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Title

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Abstract

Macroeconomic stagnation in emerging economies is frequently associated with persistent underinvestment in innovation despite the presence of high-return technological opportunities. This paper develops a stochastic evolutionary game-theoretic model showing how macroeconomic volatility can endogenously generate innovation traps, in which decentralized firm behavior converges to a stable equilibrium dominated by imitation despite positive aggregate returns to innovation. We model a large population of firms facing a strategic choice between high-risk innovative strategies and low-risk imitative behaviors. To capture the influence of global market uncertainty and supply-chain volatility, stochastic fluctuations are incorporated directly into the payoff structure of the innovation game.

The evolution of strategic behavior is described using stochastic replicator dynamics driven by Brownian motion, where the state variable represents the proportion of firms adopting innovative strategies. Analytical results show that sufficiently high environmental volatility can alter the stability structure of the evolutionary system, causing the dynamics to converge toward a suboptimal equilibrium dominated by imitation. This equilibrium represents an innovation trap in which decentralized strategic incentives suppress innovation even when aggregate economic returns remain positive.

We derive explicit volatility thresholds that determine the stability of innovative equilibria and identify parameter regions where targeted risk-mitigation policies can destabilize the stagnation state and induce a transition toward innovation-driven growth. The results provide a formal mechanism explaining how macroeconomic volatility can endogenously generate persistent innovation failure and demonstrate how strategic interventions can restore self-sustaining innovative equilibria.

Keywords: Evolutionary Game Theory; Stochastic Replicator Dynamics; Innovation Traps; Macroeconomic Volatility; Strategic Innovation; Equilibrium Stability

1. Introduction

Persistent macroeconomic stagnation remains one of the central development challenges facing many emerging economies. Numerous countries have successfully transitioned from low-income production structures based on labor-intensive manufacturing to middle-income industrialization, yet subsequently experience prolonged slowdowns in productivity growth and technological advancement. This phenomenon, often described as the middle-income trap, has been widely documented in the empirical growth literature (Eichengreen et al., 2013; Felipe et al., 2012; Bulman et al., 2017). In such environments, firms continue to rely primarily on imitation and incremental improvements rather than pursuing high-risk technological innovation, thereby limiting the economy's ability to progress toward innovation-driven growth.

Most existing explanations for this stagnation emphasize structural and institutional constraints. Studies highlight factors such as human capital accumulation, financial development, and institutional quality as determinants of whether an economy successfully transitions to the technological frontier (Acemoglu, 2009; Benhabib and Spiegel, 1994; Gill and Kharas, 2007). While these perspectives provide valuable insights, they typically treat stagnation as the result of exogenous structural deficiencies that require external policy interventions to resolve. In contrast, relatively little attention has been devoted to understanding whether stagnation may arise endogenously from decentralized strategic interactions among firms operating in uncertain environments.

From the perspective of firm-level decision-making, the transition from imitation-based production to innovation-driven growth involves a fundamental strategic dilemma. Innovation typically requires substantial sunk investments, uncertain research outcomes, and long development horizons. In contrast, imitation strategies, such as adopting existing technologies or incremental process improvements offer more predictable returns and lower risk exposure. When firms operate in volatile economic environments characterized by fluctuating demand, supply-chain disruptions, and uncertain financial conditions, the incentives to undertake risky innovation may become significantly weakened. The literature on investment under uncertainty demonstrates that firms often delay irreversible investments when volatility increases, as the value of waiting rises with uncertainty (McDonald and Siegel, 1986; Pindyck, 1991; Dixit and Pindyck, 1994).

These insights suggest that macroeconomic volatility may generate systemic coordination failures in innovation decisions. If many firms simultaneously adopt cautious strategies focused on imitation, the aggregate level of technological experimentation declines, reducing the likelihood of breakthrough innovations. Over time, this collective behavior can lock the economy into a low-innovation equilibrium, even when the expected long-run gains from innovation remain positive. Such outcomes represent what we term an innovation trap, a state in which decentralized decision-making leads to persistent underinvestment in innovation despite favorable aggregate incentives.

This paper proposes a formal framework for analyzing this phenomenon using stochastic evolutionary game theory. We model the economy as a large population of firms repeatedly interacting in a strategic environment in which each firm chooses between two behavioral strategies: innovation and imitation. Innovation offers potentially high rewards but is subject to substantial uncertainty, while imitation provides lower but more stable returns. The distribution of strategies within the firm population evolves over time according to stochastic replicator dynamics, which capture how successful strategies proliferate while unsuccessful ones decline (Hofbauer and Sigmund, 1998; Sandholm, 2010).

