

Adaptive fuzzy Sliding Mode Trajectory Tracking Control of a 7-DOF Redundant Hydraulic Manipulator

Zhilin Wang

Donghai Su

sudonghai2025@126.com

Zhengwen Li

Article

Keywords: redundant hydraulic manipulator, Trajectory tracking control, Sliding mode disturbance observer, Adaptive fuzzy logic system, Trajectory planning, Fast reaching law

Posted Date: April 9th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-9228660/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Adaptive fuzzy Sliding Mode Trajectory Tracking Control of a 7-DOF Redundant Hydraulic Manipulator

Zhilin Wang^{1,2}, Donghai Su^{1,*}, Zhengwen Li²

¹ College of Mechanical Engineering, Shenyang University of Technology, Shenyang, Liaoning Province, 110870, China

² Liaodong University, Dandong, Liaoning Province, 118001, China

* Corresponding Author: sudonghai2025@126.com

Abstract: For trajectory tracking control problem of a 7-DOF redundant hydraulic manipulator, an adaptive fuzzy sliding mode control method based on a novel fast reaching law is proposed. Based on the kinematic analysis of the manipulator, the dynamic equation of the manipulator is constructed by using the Lagrange dynamic equation. A trajectory planning method for the manipulator integrating seventh-order polynomial interpolation and genetic algorithm optimization is proposed. Taking the planned trajectory as the expected trajectory, the sliding mode controller is designed to achieve trajectory tracking control of the manipulator. A sliding mode disturbance observer is used to observe the system modeling error, and an adaptive fuzzy logic system is designed to estimate the observation error of the disturbance observer. The sliding mode control law is deduced based on the fast reaching law, which can achieve global fast convergence of the sliding mode function while reducing the chattering of the controller. The simulation results show that the proposed control method has good tracking performance and strong robustness.

Keywords: redundant hydraulic manipulator, Trajectory tracking control, Sliding mode disturbance observer, Adaptive fuzzy logic system, Trajectory planning, Fast reaching law

1. Introduction

The 7-DOF redundant hydraulic manipulator adopts hydraulic transmission, which offers the advantages of high strength and high rigidity, and features simple operation and fast movement. This manipulator can operate stably under complex working conditions, thereby effectively enhancing production efficiency and alleviating labor intensity. However, as a complex nonlinear and coupled system, this kind of manipulator is subject to external disturbances and unmodeled dynamics during actual operation, which increases the difficulty of its precise control. To address this problem, many scholars have proposed control methods such as PID control [1], fuzzy control [2], sliding mode control [3], and adaptive control [4]. However, in the face of complex external disturbances and unmodeled dynamics, using the aforementioned methods alone can hardly meet the control requirements of interference suppression capability, and cannot meet the stringent performance standards of modern industry for manipulator trajectory tracking control.

A. De Luca et al. proposed a PD control method incorporating online gravity estimation compensation to address the stability issue in flexible manipulator control, effectively enhancing the dynamic response performance [5]. Z. Song et al. integrated computed torque control with fuzzy control to approximate unmodeled uncertainty, and verified the stability of the closed-loop system and desired tracking accuracy via Lyapunov theory [6]. M. Zhihong et al. proposed a terminal sliding mode controller, which achieved accurate trajectory tracking within finite time for the second-order system of multi-joint

manipulators and exhibited strong anti-interference ability [7]. M. I. Azeez et al. proposed a PID controller that improved trajectory tracking performance and robustness by leveraging Lyapunov stability theory and Golden Jackal optimization algorithm [8]. D. T. Tran et al. combined a terminal sliding mode controller with a linear extended state observer, and achieved control efficiency improvement and chattering suppression via uncertainty estimation [9]. L. Yang et al. designed a non-singular fast terminal sliding mode controller, which met the high-performance control requirements for manipulator trajectory tracking and effectively suppressed system disturbances [10]. K. Gou et al. proposed a novel sliding mode control strategy based on a nonlinear observer, which effectively improved the anti-interference capability of the system [11].

In order to achieve accurate trajectory tracking control of a 7-DOF redundant hydraulic manipulator, this paper proposes an adaptive fuzzy sliding mode control method based on a novel fast reaching law. This paper is structured as follows: In Section II, the kinematics analysis of the 7-DOF redundant hydraulic manipulator is conducted, including the forward kinematics, inverse kinematics and Jacobian matrix derivation of the manipulator. In Section III, the dynamic model of the manipulator is established via the Lagrange method on the basis of kinematics modeling, and the relationship between the output force of hydraulic cylinders and joint torque of the manipulator is analyzed. In Section IV, a trajectory planning method for the manipulator that integrates seventh-order polynomial interpolation and genetic algorithm optimization is proposed. In Section V, an adaptive fuzzy sliding mode control method is proposed for trajectory tracking control of the manipulator, which mainly includes the design of a sliding mode disturbance observer, a fuzzy logic system and a fast reaching law.

2 Kinematics Analysis Of A 7-Dof Redundant Hydraulic Manipulator

The basic structure of the 7-DOF redundant hydraulic manipulator is shown in Fig. 1. This manipulator primarily consists of a fixed base, shoulder rotation arm, upper arm, elbow arm, lower arm, wrist pitch arm, wrist yaw arm, and wrist rotation arm. The first to the seventh joints correspond sequentially to the rotary joints from the rotary base to the rotary end-effector. The first and seventh joints are driven by hydraulic motors, while the second to the sixth joints are driven by hydraulic cylinders. The first joint controls the end-effector orientation, the second to the fourth joints enable manipulator extension or retraction as well as end-effector lifting, lowering and distance adjustment relative to the target position. The fifth to the seventh joints adjust the end-effector pose to reach the target point.

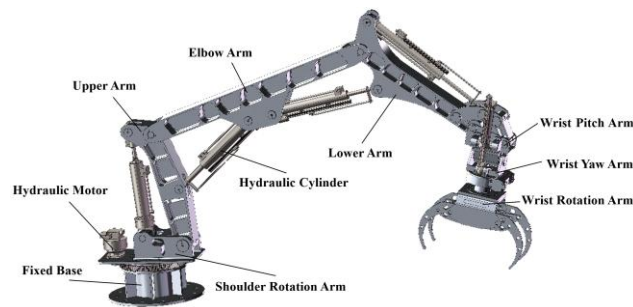


Figure 1. Structure diagram of the 7-DOF manipulator

A. Forward Kinematics Analysis of the Manipulator

For the forward kinematics analysis of the manipulator, the modified D-H method is employed [12]. Compared to the standard D-H method, this approach features clearer coordinate system definitions and avoid coordinate ambiguity. Meanwhile, it renders the coordinate transformation matrix between adjacent links dependent on the current joint parameters, thus being more suitable for modeling manipulators with complex configurations.

A simplified model is firstly established according to the basic structure of the manipulator, followed by the construction of its link coordinate system via the improved D-H method. The link coordinate system of the manipulator is shown in Fig. 2. The origin of the base coordinate system $\{0\}$ is located at the center of the base bottom surface, with Z_0 pointing vertically upward. For the link coordinate systems $\{i\}$ ($i=1,2,\dots,6$), the origin is set at the centers of the i -th joints, Z_i is aligned with the joint axes, X_i is the common normal of Z_i and Z_{i+1} , and Y_i is determined by the right-hand rule. The origin of the tool coordinate system $\{7\}$ is located at the tool center, with the direction of the axes shown in the Fig. 2.

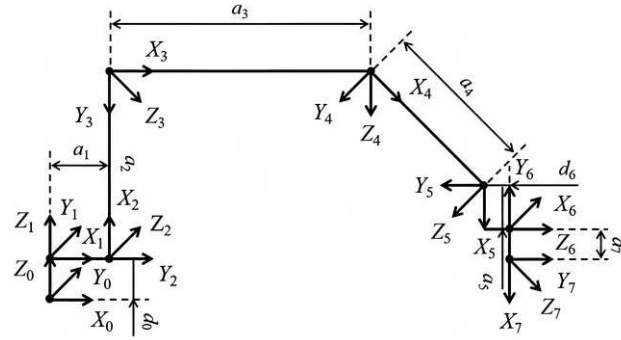


Figure 2. Link coordinate systems of the manipulator

After establishing the link coordinate systems of the manipulator, the D-H parameter can be obtained as shown in Table I. In Table I, the four parameters link twist α_{i-1} , link length a_{i-1} , joint angle q_i , and link offset d_i collectively describe the relative position and attitude relationship between adjacent link coordinate systems.

Tab.1 D-H parameter of the manipulator

i	$\alpha_{i-1}/^\circ$	a_{i-1}/mm	$q_i/^\circ$	d_i/mm
1	0	0	q_1	143
2	-90	270	q_2	0
3	0	870	q_3	0
4	0	1787	q_4	0
5	0	1020	q_5	0
6	90	210	q_6	130
7	90	0	q_7	760

The coordinate transformation matrix from link coordinate system $\{i\}$ to $\{i-1\}$, denoted as ${}^{i-1}_i\mathbf{A}$ ($i=1,2,\dots,7$), is defined as

$${}^{i-1}_i\mathbf{A} = \begin{bmatrix} c q_i & -s q_i & 0 & a_{i-1} \\ s q_i c \alpha_{i-1} & c q_i c \alpha_{i-1} & -s \alpha_{i-1} & -s \alpha_{i-1} d_i \\ s q_i s \alpha_{i-1} & c q_i s \alpha_{i-1} & c \alpha_{i-1} & c \alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where cq_i denotes $\cos q_i$, ca_i denotes $\cos \alpha_i$, sq_i denotes $\sin q_i$, sa_i denotes $\sin \alpha_i$.

Using (1), the transformation matrix ${}^0_7\mathbf{A}$ of the manipulator coordinate system $\{7\}$ relative to the base coordinate system $\{0\}$ can be derived as

$${}^0_7\mathbf{A} = {}^0_1\mathbf{A} {}^1_2\mathbf{A} {}^2_3\mathbf{A} {}^3_4\mathbf{A} {}^4_5\mathbf{A} {}^5_6\mathbf{A} {}^6_7\mathbf{A} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where

$$\begin{aligned} n_x &= (cq_1cq_{2345}cq_6 - sq_1sq_6)cq_7 + cq_1sq_{2345}sq_7 \\ n_y &= (sq_1cq_{2345}cq_6 + cq_1sq_6)cq_7 + sq_1sq_{2345}sq_7 \\ n_z &= sq_{2345}cq_6cq_7 - cq_{2345}sq_7 \\ o_x &= -(cq_1cq_{2345}cq_6 - sq_1sq_6)sq_7 + cq_1sq_{2345}cq_7 \\ o_y &= -(sq_1cq_{2345}cq_6 + cq_1sq_6)sq_7 + sq_1sq_{2345}cq_7 \\ o_z &= -cq_{2345}cq_7 - sq_{2345}cq_6sq_7, a_x = -sq_1cq_6 - cq_1cq_{2345}sq_6 \\ a_y &= -sq_1cq_{2345}sq_6 + cq_1cq_6, a_z = -sq_{2345}sq_6 \\ p_x &= cq_1(a_1 + a_2cq_2 + a_3cq_{23} + a_4cq_{234} + a_5cq_{2345} - d_6sq_{2345}) \\ p_y &= sq_1(a_1 + a_2cq_2 + a_3cq_{23} + a_4cq_{234} + a_5cq_{2345} - d_6sq_{2345}) \\ p_z &= d_1 + d_6cq_{2345} + a_2sq_2 + a_3sq_{23} + a_4sq_{234} + a_5sq_{2345} \end{aligned}$$

where cq_{23} , sq_{23} , cq_{234} , sq_{234} , cq_{2345} , sq_{2345} respectively represent $\cos(q_2 + q_3)$, $\sin(q_2 + q_3)$, $\cos(q_2 + q_3 + q_4)$, $\sin(q_2 + q_3 + q_4)$, $\cos(q_2 + q_3 + q_4 + q_5)$, $\sin(q_2 + q_3 + q_4 + q_5)$

B. Inverse Kinematic Analysis of the Manipulator

The manipulator has three parallel adjacent joint axes and satisfies the Peiper criterion, thus admitting a closed-form inverse kinematics solution [13]. Based on this criterion, an analytical algorithm with a dynamic pose constraint matrix is adopted. By fixing the DOF of the second joint (i. e., the joint angle is set as a constant), the redundant manipulator is reduced to a 6-DOF manipulator for solving inverse kinematics.

