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# Dual Dimensions of Productivity in Cross-Community Mathematics Teacher Dialogue

**Alon Pinto**

alon.pinto@weizmann.ac.il

weizmann institute of science

**Myriam Goor**

weizmann institute of science

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**Dual Dimensions of Productivity in Cross-Community Mathematics Teacher Dialogue**

Alon Pinto and Myriam Goor

Weizmann Institute of Science, Israel

While scholars have been calling for cross-community teacher dialogue, it remains relatively rare, both in research and in practice, and its potential productivity is unclear. We propose a dual framework distinguishing between pedagogical productivity and boundary productivity. The former dimension relates to the advancement of instructional reasoning and pedagogical judgment and the latter to articulation, negotiation, and transformation of professional boundaries. We conceptualize these forms of productivity as emerging through processes of reflection, understood as the collective examination and reinterpretation of instructional practice and professional perspectives in interaction. Drawing on four cases from a secondary-tertiary video-club setting (M-Cubed), we show that these dimensions are analytically distinct and variably related. Cross-community deliberation may stall along both dimensions (boundary-avoidance deliberation), advance pedagogical reasoning without boundary reflection (boundary-mediated deliberation), clarify professional differences without instructional advancement (boundary-reflective deliberation), or integrate both forms of productivity (boundary-integrative deliberation). We further show that these configurations are shaped by distinct reflection mechanisms, including problematization, defamiliarization, avoidance, and translation. These findings provide a framework for analyzing the generative force of cross-community teacher deliberation and clarify the conditions under which dialogues between secondary and university mathematics teachers advance pedagogical reasoning and judgment, boundary reflexivity, or both.

Keywords: Teacher discourse, pedagogical productivity, boundary encounters, dialogical learning, reflection.

## 1. Rationale

For more than a century, scholars and mathematics educators have been concerned with discontinuities that divide secondary and tertiary mathematics education. Felix Klein's (1908) notion of the *double discontinuity* remains one of the most enduring formulations of this problem: students encounter a first discontinuity when transitioning from school to university, encountering mathematics that often appears to them as an entirely new discipline with unfamiliar norms of

rigor, explanation, and argumentation; prospective teachers encounter a second discontinuity when returning to school and find it difficult to connect their teaching to the mathematics they practiced and experienced at university (Kilpatrick, 2019). Research across many countries shows that transitions across these two discontinuities are still frequently accompanied by frustration, alienation, and disengagement, turning tertiary mathematics education into a gatekeeper rather than a gateway to further study and participation in mathematically intensive fields (Biza et al., 2022; Di Martino et al., 2023; Gueudet, 2008, 2023; Pinto & Koichu, 2023).

Despite decades of extensive research and numerous interventions, instructional innovations, and professional development initiatives, the secondary-tertiary divide remains a persistent challenge (Pinto & Koichu, 2023). Much of this work has approached the divide as resulting from a curricular gap, a deficit in student preparation, or ineffective pedagogies, and has attempted to address these issues on one side of the secondary-tertiary divide. Consequently, research mostly engaged separately the two professional communities most closely positioned to the double discontinuity – secondary mathematics teachers (SMTs) and university mathematics teachers (UMTs). The premise of this study is that such separation, however well-intentioned, risks reinforcing the very structures that divide secondary and tertiary mathematics education (Pinto, in press).

In this paper, we reconceptualize the secondary-tertiary divide as a boundary phenomenon that is maintained by historically entrenched professional, epistemological, and institutional divisions between SMTs and UMTs. Recent research suggests that collaboration between SMTs and UMTs may serve as a valuable resource for understanding and addressing the secondary-tertiary divide (Hochmuth et al., 2021; Pinto & Koichu, 2023; Winsløw et al., 2021). However, collaboration between these communities remains relatively rare, and when it does occur it is often derailed by asymmetrical relationships and deficit narratives (Pinto, in press). While UMTs and SMTs operate within partially overlapping instructional environments and both share a commitment to mathematics and to students' mathematical learning, they operate within distinct professional jurisdictions that limit both opportunity and capacity for sustained meaningful professional dialogue across the secondary-tertiary divide (Fried, 2014; Pinto & Lai, 2025; Sfard, 1998).

To address these challenges, we draw on the literature on *boundary encounters* and *dialogical learning* to conceptualize cross-community professional deliberation as a space where professional differences and tensions can be leveraged as resources for mutual learning (Akkerman & Bakker,

2012; Goos & Bennison, 2018; Pinto & Cooper, 2022, 2023a). In such interactions, participants articulate and engage with distinctions across professional discourses rather than suppressing them. These processes can be understood as forms of reflection, broadly conceived as the collective examination, clarification, and negotiation of instructional practices and professional jurisdictions.

While prior research has emphasized the importance of reflection in teacher discourse and in cross-community interaction, it has paid less attention to how it unfolds in cross-community teacher talk. This study investigates how such reflection develops in cross-community deliberation between SMTs and UMTs and how it shapes professional learning, addressing the following question:

*How does cross-community professional deliberation between university and secondary mathematics teachers generate learning about their instructional practice and about differences between their professional practices?*

## 2. Unpacking the Productivity of Cross-Community Professional Deliberation

In this section, we discuss *teacher professional deliberation* as the broad interactional category in our study and conceptualize cross-community deliberation as a form of such discourse in which learning may unfold both pedagogically and across professional boundaries, through processes of reflection on instructional practice and professional perspectives.

### *2.1. Discourse as a professional practice*

Research on teacher professional learning increasingly treats teachers' talk about practice as a central site for professional learning. Through discussions of instructional situations, teachers notice and interpret events, evaluate pedagogical decisions, and deliberate about teaching alternatives. Such discursive activities enable teachers to articulate and refine the professional discourse through which they make sense of the complex demands of teaching (Borko et al., 2008; Lefstein et al., 2020). From this perspective, professional knowledge is not a static body of information but a cultural and situated practice that is constituted in shared discourses and developed through interaction, argumentation, and collective reasoning (Stigler & Hiebert, 1999).

