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Derivations and Structural Consequences of the R-Universe Framework

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Research Article

Keywords: cosmology, dark energy, cosmological constant, renormalization group, quantum gravity, vacuum energy, cosmic acceleration

Posted Date: March 13th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-9097693/v1

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Additional Declarations: The authors declare no competing interests.

Derivations and Structural Consequences

of the R-Universe Framework

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March 11, 2026

Abstract

Despite its remarkable phenomenological success, the ΛCDM model does not provide a microscopic explanation for the observed value of the cosmological constant. In particular, the origin of the present dark-energy fraction $\Omega_\Lambda \simeq 0.69$ remains unexplained within standard cosmology.

Here we propose a framework in which the late-time cosmological evolution can be interpreted as an effective renormalization–group (RG) flow of the vacuum sector driven by the Hubble scale. Within this description the dimensionless dark-energy fraction obeys a closed evolution equation:

$$\beta(\Omega_\Lambda) \equiv \frac{d\Omega_\Lambda}{d\ln a} = \frac{3}{2}\theta\,\Omega_\Lambda(1-\Omega_\Lambda),$$

which takes the form of a logistic RG flow connecting the fixed points $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1$, corresponding to matter-dominated and vacuum-dominated cosmological regimes.

Consistency between the RG scaling of the cosmological coupling and the Friedmann background dynamics implies a non-trivial stationary solution for the dark-energy fraction:

$$\Omega_\Lambda = \frac{\theta}{\theta+1},$$

which relates the present cosmological composition directly to the relevant critical exponent θ of the underlying gravitational fixed point. Using the observed value $\Omega_\Lambda \simeq 0.69$ yields $\theta \simeq 2.2$, numerically close to critical exponents obtained in functional renormalization–group studies of asymptotically safe gravity.

The resulting dynamics possesses a global phase-space invariant:

$$H\,\Omega_\Lambda^{1/\theta} = \text{const},$$

which constrains the cosmological trajectory in the reduced phase space (H, Ω_Λ) and provides a dynamical description of the transition from matter domination to late-time accelerated expansion.

These results suggest that the observed cosmological composition may encode information about the renormalization-group structure of quantum gravity, with late-time cosmological observables acting as possible probes of gravitational critical exponents.

1 Introduction

Despite the empirical success of the ΛCDM model, the physical origin of the cosmological constant remains unresolved [1, 2]. Observations of Type Ia supernovae, cosmic microwave background anisotropies, and baryon acoustic oscillations show that the present Universe is undergoing accelerated expansion and that approximately 69% of its current energy budget is stored in a dark-energy component consistent, at leading order, with a cosmological constant [3, 4, 5]. Within ΛCDM this component is parametrized by Λ, but the observed value is extraordinarily small compared with natural quantum-field-theoretic expectations and is radiatively unstable under additive vacuum-energy corrections [1].

This tension suggests that the observed cosmological constant may not be a fundamental microscopic parameter. A different possibility is that the late-time vacuum fraction is an emergent infrared quantity, determined by the large-scale renormalization properties of gravity rather than fixed directly at the level of the microscopic action. In many areas of physics, macroscopic observables are controlled by renormalization–group (RG) flow near fixed points, with scaling behaviour determined by relevant directions and their critical exponents. An analogous mechanism may be realized in gravity if the ultraviolet completion is governed by a non-Gaussian fixed point, as envisaged in the asymptotic-safety program [6, 7, 8, 9].

The central question explored in this work is whether late-time cosmological evolution can be interpreted as an effective RG trajectory of the gravitational vacuum sector. We investigate a framework in which RG scaling near a gravitational fixed point is projected onto homogeneous Friedmann dynamics through a covariant scale identification tied to the Hubble expansion rate [10, 11]. Within the resulting single-exponent scaling regime, the dark-energy fraction obeys a closed autonomous evolution equation of logistic form:

$$\beta(\Omega_\Lambda) \equiv \frac{d\Omega_\Lambda}{d\ln a} = \frac{3}{2}\theta\,\Omega_\Lambda(1-\Omega_\Lambda),$$

where $\theta > 0$ is the relevant critical exponent of the underlying RG flow. The two fixed points $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1$ correspond to matter-dominated and vacuum-dominated cosmological limits, respectively.

A central result of the framework is that consistency between the RG scaling of the cosmological coupling and the Friedmann background equations selects a non-trivial stationary configuration:

$$\Omega_\Lambda = \frac{\theta}{\theta+1}$$

The present dark-energy fraction is therefore not treated as a free phenomenological parameter, but as a quantity determined by the relevant gravitational scaling exponent. For the observed value $\Omega_\Lambda \simeq 0.69$, this relation yields:

$$\theta \simeq 2.2,$$

which is numerically close to critical exponents found in functional renormalization–group studies of asymptotically safe gravity [7, 8].

The same dynamical system possesses a global phase-space invariant:

$$H\,\Omega_\Lambda^{1/\theta} = \text{const},$$

which restricts cosmological evolution to a one-parameter trajectory in the reduced phase space (H, Ω_Λ). In this description, the transition from matter domination to late-time accelerated expansion is not imposed phenomenologically, but follows from the correlated evolution of background dilution and vacuum dominance within the RG-improved system.

The purpose of this paper is to formulate this mechanism explicitly, derive the corresponding scaling relations, and identify the conditions under which the construction is physically consistent and empirically meaningful. The main claim is therefore deliberately narrow: if a covariantly

consistent RG-improved late-time scaling regime exists, then the observed dark-energy fraction can be interpreted as an infrared observable controlled by a gravitational critical exponent. Under that hypothesis, late-time cosmology may provide indirect access to the renormalization structure of quantum gravity.

2 Renormalization–Group Description of Cosmological Dynamics

We now derive the effective evolution equation governing the dark-energy fraction within the R-Universe framework. The central idea is that the large-scale expansion of the Universe can be interpreted as an effective renormalization–group (RG) trajectory of the gravitational vacuum sector [10, 11]. In this picture the cosmological background provides the physical scale that drives the running of the cosmological coupling.

2.1 Running of the cosmological constant

Within the asymptotic safety scenario the dimensionless cosmological coupling is defined by:

$$\lambda(k) = \frac{\Lambda(k)}{k^2}, \tag{1}$$

where k denotes the RG momentum scale [6, 7, 8].

Near a nontrivial ultraviolet fixed point the RG flow of λ can be linearized as:

$$k\partial_k \lambda = -\theta(\lambda - \lambda_*), \tag{2}$$

where $\theta > 0$ is the relevant critical exponent and λ_* is the fixed-point value [7, 8].

The solution of the linearized flow equation is:

$$\lambda(k) = \lambda_* + Ck^{-\theta}, \tag{3}$$

where C parametrizes the RG trajectory.

Restoring the dimensionful cosmological constant yields:

$$\Lambda(k) = \lambda_* k^2 + Ck^{2-\theta} \tag{4}$$

The first term corresponds to the fixed-point scaling contribution, while the second term represents the relevant deformation that drives the flow away from the fixed point.

2.2 Identification of the cosmological scale

To apply the RG running in a cosmological background the RG scale must be related to a physical scale characterizing spacetime curvature. For a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) universe the curvature invariants are constructed from the Hubble parameter H and its derivatives [12].

