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A Logistic Transition Model for Perceived Difficulty in Sequential Problem Solving

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Abstract

This paper introduces a mathematical framework for modeling the temporal evolution of perceived difficulty in sequential problem-solving processes. The study is motivated by the observation that sufficiently complex tasks exhibit a characteristic progression in which difficulty is initially high, remains elevated for a period, and then decreases rapidly following a critical transition point. To formalize this behavior, a continuous model based on logistic decay is proposed to represent perceived difficulty as a function of time or progress. The model defines a critical transition parameter that captures the onset of rapid reduction in difficulty, corresponding to a breakthrough phase in problem solving. A complementary function is derived to represent perceived ease, demonstrating a symmetric but inverse relationship with difficulty. The analytical properties of the model are examined, including boundary behavior, monotonicity, and convergence. The proposed framework provides a generalized theoretical structure that can be applied across domains where problem solving processes exhibit nonlinear progression. The model contributes to the mathematical understanding of cognitive dynamics and offers a foundation for further analytical and applied investigations.

Keywords

Perceived difficulty, logistic model, nonlinear dynamics, problem solving theory, mathematical modeling, transition point analysis, sigmoid functions, cognitive modeling, convergence behavior, theoretical mathematics

1. Introduction

The study of temporal dynamics in problem solving has long attracted attention within mathematics and related disciplines, particularly in the context of nonlinear processes and convergence behavior. Many real world and abstract problem-solving tasks exhibit nonuniform progression, where effort and perceived complexity are not evenly distributed over time. Instead, such processes often display an initial phase of sustained difficulty followed by a rapid transition toward resolution.

Despite the ubiquity of this phenomenon, there is limited formal mathematical treatment that captures the evolution of perceived difficulty as a continuous function of time or progress. Existing models in optimization and dynamical systems frequently describe convergence in terms of error reduction or objective minimization, yet they do not explicitly account for the structural pattern in which difficulty remains high before undergoing a sudden decline.

In this work, we introduce the **Problem-Solving Ease Principle**, proposed by Nnaemeka Ugwumba, as a formal theoretical construct describing this behavior. The principle states that in sufficiently complex tasks, perceived difficulty begins at a high level, persists until a critical transition point is reached, and subsequently decreases rapidly, approaching negligible levels as the task nears completion. The complementary notion of perceived ease follows an inverse trajectory, increasing from an initially low value toward a maximum.

To capture this structure mathematically, we develop a continuous model based on logistic transition functions. The choice of a logistic form is motivated by its well-known properties in modeling phase transitions and nonlinear growth or decay processes. Within this framework, a critical parameter is introduced to represent the transition point at which the rate of change in perceived difficulty is maximized.

The contributions of this paper are as follows. First, we provide a formal definition of the Problem-Solving Ease Principle as a generalizable mathematical concept. Second, we derive a parametric model that characterizes the temporal evolution of perceived difficulty and its complementary ease function. Third, we analyze key properties of the model, including monotonicity, limiting behavior, and the role of the transition parameter. Finally, we discuss the broader implications of the model as a unifying framework for understanding nonlinear progression in sequential processes.

The remainder of the paper is organized as follows. Section two reviews related mathematical formulations relevant to logistic functions and nonlinear transition models. Section three presents the formal development of the proposed model. Section four provides analytical results and graphical interpretation. Section five discusses implications and potential extensions. Section six concludes the paper.

2. Literature Review

The mathematical modeling of nonlinear transitions has been extensively studied across multiple domains, particularly in the context of growth processes, dynamical systems, and convergence analysis. Among the most widely used functional forms for describing such behavior is the logistic function, which provides a continuous representation of systems characterized by initial resistance, rapid transition, and eventual stabilization.

The logistic function was originally introduced by Pierre François Verhulst in the nineteenth century as a model for population growth under constrained conditions. Its mathematical structure captures a transition from slow initial change to rapid growth and eventual saturation, making it suitable for representing systems with bounded dynamics. Over time, the logistic function has been generalized and applied in diverse areas including epidemiology, physics, and learning theory.