To incorporate macroeconomic uncertainty, stochastic volatility is introduced into the payoff structure governing innovation incentives. Environmental shocks representing fluctuations in global demand, financial conditions, and supply-chain disruptions are modeled as stochastic perturbations driven by Brownian motion. This approach follows the stochastic evolutionary game literature, where random fluctuations influence the long-run stability of strategic equilibria (Fudenberg and Harris, 1992; Cabrales, 2000; Imhof, 2005; Choudhury and Aydinyan, 2023).

The analysis shows that macroeconomic volatility can fundamentally reshape the stability structure of the innovation game. When volatility exceeds a critical threshold, the stochastic evolutionary dynamics converge almost surely to an imitation-dominated equilibrium. In this regime, innovation becomes systematically suppressed even though the expected payoff from innovation remains positive at the aggregate level. The result is a mathematically defined innovation trap, representing an endogenous mechanism of macroeconomic stagnation.

The contributions of this paper are threefold. First, we introduce a stochastic evolutionary game-theoretic framework that models technological competition between innovation and imitation in

volatile economic environments. Second, we derive analytical conditions under which stochastic volatility destabilizes innovation equilibria and generates imitation-dominated outcomes. Third, we demonstrate how targeted interventions, such as innovation subsidies or risk-sharing mechanisms can alter the geometry of the evolutionary dynamics and induce a transition toward innovation-driven equilibria.

The remainder of the paper proceeds as follows. Section 2 presents the stochastic evolutionary game model and formulates the strategic environment faced by firms. Section 3 analyzes the equilibrium properties of the system and derives the conditions under which volatility generates innovation traps. Section 4 introduces control mechanisms capable of destabilizing stagnation equilibria and restoring innovation incentives. Section 5 presents numerical experiments and policy phase analysis of the model using macroeconomic data. Section 6 concludes with policy implications and directions for future research.

2. The Stochastic Evolutionary Game Model

2.1 Firms and Strategy Space

We consider an economy composed of a large population of firms that repeatedly interact in a competitive environment. Each firm must choose between two strategic behaviors governing its technological development strategy. Let the strategy space be defined as

$$S = \{\text{Innovation, Imitation}\}.$$

The innovation strategy represents investment in new technologies, research and development, or disruptive production processes. This strategy offers the possibility of substantial technological gains but involves significant uncertainty, high fixed costs, and long development horizons (Romer, 1990; Grossman and Helpman, 1991). Successful innovation can generate large productivity improvements and create new technological opportunities for the broader economy.

In contrast, the imitation strategy corresponds to the adoption or incremental improvement of existing technologies. Firms pursuing imitation rely on established production techniques and existing supply-chain structures, allowing them to achieve relatively stable returns with lower risk exposure. However, imitation strategies typically face diminishing returns as market

competition intensifies and technological opportunities become exhausted (Segerstrom, 1991; Mukoyama, 2003).

Let $x(t) \in [0,1]$ denote the proportion of firms adopting the innovation strategy at time t. The remaining fraction $1 - x(t)$ represents firms following the imitation strategy. The evolution of $x(t)$ reflects the dynamic adaptation of firms as they respond to the relative success of competing strategies.

2.2 Baseline Payoff Structure

We begin by specifying the deterministic payoff structure governing interactions between firms. Let the payoff matrix of the innovation–imitation game be represented as

	Innovation	Imitation
Innovation	a	b
Imitation	c	d

where the entries represent expected economic returns from strategic interactions between firms.

The parameters capture several key economic mechanisms. First, innovation typically yields high potential returns when technological breakthroughs occur, implying that $a$may exceed imitation-based payoffs under favorable conditions. However, innovation also involves substantial costs and risks associated with uncertain research outcomes. Second, imitation provides stable but limited gains, reflecting incremental improvements within established technological regimes.

This payoff structure captures the central trade-off faced by firms: innovation offers potentially higher rewards but exposes firms to greater risk, while imitation provides safer but lower returns.

2.3 Incorporating Macroeconomic Volatility

Real-world economic environments are characterized by substantial uncertainty arising from fluctuations in global demand, financial markets, and supply-chain conditions. Such volatility affects the expected profitability of innovation by altering the likelihood of successful technological development and the value of resulting innovations.

To capture this phenomenon, we introduce stochastic volatility into the payoff structure of the innovation strategy. Specifically, the payoff advantage of innovation is assumed to be subject to

random environmental fluctuations represented by a stochastic process. These fluctuations capture macroeconomic shocks such as commodity price volatility, financial crises, or disruptions in global production networks.