Not all talk about practice is conducive to professional learning. Research on teacher learning communities and video-based professional development show that discourse becomes *pedagogically productive* when it is collaborative, reflective, reasoned, grounded in sensemaking, and anchored in teachers' instructional contexts (Arcavi & Schoenfeld, 2008; Borko et al., 2008;

Ehrenfeld, 2023; Horn & Garner, 2022; Lefstein et al., 2020). Such discourse typically centers representations of practice – classroom episodes, student work, or instructional artifacts – which provide shared reference points for collective inquiry (Buchbinder & Kuntze, 2018; Karsenty & Arcavi, 2018). These processes support teachers in developing greater sensitivity to critical lesson events, refine their interpretations, sharpen their pedagogical judgment, and enrich their repertoire of instructional responses (Lefstein et al., 2020).

While much of this work focuses on discourse within relatively homogeneous communities, the present study examines discourse across distinct professional communities, where interaction becomes a site not only for instructional reasoning and pedagogical judgment but also for articulating and negotiating differences in professional perspectives (Hollingsworth & Clarke, 2017; Schwarts and Karsenty, 2020). This draws attention to the role of *boundary crossing* in shaping teachers' professional discourses.

### *2.2. Professional perspectives in cross-community dialogue*

Teaching is inherently a cultural activity. Teachers working in different institutional contexts are socialized into distinct professional discourses that shape how they interpret teaching situations (Stigler & Hiebert, 1999). As such, research on teacher discourse underlines the important benefits of teachers' collective engagement with professional practices from "distant" cultures that support articulation, negotiation and reconsideration of what is expected and accepted in classes (Arcavi & Schoenfeld, 2008; Hollingsworth & Clarke, 2017; Schwarts and Karsenty, 2020).

These dynamics are particularly pronounced in interactions between secondary mathematics teachers (SMTs) and university mathematics teachers (UMTs), who operate within distinct yet partly overlapping professional discourses and jurisdictions (Fried, 2014; Pinto & Lai, 2025). As such, their interactions can be meaningfully conceptualized and studied as *boundary encounters,* in which professional distinctions become salient and create opportunities for *dialogical learning* (Goos & Bennison, 2018; Hernandez-Martinez et al., 2023; Pinto & Cooper, 2022, 2023a).

*Boundaries* in this context are understood as distinctions in how actions and interactions are conducted, reasoned, and coordinated across professional communities (Akkerman & Bakker, 2011; Wenger, 1998). In our context, boundaries are constituted as relations between professional discourses across teacher communities. They become visible when SMTs and UMTs interpret the same instructional situation differently, prioritize different concerns, or rely on different forms of

professional justification. For example, SMTs may emphasize student understanding or curricular pacing, whereas UMTs may foreground mathematical structure and disciplinary authenticity (Mytlis & Pinto, 2025; Pinto & Cooper, 2023a). These differences reflect distinct professional obligations and evaluative frameworks within these two communities (Herbst & Chazan, 2020).

The potential for learning in boundary encounters depends on the ways in which participants engage with boundaries. Akkerman and Bakker (2011) found that in productive boundary encounters, participants *identify* and *legitimize* professional distinctions, *coordinate* joint inquiries without necessarily reaching consensus about their precise nature, develop *shared* and *hybrid* perspectives that integrate elements from multiple professional discourses.

In addition, *reflection* is recognized in both the literature on teacher discourse and the literature on boundary crossing as a central process through which professional differences become a catalyst and a resource for professional learning (Akkerman & Bakker, 2011; Arcavi & Schoenfeld, 2008; Lefstein et al., 2020; Schwarts & Karsenty, 2020). According to Akkerman and Bakker (2011), boundaries support reflection by creating the "possibility to look at oneself through the eyes of other worlds" (p. 145) that may result in "an expanded set of perspectives and thus a new construction of identity that informs future practice" (p. 146). The literature on video-based teacher learning describes reflection as a process that entails 'stepping into the shoes' of other teachers and inspecting their actions and decision-making (Arcavi and Schoenfeld, 2008), with the intention of "pursuing and seizing opportunities to learn from such scrutiny in order to reencounter the professional practice with new tools, insights, and horizons" (Karsenty and Arcavi, 2017, p. 435).

Prior research points to reflection mechanisms that may unfold when teachers engage with the professional perspectives of other teacher communities. Most pertinent for this study are the mechanisms of *problematization* (Arcavi & Schoenfeld, 2008) and *defamiliarization* (Schwarts & Karsenty, 2020) of routine situations and practices that "help teachers regain the capacity to question the obvious" (Cooper, 2016, p. 225). However, such reflection is not automatic. Interactions between different professional communities often unfold through *incommensurable* forms of reasoning (Sfard, 2008) that not only lead to disagreement but also prevent deliberation as the two sides implicitly draw on different understandings of what counts as valuable mathematical learning activities (Pinto & Cooper, 2018, 2023b). When differences remain implicit or are treated as errors to be corrected or obstacles to overcome, dialogue may stall or become

adversarial. Conversely, when surfaced and explored, they can trigger reflective processes that support articulation of tacit assumptions and generate new forms of understanding, and transform practice (Goos & Bennison, 2018; Hernandez-Martinez et al., 2023; Pinto & Lai, 2025).

This view of professional development resonates with broader theories of dialogical learning, which emphasize that understanding develops through sustained engagement with a multitude of voices without assertion of authority (Bakhtin, 1981; Hernandez-Martinez et al., 2023; Star, 1989; Wegerif, 2011). From this perspective, dialogues between SMTs and UMTs are important not only for advancing their professional discourses but also for exploration and negotiation of their overlapping *professional jurisdictions* that will clarify the responsibilities of each community (Abbott, 1988; Blok et al., 2019; Herbst & Chazan, 2020; Liu, 2018; Pinto & Cooper, 2023a).