The dynamic pose matrix ${}^1_7\mathbf{A}_i$ is obtained by left-multiplying (2) by matrix ${}^0_1\mathbf{A}^{-1}$. The dynamic pose matrix ${}^1_7\mathbf{A}_f$ can be derived from the end-effector pose. By matching each corresponding element in matrix ${}^1_7\mathbf{A}_i$ with matrix ${}^1_7\mathbf{A}_f$, the joint angles q_1 , q_6 , and q_7 , as well as angles q_{23} , q_{234} and angle q_{2345} , can be calculated as

$$\begin{aligned} q_1 &= \text{atan}(p_y - d_7a_y/p_x - d_7a_x) \\ q_6 &= \text{acos}(s_1a_x - c_1a_y) \\ q_7 &= \text{atan}(s_1o_x - c_1o_y/c_1n_y - s_1n_x) \\ \begin{cases} q_{23} &= \text{asin}[(O^2 + U^2 + a_3^2 - a_4^2)/2a_3\sqrt{O^2 + U^2}] - \varphi \\ q_{23} &= -\pi - \text{asin}[(O^2 + U^2 + a_3^2 - a_4^2)/2a_3\sqrt{O^2 + U^2}] - \varphi \end{cases} \\ q_{234} &= \text{atan}(((O + a_3sq_{23})/a_4)/((U - a_3cq_{23})/a_4)) \\ \begin{cases} q_{2345} &= \text{atan}((o_zs_7/c_6 - n_zc_7/c_6)/(o_zc_7 + n_zs_7)) \quad \cos q_6 \neq 0 \\ q_{2345} &= \text{atan}((a_z/s_6)/(o_zc_7 + n_zs_7)) \quad \cos q_6 = 0 \end{cases} \end{aligned} \quad (3)$$

$$\begin{aligned} \text{where } O &= p_z - a_2s_2 - a_5s_{2345} - d_1 - d_6c_{2345} \\ U &= c_1p_x + s_1p_y - a_1 - a_2c_2 - a_5c_{2345} + d_6s_{2345} \varphi = \arctan(O/U) \end{aligned}$$

The second joint serves as the constrained joint, and its angle is held as a constant.

Using (3), the joint angle q_3 can be obtained as

$$q_3 = q_{23} - q_2 \quad (4)$$

The joint angles q_4 and q_5 are obtained by

$$\begin{aligned} q_4 &= q_{234} - q_{23} \\ q_5 &= q_{2345} - q_2 - q_3 - q_4 \end{aligned} \quad (5)$$

The kinematic inverse solution analysis yields four sets of inverse solutions for the manipulator, which need to be selected according to the joint angle limits. If any joint angle exceeds the limit, the solution is defined as invalid, and feasible solutions can be obtained by adjusting the angle of redundant joint. When multiple solutions exist, the one with the shortest path and lowest energy consumption is selected [14].

C. Jacobian Matrix of the Manipulator

When the manipulator operates at a desired velocity in the Cartesian space, it is prone to singular configurations that cause joint velocities tend to infinity. The Jacobian matrix establishing the mapping relationship between the end effector velocity and joint velocities, provides a core basis for solving this problem. The Jacobian matrix of the 7-DOF redundant hydraulic manipulator is a 6×7 non-invertible matrix and direct derivation according to the definition is complex, thus the vector product method is adopted for derivation [15].

For the i -th joint of the manipulator, the following relationship holds:

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} Z_i \times {}^iP_7^0 \\ Z_i \end{bmatrix} \dot{q}_i \quad (6)$$

where v_i denotes the linear velocity component of the end-effector due to the i -th joint motion, ω_i denotes the angular velocity component of the end-effector due to the i -th joint motion, Z_i represents of the unit vector of the Z-axis of coordinate system $\{i\}$ in the base coordinate system $\{0\}$, which can be obtained from the matrix ${}^0A_i$. ${}^iP_7^0 = {}^0R_i {}^iP_7$, and ${}^iP_7^0$ denotes the representation of the position vector of the origin of the tool coordinate system relative to coordinate system $\{i\}$ expressed in the base coordinate system $\{0\}$. ${}^0R_i$ denotes the attitude transformation matrix of coordinate system $\{i\}$ relative to the base coordinate system $\{0\}$, which can be obtained from the matrix ${}^0A_i$. ${}^iP_7$ denotes the position vector of the origin of the tool coordinate system relative to the coordinate system $\{i\}$, which can be calculated from the matrix ${}^0A_i$. $\dot{q}_i$ denotes the angular velocity of the i -th joint.

Using (6), one can obtain

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = J_i \dot{q}_i \quad J_i = \begin{bmatrix} Z_i \times ({}^0R_i {}^iP_7) \\ Z_i \end{bmatrix} \quad (7)$$

where J_i is the i -th column of the Jacobian matrix. The Jacobian matrix of the manipulator can be expressed as

$$J = [J_1, J_2, \dots, J_7] \quad (8)$$

The singularity analysis of the Jacobian matrix based on the vector product method provides a crucial theoretical basis for avoiding singular configurations and preventing joint velocities from approaching infinity in manipulator trajectory planning.

3 Dynamic Analysis Of The 7-Dof Redundant Hydraulic Manipulator

A. Dynamic Model of the Manipulator

Based on the kinematic analysis, the dynamic analysis of the manipulator is performed using the Lagrange method from the energy perspective [16]. The total kinetic energy of the manipulator can be expressed as

$$\frac{1}{2} \sum_{j=1}^7 \sum_{k=1}^7 \left[\sum_{i=\max\{j,k\}}^7 \text{tr} \left(\frac{\partial^0 \mathbf{A}}{\partial q_j} \mathbf{I}_i \frac{\partial^0 (\mathbf{A})^T}{\partial q_k} \right) \right] \dot{q}_j \dot{q}_k = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{H}(\mathbf{q}) \dot{\mathbf{q}} \quad (9)$$

where $\dot{q}_j$ denotes the angular velocity of the j-th joint, $\mathbf{I}_i$ is the pseudo-inertia matrix of the i-th link, and the pseudo-inertia matrices of each link are shown in Appendix I. The element in the j-th row and k-th column of the inertia matrix $\mathbf{H}(\mathbf{q})$ is expressed as $h_{jk} = \sum_{i=\max\{j,k\}}^7 \text{tr} \left(\frac{\partial^0 \mathbf{A}}{\partial q_j} \mathbf{I}_i \frac{\partial^0 (\mathbf{A})^T}{\partial q_k} \right)$

The total potential energy of the manipulator can be expressed as

$$V = - \sum_{i=1}^7 m_i \bar{\mathbf{g}}^T \mathbf{A} \widetilde{\mathbf{r}}_{Ci} \quad (10)$$

where m_i denotes the mass of the i-th link of the manipulator, and the mass of each link is shown in Appendix I. $\bar{\mathbf{g}} = [0, 0, -g, 0]^T$ represents the vector for gravitational acceleration in the coordinate system $\{0\}$, and g is the acceleration due to gravity. $\widetilde{\mathbf{r}}_{Ci} = [{}^i x_{Ci}, {}^i y_{Ci}, {}^i z_{Ci}, 1]^T$ is the homogeneous coordinate of the centroid of the i-th link in the coordinate system $\{i\}$, and the coordinates of the centroid of each link are shown in Appendix I.

The Lagrange function of the manipulator is $L = K - V$, and substituting the Lagrange function into the Lagrange equations of motion

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = \tau_j \quad (11)$$

where τ_j is the driving torque of the j-th joint.

The dynamic equation can be obtained as

$$\mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{T} \quad (12)$$

where $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbf{R}^{7 \times 1}$ is the centrifugal force and Coriolis force term, $\mathbf{G}(\mathbf{q}) \in \mathbf{R}^{7 \times 1}$ is the gravitational force term, $\mathbf{T} = [\tau_1, \tau_2, \dots, \tau_7]^T$ is the joint driving torque vector.

The j-th row and k-th column element of the centrifugal force and the Coriolis force term $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is expressed as

$$c_{jk} = \sum_{i=1}^7 \frac{1}{2} \left(\frac{\partial h_{jk}}{\partial q_i} + \frac{\partial h_{ij}}{\partial q_k} - \frac{\partial h_{ik}}{\partial q_j} \right) \dot{q}_i \quad (13)$$

Similarly, the j-th element of the gravity term $\mathbf{G}(\mathbf{q})$ as

$$g_j = - \sum_{k=1}^7 m_k \bar{\mathbf{g}}^T \frac{\partial^0 \mathbf{A}}{\partial q_j} \widetilde{\mathbf{r}}_{Ck} \quad (14)$$

B. Relation Between Cylinder Force and Joint Torque

Hydraulic cylinders output force along their own axes, while the dynamic equation requires joint torque as input. Since the second to the sixth joints of the manipulator are driven by hydraulic cylinders, it is necessary to establish the relationship between the hydraulic cylinder output force and joint torque.

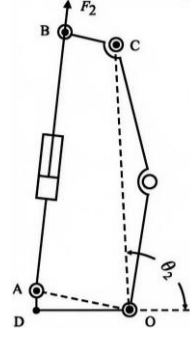


Figure 3. Structural schematic of the second joint.

The structural schematic of the second joint is shown in Fig. 3. The upper arm hydraulic cylinder force generates the torque at the second joint

$$\tau_2 = \mathbf{r}_{OB} \times \mathbf{F}_2 \quad (15)$$

where $\mathbf{r}_{OB} = \mathbf{P}_B - \mathbf{P}_O$ denotes the force arm of the upper arm actuated by the upper arm hydraulic cylinder, and is defined in the link coordinate system {2}. $\mathbf{P}_B$ denotes the position vector of connection point B between the end of the hydraulic cylinder piston rod and the upper arm, defined in the link coordinate system {2}. $\mathbf{P}_O$ denotes the position vector of the rotation center of the second joint, defined in the link coordinate system {2}. $\mathbf{F}_2 = F_2(\mathbf{P}_B - \mathbf{P}_A)$ denotes the force vector generated by the upper arm hydraulic cylinder in the link coordinate system {2}. $\mathbf{P}_A$ denotes the position vector of connection point A between the end of the hydraulic cylinder and the rotary base in the link coordinate system {2}.