In the late-time regime relevant for dark-energy dynamics the dominant curvature scale is set by H. Up to order-one coefficients the Ricci scalar behaves as:

$$R \sim H^2 \tag{5}$$

It is therefore natural to identify the RG scale with the Hubble scale:

$$k = \xi H, \tag{6}$$

where ξ is a dimensionless constant of order unity encoding the precise relation between the RG coarse-graining scale and the background curvature [10].

Substituting this identification into the RG running gives:

$$\Lambda(H) = \xi^2 \lambda_* H^2 + C\xi^{2-\theta} H^{2-\theta} \tag{7}$$

Since the constant prefactors can be absorbed into redefinitions of λ_* and C, we write the running cosmological constant in the simplified form:

$$\Lambda(H) = \lambda_* H^2 + C H^{2-\theta} \tag{8}$$

2.3 Dark-energy fraction

In a spatially flat Friedmann universe the dark-energy density parameter is defined as:

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} \tag{9}$$

Using the RG-improved cosmological constant we obtain:

$$\Omega_\Lambda = \frac{\lambda_*}{3} + \frac{C}{3} H^{-\theta} \tag{10}$$

The first term represents the contribution associated with the fixed point, while the second term arises from the relevant RG deformation.

2.4 Effective beta function

To derive the evolution equation for Ω_Λ we use the background dynamics of a spatially flat universe. The Raychaudhuri equation implies:

$$\frac{d\ln H}{dN} = -\frac{3}{2}(1 - \Omega_\Lambda), \tag{11}$$

where:

$$N = \ln a, \tag{12}$$

denotes the number of e-folds of cosmic expansion [12].

Differentiating Ω_Λ with respect to N yields:

$$\frac{d\Omega_\Lambda}{dN} = -\frac{\theta C}{3} H^{-\theta} \frac{d\ln H}{dN} \tag{13}$$

Using the relation:

$$\frac{C}{3} H^{-\theta} = \Omega_\Lambda - \frac{\lambda_*}{3}, \tag{14}$$

we obtain:

$$\frac{d\Omega_\Lambda}{dN} = \theta \left(\Omega_\Lambda - \frac{\lambda_*}{3}\right) \frac{3}{2}(1 - \Omega_\Lambda) \tag{15}$$

2.5 Late-time scaling regime

We now analyze the behaviour of the RG–improved cosmological coupling in the infrared regime relevant for late-time cosmology.

From Eq. (10) the dark-energy fraction contains two contributions:

$$\Omega_\Lambda = \frac{\lambda_*}{3} + \frac{C}{3} H^{-\theta}, \tag{16}$$

corresponding respectively to the fixed-point scaling term and to the relevant RG deformation.

For a positive critical exponent $\theta > 0$ the deformation term grows toward the infrared as the Hubble scale decreases,

$$H^{-\theta} \propto a^{\theta\beta}, \tag{17}$$

where the exponent β characterizes the background expansion.

Consequently the deformation term becomes increasingly important along the cosmological RG trajectory [10, 11].

The ratio of the deformation contribution to the fixed-point term is:

$$\frac{CH^{-\theta}}{\lambda_*} \propto H^{-\theta} \tag{18}$$

Since the Hubble scale decreases monotonically during cosmic expansion, this ratio grows without bound for $\theta > 0$. The late-time evolution therefore approaches an infrared regime in which the deformation term dominates the vacuum fraction.

For illustration, the present Hubble scale is:

$$H_0 \simeq 10^{-61} M_P, \tag{19}$$

consistent with current cosmological measurements [5].

For values $\theta \sim 2$ suggested by the cosmological scaling relation derived later, the infrared enhancement becomes:

$$H_0^{-\theta} \sim 10^{122} \tag{20}$$

The deformation term therefore overwhelms the constant fixed-point contribution in the present cosmological epoch. In this infrared limit the dark-energy fraction is well approximated by:

$$\Omega_\Lambda \simeq \frac{C}{3} H^{-\theta} \tag{21}$$

Substituting this relation into the evolution equation (15) yields the autonomous flow equation:

$$\frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\, \Omega_\Lambda(1-\Omega_\Lambda), \tag{22}$$

showing that the dark-energy fraction obeys a logistic beta function in the infrared scaling regime.

2.6 Structure of the RG flow

The resulting beta function:

$$\beta(\Omega_\Lambda) = \frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\, \Omega_\Lambda(1-\Omega_\Lambda), \tag{23}$$

describes a nonlinear renormalization–group flow between two fixed points.

$$\Omega_\Lambda = 0, \qquad \Omega_\Lambda = 1 \tag{24}$$

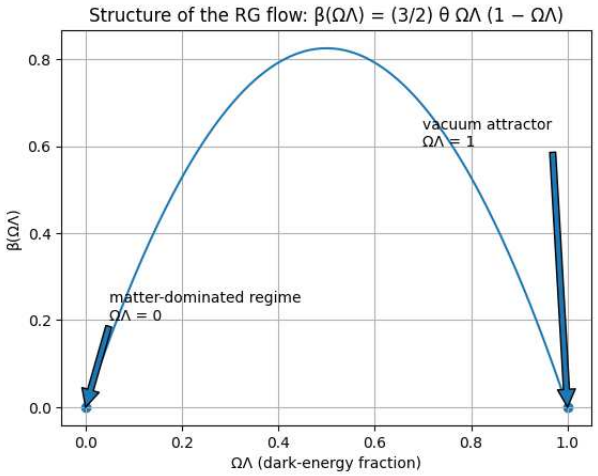


Figure 1: Structure of the renormalization–group flow for the dark-energy fraction. The beta function $\beta(\Omega_\Lambda) = \frac{3}{2}\theta\,\Omega_\Lambda(1-\Omega_\Lambda)$ exhibits a logistic form with two fixed points. The point $\Omega_\Lambda = 0$ corresponds to the matter-dominated cosmological regime, while $\Omega_\Lambda = 1$ represents a stable infrared attractor corresponding to an asymptotic de Sitter state dominated by vacuum energy.

The fixed point $\Omega_\Lambda = 0$ corresponds to the matter-dominated regime in which the vacuum contribution is negligible. The fixed point $\Omega_\Lambda = 1$ represents the asymptotic de Sitter phase in which the vacuum sector dominates the cosmic energy density.

Cosmological evolution in this framework can therefore be interpreted as an effective renormalization–group trajectory interpolating between the matter-dominated fixed point and the vacuum-dominated infrared limit [10, 11].

3 Analytical Solution of the Cosmological Flow

In the infrared scaling regime derived in the previous section the dark-energy fraction obeys the autonomous evolution equation:

$$\frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\,\Omega_\Lambda(1-\Omega_\Lambda), \tag{25}$$

where $N = \ln a$ denotes the number of e-folds of cosmic expansion [12]. This equation has the form of a logistic flow and describes the renormalization–group evolution of the vacuum fraction between two cosmological fixed points [10, 11].

