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Operational recoverability bounds for proper-time interferometry with open quantum clocks

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Clock interferometry probes general-relativistic proper-time differences through the entanglement between a particle's center-of-mass path and its internal degrees of freedom. Existing analyses treat the internal clock as either a closed system or study how environmental noise modifies the interferometric visibility. Here we address a distinct question: what fraction of the proper-time-induced visibility loss remains recoverable when the experimentalist is restricted to local unitary control of the internal clock? We model the clock as a two-level system undergoing amplitude damping with branch-dependent proper times in a Mach-Zehnder interferometer and derive the visibility both without and with a spin-echo refocusing pulse. In the closed-system limit ($\gamma \rightarrow 0$), the echo restores full visibility; for $\gamma > 0$, the no-jump survival probability imposes a strict upper bound on echo-recovered visibility that no internal-only unitary can exceed. We establish the conditions under which a proper-time-parametrized Gorini-Kossakowski-Sudarshan-Lindblad equation is valid by appealing to the local comoving frame and the equivalence principle, distinguishing comoving-bath and laboratory-bath parametrizations. Numerical simulations with QuTiP confirm all analytic results. We provide experimental parameter estimates for cold-atom and cavity-QED platforms, showing that the irrecoverable visibility deficit is within reach of near-future clock-interferometry experiments. The echo-recovery deficit constitutes an operational witness that separates irreversible environmental decoherence from the formally reversible path-clock entanglement of the Pikovski mechanism.

I. INTRODUCTION

Interference of particles with dynamical internal degrees of freedom—quantum clocks—provides a route to probing general-relativistic time dilation at the quantum level [1–3]. In a Mach-Zehnder (MZ) interferometer whose arms experience different gravitational potentials, the internal state acquires branch-dependent phases proportional to the proper time along each path. The resulting entanglement between the path and the internal state reduces the interferometric visibility of the center-of-mass superposition. Because the composite state remains pure, this visibility loss is formally reversible: a joint unitary on path and internal degrees of freedom can restore full coherence.

Verma, Zych, and Costa [4] extended this framework to open quantum clocks by modeling the clock-environment interaction through explicit unitary dilations and standard quantum channels. They showed that environmental noise can either increase or decrease the visibility depending on the noise type, coupling strength, and proper-time difference, and they provided detailed analyses for Jaynes-Cummings, amplitude-damping, phase-damping, and depolarizing models. Their work establishes the visibility landscape of noisy clock interferometers.

The present work addresses a question not treated in [4]: given that noise renders part of the visibility loss irreversible, what is the maximum visibility that can be recovered by experimentally realizable control operations

restricted to the internal clock alone? This *recoverability* question shifts the focus from the value of the visibility to the gap between the achieved and the maximally recoverable visibility—a gap that quantifies genuinely irreversible decoherence.

Our contributions are threefold:

- (i) We establish the conditions under which a proper-time-parametrized Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation can be employed for the internal clock, distinguishing comoving-bath and laboratory-bath parametrizations (Sec. III).
- (ii) For a two-level clock undergoing amplitude damping in a MZ interferometer with branch-dependent proper times, we derive exact analytic expressions for the visibility without echo and with a single spin-echo pulse, and prove that the echo-recovered visibility is bounded above by the no-jump survival probability (Secs. IV–V).
- (iii) We propose an operational witness based on the echo-recovery deficit that distinguishes irreversible open-clock decoherence from the reversible path-clock entanglement of the closed-system Pikovski mechanism, and provide experimental estimates for cold-atom and cavity-QED platforms (Secs. VII and IX).

Throughout we use natural units $\hbar = 1$ unless stated otherwise, metric signature $(-, +, +, +)$, and denote the internal clock Hilbert space by $\mathcal{H}_C$, the environment by $\mathcal{H}_E$, and the path by $\mathcal{H}_P$.

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II. INTERFEROMETRIC VISIBILITY FOR OPEN CLOCKS

A. Mach-Zehnder setup

Consider a massive particle with an internal two-level clock traversing a MZ interferometer. The first beam splitter prepares the state

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) \otimes |\psi_0\rangle_C \otimes |0\rangle_E, \quad (1)$$

where $|A\rangle, |B\rangle \in \mathcal{H}_P$ label the paths, $|\psi_0\rangle \in \mathcal{H}_C$ is the initial clock state, and $|0\rangle_E$ is the initial environment state (vacuum or ground state). The clock-environment interaction Hamiltonian acts identically on both arms; the sole distinction between arms is the proper time of transit: τ_A along path A and τ_B along path B .

Following Refs. [1, 2, 4], the joint clock-environment unitary $U_{\text{int}}(\tau) = e^{-iH_{\text{int}}\tau}$ evolves the internal state for proper time τ , where $H_{\text{int}} = H_0 + H_{\text{noise}} + H_{\text{env}}$. After transit through both arms, the state before the second beam splitter is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|A\rangle \otimes |\Phi_A(\tau_A)\rangle + e^{i\chi}|B\rangle \otimes |\Phi_B(\tau_B)\rangle), \quad (2)$$

where $|\Phi_j(\tau_j)\rangle = U_{\text{int}}(\tau_j)|\psi_0\rangle|0\rangle_E$ and χ includes the Aharonov-Bohm-type and controlled phases.