### *2.3. The dual productivity of cross-community teacher professional deliberation*

When SMTs and UMTs jointly analyze teaching situations, their professional deliberations may include various forms of reflection, depending on how participants engage with teaching situations and professional boundaries. In some cases, participants may converge on shared interpretations of the situation and engage in pedagogical deliberations where they interpret classroom events and evaluate possible instructional responses. In other cases, differences in perspective may persist, prompting an exploration and clarification of implicit assumptions and priorities, shifting attention to the participants' diverse teaching contexts and professional discourses. Understanding such professional deliberations therefore requires attention to how such processes emerge and interact. Accordingly, in this paper, we distinguish between two dimensions of productivity:

*Pedagogical productivity* captures the ways in which collective professional deliberations can advance participants' instructional reasoning and pedagogical judgment. This dimension becomes visible in reasoning around concrete instructional situations, including how participants notice and interpret classroom events, evaluate decisions, and formulate and justify possible alternatives.

*Boundary productivity* captures the ways in which collective professional deliberations can advance understanding and negotiation of professional boundaries and jurisdictions. This dimension becomes visible when participants articulate divergent rationales, make different evaluative criteria explicit, clarify professional roles or responsibilities, or develop shared ways of interpreting differences that support translation and transition across professional discourses.

These two dimensions should not be assumed to co-occur. In this paper, we examine whether and how cross-community deliberation may advance one without the other, support both, or stall along both. We further explore how these processes emerge through different mechanisms of reflection. These mechanisms are not taken as causal determinants of productivity, but as analytic constructs that describe recurrent interactional processes through which pedagogical and boundary productivity unfold and through which different forms of professional deliberation are constituted.

Our analysis of how these two dimensions unfold and interact addresses the following questions:

RQ1. What forms of cross-community professional deliberation emerge when university and secondary mathematics teachers jointly examine instructional situations?

RQ2. How do these forms of deliberation differ in their pedagogical productivity and boundary productivity?

RQ3. What reflection mechanisms can be identified that support or constrain dual productivity in cross-community professional deliberations?

## 3. Methodology and Analytic Approach

### *3.1. Research design and setting: the M-Cubed Lab*

This study is part of a long-term design-based research program – M-Cubed – dedicated to studying and fostering dialogical learning across the secondary-tertiary divide in mathematics education. Within this broader program, this paper adopts a qualitative, theory-informed comparative case-study design to develop and refine an analytical framework for understanding how pedagogical and boundary productivity emerge in cross-community professional deliberation.

The cases were drawn from the **M**-Cubed Lab (**M**athematicians, **M**athematics teachers, **M**athematics teaching), established in 2019 at the Weizmann Institute of Science to study how university mathematics teachers (UMTs) and secondary mathematics teachers (SMTs) communicate, collaborate, and learn from and with one another across professional boundaries (Pinto, in press). M-Cubed convenes small groups of experienced UMTs and SMTs in structured video-club sessions, where participants jointly examine artifacts and excerpts from authentic secondary mathematics lessons and discuss the instructional dilemmas they reveal.

The design of M-Cubed draws on two complementary strands of research: video-based teacher learning (Karsenty & Arcavi, 2018) and *boundary encounters* (Sztajn et al., 2014). Videotaped

lessons serve both as *rich representations of practice* that anchor joint inquiry and invite instructional reasoning and pedagogical judgment (Lefstein et al., 2020), and as *boundary objects* that are robust enough to support engagement without prior consensus yet flexible enough to accommodate different professional perspectives (Star, 1989, 2010; Pinto & Lai, 2025).

Sessions were facilitated by mathematics education researchers with strong disciplinary backgrounds, who acted as *brokers* between professional discourses (Pinto & Cooper, 2022, 2023a). Their role was not to eliminate differences, but to support reflective exchange by helping participants articulate tacit assumptions and explore alternative interpretations when discontinuities emerged. In this sense, M-Cubed functions as an empirical setting for examining how professional perspectives and jurisdictions are negotiated through interaction.

Since 2019, M-Cubed has generated over 100 hours of recorded joint explorations. Prior analyses of these interactional data have documented multiple forms of professional learning emerging from these discussions, illustrating how disciplinary knowledge, professional obligations, and pedagogical reasoning are utilized, translated, resisted, refined, and sometimes hybridized in cross-community deliberation (Goor & Pinto, in press; Mytlis & Pinto, in press; Pinto, in press; Pinto & Cooper, 2022, 2023a; Pinto & Lai, 2025). However, these analyses have not explicitly distinguished different dimensions of productivity, often treating the two as a single construct.

### *3.2. Data sources and case selection*

The present study draws on the broader M-Cubed corpus but focuses on four cases, chosen not for representativeness but for their analytic value in illustrating contrasting forms of cross-community deliberation. Case selection proceeded through iterative review of the corpus in relation to the study's overarching question: *how cross-community deliberation generates learning about instructional practice and professional differences*. Specifically, we selected discussions that:

- Involved sustained engagement with an instructional dilemma
- Included meaningful participation from both UMTs and SMTs
- Exhibited interactional patterns that could be analyzed in terms of pedagogical and/or boundary productivity

As summarized in Table 1, together, the cases illuminate distinct configurations of the two dimensions of productivity (low/low, high/low, low/high, and high/high). Participants included

highly experienced SMTs and UMTs, including professional mathematicians and physicists, some of whom had experience teaching secondary teachers.

The primary data consist of video recordings of M-Cubed sessions. Supplementary data include the videotaped lessons under discussion and participants' written reflections, which were used to clarify how interactions were interpreted and elaborated over time. All meetings were conducted in Hebrew and selectively translated by the authors, with analytic claims grounded in repeated engagement with the original recordings.