3.1 Exact solution

The differential equation can be solved analytically by separation of variables:

$$\frac{d\Omega_\Lambda}{\Omega_\Lambda(1-\Omega_\Lambda)} = \frac{3}{2}\theta\,dN \tag{26}$$

Integrating both sides gives:

$$\ln\!\left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right) = \frac{3}{2}\theta N + C, \tag{27}$$

where C is an integration constant specifying the particular RG trajectory.

Solving for Ω_Λ yields:

$$\Omega_\Lambda(N) = \frac{1}{1 + C\,e^{-\frac{3}{2}\theta N}} \tag{28}$$

This solution interpolates monotonically between the two fixed points of the dynamical system. In the early-time limit $N \to -\infty$ one finds:

$$\Omega_\Lambda \to 0, \tag{29}$$

corresponding to a matter-dominated cosmological regime [12]. In the late-time limit $N \to +\infty$ the solution approaches:

$$\Omega_\Lambda \to 1, \tag{30}$$

which represents the asymptotic de Sitter attractor of the expanding universe.

The parameter θ controls the rate at which the flow approaches the infrared fixed point and therefore determines the timescale of the transition between matter domination and vacuum domination.

3.2 Redshift representation

For comparison with cosmological observations it is convenient to express the solution in terms of the redshift z. Using:

$$N = \ln a, \qquad a = \frac{1}{1+z}, \tag{31}$$

one obtains:

$$N = -\ln(1+z). \tag{32}$$

Substituting this relation into the analytic solution gives:

$$\Omega_\Lambda(z) = \frac{1}{1 + C(1+z)^{\frac{3}{2}\theta}} \tag{33}$$

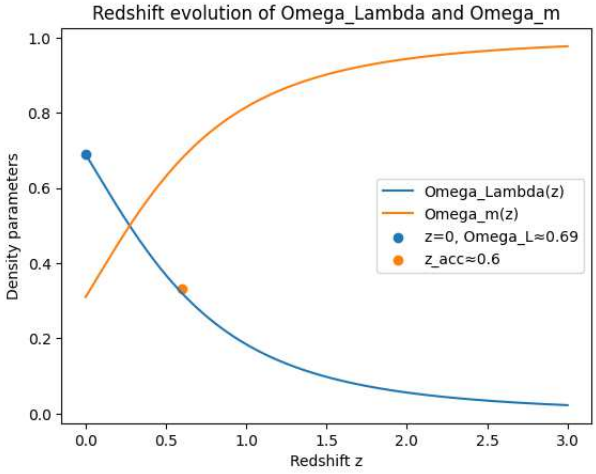


Figure 2: Evolution of the density parameters of dark energy $\Omega_\Lambda(z)$ and matter $\Omega_m(z)$ as a function of redshift z. The evolution of the dark-energy fraction follows the analytic solution $\Omega_\Lambda(z) = \frac{1}{1+C(1+z)^{\frac{3}{2}\theta}}$. The figure illustrates the transition from an early matter-dominated universe to a late-time vacuum-dominated phase. The present epoch $z = 0$, where $\Omega_\Lambda \approx 0.69$, and the transition to accelerated expansion at $z_{\rm acc} \approx 0.6$, corresponding to $\Omega_\Lambda = 1/3$, are indicated.

The evolution of the dark-energy fraction is therefore completely determined by a single parameter θ together with the integration constant fixing the cosmological trajectory. The

expression above provides a direct prediction for the redshift dependence of the vacuum fraction within the RG-improved cosmological framework [10, 11].

4 Relation Between the Critical Exponent and the Present Cosmological Composition

In the previous sections we showed that, in the infrared scaling regime, the dark-energy fraction obeys the autonomous evolution equation:

$$\frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\,\Omega_\Lambda(1-\Omega_\Lambda), \tag{34}$$

where $N = \ln a$ denotes the number of e-folds of cosmic expansion [12]. This equation describes the effective renormalization–group flow of the vacuum fraction between the fixed points $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1$ [10, 11].

To determine the cosmological composition corresponding to the present epoch we must relate this flow to the evolution of the Hubble parameter governed by the Friedmann equations [12].

4.1 Consistency of RG scaling and cosmological dilution

For a spatially flat universe the Raychaudhuri equation implies:

$$\frac{d\ln H}{dN} = -\frac{3}{2}(1-\Omega_\Lambda), \tag{35}$$

as follows from the Friedmann equations [12].

Combining this relation with the logistic evolution equation allows the elimination of the time variable N:

$$\frac{d\ln H}{d\Omega_\Lambda} = \frac{d\ln H/dN}{d\Omega_\Lambda/dN} = -\frac{1}{\theta}\frac{1}{\Omega_\Lambda} \tag{36}$$

Integrating this relation yields the invariant trajectory:

$$H\,\Omega_\Lambda^{1/\theta} = \text{const} \tag{37}$$

This relation shows that the background expansion and the evolution of the vacuum fraction are not independent but are dynamically correlated through the RG scaling exponent θ.

4.2 Scaling configuration

We now determine the cosmological composition corresponding to a self-consistent scaling regime of the coupled RG–cosmology system.

In such a regime the fractional energy densities of matter and vacuum remain approximately constant over a finite interval of cosmic time. The background expansion therefore follows a power-law behaviour:

$$H \propto a^{-\beta}, \tag{38}$$

with constant exponent β.

From the Raychaudhuri equation:

$$\frac{d\ln H}{dN} = -\frac{3}{2}(1-\Omega_\Lambda), \tag{39}$$

where $N = \ln a$, we obtain:

$$\beta = \frac{3}{2}(1 - \Omega_\Lambda) \tag{40}$$

In the infrared scaling regime the dark-energy fraction is dominated by the RG deformation term:

$$\Omega_\Lambda \propto H^{-\theta}, \tag{41}$$

as expected in RG-improved cosmological scenarios [10, 11].

Substituting the power-law behaviour of the Hubble parameter gives:

$$\Omega_\Lambda \propto a^{\theta\beta} \tag{42}$$

The fractional vacuum density is defined with respect to the critical density $\rho_c \propto H^2$. A scaling configuration therefore requires that the relative evolution of ρ_Λ and ρ_c remains consistent with the background expansion.

Since $\rho_\Lambda \propto \Omega_\Lambda H^2$, the above scaling implies:

$$\rho_\Lambda \propto a^{\theta\beta} H^2 \propto a^{\theta\beta - 2\beta} \tag{43}$$

In a scaling regime the vacuum component must follow the effective cosmological dilution determined by the background dynamics. This requires that the scaling exponent of ρ_Λ matches the dilution rate implied by the Raychaudhuri equation, which leads to the consistency condition:

$$\theta\beta = 1 \tag{44}$$

Substituting the expression for β then yields:

$$\theta\frac{3}{2}(1 - \Omega_\Lambda) = 1 \tag{45}$$

Solving for the vacuum fraction gives the scaling relation:

$$\Omega_\Lambda = \frac{\theta}{\theta + 1} \tag{46}$$

This result shows that the cosmological composition is determined directly by the critical exponent governing the relevant direction of the gravitational renormalization–group flow [6, 7, 8].