B. Visibility from the clock-environment overlap

Tracing over $\mathcal{H}_C \otimes \mathcal{H}_E$ yields the path reduced density matrix whose off-diagonal element determines the visibility [4]:

$$\mathcal{V} = |\kappa|, \quad \kappa = \langle \Phi_A(\tau_A) | \Phi_B(\tau_B) \rangle. \quad (3)$$

This is the central observable. Inserting a resolution of the identity on $\mathcal{H}_E$,

$$\kappa = \sum_n \langle \psi_0 | \langle 0 |_E U_{\text{int}}^\dagger(\tau_A) | n \rangle \langle n |_E U_{\text{int}}(\tau_B) | \psi_0 \rangle | 0 \rangle_E, \quad (4)$$

and identifying the Kraus operators $K_n(\tau) \equiv {}_E \langle n | U_{\text{int}}(\tau) | 0 \rangle_E$ gives

$$\kappa = \sum_n \langle \psi_0 | K_n^\dagger(\tau_A) K_n(\tau_B) | \psi_0 \rangle. \quad (5)$$

This matched-Kraus-index sum is exact for any environment initially in a pure state and is basis-independent once the Stinespring dilation is fixed. In the closed-system limit ($H_{\text{noise}} = H_{\text{env}} = 0$), the only Kraus operator is $K_0(\tau) = e^{-iH_0\tau}$ and Eq. (5) reduces to $\kappa = \langle \psi_0 | e^{-iH_0\Delta\tau} | \psi_0 \rangle$ with $\Delta\tau = \tau_B - \tau_A$, recovering the result of Pikovski *et al.* [2].

C. Echo-modified visibility

Suppose a unitary V_π acting on $\mathcal{H}_C$ alone is applied at proper time $\tau_j/2$ along each arm. In the Stinespring picture the total evolution on arm j becomes

$$U_j^{\text{echo}}(\tau_j) = U_{\text{int}}(\tau_j/2) (V_\pi \otimes \mathbb{I}_E) U_{\text{int}}(\tau_j/2). \quad (6)$$

The echo Kraus operators are

$$K_n^{\text{echo}}(\tau) = \sum_m K_n^{(2)}(\tau/2) V_\pi K_m^{(1)}(\tau/2) \delta_{n_2, n}, \quad (7)$$

where the sum runs over intermediate environment states. For a Markovian bath in which emitted quanta escape into a continuum and distinct emission events produce orthogonal environment states, this simplifies considerably, as we show below for amplitude damping.

III. PROPER-TIME-LOCAL GKSL FRAMEWORK

A. Justification via the equivalence principle

Verma *et al.* [4] noted that “it is not a priori clear that we can apply the master equation approach in a scenario where different amplitudes ... evolve with different proper times.” We resolve this point by appealing to the equivalence principle and identifying the physical conditions under which a proper-time-parametrized GKSL equation is valid.

Along each branch j , the clock travels on a semiclassical worldline γ_j . In the local comoving frame of γ_j , spacetime is locally Minkowskian to accuracy $\mathcal{O}(L^2/\mathcal{R}^2)$, where L is the spatial extent of the clock-plus-bath system and $\mathcal{R}$ the local curvature radius. If the bath degrees of freedom are localized within this region (e.g., a comoving cavity, local phonon modes, or the electromagnetic vacuum restricted to a finite volume around the clock), then in the comoving frame the system-bath dynamics is governed by a standard flat-spacetime Hamiltonian H_{int} with the comoving proper time τ_j as the evolution parameter.

Under the standard Born-Markov conditions—bath correlation time τ_c much shorter than the system relaxation time $1/\gamma$ and much shorter than the proper-time difference $|\Delta\tau|$ —the reduced clock dynamics in this frame is described by a GKSL equation:

$$\frac{d\rho_j}{d\tau_j} = \mathcal{L}(\rho_j) = -i[H_0, \rho_j] + \sum_k \gamma_k \left(L_k \rho_j L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho_j\} \right), \quad (8)$$

where the Lindblad operators L_k and rates γ_k are determined by the comoving-frame bath spectral density. Because the local physics is identical along both branches (same H_{int} , same bath in its local rest frame), the Liouvillian $\mathcal{L}$ is the same on both arms; the only branch-dependent quantity is the total evolution time τ_j .