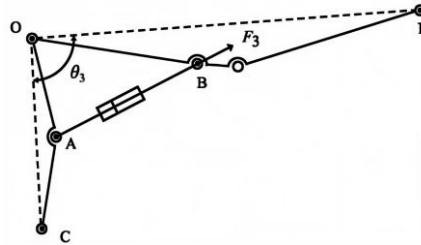


Figure 4. Structural schematic of the third joint.

The structural schematic of the third joint is shown in Fig. 4. The elbow arm hydraulic cylinder force generates the torque at the third joint

$$\tau_3 = \mathbf{r}_{OB} \times \mathbf{F}_3 \quad (16)$$

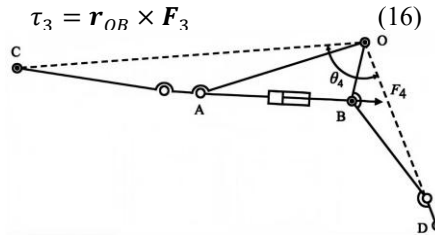


Figure 5 Structural schematic of the fourth joint.

The structural schematic of the fourth joint is shown in Fig. 5. The lower arm hydraulic cylinder force generates the torque at the fourth joint

$$\tau_4 = \mathbf{r}_{OB} \times \mathbf{F}_4 \quad (17)$$

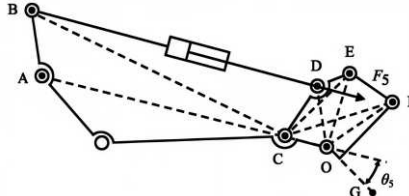


Figure 6 Structural schematic of the fifth joint.

The structural schematic of the fifth joint is shown in Fig. 6. The wrist pitch arm hydraulic cylinder force generates the torque at the fifth joint

$$\tau_5 = \mathbf{r}_{OD} \times \mathbf{F}_5 \quad (18)$$

where $\mathbf{r}_{OD} = \mathbf{P}_D - \mathbf{P}_O$ denotes the force arm of the wrist pitch arm actuated by the wrist pitch arm hydraulic cylinder, defined in the link coordinate system {5}. PD denotes the position vector of connection point D between the end of the hydraulic cylinder piston rod and the wrist pitch arm, defined in the link coordinate system {5}.

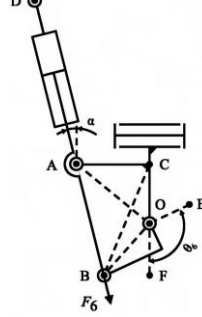


Figure 7 Structural schematic of the sixth joint.

The structural schematic of the sixth joint is shown in Fig. 7. The wrist yaw arm hydraulic cylinder force generates the torque at the sixth joint

$$\tau_6 = \mathbf{r}_{OB} \times \mathbf{F}_6 \quad (19)$$

4 Trajectory Planning Of The 7-Dof Redundant Hydraulic Manipulator

A. Seventh-Order Polynomial Trajectory Planning

Polynomial interpolation is a commonly used method for manipulator joint space trajectory planning [17], and it employs polynomial functions to describe the manipulator motion process from the start point to the end point. Among the commonly used 3rd-order, 5th-order and 7th-order polynomial interpolations, the 7th-order polynomial can satisfy higher order derivative continuity conditions (e.g., jerk continuity), and thus exhibits superior trajectory stability and motion performance to the 3rd-order and 5th-order polynomials. Therefore, the 7th-order polynomial is adopted for joint trajectory planning, and the expression of the 7th-order polynomial for one joint is defined as follows

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 \quad (20)$$

where $a_0, a_1, \dots, a_7$ denotes the coefficients of the seventh-order polynomial, and t denotes time. Taking the derivative of (20) gives the joint angular velocity

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 + 6a_6 t^5 + 7a_7 t^6 \quad (21)$$

Taking the derivative of (21) gives the joint angular acceleration

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + 30a_6 t^4 + 42a_7 t^5 \quad (22)$$

Taking the derivative of (22) gives the joint jerk

$$\ddot{\theta}(t) = 6a_3 + 24a_4 t + 60a_5 t^2 + 120a_6 t^3 + 210a_7 t^4 \quad (23)$$

At the initial time t_0 , the joint angle is denoted by θ_0 . At the final time t_f , the joint angle is denoted by θ_f . The seventh-order polynomial must satisfy the constraints for the joint angle, angular velocity, angular acceleration, and angular jerk at the start and end points

$$\left\{ \begin{array}{l} \theta_0 = a_0 \\ \theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5 + a_6 t_f^6 + a_7 t_f^7 \\ \dot{\theta}_0 = a_1 \\ \dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4 + 6a_6 t_f^5 + 7a_7 t_f^6 \\ \ddot{\theta}_0 = 2a_2 \\ \ddot{\theta}_f = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3 + 30a_6 t_f^4 + 42a_7 t_f^5 \\ \ddot{\theta}_0 = 6a_3 \\ \ddot{\theta}_f = 6a_3 + 24a_4 t_f + 60a_5 t_f^2 + 120a_6 t_f^3 + 210a_7 t_f^4 \end{array} \right. \quad (24)$$

where $\dot{\theta}_0$ and $\dot{\theta}_f$ denote the joint angular velocity at the initial time and final time of the path segment, respectively. $\ddot{\theta}_0$ and $\ddot{\theta}_f$ denote the joint angular acceleration at the initial time and final time of the path segment, respectively. $\ddot{\theta}_0$ and $\ddot{\theta}_f$ denote the joint jerk at the initial time and final time of the path segment, respectively.

Using (24), the coefficients of the 7th-order polynomial function can be obtained as

$$\left\{ \begin{array}{l} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \ddot{\theta}_0/2 \\ a_3 = \ddot{\theta}_0/6 \\ a_4 = \frac{(210\theta_f - 210\theta_0) - (90\dot{\theta}_f + 120\dot{\theta}_0)t_f + (15\ddot{\theta}_f - 30\ddot{\theta}_0)t_f^2 - (\ddot{\theta}_f + 4\ddot{\theta}_0)t_f^3}{6t_f^4} \\ a_5 = \frac{-(168\theta_f - 168\theta_0) + (78\dot{\theta}_f + 90\dot{\theta}_0)t_f - (14\ddot{\theta}_f - 20\ddot{\theta}_0)t_f^2 + (\ddot{\theta}_f + 2\ddot{\theta}_0)t_f^3}{2t_f^5} \\ a_6 = \frac{(420\theta_f - 420\theta_0) - (204\dot{\theta}_f + 216\dot{\theta}_0)t_f + (39\ddot{\theta}_f - 45\ddot{\theta}_0)t_f^2 - (3\ddot{\theta}_f + 4\ddot{\theta}_0)t_f^3}{6t_f^6} \\ a_7 = \frac{-(210\theta_f - 210\theta_0) + (60\dot{\theta}_f + 60\dot{\theta}_0)t_f - (12\ddot{\theta}_f - 12\ddot{\theta}_0)t_f^2 + (\ddot{\theta}_f + \ddot{\theta}_0)t_f^3}{6t_f^7} \end{array} \right. \quad (25)$$

B. Genetic Algorithm Trajectory Optimization

To improve the manipulators working efficiency, ensure the smoothness of the entire trajectory, minimize motion energy consumption, and avoid singular configurations during movement, the total operation time, trajectory smoothness, energy consumption, and singularity avoidance are taken as the core optimization objectives. The objective function for total operation time S1, the objective function for trajectory smoothness S2, and the objective function for energy consumption S3 are defined respectively as

$$S_1 = \sum_{k=1}^7 t_k \quad S_2 = \sum_{k=1}^7 \sqrt{\frac{\int_0^{\sum_{k=1}^7 t_k} \ddot{\theta}_{ik}(t) dt}{\sum_{k=1}^7 t_k}} \quad S_3 = \sum_{k=1}^7 \sqrt{\frac{\int_0^{\sum_{k=1}^7 t_k} \ddot{\theta}_{ik}(t) dt}{\sum_{k=1}^7 t_k}} \quad (26)$$

where t_k denotes the time of the k -th path segment. $\ddot{\theta}_{ik}(t)$ denotes the jerk of the i -th joint in the k -th path segment. $\ddot{\theta}_{ik}(t)$ denotes the angular acceleration of the i -th joint in the k -th path segment.

The objective function for avoiding singularities S4 is defined as

$$S_4 = \frac{1}{\max\{u=0.01, \frac{1}{\sum_{k=1}^7 t_k} \int_0^{\sum_{k=1}^7 t_k} |J(q) \cdot J^T(q)| dt\}} \quad (27)$$

where u denotes the threshold constant for singularity detection, $J(q)$ represents the Jacobian matrix of the manipulator as shown in (8).

To balance the above four objective functions, a weighted sum method is adopted to construct the total objective function, where different weight coefficients are assigned to each objective to balance their importance, and the total objective function is expressed as

$$f = w_1 \cdot S_1 + w_2 \cdot S_2 + w_3 \cdot S_3 + w_4 \cdot S_4 \quad (28)$$

where w_1, w_2, w_3, w_4 denote the normalized weights of the total operation time S1, trajectory smoothness S2, energy consumption S3, and singularity avoidance S4, respectively.

The motion of the manipulator is constrained by its physical structure, and the constraints imposed on the joint angles, angular velocities, angular accelerations, and angular jerks of each joint are expressed as

$$|\dot{q}_i(t)| \leq \dot{q}_{imax} \quad |\ddot{q}_i(t)| \leq \ddot{q}_{imax} \quad |\dddot{q}_i(t)| \leq \dddot{q}_{imax} \quad (29)$$

where $\dot{q}_{imax}$, $\ddot{q}_{imax}$ and $\dddot{q}_{imax}$ denote the maximum angular velocity, maximum angular acceleration, and maximum joint jerk of the i -th joint, respectively.

Proposed by John Holland based on the laws of natural evolution, the genetic algorithm is an optimal solution search algorithm that simulates the process of natural selection and genetic variation [18]. Evolving iteratively via three core operations (selection, crossover, and mutation), it features gradient-free search, high efficiency in exploring complex solution spaces, excellent robustness, and global optimization capability, making it widely applicable to multi-constraint and nonlinear engineering optimization scenarios. Therefore, this paper applies GA to the trajectory optimization of the manipulator, by encoding joint-space trajectory parameters, designing fitness function, and implementing genetic operations, it achieves multi-objective collaborative optimization of total operation time, trajectory smoothness, energy consumption, and singularity avoidance.

C. Simulation Verification

In the simulation, the trajectory of the manipulator consists of 8 waypoints, forming 7 path segments. The angular velocity $\dot{q}_i$, angular acceleration $\ddot{q}_i$, and angular jerk $\dddot{q}_i$ at each waypoint are constrained to zero. The joint angle range, angular velocities, angular accelerations, and angular jerk for each joint are specified as constraints in Table II. The positions X_j, Y_j, Z_j and orientations $\alpha_j, \beta_j, \delta_j$ of the manipulator end-effector at the 8 waypoints are detailed in Tables III and IV.