*3.3. Analytical approach*

The analysis proceeded in two iterative stages: within-case analysis and cross-case comparison. First, each case was analyzed independently to characterize how deliberation unfolded and how it was productive. To ensure comparability across cases, each case was analyzed using a common five-step template aligned with the theoretical framework developed in Section 2 (see Figure 1):

1. **Triggering situation** – the instructional event, task, or artifact that initiated the discussion.
2. **Initial framing of the problem** – how participants initially interpreted and evaluated the situation.
3. **Interactional dynamics** – how participants responded to one another, including the emergence of reflective mechanisms, shared or competing evaluative frameworks, moments of uptake, reframing, and reinterpretation.
4. **Productivity outcomes** – whether and how the discussion advanced pedagogical reasoning and/or reflection on professional differences.
5. **Interpretation** – the overall form of deliberation exemplified by the case.

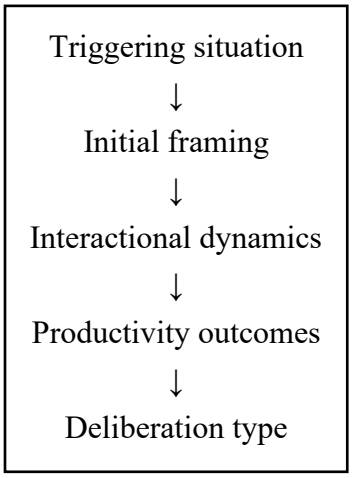


Figure 1 – Five-step analysis

Particular attention was given to interactional dynamics, understood as the processes through which participants engaged with instructional situations and professional differences, including moments of uptake, reframing, and reflection.

Second, the four cases were compared in order to identify patterned differences in how pedagogical and boundary productivity related, and how these relations were shaped by recurring reflection mechanisms. This comparative analysis led to the formulation of a typology of cross-community deliberation: *boundary-avoidance*, *boundary-mediated*, *boundary-reflective*, and *boundary-integrative*.

The analysis also supported the identification of reflection mechanisms – recurrent interactional processes through which participants collectively examine and reinterpret instructional situations and professional perspectives. These mechanisms were not imposed a priori but emerged through iterative engagement between the empirical data and existing accounts of reflection in teacher discourse and boundary crossing research. Four mechanisms emerged in the analysis: *Problematization* was identified when familiar instructional routines were treated as objects of inquiry; *defamiliarization* when participants encountered and engaged with unfamiliar interpretations of familiar practices; *avoidance* when differences were displaced through shifts in evaluative criteria; and *translation* when ideas were reformulated across professional discourses.

These categories are analytic distinctions that capture dominant interactional tendencies rather than mutually exclusive or exhaustive classifications. Rather than determining specific outcomes, they characterize how different forms of deliberation are interactionally constituted, providing a process-level account of how pedagogical and boundary productivity unfold in practice.

## 4. Dual Dimensions of Productive Teacher Talk Across the Secondary-tertiary Divide

The following analysis examines how *pedagogical productivity* and *boundary productivity* unfold in cross-community professional deliberation between university mathematics teachers (UMTs) and secondary mathematics teachers (SMTs). We interpret these processes as emerging through different forms of reflection and analyze how reflection mechanisms are taken up, sustained, or displaced in interaction, following the analytic template outlined in Section 3. We analyze four M-Cubed discussions that illustrate distinct forms of cross-community deliberation differing in their dual productivity (Table 1). Together, these cases show how variation in deliberation is constituted not only in productivity outcomes but in how reflection unfolds in interaction.

**Table 1: Different forms of cross-community professional deliberation**

| | Low boundary productivity | High boundary productivity |
|---|---|---|
| Low pedagogical productivity | *Case 1: Boundary-avoidance deliberation* | *Case 3: Boundary-reflective deliberation* |
| High pedagogical productivity | *Case 2: Boundary-mediated deliberation* | *Case 4: Boundary-integrative deliberation* |

*4.1. Case 1: Boundary-Avoidance Deliberation*

Triggering Situation

Participants viewed an excerpt from a 10th-grade calculus lesson in which the teacher, Rosemary, introduced function analysis through a graph-reconstruction activity. A student briefly exited the classroom while a graph was displayed (Figure 2) and, upon returning, attempted to reproduce the graph on the board with verbal guidance from classmates. The SMTs, who were familiar with the activity, framed it as pedagogically purposeful. For example, Cora described how she deliberately chooses functions with extrema and asymptotes in order to expose students' lack of shared conceptual vocabulary, thereby motivating the "new language we are about to study".

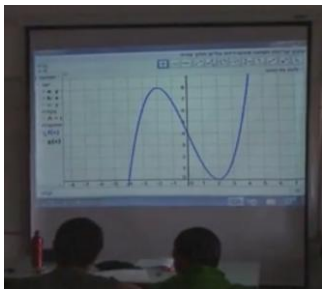


Figure 2. Rosemery's graph-reconstruction activity.

Initial Framing of the Problem

The UMTs did not object directly to Cora's pedagogical goal. However, they evaluated the activity as insufficiently mathematical. For example, one mathematician observed:

> *You just start with a figure and end with a figure so [it's] more a discussion in visual communication, not in mathematics.*

The UMTs also questioned the prohibition of informal descriptors such as gestures or metaphors. For example, they argued that if a graph looks like "a back of a crocodile" then describing it in this manner would be more natural and efficient than applying the language and tools of calculus.

They argued that the activity imposes restrictions that contradict with the proper purpose of mathematics: “We want mathematics where it is helpful.”

These contrasting evaluations reveal different evaluative criteria. Whereas SMTs framed the activity in terms of pedagogical motivation – illustrating the need for new mathematical language – the UMTs framed it in terms of disciplinary authenticity and the appropriate use of mathematics.

Interactional Dynamics

Although participants responded to one another, the underlying evaluative differences remained implicit. The SMTs did not directly engage with the UMTs’ criterion of disciplinary authenticity. Instead, their responses shifted the evaluative frame toward affective considerations, emphasizing that the activity is “fun” for students. They also deflected the UMTs’ criticism by suggesting that mathematics lessons need not always focus exclusively on mathematical content. The UMTs accepted these responses without probing further.