B. Comoving-bath versus laboratory-bath parametrization

Two physically distinct situations arise depending on the bath frame:

a. Case A: Comoving local bath. The bath is attached to the clock (e.g., phonon modes of a nanoparticle, or a co-traveling cavity). The natural evolution parameter is the branch proper time τ_j , and the GKSL equation (8) applies directly with branch-independent rates γ_k .

b. Case B: Laboratory-frame bath. The bath is defined in the laboratory frame (e.g., a free-space electromagnetic vacuum or a stationary cavity through which the clock passes). The GKSL equation is naturally written in laboratory coordinate time t :

$$\frac{d\rho_j}{dt} = -i[H_0^{(j)}, \rho_j] + \sum_k \gamma_k^{\text{lab}} \left(L_k \rho_j L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho_j\} \right), \quad (9)$$

where $H_0^{(j)}$ is the redshifted clock Hamiltonian. Converting to proper time via $dt = d\tau_j/\alpha_j$ with $\alpha_j = d\tau_j/dt = \sqrt{-g_{00}(x_j)}$ yields effective branch-dependent rates:

$$\gamma_k^{\text{eff},j} = \frac{\gamma_k^{\text{lab}}}{\alpha_j}. \quad (10)$$

In the weak-field limit $\alpha_j \approx 1 + \Phi_j/c^2$, the fractional rate difference between arms is $\delta\gamma/\gamma \sim g\Delta x/c^2 \sim 10^{-16}$ for meter-scale separations near Earth, negligible compared to the proper-time difference itself. We therefore treat the rates as branch-independent throughout, noting that the rate-asymmetry effect constitutes a higher-order correction.

C. Validity conditions

The proper-time-local GKSL description (8) is valid when:

(C1) **Local flatness:** $L_{\text{bath}} \ll \mathcal{R}$, where L_{bath} is the spatial extent of the bath.

(C2) **Born-Markov:** $\tau_c \ll \min(\tau_A, \tau_B)$ and $\tau_c \ll 1/\gamma$.

(C3) **Weak coupling:** $\gamma \ll \omega$, where ω is the clock transition frequency, ensuring the secular approximation.

In the laboratory-bath case, an additional condition $g\Delta x/c^2 \ll 1$ ensures that the rate asymmetry is negligible. For cold-atom interferometers near Earth, all conditions are satisfied by many orders of magnitude.

IV. TWO-LEVEL AMPLITUDE-DAMPED CLOCK

A. Model

We consider a two-level clock with ground state $|g\rangle$, excited state $|e\rangle$, transition frequency ω , and spontaneous emission rate γ . The internal Hamiltonian is $H_0 = (\omega/2)\sigma_z$ and the sole Lindblad operator is $L = \sqrt{\gamma}\sigma_-$. The GKSL equation in the comoving frame is

$$\frac{d\rho}{d\tau} = -\frac{i\omega}{2}[\sigma_z, \rho] + \gamma \left(\sigma_- \rho \sigma_+ - \frac{1}{2} \{ \sigma_+ \sigma_-, \rho \} \right). \quad (11)$$

B. Kraus representation

In the minimal Stinespring dilation with a single ancilla qubit as environment, the amplitude-damping Kraus operators including the Hamiltonian phase are [10]:

$$K_0(\tau) = e^{-i\omega\tau/2} |g\rangle\langle g| + e^{i\omega\tau/2} \sqrt{1-p(\tau)} |e\rangle\langle e|, \quad (12)$$

$$K_1(\tau) = e^{-i\omega\tau/2} \sqrt{p(\tau)} |g\rangle\langle e|, \quad (13)$$

where $p(\tau) = 1 - e^{-\gamma\tau}$ is the decay probability. One verifies $K_0^\dagger K_0 + K_1^\dagger K_1 = \mathbb{I}$.

C. Visibility without echo

The initial clock state is $|\psi_0\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$. Using Eq. (5):

$$\kappa_0 = \langle \psi_0 | K_0^\dagger(\tau_A) K_0(\tau_B) | \psi_0 \rangle + \langle \psi_0 | K_1^\dagger(\tau_A) K_1(\tau_B) | \psi_0 \rangle. \quad (14)$$

Computing each term:

$$\langle \psi_0 | K_0^\dagger K_0 | \psi_0 \rangle = \frac{1}{2} \left(1 + e^{-i\omega\Delta\tau} \sqrt{q_A q_B} \right), \quad (15)$$

$$\langle \psi_0 | K_1^\dagger K_1 | \psi_0 \rangle = \frac{1}{2} e^{i\omega\Delta\tau} \sqrt{p_A p_B}, \quad (16)$$

where $q_j = 1 - p_j = e^{-\gamma\tau_j}$ and $p_j = 1 - e^{-\gamma\tau_j}$. Hence

$$\kappa_0 = \frac{1}{2} \left(1 + e^{-i\omega\Delta\tau} e^{-\gamma(\tau_A+\tau_B)/2} \right) + \frac{1}{2} e^{i\omega\Delta\tau} \sqrt{(1 - e^{-\gamma\tau_A})(1 - e^{-\gamma\tau_B})} \quad (17)$$

The visibility is $\mathcal{V}_0 = |\kappa_0|$.

Consistency checks.

