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ABSTRACT

This toolkit addresses the computational modeling of three-dimensional time-dependent electric-field distributions in type-II pulsed Bessel-Gaussian second-harmonic generation within potassium titanyl phosphate (KTP) crystals, where depleted-wave dynamics dominate nonlinear optical conversion. The toolkit solves three coupled nonlinear wave equations for ordinary and extraordinary fundamental waves and extraordinary second-harmonic waves using finite difference methods in cylindrical coordinates. It accepts user-defined parameters including pulse energy, beam spot size, crystal dimensions, and optical properties as inputs, and produces spatiotemporal electric-field amplitude distributions and conversion efficiency profiles as outputs. A unified Fortran codebase provides reproducible simulation pipelines, parametric sweep capabilities for examining interaction length dependencies, and computational optimizations that enable execution on standard desktop systems despite the fine radial meshes required for Bessel-Gaussian beam fluctuations. The implementation reproduces previously published quantitative predictions, specifically confirming that for $\omega_f \approx 80$ μm spot sizes and 0.8 J pulse energies, complete energy exchange occurs over approximately 5 mm, which validates the necessity of depleted-wave formalism for crystals exceeding this interaction length. The toolkit is available as an open-source GitHub repository and is released as version v1.0.1 under the MIT License, with a permanently archived distribution identified by DOI 10.5281/zenodo.17362587.

Keywords: Bessel-Gaussian beams, second-harmonic generation, type-II phase matching, depleted-wave model, KTP, pulsed SHG

1. BACKGROUND AND MOTIVATION

Second-harmonic generation (SHG) is a nonlinear optical process in which two photons at a fundamental frequency combine within a nonlinear crystal to produce a single photon at twice the frequency,[1] effectively converting infrared laser light into visible wavelengths. This frequency-doubling mechanism finds widespread application in laser systems requiring visible output, scientific instrumentation, display technologies, and biomedical imaging where direct generation at desired wavelengths proves impractical or inefficient.[2]

Potassium titanyl phosphate (KTP, chemical formula $KTiOPO_4$) has emerged as one of the most widely adopted crystals for frequency conversion, particularly for transforming 1064 nm Nd:YAG laser output into green

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532 nm light.[3] Its dominance stems from favorable material properties: high nonlinear conversion coefficient ($d_{\text{eff}} = 7.3$ pm/V), wide acceptance angle, high thermal conductivity, small walk-off angle, and relatively high damage threshold.[4,5] These characteristics enable efficient conversion with good beam quality and operational robustness across diverse power levels and repetition rates.[6,7]

Despite decades of theoretical development and experimental deployment, accurate predictive modeling of SHG in KTP remains challenging when real-world operating conditions introduce thermal effects, temporal pulse dynamics, and spatially varying beam profiles.[4,8] The interplay between these physical phenomena creates complex dependencies that simple analytical approximations fail to capture, necessitating comprehensive computational approaches for quantitative prediction of system performance.[9]

1.1 Computational Complexity in Frequency Conversion

The SHG modeling complexity arises from coupling optical wave propagation, thermal transport, and phase-matching conditions. High-power laser absorption creates temperature gradients that alter refractive indices through the thermo-optic effect, directly affecting the phase-matching condition:[8]

$$\Delta k = k_1 + k_2 - k_3 \tag{1}$$

where k_i represents the wave vectors. Temperature rises of a few Kelvin shift the crystal from optimal phase matching, reducing conversion efficiency and introducing position-dependent phase mismatch.[4,8]

Accurate prediction requires solving coupled partial differential equations: nonlinear wave equations for electric fields and heat diffusion for temperature.[9] Boundary conditions add complexity, as experimental setups employ diverse cooling strategies (water-cooled mounts, thermoelectric controllers, air convection) with different heat extraction rates.[7] Simplified assumptions like isothermal boundaries or one-dimensional heat flow fail to capture three-dimensional thermal gradients, causing prediction errors.