A similar pattern occurred when UMTs suggested that the graph could represent a real-life phenomenon, which might give meaning to the properties students were describing. SMTs framed the suggestion as “very interesting,” yet did not pursue it further, and the proposal remained undeveloped. In both instances, participants acknowledged one another’s contributions but did not engage substantively with the reasoning behind them.

Productivity Outcomes

No sustained reflection developed along both dimensions of productivity. Pedagogically, participants did not engage in detailed interpretation of classroom events, nor analyze instructional decisions in depth, nor generate and evaluate alternatives, beyond the briefly mentioned suggestion of using real-life graphs. From a boundary perspective, differences in evaluative frameworks were neither explicitly articulated nor reflected upon.

Interpretation

We interpret this discussion as illustrating authoritative assertion of professional jurisdictions – in this case over evaluative frameworks – that resulted in *surface uptake* without *epistemic uptake* and stalled productivity along both dimensions. Participants responded interactionally to one another’s statements but did not deliberate within a shared evaluative framework, effectively reasoning in parallel while relying on distinct meanings of “motivation”: mathematical motivation

(using mathematics where it is helpful), pedagogical motivation (illustrating the need for new concepts), and affective motivation (beginning a topic with an enjoyable activity).

These distinct meanings, which correspond to different professional orientations toward the purpose of mathematics instruction, were displaced through *boundary-avoidance reframing* – implicit shifts in professional meanings and orientations.

We frame such interactional dynamics as *boundary-avoidance deliberation*, a form of cross-community interaction in which professional differences and disagreements are deflected and remain implicit and undeveloped, limiting reflective engagement.

*4.2. Case 2: Boundary-Mediated Deliberation*

Triggering Situation

This discussion emerged after watching another episode from Rosemary's lesson. Rosemary introduced the following graphing task and asked students how they might approach it:

*Sketch, as best as you can, the function:* $f(x) = 4(x-1)^3 - 3x + 4.$

The students, whose prior experience with graphing was largely limited to linear and quadratic functions, spent approximately twelve minutes substituting values. After realizing that $f(0) = 0$, $f(1) = 1$ and $f(2) = 2$ they conjectured that the graph might be linear before discovering a contradiction by computing the roots of the equation $f(x) = 0$. Rosemary eventually concluded that substitution was futile and that the class lacked the necessary tools to analyze the function.

Initial Framing of the Problem

The UMTs responded critically. They characterized students' engagement as "boring," "exhausting," and characterized teaching as emphasizing computation rather than intuition and oriented toward leading students to failure rather than to productive struggle.

In contrast to the previous case, the UMTs did not frame their critique in terms of disciplinary authenticity. Instead, their comments focused on pedagogical rationality of specific instructional decisions. They pointed to how Rosemery introduced the task and reacted to students' suggestions and argued that her actions positioned students in a situation where they repeatedly performed calculations without attempting to gain insight into the behavior of the function. For example, one

UMT highlighted Rosemary's decision not to pause when students suggested "solving the equation for x," asking whether this was a missed opportunity to probe students' understanding:

> *This was a misconception […] she just let it pass and I was wondering, wouldn't it be better to pause and ask why? What does it mean to solve for x?*

They also criticized Rosemery's framing of the activity and its conclusions. As one UMT put it:

> *[The goal] was to develop intuition, right? But at the end, it really felt like the [conclusion] was: We cannot develop intuition so let's learn to do calculus!*

These observations framed the episode as a pedagogical problem rather than a disciplinary one.

Interactional Dynamics

In response to the UMTs' critique, the group began to examine the instructional situation in detail, exploring different interpretations of Rosemary's decisions and debating what her pedagogical goals might have been. One SMT suggested that Rosemary may have intended students to recognize the limits of their current strategies:

> *[Rosemery] wanted students to realized they cannot draw the graph. She is not trying to lead them to success […] You prefer functions that will lead to success, but she has other goals.*

Although this comment implicitly revealed a difference between UMT and SMT evaluative orientations, the participants did not pursue this difference as a boundary issue. Instead, the discussion continued to focus on instructional consequences and student learning.

As the discussion progressed, participants began to articulate alternative instructional approaches. One observation concerned the structure of the task: every strategy available to students required extensive computation, leaving little time for student reflection. This prompted participants to explore possible modifications that might help students develop intuition about the function.

One proposal was to use a computer as a "black box" that could compute the values of $f(x)$ automatically, allowing students to focus on exploring different substitution strategies. Another suggestion was to replace the function with one that more readily supports conceptual reasoning, such as $f(x) = x(x-1)(x+1)$. Participants also proposed questions that might help students reason about the function qualitatively:

- How does the graph behave between two intersection points?

- What happens for very large or very small values of x?

These exchanges indicate *epistemic uptake*, as participants engaged directly with one another's reasoning and collaboratively explored pedagogical alternatives that aim to convey to students what they *can* do rather than what they *cannot* do.

Productivity Outcomes

The discussion reflects high pedagogical productivity and low boundary productivity.

Pedagogically, participants engaged in a sustained form of reflection, in which they collectively identified instructional decisions, articulated and explored their underlying pedagogical rationales, and proposed and evaluated instructional alternatives. Several SMTs explicitly noted that the discussion prompted them to reconsider how they introduce function analysis in their own classrooms. Notably, one UMT reflected:

*"I must say, I helped designing the secondary curriculum, and we never went so deep."*

From a boundary perspective, this reflection remained primarily focused on instructional practice. Although differences in professional perspectives were implicitly present – for example, whether motivation tasks should emphasize productive struggle or students' limitations – they were not explicitly taken up as objects of reflection, instead, remaining as tacit background assumptions that oriented the discussion towards pedagogical reasoning.

Interpretation

We interpret this discussion as illustrating how pedagogical productivity can emerge from a cross-community *problematizing intervention* – a critical perspective of one professional community of instructional decisions or practices of another that transforms a familiar instructional routine into an object of pedagogical inquiry. In contrast to the first case, in this discussion the UMTs' criticism was taken up epistemically and developed into collective pedagogical reasoning, presumably because it was anchored in a concrete representation of practice – the videotaped lesson episode.