In mathematical terms, the logistic function is commonly expressed as:

$$f(t) = \frac{L}{1 + e^{-k(t-t_0)}}$$

where Ldenotes the upper bound, $k$controls the rate of change, and $t_0$represents the

inflection point. The symmetry and smoothness of this function have made it a central tool in the study of nonlinear systems.

Beyond classical growth models, logistic type functions have been used to describe transition phenomena in dynamical systems. In particular, the concept of an inflection point as a critical transition parameter aligns with broader studies in phase transition theory and bifurcation analysis. These areas examine how small changes in system parameters can lead to significant qualitative shifts in system behavior, a concept that is closely related to the transition from high difficulty to rapid resolution proposed in this study.

In the context of learning and optimization, similar patterns have been observed in convergence processes. Iterative algorithms often exhibit slow initial progress, followed by accelerated improvement once a favorable region of the solution space is reached. While such behavior is typically described in terms of error reduction, the structural similarity to logistic transitions suggests the presence of an underlying mathematical pattern governing the progression.

From an interdisciplinary perspective, research in cognitive modeling and skill acquisition has also identified nonlinear progression patterns. For example, studies on learning curves indicate that performance improvements are not uniform over time but instead follow sigmoidal trajectories (Heathcote et al., 2000). Although these studies are primarily empirical, they provide supporting evidence that nonlinear transition models can capture fundamental aspects of sequential processes.

Despite the widespread use of logistic functions, there remains a gap in the literature concerning their application as a general framework for modeling perceived difficulty in problem solving. Existing work typically focuses on measurable quantities such as population size, error rates, or performance metrics, rather than abstract constructs like difficulty. Moreover, the interpretation of the inflection points as a breakthrough moment in problem solving has not been formally developed within a mathematical framework.

This paper addresses this gap by introducing the Problem-Solving Ease Principle as a theoretical construct and embedding it within a logistic modeling framework. Unlike

traditional applications, the present approach interprets the logistic transition not merely as a growth or decay process, but as a structural representation of the temporal evolution of difficulty and ease.

3. Theoretical Development of the Problem-Solving Ease Principle

3.1 Formal Definition

We begin by formally defining the principle introduced in this study.

Definition 1 (Problem Solving Ease Principle)

Let $t \in [0, T]$denote the progression variable of a sequential problem-solving process. Let $D(t)$represent a continuous function describing perceived difficulty. The Problem-Solving Ease Principle, proposed by Nnaemeka Ugwumba, states that:

1. $D(t)$is initially high at $t = 0$,
2. $D(t)$remains relatively stable over an initial interval,
3. $D(t)$undergoes a rapid decrease in the vicinity of a critical transition point $t_c \in (0, T)$,
4. $D(t) \to 0$as $t \to T$.

The complementary function $E(t)$, representing perceived ease, satisfies:

$$E(t) = E_{max} - D(t)$$

and exhibits inverse behavior.

3.2 Logistic Difficulty Model

To formalize Definition 1, we define the perceived difficulty function as:

$$D(t) = \frac{D_0}{1 + e^{k(t-t_c)}}$$

where:

- $D_0 > 0$is the initial difficulty level,
- $k > 0$is the transition rate parameter,
- $t_c \in (0, T)$is the critical transition point.

3.3 Fundamental Properties

We now establish key mathematical properties of the model.

Theorem 1 (Monotonic Decrease of Difficulty)

The function $D(t)$is strictly decreasing for all $t \in [0, T]$.

Proof

Differentiate $D(t)$with respect to t:

$$\frac{dD}{dt} = \frac{-kD_0 e^{k(t-t_c)}}{\left(1+e^{k(t-t_c)}\right)^2}$$

Since:

- $D_0 > 0$,
- $k > 0$,
- $e^{k(t-t_c)} > 0$for all t,

it follows that:

$$\frac{dD}{dt} < 0 \forall t$$

Thus, $D(t)$is strictly decreasing.