Pulsed laser operation introduces time-dependent complexity.[10,11] Pulsed beams (microsecond to femtosecond durations) exhibit peak powers far exceeding continuous-wave (CW) operation at equivalent average power.[10–12] High peak power causes rapid fundamental wave (FW) depletion as energy transfers to the second-harmonic wave (SHW), invalidating the undepleted pump approximation where fundamental field amplitude is assumed constant.[13,14] The interaction length—the characteristic distance for significant energy exchange—becomes much shorter for pulsed versus CW beams.[13] When crystal lengths exceed this interaction length, the undepleted assumption produces incorrect predictions of conversion efficiency, beam quality, and thermal load.[14]

Bessel-Gaussian beams add further challenges.[15,16] Unlike smooth Gaussian profiles, Bessel-Gaussian beams exhibit oscillatory transverse structures with multiple intensity rings.[15,17] These rapid spatial fluctuations require fine radial meshes, substantially increasing memory and execution time.[16,17] Azimuthal symmetry enables reduction to two spatial dimensions (radial and propagation) plus time, but careful numerical treatment remains essential for convergence without excessive computational cost.

1.2 Gaps in Current Methodologies

SHG computational modeling has historically relied on isolated research efforts using custom scripts for specific configurations.[1] These implementations rarely include comprehensive documentation, validated test cases, or version control, making independent verification difficult.[18] Reproducing published results requires re-implementing algorithms from mathematical descriptions, risking transcription errors, ambiguous discretization, and undocumented parameters.[18]

This lack of standardized, open implementations creates concrete problems.[19] Comparative studies across operating regimes, materials, or beam profiles require developing new codes rather than modifying validated tools.[20] Educators lack turnkey software for demonstrating thermal-optical phenomena, forcing oversimplified examples or complex commercial packages. Engineers designing SHG systems rely on vendor tools or expensive finite-element packages that may not accurately model type-II phase matching with depleted pump dynamics.[21] The absence of benchmark datasets prevents systematic validation of new methods or alternative models.[18]

1.3 Research Objectives and Scope

This toolkit provides a consolidated, reproducible solution for type-II pulsed Bessel-Gaussian SHG in KTP crystals under depleted-wave formalism. The unified Fortran implementation integrates three coupled nonlinear wave equations with finite difference methods. It accepts structured inputs (pulse energy, temporal width, beam spot size, crystal dimensions, refractive indices, absorption coefficients, nonlinear susceptibility) and solves governing equations in cylindrical coordinates, exploiting azimuthal symmetry to reduce computational cost while maintaining three-dimensional accuracy.

It targets three user communities. Academic researchers gain validated numerical solvers for exploring parameter dependencies, testing hypotheses, and generating publication figures without re-implementing algorithms. Educators obtain working examples demonstrating energy exchange, spatiotemporal field evolution, and simplified approximation breakdown. Engineers acquire predictive tools for estimating conversion efficiency, identifying thermal bottlenecks, and selecting appropriate crystal lengths.

The toolkit consolidates capabilities developed across a series of previously published articles by the authors, each addressing specific physical effects or operating regimes. Analytical heat equation solutions with continuity conditions[22] enable rapid thermal predictions for multi-layer crystal assemblies with interface boundary conditions without requiring full numerical PDE solvers for idealized geometries. Continuous-wave Gaussian beam thermal modeling[23,24] provides steady-state thermal solver validation against published CW results where absorption-driven heating reaches equilibrium, establishing benchmark cases for the more general time-dependent implementation. Pulsed-wave thermal dynamics[25,26] through time-dependent heat diffusion routines capture transient temperature evolution during pulse propagation, including thermal accumulation effects in high-repetition-rate operation where crystals do not fully cool between pulses.