Formally, the payoff differential between innovation and imitation is perturbed by stochastic noise driven by standard Brownian motion $dW(t)$. This approach follows the stochastic evolutionary game literature, in which environmental shocks influence the evolution of strategies in large populations (Fudenberg and Harris, 1992; Cabrales, 2000).

2.4 Stochastic Replicator Dynamics

The evolution of strategies within the firm population is modeled using stochastic replicator dynamics, which describe how the frequency of strategies changes over time in response to relative payoffs. Let $\pi_I(x)$ and $\pi_M(x)$ denote the expected payoffs to innovation and imitation strategies, respectively.

In deterministic environments, the replicator equation governing the evolution of the innovation share $x(t)$ takes the form

$$\dot{x} = x(1-x)(\pi_I(x) - \pi_M(x)),$$

where the growth rate of the innovation strategy depends on its payoff advantage relative to imitation (Hofbauer and Sigmund, 1998; Sandholm, 2010).

To incorporate environmental uncertainty, stochastic volatility is introduced through a diffusion term driven by Brownian motion. The resulting stochastic replicator equation can be expressed as the stochastic differential equation

$$dx(t) = x(t)(1-x(t))(\pi_I(x) - \pi_M(x))dt + \sigma x(t)(1-x(t))dW(t),$$

where σ represents the intensity of macroeconomic volatility and $W(t)$ denotes standard Brownian motion.

This stochastic differential equation governs the evolutionary trajectory of the firm population. The first term captures the deterministic evolutionary pressure arising from payoff differences between innovation and imitation. The second term represents random environmental fluctuations that perturb the evolutionary dynamics.

The introduction of stochastic volatility fundamentally alters the stability properties of the system. In particular, sufficiently large volatility may destabilize innovation equilibria and drive the population toward imitation-dominated states. The next section provides a formal mathematical analysis of these dynamics and derives the conditions under which volatility generates persistent innovation traps.

3. Mathematical Analysis of the Innovation Trap

This section analyzes the evolutionary dynamics of the innovation–imitation game introduced in Section 2. We begin by characterizing the equilibrium structure of the deterministic system. We then examine how stochastic volatility alters the stability of these equilibria and derive conditions under which the economy converges to a stagnation equilibrium dominated by imitation strategies.

Let $x(t) \in [0,1]$ denote the proportion of firms adopting the innovation strategy at time t. The remaining share $1 - x(t)$ represents firms following imitation strategies.

3.1 Deterministic Equilibria

We first consider the deterministic replicator dynamics governing the evolution of the innovation share in the absence of environmental volatility.

Let $\pi_I(x)$ and $\pi_M(x)$ denote the expected payoffs of innovation and imitation strategies, respectively. From the payoff matrix defined in Section 2, the expected payoffs are

$$\pi_I(x) = ax + b(1 - x),$$
$$\pi_M(x) = cx + d(1 - x).$$

The deterministic replicator equation describing the evolution of the innovation share is therefore

$$\dot{x} = x(1 - x)(\pi_I(x) - \pi_M(x)).$$

Substituting the payoff expressions yields

$$\dot{x} = x(1 - x)[(a - c)x + (b - d)(1 - x)].$$

Stationary states are obtained by solving $\dot{x} = 0$. Three equilibrium candidates emerge:

$$x_1^* = 0, x_2^* = 1, x_3^* = \frac{d-b}{(a-c)+(d-b)}.$$

The boundary equilibria correspond to homogeneous populations:

- $x^* = 0$: all firms adopt imitation
- $x^* = 1$: all firms pursue innovation

The interior equilibrium x_3^*represents coexistence between innovation and imitation strategies.

The stability of these equilibria depends on the relative payoff parameters. When innovation generates sufficiently large returns relative to imitation, the innovation equilibrium $x = 1$ becomes stable. Conversely, when imitation provides higher risk-adjusted returns, the imitation equilibrium $x = 0$ becomes the globally attracting state.

In deterministic environments, technological stagnation therefore arises only when imitation strategies strictly dominate innovation. However, real-world economic environments are characterized by significant uncertainty. The next subsection shows that stochastic volatility can destabilize innovation equilibria even when innovation has a positive expected payoff advantage.