Tab II: Joint constraints of the manipulator

Joint	angle ranges	Velocity	Acceleration	Jerk
1	(-3.14,3.14)	2.61	3.52	3.32
2	(-1.55,-0.87)	2.61	3.52	3.32
3	(0.69,1.39)	3.14	3.52	3.32
4	(-0.08,1.13)	3.14	3.52	3.32
5	(-0.24,1.46)	3.35	3.52	3.32
6	(1.02,2.41)	3.35	3.52	3.32
7	(-3.14,3.14)	3.35	3.52	3.32

In the Table II the unit of joint angle is rad, the unit of velocity is rad/s, the unit of acceleration is rad/s², the unit of jerk is rad/s³.

Tab III: Manipulator end pose of each waypoint

Waypoint	j=1	j=2	j=3	j=4
End-effector pose				
X_j	3942.13	3948.44	3662.69	3231.14
Y_j	131.973	960.163	1542.43	1865.5
Z_j	211.427	772.583	843.161	802.674
α_j	-96.933	-114.95	-114.06	-112.49
β_j	-145.6	-143.48	-150.33	-158.53
δ_j	-167.85	-145.76	-148.38	-166.87

Tab IV: Manipulator end pose of each waypoint

Waypoint	j=5	j=6	j=7	j=8
End-effector pose				

End-effector pose					
X_j	3087.14	2629.45	2355.02	1917.17	
Y_j	2676.88	2723.12	3011.92	3584.58	
Z_j	920.016	229.399	432.207	1183.59	
α_j	-126.76	-127.51	-138.99	-162.75	
β_j	-151.08	-169.67	-172.27	-165.34	
δ_j	-143.58	-162.08	-154.25	-125.72	

In Tables III and IV: X_j , Y_j , Z_j represent the coordinates of the origin of the tool coordinate system relative to the base coordinate system at the j -th waypoint, with units in mm. α_j , β_j , δ_j represent the roll angle, pitch angle, and yaw angle of the manipulator at the j -th waypoint, respectively, with units in rad.

Inverse kinematics solutions are derived based on the manipulator end-effector pose at each waypoint. Firstly, the redundant joint angle is selected via the genetic algorithm, then substituted into the inverse kinematics solving process, and the corresponding joint angles for each waypoint are finally obtained as shown in Tables V and VI.

Tab V: Manipulator joint angle at each waypoint

Waypoint Joint	j=1	j=2	j=3	j=4
1	0	0.1745	0.3491	0.5236
2	-1.2217	-1.3089	-1.3963	-1.3089
3	1.0472	0.9599	0.8727	0.7854
4	0.5236	0.3491	0.4363	0.5236
5	0.6109	0.6981	0.7854	1.0472
6	1.7453	1.9199	1.8326	1.5708
7	0	0.0873	0.1745	0.1745

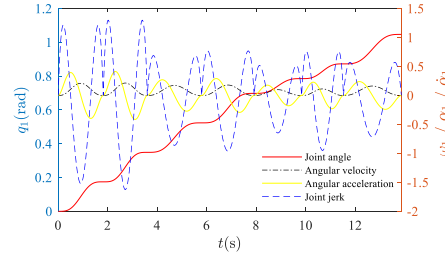
Tab VI: Manipulator joint angle at each waypoint

Waypoint Joint	j=5	j=6	j=7	j=8
1	0.6981	0.7854	0.8727	1.0472
2	-	-1.3089	-1.3963	-1.3089
3	0.9599	1.0472	0.9599	0.7854
4	0.1745	0.3491	0.4363	0.5236
5	0.8727	1.0472	0.9599	0.5236
6	1.6581	1.6581	1.7453	1.7453
7	0.2618	0.1745	0.0873	0

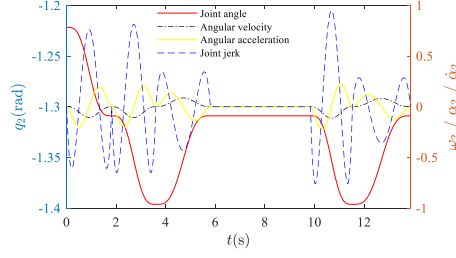
In Tables V and VI, the unit of joint angle is rad.

In the simulation, the iteration of the genetic algorithm is set to 500. Real number encoding is adopted, with a crossover operation probability $P_c=0.9$ and a mutation operation probability $P_m=0.1$. The objective function is set as the fitness function with coefficients $w_1=0.3$, $w_2=0.2$, $w_3=0.2$ and $w_4=0.2$. The objective functions and fitness function optimized by the genetic algorithm are $S_1=13.87$, $S_2=3.0977$, $S_3=10.4797$, $S_4=1.2761$, and $f=7.1317$ respectively. The optimized travel times for each path segment are $t_1=1.82s$, $t_2=1.78s$, $t_3=2.20s$, $t_4=2.12s$, $t_5=1.92s$, $t_6=1.69s$ and $t_7=2.34s$. By substituting the time duration of each path segment into (25), the coefficients of the seventh-order polynomial can be obtained. These coefficients are then

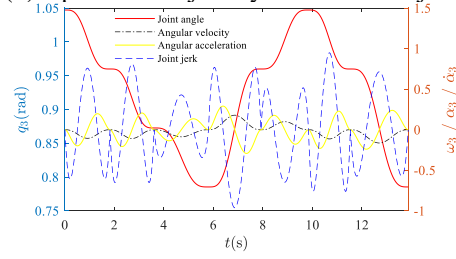
substituted into (20) to yield the optimized seventh-order polynomial interpolation trajectory. The optimized 7th-order polynomial interpolation trajectory is shown in Fig. 8.



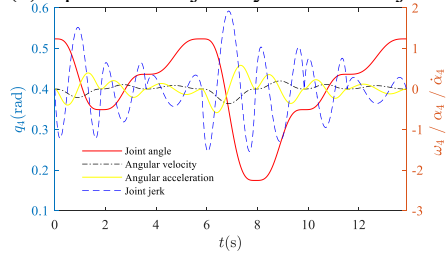
(a) Optimized trajectory of the first joint



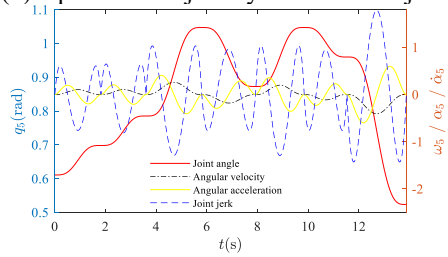
(b) Optimized trajectory of the second joint



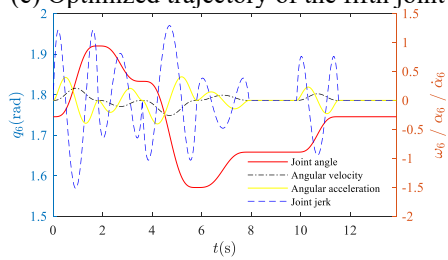
(c) Optimized trajectory of the third joint



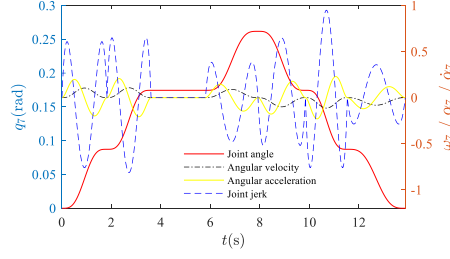
(d) Optimized trajectory of the fourth joint



(e) Optimized trajectory of the fifth joint



(f) Optimized trajectory of the sixth joint

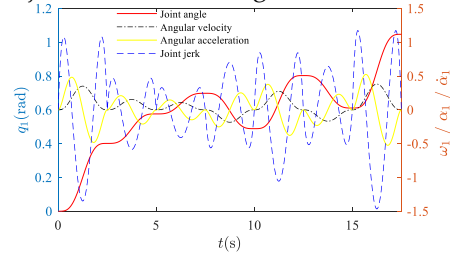


(g) Optimized trajectory of the seventh joint

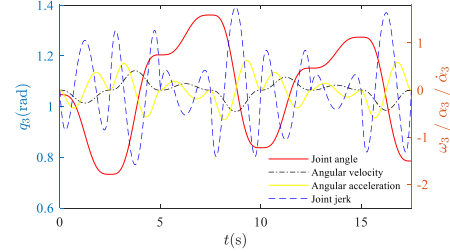
Figure 8 Optimized 7th-order polynomial interpolation trajectories of each joint.

In Fig. 8, ω_i , α_i and $\dot{\alpha}_i$ denote angular velocity, angular acceleration, and angular jerk, respectively, with the corresponding units of rad/s, rad/s² and rad/s³.

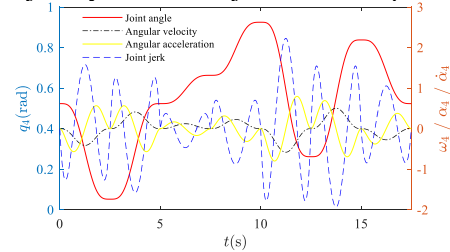
To demonstrate the superiority of the proposed trajectory planning method, a comparative analysis is conducted with the manipulator trajectory without optimization. In the manipulator trajectory without optimization, a fixed time interval of 2.5s is used between each path segment. The trajectories for each joint are obtained by (20) and (25). The redundant joint angle q_2 is set to -1.2217 rad. The constraint conditions are consistent with those in the optimized manipulator trajectory. For the seventh-order polynomial trajectories without optimization, the objective functions are $S_1=21$, $S_2=6.6975$, $S_3=23.2836$, $S_4=9.3051$. The trajectory of the seventh-order polynomial interpolation without optimization for each joint is shown in Fig. 9.



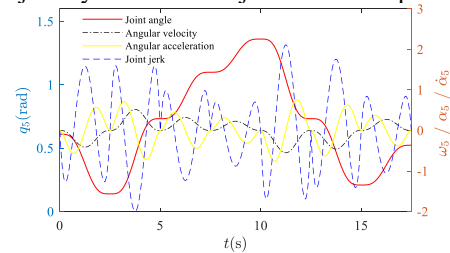
(a) Trajectory of the first joint without optimization



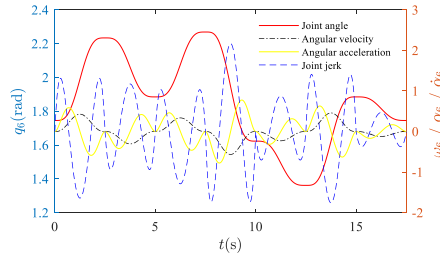
(b) Trajectory of the third joint without optimization



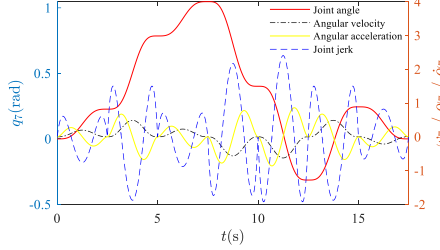
(c) Trajectory of the fourth joint without optimization



(d) Trajectory of the fifth joint without optimization



(e) Trajectory of the sixth joint without optimization



(f) Trajectory of the seventh joint without optimization

Figure 9 The 7th-order polynomial interpolation trajectories of each manipulator joint without optimization.