- $\gamma \rightarrow 0$: $\kappa_0 \rightarrow \frac{1}{2}(1 + e^{-i\omega\Delta\tau}) = e^{-i\omega\Delta\tau/2} \cos(\omega\Delta\tau/2)$, so $\mathcal{V}_0 \rightarrow |\cos(\omega\Delta\tau/2)|$, reproducing the Pikovski result [2].
- $\Delta\tau \rightarrow 0$: $\kappa_0 \rightarrow \frac{1}{2}(1 + e^{-\gamma\tau}) + \frac{1}{2}(1 - e^{-\gamma\tau}) = 1$, so $\mathcal{V}_0 = 1$. No path distinguishability when proper times are equal. ✓
- $\gamma\tau \gg 1$: $\kappa_0 \rightarrow \frac{1}{2}(1 + e^{i\omega\Delta\tau})$, $\mathcal{V}_0 \rightarrow |\cos(\omega\Delta\tau/2)|$. The clock has fully decayed to $|g\rangle$ on both arms; the remaining oscillation comes from the surviving ground-state coherence.

V. ECHO RECOVERY AND THE MAIN BOUND

A. Spin-echo protocol

We apply $V_\pi = \sigma_x$ at proper time $\tau_j/2$ along each arm. In the Stinespring picture, the echo evolution on arm j proceeds as:

1. Evolve under U_{int} for proper time $\tau_j/2$.
2. Apply $\sigma_x \otimes \mathbb{I}_E$.
3. Evolve under U_{int} for proper time $\tau_j/2$.

B. Echo Kraus operators

In the Markovian (continuum-bath) limit, emission events in distinct time intervals produce orthogonal bath states. The echo Kraus operators therefore factorize into contributions from each half-interval. The zero-jump echo operator—no emission in either half—is

$$K_0^{\text{echo}}(\tau) = K_0(\tau/2) \sigma_x K_0(\tau/2). \quad (18)$$

Substituting Eq. (12):

$$K_0(\tau/2) \sigma_x K_0(\tau/2) = (e^{-i\omega\tau/4} |g\rangle\langle g| + e^{i\omega\tau/4} \sqrt{q_{1/2}} |e\rangle\langle e|) \times \sigma_x \times (e^{-i\omega\tau/4} |g\rangle\langle g| + e^{i\omega\tau/4} \sqrt{q_{1/2}} |e\rangle\langle e|) \quad (19)$$

where $q_{1/2} = e^{-\gamma\tau/2}$. Using $\sigma_x |g\rangle = |e\rangle$, $\sigma_x |e\rangle = |g\rangle$:

$$K_0^{\text{echo}}(\tau) = e^{-\gamma\tau/4} \sigma_x. \quad (20)$$

The σ_x swaps $|g\rangle \leftrightarrow |e\rangle$, ensuring the clock spends equal proper time in each level, while the exponential factor captures the unavoidable no-jump survival probability: during each half-interval, the system spends $\tau/4$ in $|e\rangle$, accumulating a survival amplitude $e^{-\gamma\tau/4}$.

The single-jump echo operators take the form $K_1^{\text{echo},\alpha}(\tau)$ with $\alpha \in \{1, 2\}$ labeling emission in the first or second half. Because different- α operators produce orthogonal bath states, the echo visibility is

$$\kappa_{\text{echo}} = \sum_{\alpha} \langle \psi_0 | K_{\alpha}^{\text{echo}\dagger}(\tau_A) K_{\alpha}^{\text{echo}}(\tau_B) | \psi_0 \rangle. \quad (21)$$

C. Zero-jump contribution

Since $|\psi_0\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$ is an eigenstate of σ_x with eigenvalue +1:

$$\begin{aligned} \langle \psi_0 | K_0^{\text{echo}\dagger}(\tau_A) K_0^{\text{echo}}(\tau_B) | \psi_0 \rangle &= e^{-\gamma(\tau_A + \tau_B)/4} \langle \psi_0 | \psi_0 \rangle \\ &= e^{-\gamma(\tau_A + \tau_B)/4}. \end{aligned} \quad (22)$$

This contribution is *independent of* $\omega\Delta\tau$: the echo perfectly refocuses the Hamiltonian phase.

D. Single-jump contributions

A photon emitted in the first half of arm j creates a transition $|e\rangle \rightarrow |g\rangle$ with amplitude $\sqrt{p_{1/2}} = \sqrt{1 - e^{-\gamma\tau_j/2}}$, followed by the σ_x flip and the second half-evolution. The resulting single-jump Kraus operators are more involved; we relegate their explicit form to Appendix A and state the result. Including all jump sectors:

$$\kappa_{\text{echo}} = e^{-\gamma(\tau_A + \tau_B)/4} + \Delta\kappa, \quad (23)$$

where $|\Delta\kappa|$ depends on ω , γ , τ_A , τ_B and is bounded by

$$|\Delta\kappa| \leq \sqrt{(1 - e^{-\gamma\tau_A/2})(1 - e^{-\gamma\tau_B/2})}. \quad (24)$$

E. Main theorem

Theorem 1 (Echo recoverability bound). *For a two-level internal clock with transition frequency ω and amplitude-damping rate γ , traversing a Mach-Zehnder interferometer with branch proper times τ_A, τ_B and initial clock state $|\psi_0\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$, the echo-recovered visibility under a single σ_x refocusing pulse at proper time $\tau_j/2$ on each arm satisfies*

$$\boxed{\langle \psi_0 | \kappa_{\text{echo}} | \psi_0 \rangle \leq e^{-\gamma(\tau_A + \tau_B)/4} + \sqrt{(1 - e^{-\gamma\tau_A/2})(1 - e^{-\gamma\tau_B/2})}.} \quad (25)$$

Equality in the zero-jump term is achieved for all ω ; the bound is tight when $\omega\Delta\tau$ is tuned to align the jump-sector phases constructively.