3.2 Stochastic Stability and Convergence to Stagnation

Substituting the payoff expressions defined in Section 2 yields

$$\pi_I(x) - \pi_M(x) = (b-d) - [(b-d)-(a-c)]x.$$

For notational simplicity we define

$$\alpha = b - d, \beta = (b-d) - (a-c),$$

so that the payoff differential can be written compactly as

$$\pi_I(x) - \pi_M(x) = \alpha - \beta x.$$

We now analyze the stochastic evolutionary dynamics defined in Section 2. Recall that the innovation share evolves according to the stochastic replicator equation

$$dx(t) = x(t)(1-x(t))[\pi_I(x) - \pi_M(x)]dt + \sigma x(t)(1-x(t))dW(t),$$

where $\sigma > 0$ measures the intensity of macroeconomic volatility and $W(t)$ denotes standard Brownian motion.

To analyze the long-run behavior of this system, we consider the payoff differential between innovation and imitation strategies. Suppose the payoff advantage of innovation takes the linear form

$$\pi_I(x) - \pi_M(x) = \alpha - \beta x,$$

with parameters $\alpha,\ \beta > 0$. The parameter α represents the baseline advantage of innovation when the share of innovators is small, while β captures the erosion of innovation rents as the number of innovators increases.

The stochastic dynamics can then be analyzed using the log-ratio transformation

$$y(t) = \ln\left(\frac{x(t)}{1 - x(t)}\right),$$

which measures the relative prevalence of innovation strategies in the population. Applying Itô's lemma yields the stochastic differential equation

$$dy(t) = \left[\alpha - \beta x(t) - \frac{\sigma^2}{2}\right]dt + \sigma dW(t).$$

The drift term of this process determines the long-run direction of the evolutionary dynamics. In particular, when stochastic volatility becomes sufficiently large, the negative Itô correction term $\frac{\sigma^2}{2}$ dominates the deterministic payoff advantage of innovation.

This observation leads to the main theoretical result of the section.

Theorem 1 (Volatility-Induced Innovation Trap).
Consider the stochastic replicator dynamic described above. If

$$\sigma^2 > \sqrt{2\alpha},$$

then the stochastic process $x(t)$converges almost surely to the imitation equilibrium

$$x^* = 0.$$

Proof (Sketch).

Under the condition $\sigma^2 > 2\alpha$, the drift term of the transformed process $y(t)$ becomes strictly negative for all $x(t) \in (0,1)$. Consequently, the log-ratio process $y(t)$ diverges to negative infinity almost surely, implying that the innovation share satisfies $x(t) \to 0$ as $t \to \infty$.

3.3 Economic Interpretation and Sensitivity Analysis

Theorem 1 demonstrates that macroeconomic volatility can fundamentally alter the stability structure of the innovation game. Even when innovation generates a positive expected payoff advantage, sufficiently large stochastic fluctuations can bias the evolutionary dynamics toward imitation strategies.

The mechanism driving this result arises from the interaction between payoff uncertainty and evolutionary selection. Innovation strategies typically involve higher variance in returns than imitation strategies. When volatility increases, the stochastic component of the evolutionary dynamics reduces the effective growth rate of innovation strategies. As a result, the system becomes increasingly biased toward imitation, which offers more stable payoffs.

Several parameters influence the severity of the resulting innovation trap.

First, the parameter α captures the baseline payoff advantage of innovation. Larger values of α increase the expected profitability of innovation and therefore raise the volatility threshold required to trigger stagnation.

Second, the parameter β reflects competitive pressures among innovators. When technological rents are rapidly eroded by competition, innovation incentives weaken, making the system more vulnerable to stagnation.

Finally, the volatility parameter σrepresents the intensity of macroeconomic shocks. High volatility environments characterized by financial instability, fluctuating demand, or supply-chain disruptions significantly increase the likelihood that the evolutionary dynamics converge to the imitation equilibrium.

Taken together, these results show that macroeconomic stagnation can emerge as an endogenous equilibrium outcome of decentralized strategic behavior. When firms operate in highly volatile

environments, rational responses to uncertainty can collectively suppress innovation and generate persistent technological stagnation.

This finding motivates the analysis in the next section, where we investigate how targeted policy interventions and control mechanisms can alter the evolutionary dynamics and induce a transition toward innovation-driven equilibria.

4. Mechanisms for Endogenous Stabilization

Section 3 established that sufficiently large macroeconomic volatility can destabilize innovation equilibria and drive the stochastic evolutionary system toward an imitation-dominated state. In this regime, decentralized firm behavior leads to a persistent innovation trap in which the proportion of firms pursuing innovative strategies converges toward zero despite the presence of positive aggregate returns from technological experimentation.