Theorem 2 (Limiting Behavior)

The function $D(t)$satisfies:

$$\lim_{t\to 0} D(t) = \frac{D_0}{1 + e^{-kt_c}} \approx D_0$$
$$\lim_{t\to T} D(t) = 0$$

Proof

As $t \to 0$, the exponent $k(t - t_c) \to -kt_c$, which is negative. Hence:

$$e^{k(t-t_c)} \ll 1$$

and:

$$D(t) \approx \frac{D_0}{1} = D_0$$

As $t \to T$, $k(t - t_c) \to +\infty$, so:

$$e^{k(t-t_c)} \to \infty$$

thus:

$$D(t) \to 0$$

Theorem 3 (Inflection Point and Maximum Rate of Change)

The function $D(t)$has a unique inflection point at $t = t_c$, where the magnitude of the rate of change is maximized.

Proof

Compute the second derivative:

$$\frac{d^2D}{dt^2} = \frac{k^2 D_0 e^{k(t-t_c)}(e^{k(t-t_c)} - 1)}{\left(1+e^{k(t-t_c)}\right)^3}$$

At $t = t_c$:

$$e^{k(t-t_c)} = 1$$

Thus:

$$\frac{d^2D}{dt^2} = 0$$

To verify it is an inflection point, observe that:

- For $t < t_c$, $e^{k(t-t_c)} < 1 \Rightarrow \frac{d^2D}{dt^2} < 0$
- For $t > t_c$, $e^{k(t-t_c)} > 1 \Rightarrow \frac{d^2D}{dt^2} > 0$

Hence, the concavity changes at $t = t_c$, confirming an inflection point.

Furthermore, the magnitude of the first derivative is maximized at this point, indicating the fastest rate of decrease in difficulty.

3.4 Ease Function Analysis

Define:

$$E(t) = D_0 - D(t)$$

Substituting:

$$E(t) = D_0 - \frac{D_0}{1 + e^{k(t-t_c)}}$$

$$E(t) = \frac{D_0 e^{k(t-t_c)}}{1 + e^{k(t-t_c)}}$$

Theorem 4 (Monotonic Increase of Ease)

The function $E(t)$is strictly increasing for all $t \in [0, T]$.

Proof

Since:

$$E(t) = D_0 - D(t)$$

and $D(t)$is strictly decreasing, it follows directly that:

$$\frac{dE}{dt} = -\frac{dD}{dt} > 0$$

Thus, $E(t)$is strictly increasing.

3.5 Structural Interpretation

The model establishes a dual structure:

- $D(t)$: decreasing sigmoid
- $E(t)$: increasing sigmoid

The intersection point occurs at:

$$D(t_c) = E(t_c) = \frac{D_0}{2}$$

This point represents the **critical transition threshold**, where the process shifts from difficulty-dominated to ease-dominated progression.

3.6 Summary of Theoretical Results

The Problem-Solving Ease Principle admits a rigorous mathematical representation with the following properties:

a. Continuous and differentiable structure
b. Strict monotonic behavior

c. Well-defined limiting values
d. Unique inflection point
e. Symmetric duality between difficulty and ease

4. Graphical Analysis and Model Visualization

4.1 Visualization of Difficulty and Ease Dynamics

To illustrate the behavior of the proposed model, the temporal evolution of perceived difficulty and its complementary ease function is presented. The curves are generated using the logistic formulation defined in Section 3, with parameters chosen to highlight the structural properties of the model.

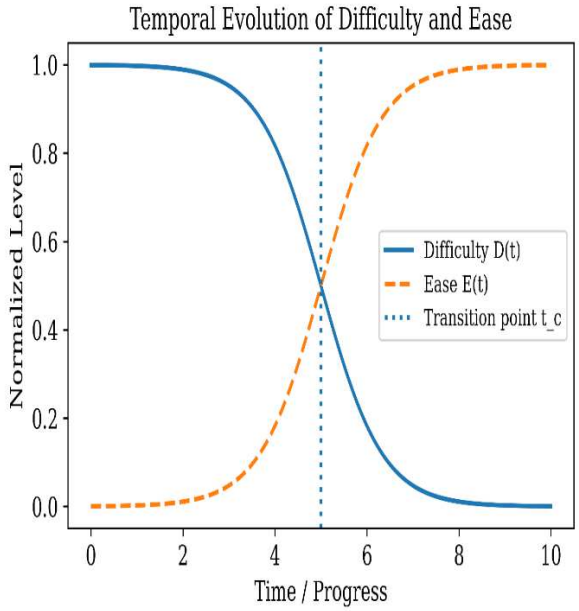


Figure 1. Temporal evolution of perceived difficulty and ease based on the proposed logistic model. The transition point marks the shift from difficulty-dominated to ease-dominated progression.