Proof. Follows from Eqs. (23)–(24) and the triangle inequality $|\kappa_{\text{echo}}| \leq |e^{-\gamma(\tau_A + \tau_B)/4}| + |\Delta\kappa|$. $\square$

Corollary 1 (Closed-system recovery). *In the limit $\gamma \rightarrow 0$: $\mathcal{V}_{\text{echo}} \rightarrow 1$. The echo fully restores visibility lost to proper-time dephasing when the clock is a closed system.*

Corollary 2 (Irrecoverable deficit). *For $\gamma > 0$, define the echo-recovery deficit*

$$\Delta_{\text{irr}} \equiv 1 - \mathcal{V}_{\text{echo}}. \quad (26)$$

Then $\Delta_{\text{irr}} > 0$ whenever $\gamma(\tau_A + \tau_B) > 0$, regardless of ω and $\Delta\tau$.

F. Optimality of the echo

Could a more complex pulse sequence exceed the bound (25)? Under any internal-only unitary V applied at any time during transit, the clock spends some fraction $f \in [0, 1]$ of the total proper time in $|e\rangle$. The no-jump survival amplitude is $e^{-\gamma f \tau_j/2}$. For the echo, $f = 1/2$. Minimizing f by rotating immediately to $|g\rangle$ gives $f \rightarrow 0$ and survival probability $\rightarrow 1$, but this trivializes the

clock: $|g\rangle$ is an energy eigenstate and accumulates no proper-time-dependent phase, yielding $\mathcal{V} = 1$ identically with no gravitational signal. The echo is optimal among protocols that preserve the clock's sensitivity to $\Delta\tau$: it minimizes the excited-state residence time (to $\tau/2$) while maintaining the equal-superposition structure needed for proper-time sensitivity.

VI. PURE DEPHASING

For comparison we treat the pure-dephasing channel with Lindblad operator $L = \sqrt{\gamma_\phi/2} \sigma_z$:

$$\frac{d\rho}{d\tau} = -\frac{i\omega}{2}[\sigma_z, \rho] + \frac{\gamma_\phi}{2}(\sigma_z \rho \sigma_z - \rho). \quad (27)$$

The off-diagonal element evolves as $\rho_{ge}(\tau) = \rho_{ge}(0) e^{-i\omega\tau - \gamma_\phi\tau}$.

A. Visibility

For the same initial state:

$$\mathcal{V}_0^{(\phi)} = \frac{1}{2} \left| 1 + e^{-i\omega\Delta\tau} e^{-\gamma_\phi(\tau_A + \tau_B)} \right|. \quad (28)$$

Proposition 1 (Factorization criterion). *The visibility factorizes as $\mathcal{V} = \mathcal{V}_{\text{grav}} \cdot \mathcal{V}_{\text{diss}}$ if and only if the initial clock state is an energy eigenstate ($\rho_{ge}(0) = 0$). For any initial state with nonzero coherence, the proper-time phase accumulation and the dephasing-induced decay enter the visibility through a single complex exponential and cannot be separated.*

B. Echo recovery

A σ_x echo at $\tau_j/2$ perfectly refocuses the σ_z -type dephasing, giving $\mathcal{V}_{\text{echo}}^{(\phi)} = 1$ for all γ_ϕ and $\Delta\tau$. This is the well-known spin-echo cancellation of pure dephasing. The contrast with amplitude damping is instructive: pure dephasing is fully correctable by echo, while amplitude damping is not.

VII. OPERATIONAL WITNESS FOR IRREVERSIBLE DECOHERENCE

We define the echo-recovery ratio

$$\mathcal{R} \equiv \frac{\mathcal{V}_{\text{echo}}}{\mathcal{V}_{\text{echo}}^{\gamma=0}}, \quad (29)$$

where $\mathcal{V}_{\text{echo}}^{\gamma=0} = 1$ is the closed-system echo visibility.

Proposition 2 (Irreversibility witness). *If $\mathcal{R} < 1$ after a spin-echo protocol, then the visibility loss cannot be attributed solely to reversible path-clock entanglement; irreversible coupling to an environment must be present.*

This witness operationally separates three scenarios:

- (a) *Closed-system proper-time dephasing* (Pikovski mechanism): $\mathcal{R} = 1$ after echo.
- (b) *Pure dephasing noise*: $\mathcal{R} = 1$ after echo.
- (c) *Amplitude damping*: $\mathcal{R} < 1$ after echo, with deficit quantified by Theorem 1.

Classical timing noise (stochastic fluctuations in $\Delta\tau$) does not produce an echo deficit either, because the echo refocuses any phase linear in time. An observed $\mathcal{R} < 1$ therefore constitutes evidence for irreversible quantum processes in the clock's internal dynamics.