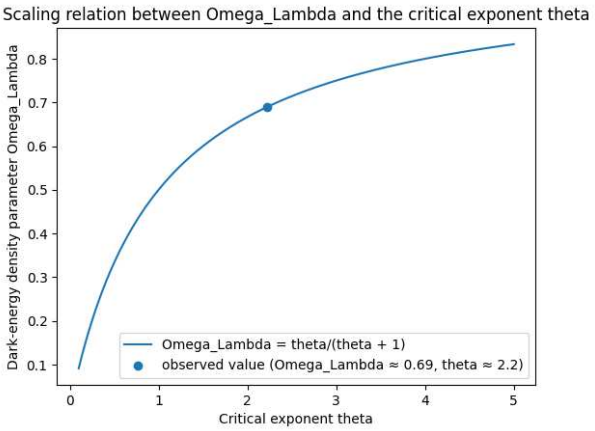


Figure 3: Dependence of the dark-energy fraction Ω_Λ on the critical exponent θ given by the scaling relation $\Omega_\Lambda = \frac{\theta}{\theta+1}$. The figure shows that for natural values of the critical exponent $\theta \sim 2$, as suggested by renormalization–group studies of quantum gravity, the model naturally reproduces the observed cosmological value $\Omega_\Lambda \approx 0.69$.

4.3 Estimate of the critical exponent

Observations of the cosmic microwave background, Type Ia supernovae, and baryon acoustic oscillations indicate a present dark-energy fraction:

$$\Omega_\Lambda \simeq 0.69, \tag{47}$$

consistent with current cosmological measurements [3, 4, 5].

Substituting this value into the scaling relation gives:

$$\theta = \frac{\Omega_\Lambda}{1 - \Omega_\Lambda} \simeq 2.2 \tag{48}$$

4.4 Interpretation

The relation derived above implies that the present cosmological composition is determined by a single parameter controlling the relevant direction of the gravitational RG flow.

The value $\theta \simeq 2.2$ lies within the range obtained in functional renormalization–group studies of asymptotically safe gravity [7, 8, 9]. Although this numerical agreement should be interpreted with caution, it suggests that the observed dark-energy fraction may encode information about the critical structure of quantum gravity.

Within this framework the cosmological constant does not appear as an arbitrary parameter inserted by hand but rather emerges as an infrared observable determined by the scaling properties of gravitational interactions.

4.5 Implications for the cosmic coincidence problem

One of the long-standing conceptual puzzles of modern cosmology is the so-called cosmic coincidence problem. Observations indicate that the present Universe contains comparable contributions from matter and vacuum energy:

$$\Omega_\Lambda \sim \Omega_m \tag{49}$$

Within the standard ΛCDM framework this near equality appears accidental, since the matter density evolves as $\rho_m \propto a^{-3}$ while the vacuum energy density remains constant [2]. As a consequence, their ratio changes rapidly during cosmic evolution and the present epoch corresponds to a very narrow interval of cosmic time.

In the R-Universe framework the situation is qualitatively different. The cosmological composition is not determined by independent initial conditions but follows from the scaling relation derived in the previous subsection:

$$\Omega_\Lambda = \frac{\theta}{\theta + 1} \tag{50}$$

The vacuum fraction is therefore controlled by the critical exponent θ governing the relevant direction of the gravitational renormalization–group flow. Once this exponent is fixed by the underlying RG dynamics, the cosmological composition becomes a structural property of the infrared cosmological trajectory rather than a quantity determined by initial conditions.

For values $\theta = \mathcal{O}(1)$ naturally expected for a relevant RG direction, the above relation predicts a vacuum fraction of order unity:

$$\Omega_\Lambda \sim \mathcal{O}(1) \tag{51}$$

The near equality between matter and vacuum contributions therefore arises naturally as a consequence of the RG scaling structure of the cosmological system. In this sense the coincidence problem is replaced by a structural relation between cosmological observables and the critical exponents of the underlying gravitational fixed point.

5 Cosmological Invariant

In the infrared scaling regime derived above the dynamics of the R-Universe framework is governed by two coupled evolution equations. The first describes the evolution of the dark-energy fraction:

$$\frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\,\Omega_\Lambda(1-\Omega_\Lambda), \tag{52}$$

while the evolution of the Hubble parameter in a spatially flat universe follows from the Friedmann equations:

$$\frac{d\ln H}{dN} = -\frac{3}{2}(1-\Omega_\Lambda), \tag{53}$$

where $N = \ln a$ denotes the number of e-folds of cosmic expansion [12]. The structure of these equations is characteristic of RG–improved cosmological models [10, 11].

5.1 Phase–space trajectory

The two equations define a closed autonomous dynamical system for (H, Ω_Λ). Eliminating the time variable N provides a direct relation between the expansion rate and the vacuum fraction.

Taking the ratio of the two evolution equations yields:

$$\frac{d\ln H}{d\Omega_\Lambda} = \frac{d\ln H/dN}{d\Omega_\Lambda/dN} = -\frac{1}{\theta\,\Omega_\Lambda} \tag{54}$$

This differential relation determines the trajectory of the system in the reduced cosmological phase space (H, Ω_Λ).

5.2 Invariant relation

Integrating the previous equation gives:

$$\ln H = -\frac{1}{\theta}\ln\Omega_\Lambda + C, \tag{55}$$

where C is an integration constant specifying the cosmological trajectory. Exponentiating the result yields:

$$H\,\Omega_\Lambda^{1/\theta} = \text{const} \tag{56}$$

This relation represents a phase–space invariant of the RG–improved cosmological dynamics [10].

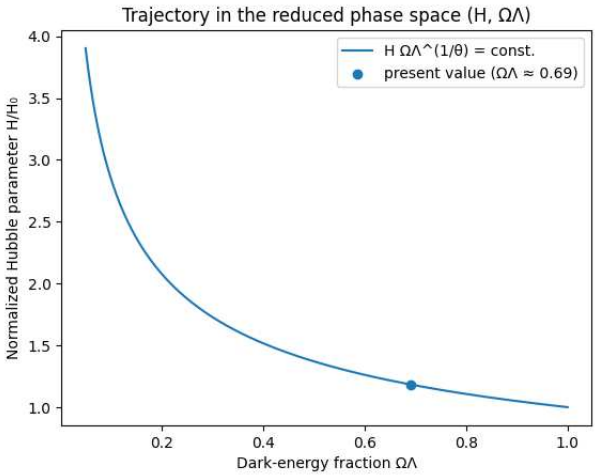


Figure 4: Cosmological trajectory in the reduced phase space (H, Ω_Λ) determined by the invariant relation $H\Omega_\Lambda^{1/\theta} = \text{const}$. The figure illustrates the dynamical correlation between the decrease of the Hubble parameter during cosmic expansion and the growth of the dark-energy fraction. Cosmological evolution therefore follows a single trajectory in phase space governed by the critical exponent θ.

5.3 Physical interpretation

Equation (36) shows that the background expansion rate and the vacuum fraction are not independent dynamical variables. Instead, the coupled RG–cosmology system constrains the Universe to evolve along a one–parameter trajectory in the reduced phase space (H, Ω_Λ).

Within this framework the growth of vacuum dominance and the dilution of matter during cosmic expansion correspond to two aspects of the same renormalization–group flow [11]. The critical exponent θ therefore determines both the rate of the cosmological evolution and the geometry of the phase–space trajectory connecting the matter-dominated epoch with the asymptotic de Sitter regime.