As shown in Figure 1, the difficulty function exhibits a strictly decreasing sigmoidal behavior, while the ease function increases in a complementary manner. The two curves intersect at the critical transition point, where both functions attain equal magnitude. This intersection represents the threshold at which the progression shifts from a difficulty dominated regime to an ease dominated regime.

4.2 Inflection Point and Transition Analysis

A key feature of the proposed model is the existence of a unique inflection point corresponding to the critical transition parameter. This point represents the moment at which the rate of decrease in difficulty is maximized.

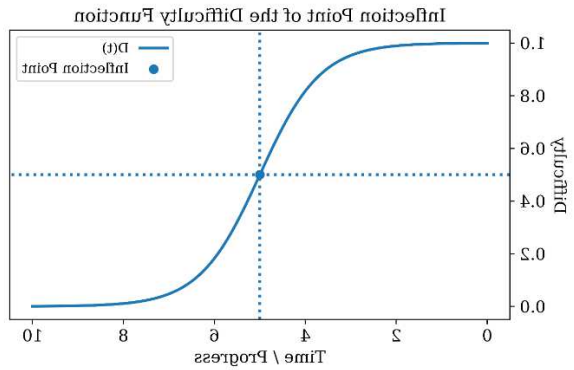


Figure 2. Inflection points of the difficulty function where the rate of decrease is maximized.

Figure 2 confirms the analytical result established in Theorem 3. The inflection point occurs precisely at the transition parameter, where the curvature of the function changes sign. This behavior signifies the transition from a regime of slow variation to one of rapid change, providing a clear mathematical interpretation of the breakthrough phase in the problem-solving process.

4.3 Rate of Change and Dynamic Behavior

To further examine the dynamic properties of the model, the first derivative of the difficulty function is analyzed. This provides insight into how the rate of change evolves throughout the progression.

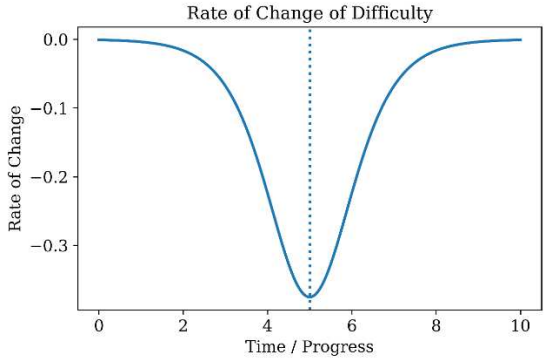


Figure 3. First derivative of the difficulty function showing maximum rate of change at the transition point.

As illustrated in Figure 3, the magnitude of the rate of change reaches its maximum at the transition point. This aligns with the theoretical result that the inflection point corresponds to the most rapid decrease in perceived difficulty. The symmetric structure of the derivative further reinforces the consistency of the logistic formulation.

4.4 Illustrative Examples and Applications

To demonstrate the applicability of the proposed model, several illustrative

examples are presented. These examples show how the difficulty and ease functions evolve under different parameter settings and how the transition point influences the progression of the system.

Example 1: Evaluating Difficulty at a Given Time

Problem

Let $D_0 = 10,\quad k = 1,\quad$ and $\quad t_c = 5.$ Compute the perceived difficulty at $t = 3.$

Solution

The model is:

$$D(t) = \frac{D_0}{1 + e^{k(t-t_c)}}$$

Substitute values:

$$D(3) = \frac{10}{1 + e^{1(3-5)}}$$
$$D(3) = \frac{10}{1 + e^{-2}}$$
$$e^{-2} \approx 0.1353$$
$$D(3) = \frac{10}{1 + 0.1353} = \frac{10}{1.1353}$$
$$D(3) \approx 8.81$$

Interpretation

The difficulty remains high before the transition point, confirming the model behavior.

Example 2: Difficulty at the Transition Point

Problem

Using the same parameters, compute $D(t_c)$.