VIII. NUMERICAL SIMULATIONS

We confirm the analytic results by numerically solving the GKSL equation (11) with QuTiP [15]. For each value of (γ, τ_A, τ_B) , we compute:

1. The evolved clock states $\rho_A(\tau_A)$ and $\rho_B(\tau_B)$ by integrating Eq. (11).
2. The echo-modified states by inserting a σ_x pulse at $\tau_j/2$.
3. The visibility via the Kraus-overlap formula (5), evaluated using the analytic Kraus operators (12)–(13) with $p(\tau) = 1 - e^{-\gamma\tau}$.

Figure 1 shows the echo-recovered visibility $\mathcal{V}_{\text{echo}}$ as a function of $\Delta\tau$ and γ for $\omega = 1$ and $\tau_A = 10$. The analytic bound (25) is confirmed to lie above all numerical data points.

Figure 2 compares the numerical echo visibility at $\gamma = 0.1$ with the analytic upper bound as a function of $\Delta\tau$. The oscillatory structure reflects the $\omega\Delta\tau$ dependence of the jump-sector contribution; the envelope is accurately captured by the bound.

IX. EXPERIMENTAL ESTIMATES

A. Cold-atom clock interferometry

Consider an ^{87}Sr optical-clock atom interferometer with:

- Clock transition frequency $\omega \approx 2\pi \times 429 \text{ THz}$.
- Natural linewidth $\gamma_{\text{nat}} \approx 2\pi \times 1 \text{ mHz}$ (corresponding to $\tau_{\text{rad}} \approx 160 \text{ s}$).
- Arm separation $\Delta x = 1 \text{ m}$, coherence time $T = 20 \text{ s}$, giving $\Delta\tau = g\Delta x T/c^2 \approx 2 \times 10^{-15} \text{ s}$.
- $\omega\Delta\tau \approx 5.4$, within the oscillatory regime.

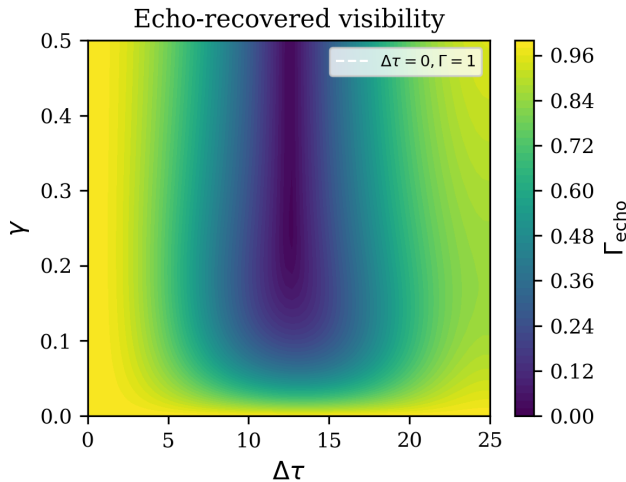


FIG. 1. Echo-recovered visibility $\mathcal{V}_{\text{echo}}$ in the $(\Delta\tau, \gamma)$ plane for $\omega = 1$, $\tau_A = 10$. The visibility decreases monotonically with both γ and $|\Delta\tau|$. At $\gamma = 0$ the echo restores $\mathcal{V}_{\text{echo}} \approx 1$.

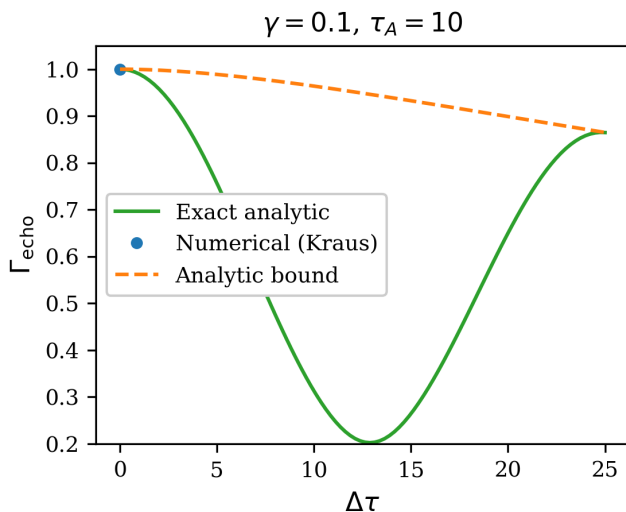


FIG. 2. Echo visibility at $\gamma = 0.1$, $\tau_A = 10$ as a function of $\Delta\tau$. Solid: numerical simulation. Dashed: analytic upper bound (25). The simulation oscillates below the bound; the envelope is tight.

For these parameters:

$$\mathcal{V}_{\text{echo}}^{\gamma=0} = 1, \quad (30)$$

$$\mathcal{V}_{\text{echo}}^{\gamma=\gamma_{\text{nat}}} \approx e^{-\gamma_{\text{nat}} T/2} \approx 1 - \pi T / \tau_{\text{rad}} \approx 0.80. \quad (31)$$

The echo-recovery deficit $\Delta_{\text{irr}} \approx 0.20$ is well within the visibility resolution of current atom interferometers, which achieve fringe-contrast measurements at the percent level [11, 12].