Phase-mismatch modeling for pulsed operation[27,28] implements temperature-dependent refractive index formulas and resulting phase-mismatch calculations that reproduce experimentally observed efficiency rollover at high pump powers. Ideal continuous-wave Gaussian SHG[29,30] extends standard treatments to include depleted-wave modeling in double-pass cavity configurations, demonstrating conversion efficiency predictions under fundamental beam depletion conditions. Ideal pulsed-wave Bessel-Gaussian SHG[31,32] through the core three-equation coupled solver implements the depleted-wave formalism validated in Sabaeian et al.,[31,32] accurately predicting fundamental wave depletion and confirming the inapplicability of undepleted approximations for moderate pulse energies and beam spot sizes. While the present toolkit focuses on pulsed Bessel-Gaussian beam propagation, the methodology developed in Sabaeian et al.[33] for coupled continuous-wave thermal and optical fields demonstrates the full thermal-optical coupling framework where heat and wave equations are solved iteratively with mutual feedback, capturing self-consistent temperature distributions and phase-matching conditions in high-average-power CW systems.

1.4 Key Advances

This work provides a unified open-source Fortran codebase for depleted-wave type-II SHG with pulsed Bessel-Gaussian beams, featuring validated numerical methods and documented compilation procedures. Key contributions include: reproducible simulations through version control (GitHub v1.0.1), permanent archival (Zenodo DOI 10.5281/zenodo.17362587), and MIT licensing; parametric sweep capabilities for systematic exploration of pulse energy, beam spot size, crystal length, and material parameters mapping undepleted to depleted-wave transitions; benchmark datasets validating against published solutions and measurements; performance optimization through cylindrical coordinate reduction and efficient finite difference stencils enabling desktop execution without high-performance computing infrastructure; and comprehensive documentation lowering adoption barriers.

The following sections present the mathematical formulation, numerical implementation and architecture, validation against benchmarks, usage examples, and computational performance characteristics and limitations.

2. COMPUTATIONAL FRAMEWORK

The toolkit employs a modular architecture separating physical modeling from numerical implementation, enabling parameter configuration while maintaining efficiency. The structure reflects coupled nonlinear wave equations solved via finite difference methods in cylindrical coordinates, with distinct modules for field propagation, material properties, boundary conditions, and post-processing.

2.1 Solver Components

The **Core Solver** implements three coupled PDE solvers for ordinary and extraordinary fundamental waves and extraordinary second-harmonic wave in type-II phase matching. It uses second-order centered finite differences radially and first-order forward differences in propagation direction with explicit time-stepping. The module supports steady-state CW solutions via iterative convergence and transient pulsed operation with adaptive or fixed time steps, accounting for optical absorption and depleted wave dynamics by simultaneously updating all field components.

The **Geometry and Materials** module manages KTP properties and temperature dependencies, storing refractive indices via Sellmeier equations or tabulated dispersion. Temperature-dependent indices follow:

$$n(T) = n_0 + \frac{dn}{dT}(T - T_0) \tag{2}$$

where dn/dT represents thermo-optic coefficients. The module maintains absorption coefficients γ_1, γ_2, γ_3 for the three waves, effective susceptibility d_{eff} based on phase-matching angle, and thermal properties (conductivity, heat capacity, density) for heat diffusion coupling with anisotropic crystallographic variations.

The **Boundary and Cooling** module defines initial and boundary conditions for optical fields and thermal distributions. For electric fields, it applies input beam profiles (Gaussian, Bessel-Gaussian, or custom radially symmetric) at crystal entrance with user-defined spot sizes and temporal shapes, implementing zero-field or transparent radial boundaries for forward-propagating components. For thermal modeling, it provides cooling presets (water-cooled holders, thermoelectric controllers, air convection, adiabatic boundaries), translating these into Robin or Dirichlet boundary conditions with heat transfer coefficients, emissivity values, or fixed surface temperatures.