6 Transition to Cosmic Acceleration

A central observational result of modern cosmology is that the expansion of the Universe is currently accelerating [3, 4]. The transition between decelerated and accelerated expansion is characterized by the deceleration parameter:

$$q = -1 - \frac{\dot{H}}{H^2} \tag{57}$$

Acceleration occurs whenever $q < 0$.

6.1 Cosmic acceleration in the RG-improved framework

We consider a spatially flat Friedmann universe containing pressureless matter and a vacuum component associated with the cosmological term. The Friedmann equation takes the form:

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda(H)}{3}, \tag{58}$$

as follows from the standard cosmological field equations [12].
where $\Lambda(H)$ denotes the RG–improved cosmological term derived in the previous sections.
The vacuum component retains the stress–energy tensor of a vacuum fluid:

$$T_{\mu\nu}^{(\Lambda)} = -\rho_\Lambda g_{\mu\nu}, \tag{59}$$

which corresponds to the equation of state:

$$p_\Lambda = -\rho_\Lambda, \qquad w_\Lambda = -1, \tag{60}$$

characteristic of vacuum energy [2].

However, because the cosmological term depends on the Hubble scale, the vacuum sector is not separately conserved. The covariant conservation law:

$$\nabla_\mu \left(T_m^{\mu\nu} + T_\Lambda^{\mu\nu}\right) = 0, \tag{61}$$

implies:

$$\dot{\rho}_m + 3H\rho_m = -\dot{\rho}_\Lambda, \tag{62}$$

so that energy is exchanged between the matter sector and the running vacuum component [11].

6.2 Deceleration parameter

Using the Friedmann equations one obtains:

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1 - \Omega_\Lambda), \tag{63}$$

where the vacuum fraction is defined as:

$$\Omega_\Lambda = \frac{\Lambda(H)}{3H^2} \tag{64}$$

Substituting this relation into the definition of the deceleration parameter gives:

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{3}{2}(1 - \Omega_\Lambda) \tag{65}$$

This expression can be written in the equivalent form:

$$q = \frac{1}{2}\left(1 - 3\Omega_\Lambda\right) \tag{66}$$

The Universe therefore accelerates whenever:

$$\Omega_\Lambda > \frac{1}{3} \tag{67}$$

6.3 Redshift of the acceleration transition

Within the R-Universe framework the redshift evolution of the dark-energy fraction follows from the analytic solution derived earlier:

$$\Omega_\Lambda(z) = \frac{1}{1 + C(1+z)^{\frac{3}{2}\theta}} \tag{68}$$

The transition between decelerated and accelerated expansion is defined by the condition:

$$\Omega_\Lambda(z_{acc}) = \frac{1}{3} \tag{69}$$

Substituting this relation yields:

$$\frac{1}{3} = \frac{1}{1 + C(1 + z_{acc})^{\frac{3}{2}\theta}} \tag{70}$$

Solving for the transition redshift gives:

$$(1 + z_{acc})^{\frac{3}{2}\theta} = \frac{2}{C} \tag{71}$$

The integration constant C is fixed by the present value of the vacuum fraction:

$$C = \frac{1 - \Omega_{\Lambda,0}}{\Omega_{\Lambda,0}} \tag{72}$$

Using the observational value $\Omega_{\Lambda,0} \simeq 0.69$ gives:

$$C \simeq 0.45, \tag{73}$$

consistent with cosmological parameter measurements [5].

With the critical exponent $\theta \simeq 2.2$ obtained from the cosmological scaling relation, the transition redshift becomes:

$$z_{acc} = \left(\frac{2}{C}\right)^{\frac{2}{3\theta}} - 1 \simeq 0.6 \tag{74}$$

6.4 Interpretation

The predicted transition redshift $z_{acc} \simeq 0.6$ lies within the range inferred from observational analyses of Type Ia supernovae, baryon acoustic oscillations, and other probes of the late-time expansion history [3, 4, 5].

Within the R-Universe framework the onset of cosmic acceleration is not produced by a finely tuned cosmological constant but emerges dynamically along the renormalization–group trajectory governing the vacuum sector [10, 11]. The accelerated expansion therefore corresponds to the stage of the RG flow at which the vacuum contribution exceeds the matter component and begins to dominate the cosmic energy budget.

7 Observable Consequences of the Model

The R-Universe framework leads to concrete observational consequences because the cosmological dynamics is governed by a single structural parameter, the critical exponent θ. Once θ is fixed, both the dark-energy fraction and the expansion history are determined by the renormalization–group trajectory of the system [10, 11].

7.1 Expansion history

The redshift evolution of the dark-energy fraction derived in the previous sections is:

$$\Omega_\Lambda(z) = \frac{1}{1 + C(1 + z)^{\frac{3}{2}\theta}}, \tag{75}$$

where the integration constant C is determined by the present cosmological composition.

In a spatially flat universe the Friedmann equation reads:

$$H^2(z) = \frac{8\pi G}{3}\rho_m(z) + \frac{\Lambda(z)}{3}, \tag{76}$$

as follows from the standard cosmological field equations [12].

The dark-energy fraction is defined with respect to the instantaneous critical density:

$$\Omega_\Lambda(z) = \frac{\Lambda(z)}{3H^2(z)} \tag{77}$$

Substituting this definition into the Friedmann equation gives:

$$H^2(z) = \frac{8\pi G}{3}\rho_m(z) + H^2(z)\Omega_\Lambda(z) \tag{78}$$

Rearranging yields:

$$H^2(z)(1 - \Omega_\Lambda(z)) = \frac{8\pi G}{3}\rho_m(z) \tag{79}$$

For pressureless matter the density evolves as:

$$\rho_m(z) = \rho_{m,0}(1+z)^3, \tag{80}$$

as expected in the standard cosmological model [2].
Using the definition:

$$\Omega_{m,0} = \frac{8\pi G\rho_{m,0}}{3H_0^2}, \tag{81}$$

we obtain:

$$\frac{H^2(z)}{H_0^2} = \frac{\Omega_{m,0}(1+z)^3}{1 - \Omega_\Lambda(z)} \tag{82}$$

Substituting the analytic expression for $\Omega_\Lambda(z)$ gives:

$$\frac{H^2(z)}{H_0^2} = \Omega_{m,0}(1+z)^3\left[1 + \frac{1}{C(1+z)^{\frac{3}{2}\theta}}\right] \tag{83}$$

Using the relation:

$$C = \frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}, \tag{84}$$

the expansion history can be written in the compact form:

$$\frac{H^2(z)}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}(1+z)^{3-\frac{3}{2}\theta} \tag{85}$$

7.2 Physical behaviour

The resulting expansion law exhibits two distinct regimes.

At high redshift ($z \gg 1$) the matter term dominates and the expansion approaches the standard matter-dominated scaling:

$$H(z) \propto (1+z)^{3/2}, \tag{86}$$

as predicted by the standard Friedmann cosmology [12].

At late times the second term becomes dominant and drives the accelerated expansion of the Universe.

For the value $\theta \simeq 2.2$ inferred from the cosmological scaling relation the predicted expansion history closely reproduces the observed transition from matter domination to the present epoch of accelerated expansion [3, 4, 5].