We frame such interactional dynamics as *boundary-mediated deliberation*, a form of cross-community interaction in which professional differences problematize instruction and implicitly orient pedagogical reflection without becoming an object of explicit boundary reflection.

*4.3. Case 3: Boundary-reflective deliberation*

Triggering Situation

This case concerns a discussion of a task from an Israeli high-track matriculation exam:

$$Let\ f(x) = x + ln\ (x^2 - 3).\ Analyze\ f(x)\ and\ sketch\ its\ graph.$$

A physicist in the group immediately observed that the function "behaves asymptotically like a line" because the logarithmic term becomes negligible relative to $x$ as $x$ tends to $\pm\infty$. This claim surprised the SMTs, as it contradicted the standard solution taught in secondary schools, according to which the function has no asymptote.

Initial Framing of the Problem

In contrast to the previous cases, rather than deflecting or ignoring the discrepancy, participants made it an explicit object of deliberation, revealing two distinct definitions of line asymptote.

For the SMTs, the line asymptote was conceptualized in terms of *difference*:

$$|f(x) - x| \xrightarrow[x\to\pm\infty]{} 0$$

However, for the physicists, asymptotic behavior of functions was conceptualized in terms of ratio:

$$\left|\frac{f(x)}{x}\right| \xrightarrow[x\to\pm\infty]{} 1$$

The revelation of incompatible yet mathematically legitimate interpretations of asymptotic behavior explained why the UMTs and SMTs reached different conclusions regarding the function. The discussion then shifted to examining pedagogical implications of this difference. Initially, the use of distinct definitions in secondary and tertiary education risked being framed as a matter of right and wrong. For example, one UMT posed the provocative question:

> *Why are secondary students engaged with asymptotic behavior in a way that is evidently useless outside of secondary education?*

However, the SMTs resisted this framing of a problem, rejecting the implication that the school curriculum was "useless" or deficient. They further rejected the UMTs' suggestion of addressing both definitions in class. At this juncture, it is not difficult to imagine the UMTs and SMTs become entrenched in their respective conflicting positions, each trying to assert the correctness of their own perspective within their distinct professional jurisdictions.

Interactional Dynamics

Rather than escalating into jurisdictional disagreement, the discussion gradually shifted towards joint exploration of different mathematical and pedagogical purposes served by each definition.

This shift marked a moment of *defamiliarization*. Participants encountered unfamiliar interpretations of a concept they believed they understood, prompting them to reflect upon and articulate their own tacit assumptions about asymptotic behavior. The SMTs explained that the difference-based definition supports students' early encounters with function analysis and helps expand their repertoire of functions in manageable ways. UMTs and physicists clarified how ratio-based reasoning becomes more relevant in advanced mathematics and physics.

Through this exchange, participants moved from defending positions to reflecting on the professional contexts in which each definition operates (For further details see Pinto & Lai, 2025).

Productivity Outcomes

This interaction reflects high boundary productivity and low pedagogical productivity.

From a boundary perspective, the discussion surfaced tacit differences between professional discourses, articulating their underlying rationales, and legitimizing their coexistence. Thus, professional diversity was not erased but rather clarified and rendered intelligible, transforming how participants understand the secondary-tertiary divide.

At the same time, from a pedagogical perspective, reflection remained focused on disciplinary meanings and perspectives rather than extending into formulation and evaluation of concrete instructional strategies or actionable teaching moves.

Interpretation

We interpret this episode as illustrating the role of *defamiliarization* – encountering unfamiliar interpretations of a familiar concept – as a reflection mechanism in cross-community dialogue. Comparison of conflicting pedagogical perspectives prompts and orients sustained reflection on professional differences, preventing them from collapsing into a jurisdictional dispute.

We frame such interactional dynamics as *boundary-reflective deliberation*, a form of cross-community interaction in which differences between professional perspectives become explicit objects of reflection in isolation from specific instructional contexts and pedagogical implications.

*4.4. Case 4: Boundary-Integrative Deliberation*

Triggering Situation

This discussion emerged from a videotaped lesson in Naomi's advanced-track 11th-grade class, in which students presented six alternative proofs to Martin Gardner's Three-Squares Problem:

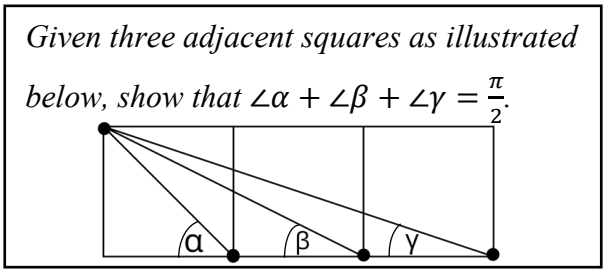



**Figure 3: The Three-Squares Problem**

Three of these proofs, which are the focus of this analysis, were in planar geometry[1]. Participants observed that Naomi did not make explicit why the class was engaging with multiple student proofs. This prompted a discussion of the pedagogical value of presenting alternative proofs and how a teacher might support students in learning from them.

Initial Framing of the Problem

Participants initially framed the pedagogical value of alternative proofs in different ways. Some SMTs emphasized affective and identity-related considerations, such as student voice, ownership, and participation. For example, Giselle noted that creating space for multiple student solutions allows students "to be heard, to be seen."

Others emphasized content-oriented considerations, questioning what mathematical learning might result from presenting multiple proofs. One SMT asked whether the lesson offered any "content" beyond showcasing student solutions.

As in the previous cases, the discussion initially risked fragmenting into parallel evaluative frameworks. The turning point came when Benjamin, a mathematician, introduced a new criterion:

> *I don't think we actually saw alternative proofs. I think we saw the same proof in different costumes.*

[1] The proofs are presented in detail here: https://stwis.org/5gylx8.

Benjamin further argued that there is no 'principal difference' between the three planar proofs ("one student looked at this similar triangle, while another looked at another"), and that presenting more than one proof was repetitive and of limited value. This reframed the discussion from the general value of student participation to the mathematical and pedagogical significance of the apparent "sameness" of the proofs.