The comparison using Fig. 8 and Fig. 9 shows that the 7th-order polynomial joint trajectory curves optimized by genetic algorithm are smoother than the trajectory curves without optimization, and the impact (i. e., joint jerk) is also reduced to a certain extent. Comparing with the objective functions of the trajectory without optimization, the total operation time S1, trajectory smoothness S2, energy consumption S3 and singularity avoidance S4 all decrease significantly through optimization. In summary, the optimized 7th-order polynomial interpolation trajectory enables the manipulator to achieve shorter operation time, smoother motion, lower energy consumption, and more reasonable joint postures, thus verifying the superiority of the proposed trajectory planning method.

5 Design Of Manipulator Trajectory Tracking Controller

The motion of the manipulator is affected by modeling error such as unmodeled joint friction, and the actual dynamic model of the manipulator considering modeling error is expressed as

$$\mathbf{D}(\mathbf{q}) \cdot \ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) \cdot \dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{T} \quad (30)$$

where $\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}})$ represents bounded modeling error including unmodeled joint friction, etc.

For the 7-DOF hydraulic redundant manipulator described in (1), the trajectory tracking error of each joint is defined as

$$\tilde{q}_i(t) = q_i(t) - q_{id}(t) \quad (31)$$

where $q_{id}(t)$ is the expected trajectory of each joint, $i=1,2,\dots,7$. In this paper, the expected trajectory of each joint is consistent with the optimized planned trajectory in Section IV.

The trajectory tracking control objective is to rationally design the control inputs $\tau_i (i = 1, 2, \dots, 7)$ such that the angles of each joint $q_i(t)$ converge to their desired trajectories $q_{id}(t)$, thereby the trajectory tracking error $\tilde{q}_i(t)$ of each joint is minimized to zero.

Define the sliding mode vector of the manipulator system as

$$\mathbf{s} = \ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} = \ddot{\mathbf{q}} - \ddot{\mathbf{q}}_d + \mathbf{C}\dot{\mathbf{q}} \quad (32)$$

where, $\mathbf{C}$ is a 7th-order positive definite diagonal matrix and

$$\begin{aligned} \mathbf{s} &= [s_1, s_2, s_3, s_4, s_5, s_6, s_7]^T \dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6, \dot{q}_7]^T \dot{\mathbf{q}}_d \\ &= [\dot{q}_{1d}, \dot{q}_{2d}, \dot{q}_{3d}, \dot{q}_{4d}, \dot{q}_{5d}, \dot{q}_{6d}, \dot{q}_{7d}]^T \\ \tilde{\mathbf{q}} &= [\tilde{q}_1, \tilde{q}_2, \tilde{q}_3, \tilde{q}_4, \tilde{q}_5, \tilde{q}_6, \tilde{q}_7]^T \end{aligned}$$

Considering the modeling errors such as joint friction existing in the actual manipulator system, to ensure that the proposed trajectory tracking controller has high control accuracy, a sliding mode disturbance observer is designed to estimate the modeling errors of the system. Meanwhile, an adaptive fuzzy logic system is used to design a compensation system for the estimation error of the disturbance observer.

A. Design of Sliding Mode Disturbance Observer and Adaptive fuzzy system

The sliding function (31) is rewritten as $\mathbf{s} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r$ by defining $\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \mathbf{C}\tilde{\mathbf{q}}$. Note that the inertia matrix $\mathbf{D}(\mathbf{q})$ is invertible, taking the derivative of $\mathbf{s}$ and using (30) yields

$$\dot{\mathbf{s}} = \ddot{\mathbf{q}} - \ddot{\mathbf{q}}_r = -\mathbf{D}^{-1}(\mathbf{H}\dot{\mathbf{q}} + \mathbf{G}) - \ddot{\mathbf{q}}_r + \mathbf{D}^{-1}\mathbf{T} - \mathbf{D}^{-1}\mathbf{F} \quad (33)$$

(33) can be expressed as

$$\dot{s}_i = n_i - \ddot{q}_{ir} + \tilde{\tau}_i + f_i \quad i = 1, 2, \dots, 7 \quad (34)$$

where n_i , $\ddot{q}_{ir}$ and $\tilde{\tau}_i$ represent the i -th element of vectors $-\mathbf{D}^{-1}(\mathbf{H}\dot{\mathbf{q}} + \mathbf{G} + \mathbf{F})$, $\ddot{\mathbf{q}}_r$ and $\mathbf{D}^{-1}\mathbf{T}$ respectively, f_i is the i -th element of the modeling error vector $-\mathbf{D}^{-1}\mathbf{F}$. f_i is assumed to be bounded by $|f_i| \leq D_i$, here D_i is a positive constant.

Define the auxiliary sliding variable $\sigma_i(t)$ as [19]

$$\sigma_i(t) = s_i(t) + z_i(t) \quad \dot{z}_i(t) = \ddot{q}_{ir} - n_i - \tilde{\tau}_i - u_i \quad (35)$$

where ($i=1,2,\dots,7$), z_i is the internal variable of the i -th sliding mode disturbance observer, u_i is the injection term of the i -th sliding mode disturbance observer.

The injection term $u_i(t)$ for the i -th sliding mode disturbance observer is defined as $u_i(t) = \varepsilon_i \text{sign}(\sigma_i)$, here ε_i is a positive constant satisfying $\varepsilon_i > D_i$.

Using (34) and (35), one can obtain $\dot{\sigma}_i(t) = \dot{s}_i + \dot{z}_i = f_i - u_i$. Then, one can further derive that $\sigma_i \dot{\sigma}_i \leq |\sigma_i|(|f_i| - \varepsilon_i) < 0$. This means that the auxiliary sliding variable $\sigma_i(t)$ can converge to zero within a finite time.

According to the principle of equivalent control, the equivalent control of the sliding mode disturbance observer injection term can accurately estimate the unknown disturbance $f_i(t)$. The equivalent control $u_{eq,i}(t)$ can be calculated as the first-order low-pass filtered average of the disturbance observer injection term $u_i(t)$

$$\gamma_i \dot{u}_{eq,i}(t) + u_{eq,i}(t) = u_i(t) \quad (36)$$

where γ_i is a small filter time constant.

Therefore, the sliding mode disturbance observer $\hat{\mathbf{F}}$ for the system modeling error $\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}})$ can be expressed as

$$\hat{\mathbf{F}} = -\mathbf{D}U_{eq} \quad (37)$$

where

$$\hat{\mathbf{F}} = [\hat{F}_1, \hat{F}_2, \dots, \hat{F}_7]^T \quad U_{eq} = [u_{eq,1}, u_{eq,2}, \dots, u_{eq,7}]^T$$

Due to observation errors $\tilde{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) - \hat{\mathbf{F}}$ during the operation of the disturbance observer, to suppress the adverse effect of observation error in the disturbance observer on the trajectory tracking control accuracy of the manipulator, an adaptive fuzzy logic system is employed to design an observation error estimator.

The fuzzy logic system $\hat{F}_i(\mathbf{x}_i|\hat{\boldsymbol{\eta}}_i)$ ($i = 1, 2, 3, \dots, 7$) and adaptive law of parameter column vector $\hat{\boldsymbol{\eta}}_i^T$ are designed as

$$\hat{F}_i(\mathbf{x}_i|\hat{\boldsymbol{\eta}}_i) = \hat{\boldsymbol{\eta}}_i^T \boldsymbol{\xi}_i(\mathbf{x}_i) \quad \dot{\hat{\boldsymbol{\eta}}}_i = -\frac{1}{\rho_i} s_i \boldsymbol{\xi}_i(\mathbf{x}_i) \quad (38)$$

where $\boldsymbol{\xi}_i(\mathbf{x}_i) = (\xi_i^1(\mathbf{x}_i), \xi_i^2(\mathbf{x}_i), \dots, \xi_i^l(\mathbf{x}_i))^T$ is the fuzzy basis vector, and ρ_i is a positive constant.

The l -th element in the fuzzy basis vector $\boldsymbol{\xi}_i(\mathbf{x}_i)$ is

$$\xi_l(\mathbf{x}_i) = \frac{\prod_{j=1}^n \mu_{A_j^l}(\mathbf{x}_i)}{\sum_{j_1=1}^{N_1} \sum_{j_2=1}^{N_2} \dots \sum_{j_n=1}^{N_n} \prod_{i=1}^n \mu_{A_i^l}(\mathbf{x}_i)} \quad (39)$$

where n represents the element number of the input $\mathbf{x}_i$, $\mu_{A_i^{j_l}}$ represents the membership function, $A_i^{j_l} (j_l = 1, 2, \dots, N_i)$ represents the fuzzy set, N_i is the number of fuzzy rules, $x_i^{j_l}$ denotes the l -th element of the input $\mathbf{x}_i$. The input $x_i^{j_l}$ is fuzzyfied using Gaussian membership function

$$\mu_{A_i^{j_l}}(x_i^{j_l}) = \exp\left(-\left(\frac{x_i^{j_l} - c_i^{j_l}}{\sqrt{2}\sigma_i^{j_l}}\right)^2\right) \quad (40)$$

where $\sigma_i^{j_l}$ is a positive constant, $c_i^{j_l}$ is used to determine the center of the Gaussian membership function, $j_l = 1, 2, \dots, N$, here N denotes the number of fuzzy rules. At this time, the parameter vectors are $\boldsymbol{\sigma}_i^l = (\sigma_i^1, \sigma_i^2, \dots, \sigma_i^{j_l}, \dots, \sigma_i^N)$, $\mathbf{c}_i^l = (c_i^1, c_i^2, \dots, c_i^{j_l}, \dots, c_i^N)$.

B. Design of Sliding Mode Controller

Considering the advantages of the double power reaching law [20] in terms of fast sliding variable convergence and control input chattering elimination, a novel fast reaching law based on the double power reaching law is proposed to achieve global fast convergence of the sliding variable while reducing the chattering of the controller

$$s_i = \begin{cases} -k_{1i}|s_i|^{\lambda_{1i}}\text{sign}(s_i) - k_{2i}|s_i|^{\mu_{1i}}\text{sign}(s_i) - k_{3i}s_i, & |s_i| \geq 1 \\ -k_{1i}|s_i|^{\lambda_{2i}}\text{sign}(s_i) - k_{2i}|s_i|^{\mu_{2i}}\text{sign}(s_i) - k_{3i}s_i, & |s_i| < 1 \end{cases} \quad (41)$$

where, k_{1i} , k_{2i} and k_{3i} are positive constants, and $k_{1i} > k_{2i}$, $\lambda_{1i} > \mu_{1i} > 1$, $0 < \lambda_{2i} < \mu_{2i} < 1$.