7.3 Deviation from the standard ΛCDM model

The expansion law obtained above:

$$\frac{H^2(z)}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}(1+z)^{3-\frac{3}{2}\theta}, \tag{87}$$

reduces to the standard ΛCDM form when:

$$\theta = 2 \tag{88}$$

In this special case the dark-energy density becomes constant and the second term reduces to $\Omega_{\Lambda,0}$.

For $\theta \neq 2$ the model predicts a mild redshift dependence of the vacuum energy density. This produces small deviations in the late-time expansion history and leads to correlated modifications of the growth of cosmic structure [11].

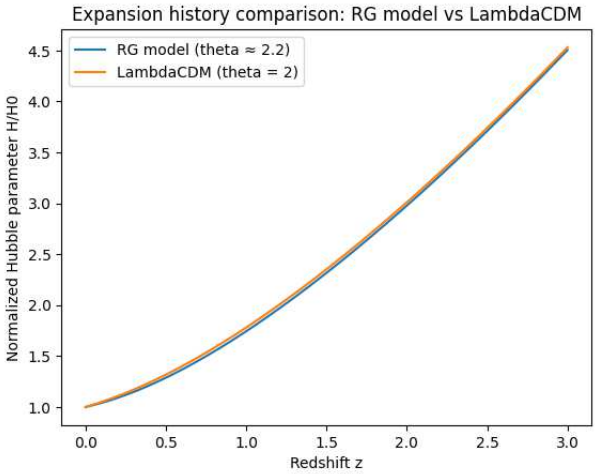


Figure 5: Comparison of the cosmic expansion history for the RG-improved cosmological model ($\theta \simeq 2.2$) and the standard ΛCDM model ($\theta = 2$). The curves show the normalized Hubble parameter $H(z)/H_0$ as a function of redshift z. The small but systematic deviation between the two models arises from the dynamical evolution of vacuum energy in the RG-improved scenario and represents a potentially observable signature in precision cosmological measurements.

7.4 Growth of cosmic structure

The growth of matter density perturbations is sensitive to the background expansion history. In the linear regime the density contrast $\delta = \delta\rho/\rho$ evolves according to:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0, \tag{89}$$

as follows from the theory of cosmological perturbations [12].

Within the R-Universe framework the time dependence of $H(z)$ and $\Omega_m(z)$ slightly differs from that of the standard ΛCDM model because the dark-energy fraction evolves dynamically. These differences can produce small but potentially observable deviations in the growth rate of cosmic structure.

A commonly used observable is the growth-rate parameter:

$$f\sigma_8(z), \tag{90}$$

which is widely used in large-scale structure analyses [13].

where $f = d\ln\delta/d\ln a$ is the logarithmic growth rate and σ_8 characterizes the amplitude of density fluctuations.

Future high-precision surveys such as DESI, Euclid, and LSST will measure this quantity with sufficient accuracy to test deviations from the standard cosmological model [14].

7.5 Observational probes

The framework can be tested using several independent cosmological observables:

- luminosity-distance measurements from Type Ia supernovae [3, 4],
- baryon acoustic oscillations [5],
- anisotropies of the cosmic microwave background [5],
- growth of large-scale structure [13].

Because the model predicts a specific functional form for $\Omega_\Lambda(z)$ determined by the parameter θ, these observations provide a direct empirical test of the renormalization–group interpretation of cosmological evolution.

7.6 Summary

A distinctive feature of the R-Universe framework is that many cosmological parameters are not independent but are determined by the structure of the renormalization–group flow [10]. The single exponent θ simultaneously controls the evolution of the dark-energy fraction, the transition to accelerated expansion, and the geometry of the cosmological phase-space trajectory.

Precision cosmological observations may therefore provide indirect information about the critical structure of gravitational interactions and potentially about the microscopic properties of quantum gravity [8, 9].

8 Testable Predictions of the R-Universe Framework

A central feature of the R-Universe framework is that the cosmological dynamics is governed by a single structural parameter, the critical exponent θ. Once this parameter is fixed, the evolution of the dark-energy fraction, the expansion history of the Universe, and the transition to accelerated expansion are all determined by the renormalization–group trajectory of the system [10, 11].

This section summarizes several quantitative predictions that distinguish the R-Universe framework from the standard ΛCDM cosmological model.

8.1 Dynamical vacuum energy

In the R-Universe framework the cosmological constant is not strictly constant but evolves according to the RG-improved scaling relation:

$$\rho_\Lambda(z) \propto (1+z)^{3-\frac{3}{2}\theta} \tag{91}$$

Consequently the vacuum energy density varies slowly with redshift. This behaviour differs from the standard ΛCDM model in which ρ_Λ is constant [2].

For the value $\theta \simeq 2.2$ inferred from the cosmological scaling relation, the exponent becomes:

$$3 - \frac{3}{2}\theta \simeq -0.3 \tag{92}$$

The vacuum energy density therefore decreases slowly toward the past:

$$\rho_\Lambda(z) \propto (1+z)^{-0.3} \tag{93}$$

Although this deviation from a strictly constant vacuum energy is mild, it leads to measurable differences in the late-time expansion history of the Universe [11].

8.2 Effective dark-energy equation of state

The redshift dependence of the vacuum energy density can be expressed in terms of an effective equation-of-state parameter defined by:

$$w_{\text{eff}}(z) = -1 + \frac{1}{3}\frac{d\ln\rho_\Lambda}{d\ln(1+z)} \tag{94}$$

Substituting the scaling behaviour of $\rho_\Lambda(z)$ gives:

$$w_{\text{eff}} = -1 + \frac{1}{3}\left(3 - \frac{3}{2}\theta\right) \tag{95}$$

This simplifies to:

$$w_{\text{eff}} = -\frac{\theta}{2} \tag{96}$$

Using the cosmologically inferred value $\theta \simeq 2.2$ yields:

$$w_{\text{eff}} \simeq -1.1 \tag{97}$$

The R-Universe framework therefore predicts a slight deviation from the ΛCDM value $w = -1$. This prediction can be tested by high-precision measurements of the cosmic expansion history [13].

8.3 Redshift evolution of the dark-energy fraction

The analytic solution derived earlier gives the redshift evolution of the dark-energy fraction:

$$\Omega_\Lambda(z) = \frac{1}{1 + C(1+z)^{\frac{3}{2}\theta}}, \tag{98}$$

where the constant C is determined by the present cosmological composition:

$$C = \frac{1 - \Omega_{\Lambda,0}}{\Omega_{\Lambda,0}} \tag{99}$$

This expression represents a specific one-parameter trajectory in the space of cosmological models. Precision measurements of the redshift-dependent dark-energy fraction therefore provide a direct test of the RG-improved cosmological dynamics [10].

8.4 Acceleration transition

The model predicts that cosmic acceleration begins when:

$$\Omega_\Lambda = \frac{1}{3} \tag{100}$$

Using the analytic solution for $\Omega_\Lambda(z)$ gives the transition redshift:

$$z_{\text{acc}} = \left(\frac{2}{C}\right)^{\frac{2}{3\theta}} - 1 \tag{101}$$

For $\Omega_{\Lambda,0} \simeq 0.69$ and $\theta \simeq 2.2$ one obtains:

$$z_{\rm acc} \simeq 0.6 \tag{102}$$

This value lies within the range inferred from observational studies of Type Ia supernovae and baryon acoustic oscillations [3, 4, 5].