The **Studies and Reporting** module automates parametric investigations and visualization. It supports parameter sweeps (pulse energy, beam spot size, crystal length, temperature) with basic parallelization. Post-processing computes derived quantities (conversion efficiency, peak intensity evolution, radial profiles, interaction lengths) and generates contour and line plots. The module exports data to plain-text columnar formats with documented headers for reproducibility.

2.2 Processing Pipeline

The toolkit employs structured data flow from configuration files to analysis-ready outputs. Input files organize parameters hierarchically (crystal geometry, material properties, beam specifications, solver controls), with validation against physically reasonable ranges checking for errors like negative energies or insufficient mesh resolution.

Intermediate artifacts include discretized temperature fields, phase-mismatch distributions $\Delta k(T, z)$, and time-evolved field envelopes $\psi_1(r, z, t)$, $\psi_2(r, z, t)$, $\psi_3(r, z, t)$ at all grid points, remaining in memory with optional checkpoint files for restart or animation. Outputs include intensity distributions $|\psi_i(r, z)|^2$, one-dimensional profiles, and scalar statistics. Files follow consistent naming, e.g., `E_08_f_4000_Np_1_tp_50_Er.plt` indicating pulse energy 0.8 J, fundamental frequency 4000, single pulse, duration 50 μs, and radial field component, with headers labeling quantities and units.

2.3 Command-Line Operation

The toolkit is operated via command-line interface. To run a simulation, use the command:

```
./shg_pw_bg_ideal --config=simulation.conf --output=results/ --verbose
```

This command executes the program with the specified configuration file, sets the output directory, and enables verbose progress messages. Additional flags can be added for options such as checkpoint frequency. The command-line interface allows for batch operations and parameter sweeps using shell scripts.

2.4 Computational Efficiency

The toolkit achieves practical execution times on desktop systems via several optimizations. Cylindrical symmetry allows reduction to 2D (radial and longitudinal), reducing memory usage by up to two orders of magnitude and supporting denser meshes. Compiler optimization options further improve speed through vectorization and more efficient loop handling. For example, to compile the code with Intel Fortran, run:

```
ifort -O3 -xHost -ipo -no-prec-div src/*.f90 -o shg_pw_bg_ideal
```

Here, `-O3` enables advanced optimization, `-xHost` generates instructions for the local CPU (e.g., AVX/AVX-512), and `-ipo` activates inter-procedural optimization. Similar performance can be obtained with other Fortran compilers using high-optimization flags, as detailed in the toolkit documentation.

3. NUMERICAL VERIFICATION

The validation strategy establishes numerical accuracy and physical correctness by reproducing quantitative predictions from published peer-reviewed articles. Each benchmark compares toolkit results against reference data to verify faithful implementation of depleted-wave SHG physics with Bessel-Gaussian beams. The validation suite covers fundamental wave depletion dynamics, conversion efficiency versus crystal length, and interaction length scaling with pulse energy.

3.1 Axial Energy Transfer Dynamics

The primary validation benchmark reproduces Figure 1 from Sabaeian et al.,[31] demonstrating fundamental wave efficiency evolution along the crystal axis at multiple time instants during pulse transit. The configuration uses a KTP crystal (length 20 mm, radius 2 mm) irradiated by pulsed Bessel-Gaussian beams with spot size $\omega_f = 80$ μm, pulse energy 0.8 J, duration $t_p = 50$ μs, and transverse wave number $\alpha = 51532$ m^{-1} (cone angle 1°). Material properties: refractive indices $n_{o,\omega} = 1.8296$, $n_{e,\omega} = 1.7466$, $n_{e,2\omega} = 1.7881$; absorption coefficients $\gamma_1 = \gamma_2 = 0.5$ m^{-1} at 1064 nm and $\gamma_3 = 4$ m^{-1} at 532 nm; effective nonlinear coefficient $d_{\text{eff}} = 7.3$ pm/V for phase-matching geometry $\theta = 90°$, $\phi = 22.7°$.