The proposed fast reaching law consists of the reaching term $-k_{1i}|s_i|^{\lambda_{1i}}\text{sign}(s_i)$ or $-k_{1i}|s_i|^{\lambda_{2i}}\text{sign}(s_i)$ and the reaching term $-k_{2i}|s_i|^{\mu_{1i}}\text{sign}(s_i)$ or $-k_{2i}|s_i|^{\mu_{2i}}\text{sign}(s_i)$. When the sliding variable s_i is far away from the sliding surface (i. e., $|s_i| \geq 1$), the reaching speed of the sliding variable is primarily determined by the reaching term $-k_{1i}|s_i|^{\lambda_{1i}}\text{sign}(s_i)$. Since $\lambda_{1i} > \mu_{1i} > 1$, the sliding variable can achieve greater convergence effects when it is far away from the sliding surface. As the sliding variable is near to the sliding surface (i. e., $|s_i| < 1$), its reaching speed is primarily determined by the reaching term $-k_{1i}|s_i|^{\lambda_{2i}}\text{sign}(s_i)$. Since $0 < \lambda_{2i} < \mu_{2i} < 1$, the sliding function can also achieve significant reaching effects when it is near to the sliding surface. Moreover, the reaching term $-k_{3i}s_i$ plays the role in accelerating the reaching phase.

Based on the reaching law (41), the sliding mode disturbance observer (37), and the adaptive fuzzy system (38), the the sliding mode controller can be obtained as

$$\mathbf{T} = \mathbf{D}\ddot{\mathbf{q}}_r + \mathbf{H}\dot{\mathbf{q}}_r + \mathbf{G} + \hat{\mathbf{F}} + \hat{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}}|\hat{\boldsymbol{\eta}}) + \mathbf{FRL} \quad (42)$$

where

$$\begin{aligned} \hat{\mathbf{F}} &= [\hat{F}_1, \hat{F}_2, \dots, \hat{F}_7]^T \quad \mathbf{FRL} = [rl_1, rl_2, rl_3, \dots, rl_7]^T \\ \hat{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}}|\hat{\boldsymbol{\eta}}) &= [\hat{F}_1, \hat{F}_2, \dots, \hat{F}_7]^T = [\hat{\boldsymbol{\eta}}_1^T \boldsymbol{\xi}_1(q_1, \dot{q}_1), \hat{\boldsymbol{\eta}}_2^T \boldsymbol{\xi}_2(q_2, \dot{q}_2), \dots, \hat{\boldsymbol{\eta}}_7^T \boldsymbol{\xi}_7(q_7, \dot{q}_7)]rl_i \\ &= \begin{cases} -k_{1i}|s_i|^{\lambda_{1i}}\text{sign}(s_i) - k_{2i}|s_i|^{\mu_{1i}}\text{sign}(s_i) - k_{3i}s_i, & |s_i| \geq 1 \\ -k_{1i}|s_i|^{\lambda_{2i}}\text{sign}(s_i) - k_{2i}|s_i|^{\mu_{2i}}\text{sign}(s_i) - k_{3i}s_i, & |s_i| < 1 \end{cases} \end{aligned}$$

Theorem 1: For the manipulator system (30), if the sliding mode controller (42) is adopted, with the sliding function designed in (32), the sliding mode disturbance observer designed in (37), and the adaptive fuzzy system designed in (38), then the sliding function can converge to zero in finite time.

Proof: The Lyapunov function is defined as

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{D} \mathbf{s} + \frac{1}{2} \sum_{i=1}^7 \tilde{\boldsymbol{\eta}}_i^T \rho_i \tilde{\boldsymbol{\eta}}_i \quad (43)$$

where $\tilde{\boldsymbol{\eta}}_i = \boldsymbol{\eta}_i^* - \hat{\boldsymbol{\eta}}_i$, $\boldsymbol{\eta}_i^*$ is the ideal parameter vector.

When $|s_i| \geq 1 (i = 1, 2, \dots, 7)$, the derivative of the Lyapunov function V can be obtained by using (33)

$$\begin{aligned}
\dot{V} &= \mathbf{s}^T \mathbf{D} \dot{\mathbf{s}} + \frac{1}{2} \mathbf{s}^T \dot{\mathbf{D}} \mathbf{s} + \sum_{i=2}^7 \tilde{\boldsymbol{\eta}}_i^T \rho_i \dot{\tilde{\boldsymbol{\eta}}}_i \\
&= \mathbf{s}^T (\mathbf{T} - \mathbf{H} \dot{\mathbf{q}}_r - \mathbf{G} - \mathbf{F} - \mathbf{D} \ddot{\mathbf{q}}_r) - \mathbf{s}^T \mathbf{H} \mathbf{s} + \frac{1}{2} \mathbf{s}^T \dot{\mathbf{D}} \mathbf{s} + \sum_{i=1}^7 \tilde{\boldsymbol{\eta}}_i^T \rho_i \dot{\tilde{\boldsymbol{\eta}}}_i
\end{aligned} \tag{44}$$

Substituting the control law (42) into (44), and using the fact that $\mathbf{s}^T (\dot{\mathbf{D}} - 2\mathbf{H}) \mathbf{s} = 0$, one can obtain

$$\begin{aligned}
\dot{V} &= \mathbf{s}^T (\hat{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}} | \hat{\boldsymbol{\eta}}) - \bar{\mathbf{F}} - \mathbf{K}_1 \boldsymbol{\Lambda}_{\lambda 1} \text{sign}(\mathbf{s}) - \mathbf{K}_2 \boldsymbol{\Lambda}_{\mu 1} \text{sign}(\mathbf{s}) - \mathbf{K}_3 \mathbf{s}) + \sum_{i=1}^7 \tilde{\boldsymbol{\eta}}_i^T \rho_i \dot{\tilde{\boldsymbol{\eta}}}_i \tag{45}
\end{aligned}$$

where $\boldsymbol{\Lambda}_{\lambda 1} = \text{diag}(|s_1|^{\lambda_{11}}, |s_2|^{\lambda_{12}}, \dots, |s_7|^{\lambda_{17}})$ $\boldsymbol{\Lambda}_{\mu 1} = \text{diag}(|s_1|^{\mu_{11}}, |s_2|^{\mu_{12}}, \dots, |s_7|^{\mu_{17}})$
 $\mathbf{K}_1 = \text{diag}(k_{11}, k_{12}, \dots, k_{17})$ $\mathbf{K}_2 = \text{diag}(k_{21}, k_{22}, \dots, k_{27})$ $\mathbf{K}_3 = \text{diag}(k_{31}, k_{32}, \dots, k_{37})$
The fuzzy approximation error is defined as $\boldsymbol{\varsigma} = \hat{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}} | \hat{\boldsymbol{\eta}}) - \bar{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}})$. Substituting the adaptive law (38) into (45), one can obtain

$$\begin{aligned}
\dot{V} &= -\mathbf{s}^T (\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) - \hat{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}} | \hat{\boldsymbol{\eta}}) + \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}} | \boldsymbol{\eta}^*) - \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}} | \boldsymbol{\eta}^*)) \\
&\quad + \mathbf{K}_1 \boldsymbol{\Lambda}_{\lambda 1} \text{sign}(\mathbf{s}) + \mathbf{K}_2 \boldsymbol{\Lambda}_{\mu 1} \text{sign}(\mathbf{s}) - \mathbf{K}_3 \mathbf{s} + \sum_{i=2}^7 \tilde{\boldsymbol{\eta}}_i^T \rho_i \dot{\tilde{\boldsymbol{\eta}}}_i \\
&= -\hat{\mathbf{s}}^T \mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 1} - \hat{\mathbf{s}}^T \mathbf{K}_2 \hat{\mathbf{s}}_{\mu 1} - \mathbf{s}^T \mathbf{K}_3 \mathbf{s} + \mathbf{s}^T \boldsymbol{\varsigma} \\
&\leq -\hat{\mathbf{s}}^T \mathbf{K}_2 \hat{\mathbf{s}}_{\mu 1} - \hat{\mathbf{s}}^T (\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 1} - \hat{\boldsymbol{\varsigma}})
\end{aligned} \tag{46}$$

$$\begin{aligned}
\text{where } \hat{\mathbf{s}}_{\lambda 1} &= [|s_1|^{\lambda_{11}}, |s_2|^{\lambda_{12}}, \dots, |s_7|^{\lambda_{17}}]^T \hat{\mathbf{s}}_{\mu 1} = [|s_1|^{\mu_{11}}, |s_2|^{\mu_{12}}, \dots, |s_7|^{\mu_{17}}]^T \\
\hat{\mathbf{s}} &= [|s_1|, |s_2|, \dots, |s_7|]^T \hat{\boldsymbol{\varsigma}} = [|\varsigma_1|, |\varsigma_2|, \dots, |\varsigma_7|]^T
\end{aligned}$$

Here ς_i is the i -th element of $\boldsymbol{\varsigma}$.

One can see from (46) that, if $\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 1} \geq \boldsymbol{\varsigma}$ then $\dot{V} \leq -\hat{\mathbf{s}}^T \mathbf{K}_2 \hat{\mathbf{s}}_{\mu 1} - \hat{\mathbf{s}}^T (\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 1} - \boldsymbol{\varsigma}) < 0$. By selecting k_{1i} to satisfy that $k_{1i} \geq \varsigma_i$, then $\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 1} \geq \boldsymbol{\varsigma}$ is satisfied. This indicates that by appropriately selecting k_{1i} the sliding function can be reduced to $|s_i| \leq 1$ within a finite time.

When $|s_i| < 1$, the derivative of the Lyapunov function V can be similarly obtained

$$\begin{aligned}
\dot{V} &= -\mathbf{s}^T \mathbf{K}_1 \boldsymbol{\Lambda}_{\lambda 2} \text{sign}(\mathbf{s}) - \mathbf{s}^T \mathbf{K}_2 \boldsymbol{\Lambda}_{\mu 2} \text{sign}(\mathbf{s}) - \mathbf{s}^T \mathbf{K}_3 \mathbf{s} + \mathbf{s}^T \boldsymbol{\varsigma} \\
&= -\hat{\mathbf{s}}^T \mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 2} - \hat{\mathbf{s}}^T \mathbf{K}_2 \hat{\mathbf{s}}_{\mu 2} - \mathbf{s}^T \mathbf{K}_3 \mathbf{s} + \mathbf{s}^T \boldsymbol{\varsigma} \\
&\leq -\hat{\mathbf{s}}^T \mathbf{K}_2 \hat{\mathbf{s}}_{\mu 2} - \hat{\mathbf{s}}^T (\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 2} - \hat{\boldsymbol{\varsigma}})
\end{aligned} \tag{47}$$

where

$$\begin{aligned}
\boldsymbol{\Lambda}_{\lambda 2} &= \text{diag}(|s_1|^{\lambda_{21}}, |s_2|^{\lambda_{22}}, \dots, |s_7|^{\lambda_{27}}) \\
\boldsymbol{\Lambda}_{\mu 2} &= \text{diag}(|s_1|^{\mu_{21}}, |s_2|^{\mu_{22}}, \dots, |s_7|^{\mu_{27}}) \\
\hat{\mathbf{s}}_{\lambda 2} &= [|s_1|^{\lambda_{22}}, |s_2|^{\lambda_{23}}, \dots, |s_7|^{\lambda_{27}}]^T \\
\hat{\mathbf{s}}_{\mu 2} &= [|s_1|^{\mu_{21}}, |s_2|^{\mu_{22}}, \dots, |s_7|^{\mu_{27}}]^T
\end{aligned}$$

One can infer from (47) that, if $\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 2} \geq \hat{\boldsymbol{\varsigma}}$ then $\dot{V} \leq -\hat{\mathbf{s}}^T \mathbf{K}_2 \hat{\mathbf{s}}_{\mu 2} - \hat{\mathbf{s}}^T (\mathbf{K}_1 \hat{\mathbf{s}}_{\lambda 2} - \hat{\boldsymbol{\varsigma}}) < 0$. This indicates that the sliding function s_i can converge to the zero neighborhood $|s_i| \leq \left(\frac{\varsigma_i}{k_{1i}}\right)^{\frac{1}{\lambda_{2i}}}$ within a finite time. It should be noted that since $0 < \lambda_{2i} < 1$, when the estimation error ς_i of the fuzzy logic system is sufficiently small, $\left(\frac{\varsigma_i}{k_{1i}}\right)^{\frac{1}{\lambda_{2i}}}$ can be approximated to be zero. In summary, $\dot{V} < 0$ can be ensured by selecting sufficiently large values of $\mathbf{K}_1$. According to Lyapunov stability theorem, the sliding function s_i can converge to zero within finite time.