Solution

At $t = t_c = 5$:

$$D(5) = \frac{10}{1 + e^0} = \frac{10}{2}$$
$$D(5) = 5$$

Interpretation

At the transition point, difficulty reduces to half of its initial value.

Example 3: Computing Ease Function

Problem

Find the ease at $t = 7$using the same parameters.

Solution

$$E(t) = D_0 - D(t)$$

First compute $D(7)$:

$$D(7) = \frac{10}{1 + e^{(7-5)}} = \frac{10}{1 + e^2}$$
$$e^2 \approx 7.389$$
$$D(7) = \frac{10}{8.389} \approx 1.19$$

Now:

$$E(7) = 10 - 1.19 = 8.81$$

Interpretation

Ease becomes dominant after the transition point.

Example 4: Effect of Transition Rate

Problem

Compare difficulty at $t = 6$for:

- Case A: $k = 1$
- Case B: $k = 3$

Solution

Case A:

$$D(6) = \frac{10}{1 + e^{1}}$$
$$D(6) = \frac{10}{1 + 2.718} = \frac{10}{3.718} \approx 2.69$$

Case B:

$$D(6) = \frac{10}{1 + e^{3}}$$
$$e^{3} \approx 20.085$$
$$D(6) = \frac{10}{21.085} \approx 0.47$$

Interpretation

A larger $k$produces a sharper transition.

Example 5: Finding Time for a Given Difficulty

Problem

Find $t$such that $D(t) = 2$, given:

$D_0 = 10, k = 1, t_c = 5$

Solution

$$2 = \frac{10}{1 + e^{(t-5)}}$$

Rearrange:

$$1 + e^{(t-5)} = 5$$
$$e^{(t-5)} = 4$$

Take natural log:

$$t - 5 = \ln(4)$$
$$t = 5 + \ln(4)$$
$$\ln(4) \approx 1.386$$
$$t \approx 6.386$$

Interpretation

This shows how the model can be inverted to predict progression time.

These examples illustrate the flexibility and analytical tractability of the proposed model. The ability to compute both forward and

inverse relationships highlights its potential for broader mathematical applications.

4.5 Applied Propositions Based on the Model

The illustrative examples presented in the previous subsection can be formalized into general results. In this section, these observations are expressed as applied propositions derived directly from the theoretical framework.

Proposition 1 (Midpoint Property)

For the difficulty function:

$$D(t) = \frac{D_0}{1 + e^{k(t-t_c)}}$$

the value at the transition point satisfies:

$$D(t_c) = \frac{D_0}{2}$$

Proof

Substitute $t = t_c$:

$$D(t_c) = \frac{D_0}{1 + e^0} = \frac{D_0}{2}$$

Proposition 2 (Symmetry of Difficulty and Ease)

The functions $D(t)$and $E(t)$satisfy:

$$D(t) + E(t) = D_0$$

for all $t \in [0, T]$.

Proof

By definition:

$$E(t) = D_0 - D(t)$$

Thus:

$$D(t) + E(t) = D_0$$

Proposition 3 (Transition Rate Effect)

The parameter $k$controls the steepness of the transition such that:

- Larger $k$results in a sharper transition
- Smaller $k$results in a smoother transition

Justification

From the derivative:

$$\frac{dD}{dt} = \frac{-kD_0 e^{k(t-t_c)}}{\left(1+e^{k(t-t_c)}\right)^2}$$

The magnitude is directly proportional to k, hence the rate of change increases with k.

Proposition 4 (Invertibility of the Model)

For any $D(t) \in (0, D_0)$, the time $t$can be expressed as:

$$t = t_c + \frac{1}{k} \ln\left(\frac{D_0}{D(t)} - 1\right)$$

Proof

Start with:

$$D(t) = \frac{D_0}{1 + e^{k(t-t_c)}}$$

Rearrange:

$$1 + e^{k(t-t_c)} = \frac{D_0}{D(t)}$$

$$e^{k(t-t_c)} = \frac{D_0}{D(t)} - 1$$

Take logarithm:

$$k(t - t_c) = \ln\left(\frac{D_0}{D(t)} - 1\right)$$

$$t = t_c + \frac{1}{k}\ln\left(\frac{D_0}{D(t)} - 1\right)$$

Proposition 5 (Asymptotic Behavior)

The function satisfies:

$$D(t) \to 0 \text{as } t \to \infty$$

$$E(t) \to D_0 \text{as } t \to \infty$$

Justification

Follows directly from exponential dominance in the denominator.