The toolkit reproduces characteristic depletion behavior: fundamental wave depletion starts at zero at crystal entrance, builds to near 100% at pulse peak ($t = 2t_p = 100$ μs), then decays as the trailing edge enters. Along propagation direction, depletion increases monotonically, approaching complete energy transfer within approximately 5 mm, confirming the interaction distance. Quantitative comparison shows agreement within 2–5% across all time instants and positions, with larger deviations near entrance where finite difference discretization introduces numerical diffusion. The residual discrepancy traces to mesh resolution differences, as the original publication employed finer radial grids to better resolve Bessel function oscillations than default toolkit settings allow under typical computational time constraints.

3.2 Frequency-Doubled Output Characteristics

Validation of SHW generation reproduces Figures 2 and 3 from the reference, showing SHW efficiency growth along the crystal axis and comparing fundamental and harmonic efficiencies at temporal peak. The setup remains identical to the fundamental wave case. The toolkit captures rapid SHW efficiency rise from zero to near 100% at 5 mm, with subsequent gradual decrease from optical absorption during remaining propagation. Simultaneous plotting confirms energy conservation: the efficiency sum remains approximately constant when absorption is neglected, showing systematic decrease with absorption included.

Numerical agreement reaches 3–7% relative error for SHW efficiency, with largest deviations in the transition region where efficiency rapidly increases over sub-millimeter distances. This transition demands exceptionally fine propagation resolution to capture steep gradients. The default mesh (400 points along 20 mm) yields 50 μm steps introducing modest smoothing. Users requiring higher accuracy can increase mesh density at the cost of longer execution (mesh doubling improves agreement to 1–2% while quadrupling computation time due to Courant-condition time step reduction).

3.3 Transverse Intensity Distribution

The toolkit reproduces radial intensity distributions at crystal input and output faces (Figures 4, 5, 6 in reference). At entrance, the fundamental wave exhibits characteristic Bessel-Gaussian profile: central peak with oscillating rings of decreasing amplitude modulated by Gaussian envelope. Toolkit profiles match published curves within 1–3% for central lobe and primary side lobes, with larger deviations (5–10%) for outer rings where Gaussian envelope suppresses intensities below 10% of peak and numerical noise becomes prominent. At output after 20 mm propagation, fundamental wave efficiency drops to residual values below 0.5% across radial extent, confirming nearly complete depletion. The second-harmonic wave develops similar Bessel-Gaussian structure reaching peak efficiencies near 90% on axis. The reduction from theoretical 100% arises from optical absorption ($\gamma_3 = 4$ m^{-1} and path length yielding approximately 8% loss).

3.4 Energy-Dependent Conversion Distance

Critical validation of depleted-wave formalism examines interaction length variation with pulse energy, reproducing the parametric sweep in Figure 8 of the reference. The toolkit executes simulations across 0.1 J to 1.0 J while fixing other parameters. Results show higher pulse energies produce shorter interaction lengths where SHW efficiency saturates, ranging from approximately 16 mm (lowest energy) to 3 mm (highest energy). This inverse scaling follows:

$$L \propto \left(\frac{\omega_f^2}{P} \right)^{1/2} \tag{3}$$

where pulse power P in the denominator reduces characteristic distance for energy exchange. The toolkit reproduces published values within 5–10% across the energy range, with better agreement for higher energies where depletion occurs over shorter distances less sensitive to cumulative numerical errors.

The validation sweep (ten energy values) completes in approximately 150 minutes on a quad-core workstation using OpenMP parallelization. Each simulation requires 12–18 minutes, with higher energies converging faster due to quicker steady-state establishment allowing early termination when field changes fall below convergence thresholds.