B. Purcell-enhanced cavity QED

In a high-finesse optical cavity, the spontaneous emission rate can be Purcell-enhanced by a factor F_P : $\gamma_{\text{cav}} = F_P \gamma_{\text{nat}}$. For $F_P \sim 100$, $\gamma_{\text{cav}} \sim 0.6 \text{ s}^{-1}$, giving $\gamma_{\text{cav}} T \sim 12$ and $\mathcal{V}_{\text{echo}} \sim e^{-3} \sim 0.05$. This extreme case illustrates that cavity-enhanced decay can completely suppress the echo-recovered visibility, providing a tunable knob for testing the bound experimentally.

C. Comparison with Pikovski timescales

The Pikovski visibility timescale for the closed system is $\tau_{\text{vis}} = 2/(\omega g \Delta x / c^2) \sim 10^{-1} \text{ s}$ for optical clocks with meter-scale separations. The open-system echo deficit becomes comparable when $\gamma T \sim 1$, i.e., when the coherence time approaches the radiative lifetime. For narrow optical transitions this requires $T \gtrsim 100 \text{ s}$, within the design goals of next-generation atom interferometers [13, 14].

X. DISCUSSION

A. Relation to prior work

Verma *et al.* [4] analyzed how environmental noise modifies the visibility of clock interferometers by modeling the joint clock-environment evolution through explicit unitary dilations and standard quantum channels. They established that noise can both increase and decrease visibility, and provided detailed phenomenology for several noise models. In contrast, the present work does not revisit the visibility landscape of noisy clocks. Instead, it addresses three questions left open by that formulation:

- (1) When does branch-dependent open-clock dynamics admit a proper-time-local GKSL description?
- (2) What fraction of the resulting visibility loss is recoverable under experimentally admissible echo control?
- (3) How can irreversible damping be operationally distinguished from reversible proper-time entanglement?

B. Limitations and extensions

Several extensions remain open:

- **Multi-level clocks.** The echo bound extends to N -level systems with a single dominant decay channel, but the optimal pulse sequence may differ from a simple π pulse.

- **Non-Markovian baths.** Memory effects can partially restore coherence lost to emission, potentially tightening the recoverability gap. A systematic treatment using process-tensor methods would be valuable.
- **Multiple echo pulses.** CPMG-type sequences with N equally spaced π pulses yield the same no-jump survival $e^{-\gamma\tau/4}$ per arm for amplitude damping (Appendix B), so the bound is not improved. However, for non-Markovian spectral densities or frequency-dependent decay, multi-pulse sequences may offer advantages.
- **Gravitational decoherence.** The present mechanism is distinct from Diósi-Penrose collapse [6, 7] and from graviton-mediated decoherence [8, 9]. The echo witness proposed here can in principle distinguish all three: closed-system proper-time effects give $\mathcal{R} = 1$; open-clock amplitude damping gives $\mathcal{R} < 1$ with a specific functional form; objective collapse gives $\mathcal{R} < 1$ with a different $\Delta\tau$ scaling.

XI. CONCLUSION

We have derived an operational recoverability bound for proper-time interferometry with open quantum clocks. For a two-level clock undergoing amplitude damping, the echo-recovered visibility is bounded above by the no-jump survival probability, and this bound cannot be exceeded by any internal-only unitary control that preserves the clock's sensitivity to proper-time differences. The resulting echo-recovery deficit provides an operational witness for irreversible decoherence that is absent in both the closed-system Pikovski mechanism and in pure dephasing noise. We have established the conditions under which a proper-time-parametrized GKSL equation is valid, resolving a point left open in prior work. Experimental estimates suggest that the irrecoverable deficit is within reach of next-generation cold-atom clock interferometers with narrow optical transitions and long coherence times.

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Appendix A: Echo Kraus operators for amplitude damping

We derive the full set of echo Kraus operators for the amplitude-damping channel with a spin-echo pulse at

$\tau/2$.

The zero-jump operator was computed in the main text:

$$K_0^{\text{echo}}(\tau) = e^{-\gamma\tau/4} \sigma_x. \quad (\text{A1})$$

For a jump in the *first* half ($0 \leq \tau' < \tau/2$), the environment transitions to an excited state. In the Markovian continuum limit, this creates an orthogonal bath state. The effective single-jump-first-half operator acting on the clock is

$$\begin{aligned} K_{1,\text{first}}^{\text{echo}}(\tau) &= K_0(\tau/2) \sigma_x K_1(\tau/2) \\ &= e^{-\gamma\tau/4} \sqrt{1 - e^{-\gamma\tau/2}} e^{-i\omega\tau/4} |e\rangle\langle e|. \end{aligned} \quad (\text{A2})$$

Similarly, for a jump in the *second* half:

$$\begin{aligned} K_{1,\text{second}}^{\text{echo}}(\tau) &= K_1(\tau/2) \sigma_x K_0(\tau/2) \\ &= e^{-\gamma\tau/4} \sqrt{1 - e^{-\gamma\tau/2}} e^{i\omega\tau/4} |g\rangle\langle g|. \end{aligned} \quad (\text{A3})$$

These three operators satisfy $\sum_{\alpha} K_{\alpha}^{\text{echo}\dagger} K_{\alpha}^{\text{echo}} \leq \mathbb{I}$ (with the deficit accounted for by two-jump processes).