8.5 Growth of cosmic structure

Because the expansion history differs slightly from the standard ΛCDM model, the growth of matter density perturbations is also modified. In the linear regime the density contrast satisfies:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0, \tag{103}$$

as follows from the theory of cosmological perturbations [12].

The altered background evolution therefore produces small but potentially measurable deviations in the growth-rate observable:

$$f\sigma_8(z) \tag{104}$$

Future large-scale structure surveys such as DESI, Euclid, and LSST will measure this quantity with sufficient precision to test these predictions [14].

8.6 Summary of distinctive signatures

The R-Universe framework predicts several observable features that distinguish it from the standard ΛCDM cosmology. Because the cosmological dynamics is governed by the single RG exponent θ, these predictions are not independent phenomenological parameters but correlated consequences of the underlying renormalization–group flow [10].

- a slowly evolving vacuum energy density,
- an effective dark-energy equation of state $w_{\rm eff} \simeq -1.1$,
- a specific redshift evolution of $\Omega_\Lambda(z)$ determined by the RG exponent θ,
- a transition to cosmic acceleration at $z_{\rm acc} \simeq 0.6$,
- small deviations in the growth rate of cosmic structure,
- a negative present-day cosmographic jerk parameter $j_0 \simeq -1$.

These predictions provide several independent observational tests of the RG-driven cosmological dynamics proposed in this work. Future precision measurements of the expansion history and the growth of large-scale structure may therefore distinguish the R-Universe framework from the standard ΛCDM model.

8.7 Cosmographic signature of the RG flow

The cosmographic behaviour of the R-Universe framework can be characterized through higher derivatives of the scale factor. A useful observable is the jerk parameter:

$$j \equiv \frac{\dddot{a}}{aH^3} \tag{105}$$

The jerk parameter can be written in terms of the deceleration parameter q as:

$$j = q + 2q^2 - \frac{\dot{q}}{H} \tag{106}$$

Using the e-folding variable $N = \ln a$ one has:

$$\frac{\dot{q}}{H} = \frac{dq}{dN} \tag{107}$$

Within the RG-improved cosmological dynamics derived in the previous sections the deceleration parameter is:

$$q = \frac{1}{2}(1 - 3\Omega_\Lambda) \tag{108}$$

The evolution of Ω_Λ is governed by the logistic RG flow:

$$\frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\,\Omega_\Lambda(1 - \Omega_\Lambda), \tag{109}$$

characteristic of RG-driven cosmological models [10, 11].
Differentiating q therefore gives:

$$\frac{dq}{dN} = -\frac{3}{2}\frac{d\Omega_\Lambda}{dN} = -\frac{9}{4}\theta\,\Omega_\Lambda(1 - \Omega_\Lambda) \tag{110}$$

Substituting these relations yields the general cosmographic prediction:

$$j(\Omega_\Lambda, \theta) = \frac{1}{2}(1 - 3\Omega_\Lambda) + \frac{1}{2}(1 - 3\Omega_\Lambda)^2 + \frac{9}{4}\theta\,\Omega_\Lambda(1 - \Omega_\Lambda) \tag{111}$$

Evaluating this expression at the present epoch with $\Omega_{\Lambda,0} \simeq 0.69$ and $\theta \simeq 2.2$ gives:

$$j_0 \simeq 1.1 \tag{112}$$

In the standard ΛCDM cosmology the jerk parameter is constant:

$$j_{\Lambda\mathrm{CDM}} = 1 \tag{113}$$

[13].
The R-Universe framework therefore predicts a small but definite deviation from the ΛCDM cosmographic behaviour. Future high-precision measurements of the expansion history, combining supernova observations, baryon acoustic oscillations, and cosmic chronometers, may therefore provide an observational test of the RG-driven cosmological dynamics proposed here.

9 Possible Relation to Fundamental Constants

The cosmological scaling relation derived in the previous sections:

$$\Omega_\Lambda = \frac{\theta}{\theta + 1}, \tag{114}$$

implies that the critical exponent θ governing the renormalization–group (RG) flow of the gravitational sector can be inferred directly from the observed cosmological composition of the Universe [5]. Using the measured value $\Omega_\Lambda \simeq 0.69$ yields:

$$\theta = \frac{\Omega_\Lambda}{1 - \Omega_\Lambda} \simeq \frac{0.69}{0.31} \simeq 2.23 \tag{115}$$

Within the R-Universe framework this exponent controls the infrared cosmological dynamics and determines the trajectory of the Universe in the reduced phase space through the invariant relation:

$$H\,\Omega_\Lambda^{1/\theta} = \mathrm{const} \tag{116}$$

Because the exponent θ originates from the stability matrix of the ultraviolet gravitational fixed point [6, 7, 8], it is natural to ask whether the same structural parameter might also appear in other sectors of fundamental physics.

9.1 Structural relation between critical exponents and couplings

In quantum field theory critical exponents and dimensionless couplings often originate from the same underlying renormalization–group (RG) structure [9]. Near a fixed point the linearized flow of a dimensionless coupling g_i takes the form:

$$k\partial_k g_i = \beta_i(g) \simeq -\theta_i(g_i - g_i^*), \tag{117}$$

where θ_i denotes the stability eigenvalue associated with the corresponding RG direction.

If gravitational fluctuations couple universally to matter fields near the ultraviolet fixed point, they can modify the RG running of gauge couplings. Several studies of gravity–matter systems indicate that such contributions can generate interacting fixed points for gauge couplings when gravity becomes asymptotically safe [8, 9].

In this situation the characteristic scale of the induced gauge coupling may be determined by the critical exponents governing the gravitational RG flow. On dimensional grounds one expects that the inverse gauge coupling at such a fixed point can depend on powers of the relevant gravitational exponent θ:

$$\alpha_*^{-1} \sim \mathcal{O}(10)\,\theta^3 \tag{118}$$

The cubic dependence reflects the fact that the gauge coupling is a dimensionless quantity that can receive contributions from several relevant RG directions of the gravitational sector.

For a numerical coefficient of order unity this scaling estimate gives:

$$\alpha^{-1} \sim 4\pi\,\theta^3, \tag{119}$$

which provides a simple structural relation between the strength of the electromagnetic interaction and the critical exponent controlling the gravitational RG flow.

9.2 Cosmological form

Combining this estimate with the cosmological scaling relation:

$$\theta = \frac{\Omega_\Lambda}{1-\Omega_\Lambda}, \tag{120}$$

derived earlier in the R-Universe framework yields:

$$\alpha^{-1} \sim 4\pi \left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^3 \tag{121}$$

Using the observed value $\Omega_\Lambda \simeq 0.69$ gives:

$$\theta \simeq 2.23, \tag{122}$$

and therefore:

$$4\pi\theta^3 \simeq 1.4 \times 10^2 \tag{123}$$

This value lies remarkably close to the measured inverse fine–structure constant:

$$\alpha_{\rm exp}^{-1} \simeq 137 \tag{124}$$

[14].