Interactional Dynamics

Benjamin's intervention functioned simultaneously as a problematizing move and a boundary stimulus, initiating a collective reflective process that intertwined both pedagogical reasoning and boundary exploration.

Pedagogically, the discussion shifted from affective considerations toward analysis of mathematical structure and pedagogical implications. Participants examined in what sense the three proofs were indeed "the same". Sam, a mathematician, observed that all proofs relied on similarity with a common scale factor $\sqrt{2}$ and hypothesized that the problem cannot be proven by restricting to congruent triangles. Benjamin articulated a generalizable mathematical insight:

> *What could have been valuable is to observe that in geometry the way to show that angles add up to* 90° *is by embedding them in a right angle.*

At this point, *translation* across professional discourses emerged as a key reflection mechanism. Daniel, an SMT, translated these structural observations into an actionable teaching move that the group explored and developed:

> *The teacher can say, wait a minute, let's look at these three solutions. In all of them we found similar triangles; In all three, our anchor was finding a right angle.*

This translation involved more than acknowledging or rephrasing. It entailed transforming disciplinary insights into forms that were meaningful, actionable, and justifiable within a pedagogical framework.

At the same time, the observed disagreement in the group regarding whether the proofs are 'the same' prompted an explicit articulation of a professional difference between how UMTs and SMTs interpret and compare proofs – mathematically and pedagogically. For mathematicians, the three proofs represented isomorphic structures expressing the same "big idea," whereas for teachers they represented distinct processes of student reasoning that reflect different heuristics and learning trajectories. Importantly, this divergence was framed not as a matter of correctness but as reflecting

legitimate pedagogical commitments aligned with distinct professional orientations toward proofs – structure versus process – in the two communities (for further details, see Pinto, 2025).

Productivity Outcomes

This interaction reflects high pedagogical productivity and high boundary productivity.

Pedagogically, participants engaged in sustained reflection on instructional practice. They analyzed the lesson in detail, identified a meaningful mathematical pattern across the proofs, and developed concrete instructional strategies for making that pattern visible to students. The discussion shifted from general claims about the value of multiple solutions to specific pedagogical moves grounded in mathematical reasoning.

From a boundary perspective, participants articulated and examined a meaningful difference between professional interpretations of proofs – structure versus process – and reflected on how these perspectives relate to instructional goals and orientations. This difference was not only surfaced but integrated into the pedagogical discussion as a resource for reasoning about teaching.

Interpretation

We interpret this episode as illustrating how pedagogical and boundary productivity may unfold and integrate through *translation across professional discourses*, a reflection mechanism by which insights generated within one professional framework are transformed into forms that are meaningful and legitimate within another.

Benjamin's evaluative framework was not treated as a disagreement to be resolved or isolated, but as a shared, dual problem space: problematization of the assumption that engaging students with alternative proofs is inherently valuable and exposure of a difference in how mathematicians and teachers interpret and compare proofs. This duality was sustained within the discussion enabling participants to engage with both forms of reflection simultaneously.

We frame such interactional dynamics as *boundary-integrative deliberation*, a form of cross-community interaction in which reflection on instructional practice and reflection on professional differences become mutually reinforcing.

Unlike Case 2, differences between professional perspectives were not left implicit. Unlike Case 3, reflection on those differences did not remain isolated from instructional reasoning. Instead,

translation across professional discourses enabled participants to coordinate and integrate disciplinary and pedagogical perspectives within a shared line of inquiry.

## 5. Discussion

### *5.1. Cross-community deliberation as configurations of reflection processes*

A central finding of this study is that productivity in cross-community teacher deliberation unfolds along two analytically distinct dimensions: *pedagogical productivity* and *boundary productivity*.

Across cases, deliberation between secondary mathematics teachers (SMTs) and university mathematics teachers (UMTs) was shown to be variably productive – it could stall along both dimensions (Case 1), advance one without the other (Cases 2, 3), or integrate both (Case 4).

Taken together, the four forms of deliberation identified in this study – boundary-avoidance, boundary-mediated, boundary-reflective, and boundary-integrative – show that cross-community teacher deliberation can be meaningfully understood as distinct configurations of reflection, shaped by how participants engage with instructional situations and professional differences.

In this sense, the contribution of the study is not only to differentiate forms of productive teacher discourse, but to show that productivity manifests in distinct reflection processes that are structured and interactionally contingent. Rather than viewing cross-community teacher reflection as a uniform or inherently productive process, the findings demonstrate how it may take different forms, remain partial, or fail to develop altogether, depending on how differences are taken up.

### *5.2. A dual framework: reflection on practice and reflection on boundaries*

The dual-productivity framework developed in this paper contributes to existing research by distinguishing between two forms of reflection that have often been conflated. Pedagogical productivity corresponds to reflection on instructional practice, while boundary productivity corresponds to reflection on professional discourses and jurisdictions.

This distinction aligns with research on pedagogically productive teacher talk, which emphasizes reasoning about instructional decisions and the development of pedagogical judgment. It also aligns with research on boundary encounters, which conceptualizes learning as emerging through sustained engagement with differences across professional practices. The present study brings these strands together by showing that both forms of reflection may emerge within the same interaction yet remain analytically distinct.

Importantly, the findings demonstrate that these forms of reflection are variably related rather than mutually reinforcing by default. In some cases, reflection on practice develops without explicit engagement with professional differences (Case 2). In others, reflection focuses on professional differences without informing instructional reasoning (Case 3). Only in boundary-integrative deliberation (Case 4) do these forms of reflection become mutually reinforcing.

Thus, this study refines current insights regarding conditions that support pedagogically productive talk in homogeneous teacher communities (Lefstein et al., 2023). The discussions analyzed in this paper could all be characterized as respectful, reasoned, multi-voiced, focused on problems of practice, and anchored in a concrete instructional situation. Yet productivity of these deliberations varied, showing that it depends not simply on the presence or mutual acknowledgment of multiple perspectives, but on how these perspectives are taken up as objects of reflection. Differences may remain implicit, be explored in isolation, or be integrated into pedagogical reflection.