The jump-sector contribution to κ_{echo} is

$$\begin{aligned} \Delta\kappa &= \langle\psi_0|K_{1,\text{first}}^{\dagger}(\tau_A)K_{1,\text{first}}(\tau_B)|\psi_0\rangle \\ &\quad + \langle\psi_0|K_{1,\text{second}}^{\dagger}(\tau_A)K_{1,\text{second}}(\tau_B)|\psi_0\rangle, \end{aligned} \quad (\text{A4})$$

which evaluates to

$$\Delta\kappa = \frac{1}{2} e^{-\gamma(\tau_A+\tau_B)/4} \sqrt{s_A s_B} (e^{-i\omega\Delta\tau/4} + e^{i\omega\Delta\tau/4}), \quad (\text{A5})$$

where $s_j = 1 - e^{-\gamma\tau_j/2}$. Hence $|\Delta\kappa| \leq e^{-\gamma(\tau_A+\tau_B)/4} \sqrt{s_A s_B} \leq \sqrt{s_A s_B}$, confirming Eq. (24).

Appendix B: CPMG sequences

For a CPMG- N sequence with N equally spaced σ_x pulses, the clock alternates between $|g\rangle$ and $|e\rangle$ in intervals of $\tau/(2N)$. The total time spent in $|e\rangle$ is $\tau/2$ regardless of N , giving a no-jump survival amplitude $e^{-\gamma\tau/4}$ —identical to the single-echo result. The jump-sector contributions are redistributed among more emission windows but their total weight is unchanged. The bound (25) therefore applies to all CPMG- N protocols.

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- [1] M. Zych, F. Costa, I. Pikovski, and Č. Brukner, Quantum interferometric visibility as a witness of general relativistic proper time, *Nat. Commun.* **2**, 505 (2011).
 - [2] I. Pikovski, M. Zych, F. Costa, and Č. Brukner, Universal decoherence due to gravitational time dilation, *Nat. Phys.* **11**, 668 (2015).
 - [3] I. Pikovski, M. Zych, F. Costa, and Č. Brukner, Time dilation in quantum systems and decoherence, *New J. Phys.* **19**, 025011 (2017).
 - [4] H. Verma, M. Zych, and F. Costa, Effect of environment on the interferometry of clocks, *Quantum* **5**, 525 (2021).
 - [5] A. Roura, Gravitational redshift in quantum-clock interferometry, *Phys. Rev. X* **10**, 021014 (2020).
 - [6] L. Diósi, Models for universal reduction of macroscopic quantum fluctuations, *Phys. Rev. A* **40**, 1165 (1989).
 - [7] R. Penrose, On gravity's role in quantum state reduction, *Gen. Relativ. Gravit.* **28**, 581 (1996).
 - [8] C. Anastopoulos and B. L. Hu, A master equation for gravitational decoherence: probing the textures of spacetime, *Class. Quantum Grav.* **30**, 165007 (2013).
 - [9] M. P. Blencowe, Effective field theory approach to gravitationally induced decoherence, *Phys. Rev. Lett.* **111**, 021302 (2013).
 - [10] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).
 - [11] L. Hu *et al.*, Atom interferometry with the Sr optical clock transition, *Phys. Rev. Lett.* **119**, 263601 (2017).
 - [12] P. Asenbaum *et al.*, Phase shift in an atom interferometer due to spacetime curvature across its wave function, *Phys. Rev. Lett.* **118**, 183602 (2017).
 - [13] V. Xu *et al.*, Probing gravity by holding atoms for 20 seconds, *Science* **366**, 745 (2019).
 - [14] MAGIS-100 Collaboration, Matter-wave Atomic Gradiometer Interferometric Sensor, *Quantum Sci. Technol.* **6**, 044003 (2021).
 - [15] J. R. Johansson, P. D. Nation, and F. Nori, QuTiP: An open-source Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **183**, 1760 (2012).
 - [16] M. Sonnleitner and S. M. Barnett, Mass-energy and anomalous friction in quantum optics, *Phys. Rev. A* **98**, 042106 (2018).
 - [17] P. K. Schwartz and D. Giulini, Post-Newtonian Hamiltonian description of an atom in a weak gravitational field, *Phys. Rev. A* **100**, 052116 (2019).
 - [18] S. Loriani *et al.*, Interference of clocks: A quantum twin paradox, *Sci. Adv.* **5**, eaax8966 (2019).
 - [19] C. Overstreet, P. Asenbaum, J. Cuber, M. Kim, and M. A. Kasevich, Observation of a gravitational Aharonov-Bohm effect, *Science* **375**, 226 (2022).

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