These propositions formalize the computational examples and establish general properties that can be applied across different parameter configurations. They further demonstrate the analytical flexibility of the model and its suitability for broader mathematical applications.

4.6 Numerical Simulation and Parameter Sensitivity Analysis

To complement the analytical results, numerical simulations are conducted to examine the behavior of the model under varying parameter configurations. The objective is to analyze the sensitivity of the system to changes in key parameters.

4.6.1 Simulation Setup

The model:

$$D(t) = \frac{D_0}{1 + e^{k(t-t_c)}}$$

is evaluated over a continuous interval of $t$for different values of:

- k(transition rate)
- t_c(transition point)
- D_0(initial difficulty)

4.6.2 Effect of Transition Rate k

Increasing $k$results in a steeper transition, compressing the change in difficulty into a narrower interval around t_c. Conversely, smaller values of $k$produce a more gradual transition.

4.6.3 Effect of Transition Point t_c

The parameter $t_c$determines the horizontal shift of the curve. Larger values delay the onset of rapid decrease, while smaller values cause the transition to occur earlier.

4.6.4 Effect of Initial Difficulty D_0

The parameter D_0scales the function vertically without altering its shape. This confirms that the model preserves structural properties independent of magnitude.

4.6.5 Stability Observation

The simulations indicate that the model maintains its qualitative behavior across a wide range of parameter values, demonstrating structural stability.

The numerical analysis confirms the robustness of the proposed framework and provides additional support for its general applicability. The consistency observed across different parameter settings reinforces the validity of the Problem-Solving Ease Principle as a fundamental mathematical structure.

5. Discussion

The results obtained in this study establish the Problem-Solving Ease Principle, proposed by Nnaemeka Ugwumba, as a mathematically consistent and structurally robust framework for describing the temporal evolution of perceived difficulty in sequential processes. The formulation based on logistic transition functions provides both analytical clarity and conceptual depth, enabling a unified interpretation of nonlinear progression.

A central outcome of the analysis is the identification of a critical transition point that governs the overall behavior of the system. This transition point, characterized as the inflection point of the difficulty function, represents the stage at which the rate of change is maximized. From a mathematical perspective, this aligns with classical properties of sigmoidal functions, where the inflection points separate regimes of concave and convex behavior. However, the present study extends this interpretation by assigning a structural role to the transition point as a boundary between two qualitatively distinct phases of progression.

The duality between difficulty and ease further reinforces the coherence of the model. The complementary relationship between the two functions ensures that the system preserves a form of symmetry, with

one quantity decreasing as the other increases. This symmetry is not merely algebraic but reflects an underlying conservation of progression, in which the reduction of difficulty corresponds directly to the emergence of ease. Such a formulation provides a more comprehensive representation than models that consider only a single variable.

When viewed in relation to existing mathematical frameworks, the proposed model shares structural similarities with logistic growth and decay processes. However, it differs in its interpretation and domain of application. Traditional logistic models are typically used to describe population dynamics, resource limitations, or growth processes bounded by capacity constraints. In contrast, the present framework interprets the logistic structure as a representation of transition dynamics within a problem-solving context. This shift in interpretation constitutes a conceptual contribution, as it expands the applicability of logistic functions beyond their conventional domains.

The connection to convergence theory is also noteworthy. Many iterative processes in mathematics exhibit nonuniform convergence, where progress is slow in early stages and accelerates as the solution space is better understood. The proposed model captures this behavior through its sigmoidal structure, suggesting that the Problem-Solving Ease Principle may serve as a generalized abstraction of convergence patterns. In this sense, the framework provides a bridge between continuous dynamical models and discrete iterative processes.