### *5.3. Reflection mechanisms in cross-community deliberation*

A second contribution of this study is the identification of reflection mechanisms through which different forms of cross-community teacher deliberation are enacted. These mechanisms do not determine particular productive outcomes. Rather, they describe how reflection is enacted in interaction and how different forms of deliberation are constituted.

As summarized in Table 2, four mechanisms were identified: *problematization*, *defamiliarization*, *boundary-avoidance reframing*, and *translation*. These mechanisms draw on existing constructs in teacher discourse in boundary crossing research (Akkerman & Bakker, 2011; Arcavi & Schoenfeld, 2008; Schwarts & Karsenty, 2020). However, to our knowledge, this study is the first to analyze how these mechanisms configure distinct forms of dual productivity in cross-community teacher deliberation. Our findings underline and elaborate the potential of these mechanisms to advance pedagogical reasoning and understanding of professional differences.

**Table 2: Reflection mechanisms in professional deliberations of UMTs and SMTs**

| Mechanism | What participants do | Type of reflection | Contribution to productivity | Case |
|---|---|---|---|---|
| Problematization | Treat instructional routines as objects of inquiry | Practice-oriented Reflection | Advances pedagogical reasoning without explicit boundary reflection | Case 2 |
| Defamiliarization | Engage with unfamiliar interpretations of familiar situations, concepts, or practices | Difference-oriented Reflection | Advances understanding of professional differences without instructional elaboration | Case 3 |
| Boundary-avoidance reframing | Shift evaluative criteria to avoid engaging differences | Displaced reflection | Limits both pedagogical and boundary productivity | Case 1 |
| Translation | Reformulate ideas across professional discourses | Cross-boundary reflection | Enables coordination of pedagogical reasoning and boundary reflection | Case 4 |

*5.4. Reconfiguring disciplinary obligation through dialogue*

The findings also contribute to understanding how obligation to the discipline is constituted in mathematics teaching. Herbst and Chazan (2020) conceptualize teaching as governed by multiple obligations, including to the discipline, which teachers balance through practical rationality.

Our findings align with this perspective but suggest a different account of how professional discourse on disciplinary obligations is developed. Herbst and Chazan (2020) describe a process in which mathematicians share with teachers their disciplinary perspectives and practices, and teachers ultimately decide what aspects of the discipline can be defensibly sustained in classroom work. In contrast, in this study, the consideration of secondary teaching in relation to disciplinary perspectives and practices emerges as a shared and negotiated professional jurisdiction. UMTs' criticisms are not immediately taken up and applied by the SMTs, and they become pedagogically powerful only when differences are acknowledged, reinterpreted and rearticulated across

professional perspectives. Cross-community deliberation thus becomes not only a site where professional obligations are balanced, but a site where their meaning is collectively produced.

*5.5. Implications for designing cross-community collaboration*

These findings have implications for the design of professional learning environments that bring together different communities.

First, they suggest that cross-community deliberation matters not only because it may improve collaboration, but also because professional differences create opportunities for forms of reflection that are difficult to achieve within homogeneous groups.

Second, designing for productive collaboration requires supporting both dimensions of reflection. Settings that focus only on pedagogical reasoning may leave professional differences implicit, while those focusing only on boundary reflection may fail to inform practice. Productive environments require conditions in which participants can engage both as part of a shared inquiry.

The M-Cubed environment illustrates how such conditions can be supported through shared representations of practice purposely designed and used as boundary objects, heterogeneous participation, and structured facilitation that sustains differences (Pinto, in press; Pinto & Cooper, 2022, 2023a; Pinto & Lai, 2025). At the same time, the findings show that productive outcomes are not guaranteed: they depend on how interactional dynamics support or constrain reflection.

*5.6. Limitations and future directions*

This study makes analytic rather than statistical claims. Based on four theoretically selected cases, it does not aim to establish the distribution of deliberation types across the broader M-Cubed corpus or other settings. Rather, its purpose is to make visible how distinct forms of productivity and reflection mechanisms can be identified and compared in cross-community deliberation.

The findings are also specific to a particular type of professional learning environment – a sustained video-club setting involving experienced SMTs and UMTs. Further research is needed to examine whether similar configurations of reflection emerge in other forms of collaboration, such as curriculum design teams, teacher education programs, or interdisciplinary research groups.

Finally, the study focuses on interactional processes rather than long-term outcomes. Future research should investigate how different reflection mechanisms develop over time and how they relate to changes in instructional practice, professional identity, and collaboration across

communities. A particularly important direction is to examine how professional learning environments can be designed to support not only the emergence of reflection, but its development toward integrative forms that connect pedagogical reasoning with boundary understanding.

## Declarations

### *Ethics Approval*

This study was conducted in accordance with institutional ethical guidelines and was approved by the relevant Institutional Review Board (IRB) at the Weizmann Institute of Science.

### *Consent for Participation and Publication*

Informed consent was obtained from all participants involved in the study for participation and for the use of data for research and publication purposes.

### *Conflict of interest*

The authors declare that they have no competing interests.

### *Availability of Data and Materials*

The data that support the findings of this study consist of video recordings of professional learning sessions. Due to ethical and institutional review board (IRB) restrictions concerning participant privacy and consent, these data are not publicly available. De-identified transcripts of the meetings can be made available from the corresponding author upon reasonable request, subject to approval in accordance with applicable ethical guidelines and data protection regulations.

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### *Authors' Contributions*

AP conceptualized and led the M-Cubed project, facilitated the meetings, conducted the data analysis, and wrote the manuscript. MG contributed to the design of the M-Cubed environment, co-facilitated selected meetings, contributed to data generation (including two of the four analyzed cases), and provided feedback on multiple drafts. Both authors read and approved the final manuscript.

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*Clinical Trial Registration*

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