3.5 Phase-Matching Consistency Tests

Rapid sanity check verifies phase-matching implementation by comparing conversion efficiency under perfect phase matching ($\Delta k = 0$) versus deliberate mismatch ($\Delta k = 1000$ m^{-1}). The test uses short crystal (5 mm), moderate pulse energy (0.4 J), and spot size 80 μm ensuring sub-minute execution. Under perfect matching, SHW efficiency approaches 65% at crystal exit; the mismatched case yields maximum efficiency below 15% with oscillatory behavior as energy periodically exchanges over coherence length:

$$L_c = \frac{\pi}{\Delta k} \approx 3.14 \text{ mm}. \tag{4}$$

The toolkit correctly reproduces expected efficiency reduction and oscillation period, confirming proper phase-mismatch exponential factors $\exp(\pm i \Delta k z)$ implementation. This test completes in approximately 45 seconds, providing immediate feedback during code modifications.

Additional consistency check confirms energy conservation: the sum of fundamental and second-harmonic powers integrated over beam profile remains constant along propagation when absorption is disabled. The toolkit computes total power at each plane:

$$P_{\text{total}}(z) = 2\pi \int_0^{r_{\max}} [I_1(r,z) + I_2(r,z) + I_3(r,z)] r \, dr \tag{5}$$

Using trapezoidal quadrature, verification confirms conservation within 0.1–0.5% numerical integration error for typical meshes. Enabling absorption causes systematic monotonic decrease consistent with specified absorption coefficients and crystal length, validating absorption term implementation.

4. IMPLEMENTATION GUIDELINES

4.1 Platform Prerequisites

The toolkit requires Intel Fortran Compiler 14.0+ or GNU Fortran (gfortran) 9.0+ for compilation. Supported systems: Linux (Ubuntu 18.04+, CentOS 7+), macOS 10.14+, and Windows 10 with WSL.

4.2 Reproducibility Guarantees

The finite difference solver operates deterministically given identical inputs and mesh configurations, producing bit-exact reproducible results on the same hardware and compiler. The toolkit does not employ stochastic algorithms, eliminating random number seeding needs. However, floating-point arithmetic order may vary across processor architectures or compiler versions due to instruction-level parallelism and vectorization, potentially introducing minor variations at machine precision level.

5. CONSTRAINTS AND FUTURE DIRECTIONS

5.1 Present Scope Boundaries

The current implementation is restricted to type-II phase matching in KTP with fixed cutting angles optimized for 1064 nm to 532 nm conversion, and all material properties are hardcoded for KTP. Alternative crystals and more general beam profiles are not yet supported; only radially symmetric Bessel-Gaussian and Gaussian beams are implemented, and accurate treatment of asymmetric or tightly focused high-NA beams would require fully two-dimensional, vectorial electromagnetic solvers beyond the present scalar, paraxial formulation.

Dynamic and multi-pass effects are also simplified. Thermal modeling is confined to steady-state temperature fields, so high-repetition-rate pulse trains with cumulative heating and phase-mismatch drift cannot be captured, and there is no coupling between successive passes to represent double-pass or cavity-enhanced configurations. In addition, the implementation is CPU-bound, using primarily serial Fortran with limited OpenMP parallelization and no GPU acceleration; this constrains mesh and time-step sizes to desktop RAM and leads to runtimes that could be reduced by one to two orders of magnitude on modern accelerators.

5.2 Common Pitfalls

Typical user mistakes can produce unstable or misleading results. The most common issue is an overly coarse spatial mesh relative to the beam spot size, which leads to oscillatory instabilities or nonphysical negative efficiencies. Bessel-beam simulations require radial spacing fine enough to resolve oscillations (roughly 10–20 grid points per period), and spot sizes below 50 μm on the default mesh (200 radial points over 2 mm) are severely under-resolved.

Another frequent problem is choosing time steps that violate the Courant-Friedrichs-Lewy (CFL) stability condition, which causes exponential error growth and floating-point overflow within a few hundred steps. The explicit scheme requires:

$$\Delta t \leq \frac{C \min(\Delta r^2, \Delta z^2)}{2D} \tag{6}$$

where D is the diffusion coefficient and $C < 1$ is a safety factor (typically 0.5). The toolkit does not automatically enforce this bound, so users must choose Δt consistent with their spatial resolution or adopt adaptive time stepping.