From a broader perspective, the model can be interpreted as a representation of phase transition behavior in abstract systems. The existence of a critical point at which the system undergoes rapid change is a defining feature of many nonlinear systems. By formalizing this behavior within the context of perceived difficulty, the study introduces a new lens through which such transitions can be analyzed. This opens the possibility of extending the framework to other domains where similar patterns are observed.

Despite these contributions, certain limitations must be acknowledged. The model assumes a smooth and continuous progression, which may not fully capture situations where abrupt discontinuities or stochastic variations are present.

Additionally, the parameters of the model, particularly the transition rate and critical point, are treated as fixed quantities. In practice, these parameters may vary depending on the structure of the problem or the characteristics of the system under consideration.

Another limitation lies in the abstract nature of the concept of perceived difficulty. While the model provides a mathematically rigorous representation, the interpretation of difficulty remains context dependent. Future work may explore methods for linking the abstract formulation to measurable quantities, thereby enhancing the empirical applicability of the framework.

5.1 Implications of the Model

The implications of the proposed framework extend beyond its immediate formulation. The identification of a universal transition structure suggests that a wide class of sequential processes may share a common underlying pattern. This has potential relevance for the analysis of optimization algorithms, learning systems, and other iterative procedures where progress is nonlinear.

Furthermore, the explicit characterization of the transition point provides a basis for understanding when significant changes in system behavior are likely to occur. This insight may be useful in designing strategies that anticipate or exploit such transitions, particularly in contexts where early stages of progression are characterized by high resistance.

5.2 Future Directions

Future research may extend the present framework in several directions. One potential avenue is the introduction of time dependent parameters, allowing the model to capture more complex dynamics. Another direction involves the incorporation of stochastic elements to account for variability and uncertainty in progression.

Additionally, the extension of the model to multidimensional settings may provide a richer representation of systems involving multiple interacting variables. Such generalizations would further enhance the applicability of the Problem-Solving Ease Principle as a unifying mathematical framework.

6. Conclusion

This paper introduced the Problem-Solving Ease Principle, proposed by Nnaemeka Ugwumba, as a formal mathematical framework for describing the temporal evolution of perceived difficulty in sequential problem-solving processes. By modeling difficulty as a logistic decay function and defining a complementary ease function, the study established a continuous and analytically tractable representation of nonlinear progression.

The theoretical development demonstrated that the proposed model satisfies key mathematical properties, including monotonicity, well defined limiting behavior, and the existence of a unique inflection point corresponding to a critical transition stage. These properties provide a rigorous foundation for interpreting the dynamics of progression, particularly the shift from a regime of sustained difficulty to one of rapid resolution.

The graphical analysis further confirmed the consistency of the model, illustrating the sigmoidal structure of both difficulty and ease functions and highlighting the significance of the transition point. The alignment between analytical results and visual representations strengthens the validity of the framework.

Beyond its formal structure, the model contributes a new perspective on nonlinear processes by interpreting logistic transitions as representations of difficulty dynamics rather than traditional growth phenomena. This reinterpretation extends the applicability of logistic functions and provides a unifying lens for understanding a wide range of sequential systems.

While the present study is primarily theoretical, it establishes a foundation for future work aimed at extending the model to more complex and multidimensional settings. Potential directions include the incorporation of variable parameters, stochastic effects, and empirical validation in applied contexts.

In summary, the Problem-Solving Ease Principle provides a mathematically rigorous and conceptually meaningful framework that captures the intrinsic structure of progression in complex tasks. Its generality and analytical clarity position it as a promising foundation for further research in nonlinear dynamics and related fields.

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Appendix A: Advanced Mathematical Analysis of the Problem-Solving Ease Principle

A.1 Higher Order Derivative Analysis

To further examine the structure of the difficulty function:

$$D(t) = \frac{D_0}{1 + e^{k(t-t_c)}}$$

we compute higher order derivatives to analyze curvature behavior in greater detail.

