tag{A5}$$

from the leading term. Clearing the constants in A4, and using the first-order approximation in equation A5, we obtain a damped oscillator equation with an exponentially decaying forcing function

$$\frac{d^2v}{dt^2} - \frac{2}{3}\bar{\eta}_k\frac{dv}{dt} - \frac{\bar{\eta}_k\bar{\eta}^2}{3}v \approx \alpha_F^2\beta_F\left(\frac{\alpha}{\bar{\eta}}\right)^{1/2}w$$

□