Finally, incorrect phase-matching angles or missing updates to the nonlinear tensor for non-default crystal orientations can make the effective nonlinear coefficient d_{eff} vanish. In such cases simulations run without numerical errors but predict zero or negligible SHG, even when input parameters appear reasonable.

5.3 Planned Enhancements

Planned developments address these limitations through incremental extensions of the physical models and numerical implementation. Near-term changes introduce additional nonlinear crystals (LBO, BBO, lithium niobate) via runtime-configurable material property files, enabling material substitution without recompilation and simplifying comparative studies or wavelength-specific optimization. Subsequent releases couple the time-dependent heat diffusion solver to pulsed propagation to capture repetition-rate–dependent thermal accumulation and phase-mismatch dynamics, and add multi-pass algorithms for double-pass and cavity configurations that track reflections, phase relations, and mirror properties to support resonant cavity design.

A major architectural milestone is GPU acceleration (version 2.0), initially targeting NVIDIA CUDA to offload finite-difference stencil operations while retaining CPU control of initialization, boundary conditions, and I/O, with projected speedups of order 20–50× for large, stencil-dominated problems. Planned user-interface improvements (version 2.1) provide Jupyter notebook templates with interactive widgets and preconfigured visualization layouts, lowering the barrier to parameter exploration and figure generation while also serving as instructional material on depleted-wave SHG. Longer-term goals focus on realistic non-ideal boundary conditions that account for surface roughness, coating imperfections, and Fresnel reflections via ray-tracing or beam-propagation extensions, for which community contributions through peer-reviewed GitHub pull requests are anticipated.

6. ACCESS AND DISTRIBUTION

The toolkit source code, documentation, benchmark data, and validation test cases are publicly available on GitHub under release tag v1.0.1. The software is distributed under the MIT License, permitting free use, modification, redistribution, and incorporation into proprietary or open-source projects without restriction beyond copyright notice preservation and warranty disclaimer. A permanent archived snapshot (v1.0.1) is deposited at Zenodo with DOI 10.5281/zenodo.17362587, ensuring long-term accessibility independent of repository hosting. Users may download the archived release from Zenodo or clone the GitHub repository and check out the tagged version for guaranteed correspondence with the implementation described here.

7. SUMMARY AND OUTLOOK

This work presented a unified open-source toolkit for modeling three-dimensional time-dependent electric-field distributions in type-II pulsed Bessel-Gaussian SHG within KTP crystals under depleted-wave formalism. The implementation successfully reproduced quantitative predictions from published validations, confirming that for pulse energies of 0.8 J and spot sizes of 80 μm, fundamental wave depletion completes over an interaction length of approximately 5 mm, establishing the inapplicability of undepleted pump approximations for moderate-energy pulsed operation in longer crystals.

The toolkit provides researchers, educators, and engineers with validated numerical solvers, parametric sweep capabilities, and benchmark datasets through version-controlled GitHub distribution and permanent Zenodo archival under permissive MIT licensing, eliminating algorithm reconstruction from scattered literature and enabling reproducible investigations of conversion efficiency dependencies on pulse energy, beam geometry, and crystal dimensions. The modular architecture establishes a foundation for extension to alternative nonlinear crystals, asymmetric beam profiles, and coupled thermal-optical dynamics, while computational optimization strategies demonstrate practical desktop execution without specialized high-performance computing infrastructure.

Future releases addressing transient thermal accumulation, double-pass cavity configurations, and GPU acceleration represent natural progressions from this validated baseline. The authors continue developing enhanced versions incorporating time-dependent phase-mismatch evolution and multi-crystal configurations, with forthcoming publications extending the depleted-wave approach to broader classes of nonlinear frequency conversion problems where coupled field dynamics dominate system performance.

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