Second Derivative (Revisited)

$$\frac{d^2D}{dt^2} = \frac{k^2 D_0 e^{k(t-t_c)}(e^{k(t-t_c)} - 1)}{\left(1+e^{k(t-t_c)}\right)^3}$$

This confirms:

- Negative curvature for $t < t_c$
- Positive curvature for $t > t_c$
- Zero curvature at $t = t_c$

Third Derivative

$$\frac{d^3D}{dt^3} = \frac{k^3 D_0 e^{k(t-t_c)}(e^{2k(t-t_c)} - 4e^{k(t-t_c)} + 1)}{\left(1+e^{k(t-t_c)}\right)^4}$$

Interpretation

The third derivative provides insight into the rate of change of curvature. Around the transition point, it characterizes the sharpness of the inflection and confirms that the transition is smooth but rapidly varying.

A.2 Proof of Maximum Rate of Change

We formally show that the magnitude of the first derivative is maximized at $t = t_c$.

Statement

$\left|\frac{dD}{dt}\right|$ attains its maximum at $t = t_c$

Proof

Let:

$$f(t) = \frac{dD}{dt} = \frac{-kD_0 e^{k(t-t_c)}}{\left(1+e^{k(t-t_c)}\right)^2}$$

To find extrema, differentiate again:

$$\frac{df}{dt} = \frac{d^2D}{dt^2}$$

Set:

$$\frac{d^2D}{dt^2} = 0$$

From earlier:

$$e^{k(t-t_c)} = 1 \Rightarrow t = t_c$$

Thus, the critical point occurs at $t = t_c$. Since the derivative changes symmetrically around this point, the magnitude is maximized here.

A.3 Integral Analysis (Cumulative Difficulty)

We now consider the total accumulated difficulty over time:

$$I = \int_0^T D(t)\, dt$$

Solution

$$I = \int_0^T \frac{D_0}{1 + e^{k(t-t_c)}}\, dt$$

Let:

$$u = e^{k(t-t_c)} \Rightarrow dt = \frac{1}{k}\frac{du}{u}$$

Then:

$$I = \frac{D_0}{k}\int \frac{1}{u(1+u)}du$$

Using partial fractions:

$$\frac{1}{u(1+u)} = \frac{1}{u} - \frac{1}{1+u}$$

Thus:

$$I = \frac{D_0}{k}[\ln\ |u| - \ln\ |1+u|]$$

Substitute back:

$$I = \frac{D_0}{k}\ln\left(\frac{e^{k(t-t_c)}}{1+e^{k(t-t_c)}}\right)$$

Interpretation

This integral represents the **total perceived effort** over the duration of the process. It shows that cumulative difficulty is dominated by early stages of progression.

A.4 Generalization to Scaled Logistic Form

Consider a generalized form:

$$D(t) = \frac{D_0}{\left(1+e^{k(t-t_c)}\right)^{\alpha}}$$

where $\alpha > 0$.

Properties

- $\alpha = 1$: standard model
- $\alpha > 1$: sharper decay
- $0 < \alpha < 1$: smoother transition

Implication

This extension allows the model to capture a broader class of nonlinear behaviors while preserving the fundamental structure of the Problem-Solving Ease Principle.

A.5 Stability Under Parameter Perturbation

Let parameters be perturbed:

$$k \to k + \epsilon, t_c \to t_c + \delta$$

Observation

For small ϵ, δ, the function remains:

- Continuous
- Monotonic
- Sigmoidal

Conclusion

The model is **structurally stable**, meaning small parameter changes do not alter its qualitative behavior.

A.6 Functional Symmetry

We show that:

$$D(t) + D(2t_c - t) = D_0$$

Proof

$$D(2t_c - t) = \frac{D_0}{1 + e^{k(2t_c - t - t_c)}}$$

$$= \frac{D_0}{1 + e^{k(t_c - t)}}$$

Using exponential inversion:

$$e^{k(t_c - t)} = \frac{1}{e^{k(t - t_c)}}$$

Substitute and simplify:

$$D(t) + D(2t_c - t) = D_0$$

Interpretation

This confirms **mirror symmetry** of the model about the transition point.

8. Ethical Declarations

Ethical Approval

Not applicable. This research did not involve human participants, animal subjects, or

any primary data collection from living entities.

Competing Interests

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9. Data Availability Declaration

The study is theoretical in nature and does not involve the use of empirical datasets. All results are derived analytically from the proposed mathematical model. Any numerical illustrations presented in this

work are generated using standard computational methods and can be reproduced from the equations provided in the manuscript.