C. Analysis of simulation experiments

To verify the effectiveness of the proposed trajectory tracking control algorithm, trajectory tracking control simulation for the manipulator are conducted in the MATLAB simulation environment. The physical parameters of the manipulator have been presented in Section 2, Section 3, and Section 4. In addition, the gravitational acceleration is taken as $g = \frac{9.8m}{s^2}$. It is assumed that the manipulator is initially located at

the first path point of the planned trajectory, and the initial angular velocity of each joint is zero. To verify that the proposed method possesses good robustness, friction torque is applied to each joint of the manipulator. [21] proposed a nonlinear friction model that combines Coulomb, viscous, and Stribeck friction characteristics, which can eliminate the adverse effects caused by the discontinuity of the traditional friction model. This joint friction model is expressed as

$$F_f(\dot{q}_i) = \alpha_1[\tanh(\beta_1\dot{q}_i) - \tanh(\beta_2\dot{q}_i)] + \alpha_2 \tanh(\beta_3\dot{q}_i) + \alpha_3\dot{q}_i \quad (48)$$

Where $\dot{q}_i$ is the velocity of the i -th joint. α_1 , α_2 , and α_3 represent different degrees of friction, which are positive constants. β_1 , β_2 , and β_3 denote various shape coefficients used to approximate different friction effects, and are also positive constants.

The values of the parameters in the friction model for each joint of the manipulator are shown in Tables VII and VIII.

Tab. VII Parameter values α_i in joint friction models

Joint	α_1	α_2	α_3
1	0.4	0.2	0.02
2	0.4	0.2	0.02
3	0.4	0.2	0.02
4	0.4	0.2	0.02
5	0.4	0.2	0.02
6	0.4	0.2	0.02
7	0.4	0.2	0.02

Tab. VIII Parameter values β_i in joint friction models

Joint	β_1	β_2	β_3
1	2	0.4	1
2	5	1	1
3	3	0.6	1
4	4	0.5	1
5	1	0.65	1
6	3	0.55	1
7	4	0.45	1

For the friction model shown in (48), the friction torque of each joint is a function of the angular velocity of each joint. Therefore, the input to the adaptive fuzzy logic system should be determined by the joint angular velocities. In this paper, the input to the fuzzy logic system is $\mathbf{x}_i = [\dot{q}_i]$, and six fuzzy sets are designed for each input of the fuzzy logic system.

Furthermore, to demonstrate the superiority of the control method proposed in this paper, a comparative simulation with a traditional control method is conducted in the aforementioned simulation environment. In the traditional control method, the sliding mode disturbance observer designed in the proposed method is still used. However, the traditional method does not employ the adaptive fuzzy logic system to compensate for the approximation error of the disturbance observer. In addition, the controller of the traditional control method is derived using the exponential reaching law [22]. Meanwhile, to avoid chattering in the traditional controller, the saturation function is used to replace the sign function in the control law to process the continuity of the control input [22].

Fig. 10 and Fig. 11 respectively present the curves of the joint angles and joint torques variations of the manipulator under the effect of the two control methods. It can be seen from Fig. 10 that under the action of the controller proposed in this paper, even in the presence of the unmodeled friction torques in the manipulator joints, the

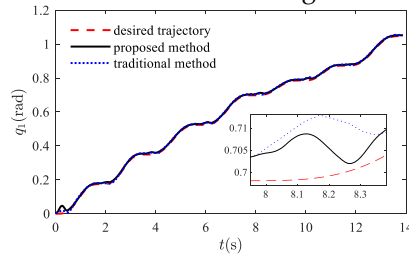
manipulator can still accurately and quickly track the desired trajectory after starting from the initial position of each joint, and maintain stable and accurate tracking throughout the subsequent simulation period. Between two adjacent path segments, due to changes in the planned trajectory, there is a short transition process near the starting position of the subsequent path segment. The simulation results indicate that the control method proposed in this paper possesses strong robustness with respect to system uncertainty. From Fig. 11, it can be seen that due to the adoption of the chattering-free fast reaching law, the output of the controller designed in this paper is relatively smooth, with no obvious chattering in the control input.

To better compare the dynamic control performance of the two control algorithms, two performance metrics E and U are used to quantitatively evaluate the dynamic control performance of the two algorithms. The definitions of the comprehensive error E and the energy consumption U are as follows

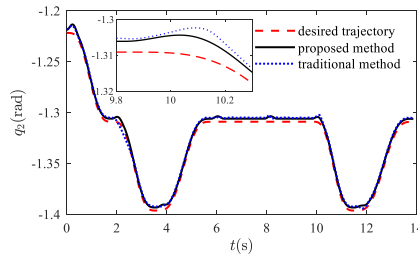
$$CE = \sum_{j=1}^n \sum_{i=1}^7 w_i |\tilde{q}_i(t_j)| \quad EC = \sum_{j=1}^n \sum_{k=1}^7 \varpi_k |\tau_k(t_j)| \quad (49)$$

where n is the number of sampling points, t_j is the j -th sampling instant, and the weights are $w_i = \frac{1}{7}$ and $\varpi_k = \frac{1}{7}$.

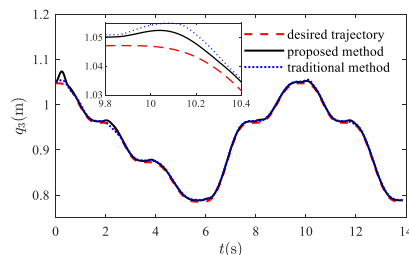
It can be computed from the simulation data in Fig. 10 that the comprehensive error of the control method proposed in this paper is $CE_1 = 5.3573\text{rad}$, and the comprehensive error of the traditional control method is $CE_2 = 6.2037\text{rad}$. From the simulation data in Fig. 11, it can be obtained that the energy consumption of the control method in this paper is $EC_1 = 2.2522 \times 10^6 \text{N}\cdot\text{m}$, while the energy consumption of the traditional control algorithm is $EC_2 = 2.2497 \times 10^6 \text{N}\cdot\text{m}$. Thus, it is evident that the performance metrics of the control method proposed in this paper are superior to those of the traditional control method. The comparison results indicate that the control algorithm proposed in this paper can provide better control performance compared to the traditional control algorithm.



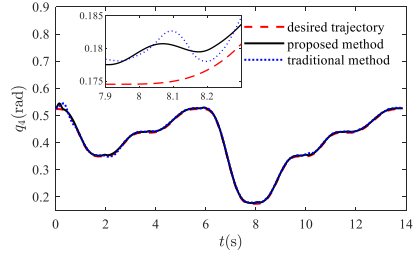
(a) The first joint



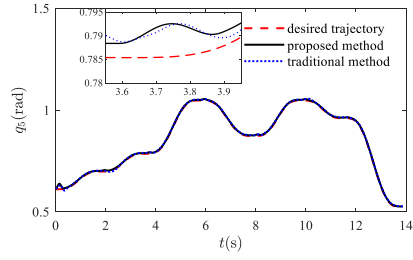
(b) The second joint



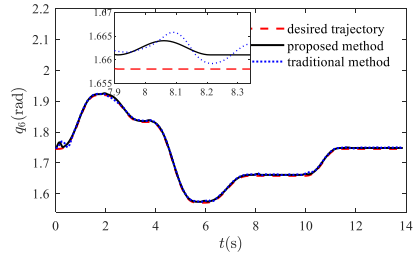
(c) The third joint



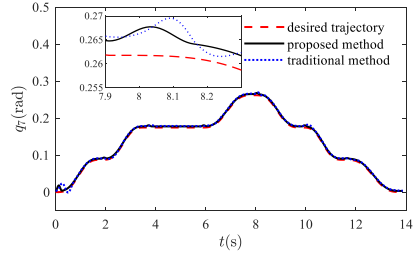
(d) The fourth joint



(e) The fifth joint

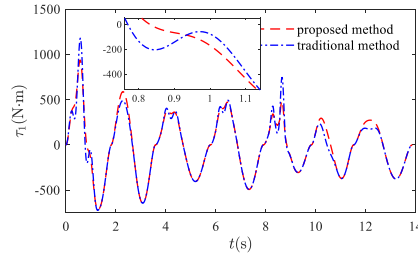


(f) The sixth joint

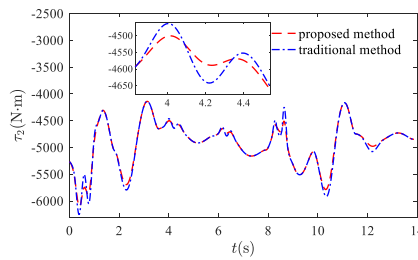


(g) The seventh joint

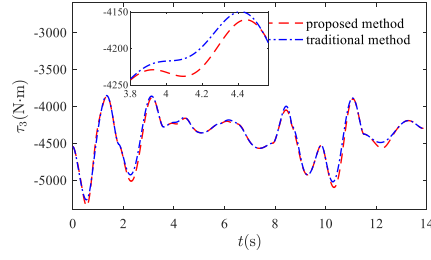
Figure 10 Trajectory tracking performance for each joint of the manipulator.