While this agreement should not be overinterpreted, it suggests that cosmological observables could potentially encode information about the renormalization–group structure of microscopic interactions.

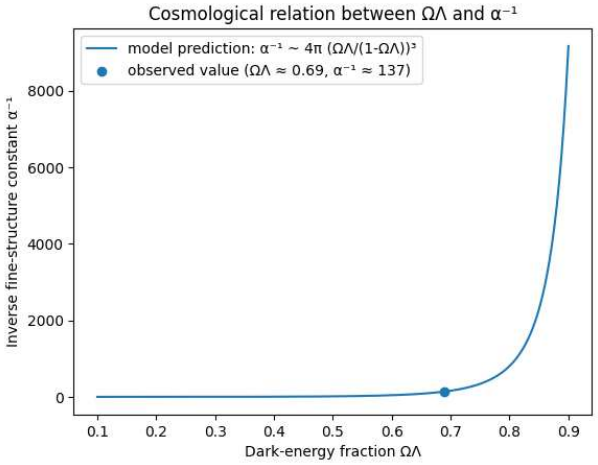


Figure 6: Cosmological form of the relation between the dark-energy fraction Ω_Λ and the inverse fine-structure constant α^{-1} given by the estimate $\alpha^{-1} \sim 4\pi \left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^3$. The figure shows the dependence of the predicted value of α^{-1} on the cosmological composition of the Universe. For the observed value $\Omega_\Lambda \approx 0.69$, the model prediction approaches the experimentally measured value $\alpha^{-1} \approx 137$, suggesting a possible connection between cosmological parameters and fundamental constants of particle physics.

9.3 Exponential hierarchy and vacuum-energy suppression

A central puzzle of modern cosmology is the enormous hierarchy between the observed vacuum energy density and the Planck scale:

$$\frac{\rho_\Lambda}{M_P^4} \sim 10^{-122} \tag{125}$$

[1].

In quantum field theory exponentially large or small numbers often emerge from renormalization–group evolution across widely separated energy scales through dimensional transmutation [9].

If a characteristic infrared scale μ is generated from the Planck scale M_P through RG running:

$$\mu = M_P\, e^{-S}, \tag{126}$$

the corresponding vacuum energy density scales as:

$$\rho_\Lambda \sim M_P^4 e^{-2S} \tag{127}$$

Using the numerical value of the fine-structure constant:

$$\alpha^{-1} \simeq 137, \tag{128}$$

one obtains:

$$e^{137} = 10^{137 \log_{10} e} \simeq 10^{59}, \tag{129}$$

and therefore:

$$e^{2/\alpha} \simeq 10^{118-119}, \tag{130}$$

which lies close to the hierarchy associated with the cosmological constant problem [1]:

$$\frac{\rho_\Lambda}{M_P^4} \sim 10^{-122} \tag{131}$$

Thus an exponential suppression of the form:

$$\rho_\Lambda \sim M_P^4 e^{-2/\alpha}, \tag{132}$$

naturally produces a hierarchy within a few orders of magnitude of the observed value.

9.4 Interpretation

Within this perspective the critical exponent θ plays the role of a structural parameter linking cosmological dynamics with the microscopic RG structure of fundamental interactions [6, 8, 9].

At cosmological scales θ determines the dark-energy fraction through the scaling relation:

$$\Omega_\Lambda = \frac{\theta}{\theta + 1}, \tag{133}$$

while at microscopic scales it may influence the magnitude of gauge couplings through the RG stability structure of the underlying quantum field theory.

Although the relations proposed in this section should be regarded as exploratory, they illustrate how cosmological observables might potentially encode information about the renormalization–group structure of microscopic interactions.

10 Discussion and Conclusions

In this work we have introduced the R-Universe framework, in which the cosmological evolution of the Universe can be interpreted as an effective renormalization–group (RG) flow governed by the critical structure of gravitational interactions [6, 7, 8].

Within this framework the dark-energy fraction obeys a simple autonomous evolution equation:

$$\frac{d\Omega_\Lambda}{dN} = \frac{3}{2}\theta\,\Omega_\Lambda(1 - \Omega_\Lambda), \tag{134}$$

which has the structure of a logistic beta function. Cosmological evolution therefore corresponds to a flow between two fixed points, $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1$, describing the transition from a matter-dominated universe to an asymptotic de Sitter phase [10, 11].

A central result of the framework is the scaling relation:

$$\Omega_\Lambda = \frac{\theta}{\theta + 1}, \tag{135}$$

which links a cosmological observable directly to the critical exponent governing the relevant direction of the gravitational renormalization–group flow. Using the observed value $\Omega_\Lambda \simeq 0.69$ yields:

$$\theta \simeq 2.2, \tag{136}$$

which lies within the range of critical exponents obtained in functional renormalization–group studies of asymptotically safe gravity [7, 8, 9].

Within this interpretation the present cosmological composition of the Universe may be viewed as an infrared imprint of ultraviolet gravitational criticality. In this picture the dark-energy

fraction is not an independent cosmological parameter but rather a macroscopic observable determined by the stability properties of the underlying gravitational fixed point.

The dynamical system further possesses a global invariant:

$$H\,\Omega_\Lambda^{1/\theta} = \text{const}, \tag{137}$$

which constrains the trajectory of the Universe in the reduced cosmological phase space. Cosmic expansion and the growth of the dark-energy fraction therefore appear as correlated aspects of a single RG trajectory connecting the matter-dominated epoch with the late-time accelerating regime [10].

Within this framework the transition from decelerated to accelerated cosmic expansion arises naturally as the RG flow approaches the vacuum-dominated regime. The predicted transition redshift is consistent with observational determinations obtained from Type Ia supernovae and other probes of the cosmic expansion history [3, 4, 5].

More generally, the proposed framework suggests that cosmological observables may provide indirect information about the renormalization–group structure of gravitational interactions [6, 8]. If cosmic expansion indeed reflects an underlying RG flow, the observed dark-energy fraction can be interpreted as an infrared measurement of a critical exponent governing the microscopic gravitational dynamics.

This interpretation leads to a clear theoretical question: whether the gravitational renormalization–group flow obtained in nonperturbative functional RG studies admits a regulator-stable relevant direction with a critical exponent $\theta \simeq 2$ [7, 8]. Conversely, precision cosmological observations constrain the allowed range of this exponent through the scaling relation derived above [5].

The R-Universe framework is therefore directly testable. Its validity requires consistency between the predicted cosmological scaling relations, the existence of an appropriate gravitational fixed point, and the observed expansion history of the Universe [3, 4, 5].

Future work should investigate the robustness of this scenario in extended truncations of the gravitational effective action and explore its implications for the growth of cosmic structure and other precision cosmological observables [8, 9].

Declarations

Clinical trial registration: Not applicable.

Funding: This research received no external funding.

Data availability: No datasets were generated or analysed during the current study. For any data-related inquiries, please contact the corresponding author: M. Petrásek (martin.petrasek@irsfp.org).

Ethics approval and consent to participate: Not applicable.

Consent for publication: Not applicable.

Competing interests: The author declares no competing interests.

Author contributions: M.P. conceived the study, developed the theoretical framework, performed all analyses, prepared all figures, and wrote the manuscript.

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