Supplementary Materials

for

Simulation of Frustrated Ising Spins Using Three Coupled Parametric Oscillators: A Ground State Energy Approach

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Supplementary Note 1: Noise Generation and Rotating Wave Approximation (RWA)

The equation of motion of the resonator with additive drive force and parametric modulation is given by:

$$\ddot{q}_1 + 2\Gamma\dot{q}_1 + \left[\omega^2 - \frac{F_p}{M}\cos(\omega_p t)\right] q_1 + \gamma q_1^3 = \frac{F_d}{M}\cos\left(\frac{\omega_p t}{2} + \phi_d\right) + \xi_1(t) \tag{1}$$

where Eq. (1) represents the displacement of the top plate. M is the moment of inertia, Γ is the damping coefficient, F_d is the amplitude of the additive drive, and F_p is the amplitude of the parametric drive term. ϕ_d represents the phase of the additive drive force. The term $\xi_1(t)$ represents the noise torque arising from thermal fluctuations or injected into the system. Specifically, $\xi_1(t)$ is a zero-mean Gaussian noise torque satisfying $\langle \xi_q(t)\xi_i(t')\rangle = 4D_i\Gamma_i\delta(t-t')$, where D is the noise intensity.

To study the system behavior, we introduce slow variables u_1 defined by $q_i = u_1 e^{i\omega_d t} + u_1^* e^{-i\omega_d t}$, and apply the Rotating Wave Approximation (RWA). The condition for RWA validity is that the damping is much smaller than the resonance frequency, allowing us to neglect rapidly rotating terms. Under this approximation, Eq. (1) is rewritten as:

$$\dot{u}_1 + I_1 u_1 + iA_1 u_1 - i\frac{3\gamma_{11}}{2\omega_1 m_1}|u_1|^2 u_1 - i\frac{\omega_1 h}{4}u_1^* = f_d \exp(i\phi_d) + \frac{[f_x(\omega) + i f_y(\omega)]}{4i\omega_1} \tag{2}$$

where the scaled force is $f_d = F_d/M$, the scaled Duffing term is $\gamma_{11} = \gamma/M$, and the scaled parametric drive is $h = F_p/M$.

Supplementary Note 2: Extraction of Switching Rates

To extract the switching rates from the simulated time traces, we analyze the residence times that the system spends in each vibrational state before transitioning to another state. The residence times for successive switching events are plotted against the event number. A typical example is shown in Fig. 1.

The switching rate W is calculated directly from the recorded residence times and switching events. For each state, the rate is obtained by:

$$W = \frac{N_{\text{switch}}}{\sum_{i=1}^{N_{\text{switch}}} \tau_i}, \tag{3}$$

where N_{switch} is the total number of switching events out of the given state, and τ_i is the residence time for the i-th event. This is mathematically equivalent to the inverse of the mean residence time, $W = 1/\langle\tau\rangle$.

For the three-resonator system, we record the residence times for each of the eight stationary states (↑↑↑, ↑↑↓, ↑↓↑, ↑↓↓, ↓↑↑, ↓↑↓, ↓↓↑, ↓↓↓) independently. The switching rate out of a given state is obtained using Eq. (3). We have verified that the measured switching rate does not depend on the choice of the phase threshold used to identify state transitions.

For uncoupled oscillators in the absence of symmetry-breaking drive, the measured switching rates out of the two states of each resonator are identical within numerical uncertainty, providing the baseline switching rate $\overline{W}_i$ for resonator i.

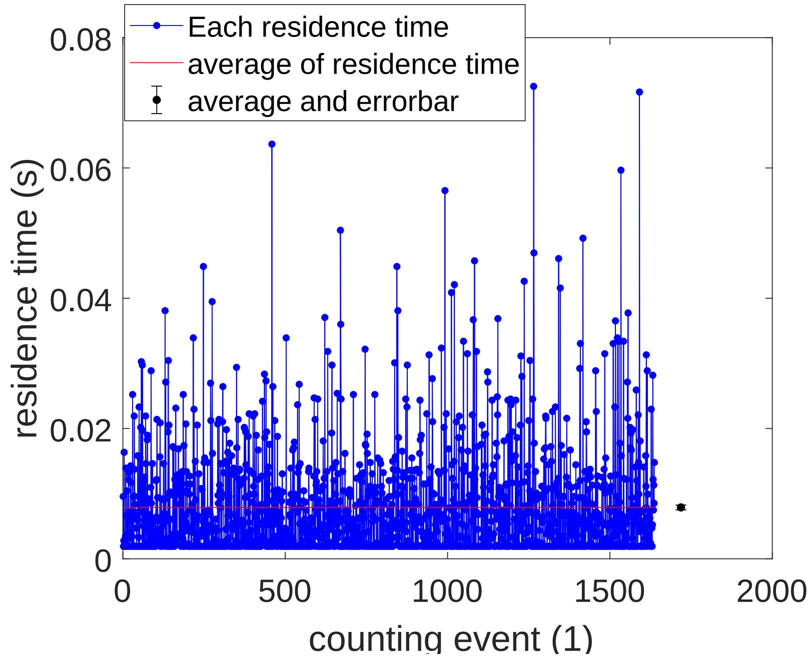

Figure 1. Residence times recorded for successive switching events. This data corresponds to transition 1 in Fig. 6(c) of the main text. Blue dots show the individual residence times τ_i (s) before each switch, plotted against the counting event number. The red horizontal line indicates the average residence time. The switching rate is calculated as $W_i = 126.90\ \mathrm{s}^{-1}$ (inverse of the mean residence time). The black circles represents the average and errorbar of the residence time for state I (↓↓↓) transition to state II (↓↓↑).

When coupling is introduced, the switching rates are modified depending on the states of the other resonators. The fractional change of the switching rates is used to extract the effective coupling parameters $J_{i,j}$ in the asymmetric Ising model mapping. For the frustrated three-spin system with antiferromagnetic coupling, we measure the switching rates for all possible transitions between the eight states to construct the complete transition rate matrix.