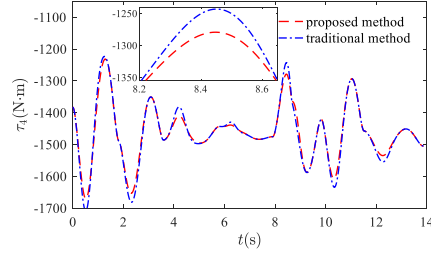
(a) The first joint



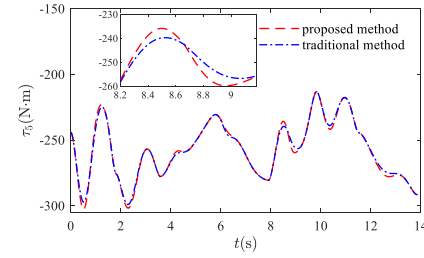
(b) The second joint



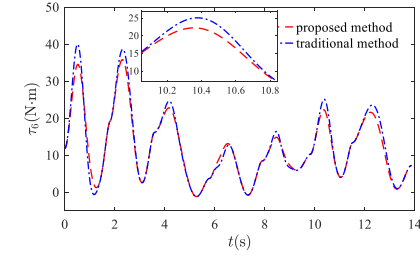
(c) The third joint



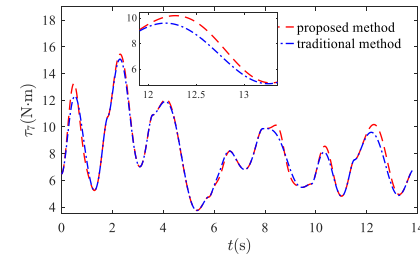
(d) The fourth joint



(e) The fifth joint



(f) The sixth joint



(g) The seventh joint

Figure 11 Torque variation for each joint of the manipulator.

6. Conclusions

For trajectory tracking control problem of a 7-DOF redundant hydraulic manipulator, an adaptive fuzzy sliding mode control method based on a novel fast reaching law is proposed in this paper. The forward kinematics equation of the manipulator is established using the improved D-H method, and the inverse kinematic solution is obtained via the explicit equation method. Based on the forward kinematics analysis of the manipulator, the velocity Jacobian matrix is derived using the vector product method, and the dynamics model of the manipulator is established via the Lagrange method. Based on the inverse

kinematics the manipulator, a trajectory planning method is proposed that integrates 7th-order polynomial interpolation with genetic algorithm optimization. On this basis, a trajectory tracking controller is designed based on a sliding mode disturbance observer and an adaptive fuzzy estimator. A first-order sliding mode disturbance observer is employed to estimate modeling errors in the manipulator system, while an adaptive fuzzy logic system is designed to estimate the observation error. A fast reaching law that dynamically adapts to the changes of the sliding variable is proposed, and the sliding mode control law for the manipulator trajectory tracking is designed based on this fast reaching law. The stability of the closed-loop control system is analyzed using the Lyapunov function method, and simulation results demonstrate that the proposed control method exhibits excellent control performance.

Funding: This work was supported by the Science and Technology Plan Joint Program of Liaoning Province (Natural Science Foundation - Surface Project) (Grant No. 2024-MSLH-179) and the Basic Scientific Research Program of Liaoning Provincial Department of Education for Universities (Grant No. LJ212511779001)..

Institutional Review Board Statement:

Informed Consent Statement:

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no competing interests.

References

- [1] M. W. Hasan and N. H. Abbas, "Disturbance rejection for underwater robotic vehicle based on adaptive fuzzy with nonlinear PID controller," *ISA Transactions*, vol. 130, pp. 360-376, 2022.
- [2] L. Lin, H. R. Wang, and H. B. Ren, "Fuzzy-based variable structure control for robotic manipulators," *Control Theory & Applications*, vol. 24, no. 4, pp. 643-645, 2007.
- [3] J. Zhai and G. Xu, "A novel non-singular terminal sliding mode trajectory tracking control for robotic manipulators," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68, no. 1, pp. 391-395, 2020.
- [4] Y. Y. He, D. L. Tang, and C. Ding, "Research on wall turning motion slip of wall-climbing robot," *China Measurement & Test*, vol. 49, no. 10, pp. 140-147, 2023.
- [5] A. De Luca, B. Siciliano, and L. Zollo, "PD control with online gravity compensation for elastic-joint robots: Theory and experiments," *Automatica*, vol. 41, no. 10, pp. 1809-1819, 2005.
- [6] Z. Song, J. Yi, D. Zhao, and X. Li, "Computed torque controller for uncertain robot manipulator systems based on fuzzy method," *Fuzzy Sets and Systems*, vol. 154, no. 2, pp. 208-226, 2005.
- [7] M. Zhihong, A. P. Paplinski, and H. R. Wu, "A robust MIMO terminal sliding mode control scheme for rigid robotic manipulators," *IEEE Transactions on Automatic Control*, vol. 39, no. 12, pp. 2464-2469, 1994.
- [8] M. I. Azeez and K. R. Atia, "Modeling of PID controlled 3DOF robotic manipulator using Lyapunov function for enhancing trajectory tracking and robustness exploiting Golden Jackal algorithm," *ISA Transactions*, vol. 145, pp. 190-204, 2024.
- [9] D. T. Tran, H. N. Tong, and K. K. Ahn, "An extended state observer-based terminal sliding mode control for 6-DOF manipulator against lumped uncertainties," *International Journal of Control, Automation and Systems*, vol. 23, pp. 810-823, 2025.
- [10] L. Yang and J. Yang, "Nonsingular fast terminal sliding mode control for nonlinear dynamical systems," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 16, pp. 1865-1879, 2011.
- [11] K. Guo, H. Zhang, C. Wei, L. Chen, and W. Jiang, "Novel sliding mode control of the manipulator based on a nonlinear disturbance observer," *Scientific Reports*, vol. 14, no. 1, pp. 690, 2024.

- [12] L. X. Sun, J. M. Gao, C. Y. Gao, Y. X. Wei, and J. Sun, "Obstacle avoidance path planning for 7-DOF manipulators based on improved RRT* algorithm," *Control Engineering of China*, vol. 30, no. 9, pp. 1592-1597, 2023.
- [13] D. L. Peiper, "The kinematics of manipulators under computer control," Ph.D. dissertation, Stanford University, Stanford, CA, USA, 1968.
- [14] X. Y. He, X. Y. Gao, H. J. Wang, Y. Z. Fu, and G. L. Sun, "Operational stability analysis of 7-DOF dual-arm collaborative robot," *Journal of Engineering Design*, vol. 26, no. 6, pp. 706-713, 2019.
- [15] P. C. Zhang and T. Zhang, "Jacobian matrix solution and singular configuration analysis of 6-DOF industrial robots based on vector product method," *Machinery Design & Manufacture*, no. 8, pp. 152-154, 2011.
- [16] B. Siciliano, O. Khatib, and T. Kröger, Eds., *Springer Handbook of Robotics*. Berlin, Germany: Springer, 2008, pp. 200.
- [17] S. L. Wang, B. K. Zhang, J. Zhou, L. Tao, and J. Yang, "Time-jerk optimal trajectory planning for industrial robots with coupled interpolation function selection," *Journal of Field Robotics*, vol. 41, no. 4, pp. 917-941, 2024.
- [18] J. H. Holland, *Adaptation in Natural and Artificial Systems*. Ann Arbor, MI, USA: University of Michigan Press, 1975.
- [19] L. Besnard, Y. B. Shtessel, and B. Landrum, "Quadrotor vehicle control via sliding mode controller driven by sliding mode disturbance observer," *Journal of the Franklin Institute*, vol. 349, no. 2, pp. 658-684, 2012.
- [20] P. Zhang and Q. Wu, "Double power reaching law for sliding mode control with unidirectional auxiliary surfaces," in *Proc. 2017 Chinese Control Conference*, Dalian, China, Jul. 2017, pp. 3798-3803.
- [21] C. Makkar, W. E. Dixon, W. G. Sawyer, and G. Hu, "A new continuously differentiable friction model for control systems design," in *Proc. IEEE/ASME Int. Conf. Adv. Intell. Mechatronics*, Monterey, CA, USA, Jul. 2005, pp. 600-605.
- [22] V. Utkin, J. Guldner, and J. M. Shi, *Sliding Mode Control in Electro-Mechanical Systems*, 2nd ed. Boca Raton, FL, USA: CRC Press, 2009.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.

Appendix I : Center of Mass and Inertia Parameters of Each Link

Link	Mass	Centroid Coordinates	Inertia Tensor		
Shoulder Rotation Arm	$m_1=158.026$	$^1x_{c1} = -0.037$	$I_{1xx} = 10.000$	$I_{1xy} = 0.001$	$I_{1xz} = -0.788$
		$^1y_{c1} = 0.000$	$I_{1yx} = 0.001$	$I_{1yy} = 12.755$	$I_{1yz} = -0.010$
		$^1z_{c1} = -0.070$	$I_{1zx} = -0.788$	$I_{1zy} = -0.010$	$I_{1zz} = 7.157$
Upper Arm	$m_2=60.595$	$^2x_{c2} = -0.132$	$I_{2xx} = 4.984$	$I_{2xy} = -1.266$	$I_{2xz} = -0.003$
		$^2y_{c2} = 0.542$	$I_{2yx} = -1.266$	$I_{2yy} = 2.182$	$I_{2yz} = -0.007$
		$^2z_{c2} = -0.001$	$I_{2zx} = -0.003$	$I_{2zy} = -0.007$	$I_{2zz} = 6.940$
Elbow Arm	$m_3=84.995$	$^3x_{c3} = 0.884$	$I_{3xx} = 1.679$	$I_{3xy} = 0.804$	$I_{3xz} = -0.011$
		$^3y_{c3} = -0.114$	$I_{3yx} = 0.804$	$I_{3yy} = 17.312$	$I_{3yz} = -0.004$
		$^3z_{c3} = 0.000$	$I_{3zx} = -0.011$	$I_{3zy} = -0.004$	$I_{3zz} = 18.492$
Lower Arm	$m_4=68.303$	$^4x_{c4} = 0.332$	$I_{4xx} = 2.076$	$I_{4xy} = -0.050$	$I_{4xz} = -0.009$
		$^4y_{c4} = 0.017$	$I_{4yx} = -0.050$	$I_{4yy} = 8.495$	$I_{4yz} = 0.001$
		$^4z_{c4} = 0.002$	$I_{4zx} = -0.009$	$I_{4zy} = 0.001$	$I_{4zz} = 10.322$
Wrist Pitch Arm	$m_5=44.202$	$^5x_{c5} = 0.062$	$I_{5xx} = 1.690$	$I_{5xy} = 0.961$	$I_{5xz} = -0.048$
		$^5y_{c5} = 0.187$	$I_{5yx} = 0.961$	$I_{5yy} = 2.428$	$I_{5yz} = 0.209$
		$^5z_{c5} = 0.061$	$I_{5zx} = -0.048$	$I_{5zy} = 0.209$	$I_{5zz} = 3.138$
Wrist Yaw Arm	$m_6=14.487$		$I_{6xx} = 0.271$	$I_{6xy} = 0.125$	$I_{6xz} = 0.042$
			$I_{6yx} = 0.125$	$I_{6yy} = 0.355$	$I_{6yz} = 0.025$

		${}^6z_{c6} = 0.028$			
Wrist Rotation Arm	$m_7=28.147$	${}^7x_{c7} = 0.000$	$I_{7xx} = 0.645$	$I_{7xy} = 0.001$	$I_{7xz} = -0.002$
		${}^7y_{c7} = 0.000$	$I_{7yx} = 0.001$	$I_{7yy} = 0.913$	$I_{7yz} = 0.001$
		${}^7z_{c7} = 0.175$	$I_{7zx} = -0.002$	$I_{7zy} = 0.001$	$I_{7zz} = 0.783$

In the appendix I the unit of mass is kg, the unit of center of mass is m, and the unit of moment of inertia is kg·m².