The existence of such traps raises a natural question: can the system be endogenously stabilized? In particular, can targeted policy mechanisms alter the incentive structure of the innovation game in a way that restores the viability of innovation strategies and prevents convergence to stagnation?

From an operations research perspective, this question can be interpreted as a control problem for stochastic evolutionary dynamics. The policymaker seeks to influence the trajectory of the evolutionary system by modifying the payoff structure governing strategic behavior. The goal of the intervention is not to dictate firm decisions directly but rather to reshape the incentive landscape so that innovation becomes evolutionarily sustainable even in the presence of stochastic volatility.

This section formalizes such interventions by introducing a control variable that captures targeted innovation-support mechanisms and analyzes how this control alters the stability properties of the stochastic evolutionary system.

4.1 Control Variable and Policy Intervention

Let $u \geq 0$ denote a policy control variable representing targeted support for innovation activities. This control variable can capture a variety of policy instruments commonly used to stimulate technological development, including:

- research and development subsidies,
- innovation tax credits,
- risk-sharing mechanisms that reduce exposure to technological uncertainty,
- infrastructure investments that lower the cost of innovation.

In the evolutionary framework developed in Section 2, the strategic interaction between firms is governed by the payoff differential between innovation and imitation strategies. The control variable modifies this payoff differential by increasing the effective return to innovation.

Specifically, the payoff advantage of innovation becomes

$$\pi_I(x) - \pi_M(x) = (\alpha + u) - \beta x,$$

where the parameter u represents the magnitude of the policy intervention.

The introduction of the control variable therefore increases the baseline profitability of innovation while leaving the competitive erosion parameter β unchanged. Economically, this specification captures the idea that policy interventions reduce the effective cost of innovation or increase its expected return without altering the fundamental competitive structure of the market.

Substituting the modified payoff differential into the stochastic replicator dynamics yields the controlled evolutionary system

$$dx(t) = x(t)(1 - x(t))[(\alpha + u) - \beta x(t)]dt + \sigma x(t)(1 - x(t))dW(t).$$

The control variable thus shifts the deterministic drift of the stochastic evolutionary system while leaving the volatility structure unchanged.

The control variable $u$represents policy support for innovation activities and is assumed to be financed through public resources. Let B denote the available policy budget and c_u the unit cost of providing innovation support. The feasible intervention level therefore satisfies the budget constraint

$$c_u u \leq B.$$

This constraint introduces an implicit trade-off between the strength of policy intervention and available fiscal resources. The policymaker's objective is therefore to determine the minimal

intervention level u capable of destabilizing the stagnation equilibrium while respecting the available budget.

4.2 Controlled Evolutionary Dynamics

To analyze the stability properties of the controlled system, we again consider the log-ratio transformation

$$y(t) = \ln\left(\frac{x(t)}{1-x(t)}\right),$$

which measures the relative prevalence of innovation strategies within the firm population.

Applying Itô's lemma to the controlled stochastic replicator equation yields the stochastic differential equation

$$dy(t) = \left[(\alpha + u) - \beta x(t) - \frac{\sigma^2}{2}\right]dt + \sigma dW(t).$$

This expression reveals how policy intervention interacts with stochastic volatility. The deterministic drift of the transformed process now depends on the adjusted innovation advantage $(\alpha+u)$ as well as the stochastic correction term $\frac{\sigma^2}{2}$.

The key insight is that policy intervention effectively counteracts the volatility penalty that suppresses innovation incentives. When the control variable is sufficiently large, the deterministic component of the evolutionary dynamics can dominate the stochastic diffusion term, preventing the system from drifting toward the imitation boundary.

This observation allows us to characterize the conditions under which policy intervention destabilizes the stagnation equilibrium and restores innovation viability.

4.3 Phase Transition to Innovation

We now derive the conditions under which policy intervention eliminates the innovation trap identified in Section 3.

Theorem 2 (Policy-Induced Escape from the Innovation Trap).

Consider the controlled stochastic replicator dynamic

$$dx(t) = x(t)(1 - x(t))[(\alpha + u) - \beta x(t)]dt + \sigma x(t)(1 - x(t))dW(t).$$

If the policy control satisfies

$$u > \frac{\sigma^2}{2} - \alpha,$$

then the imitation equilibrium $x = 0$ becomes stochastically unstable, and the controlled evolutionary system exhibits persistent positive innovation activity with high probability.

Proof (Sketch).

Applying Itô's lemma to the log-ratio transformation

$$y(t) = \ln\left(\frac{x(t)}{1 - x(t)}\right)$$

yields

$$dy(t) = \left[(\alpha + u) - \beta x(t) - \frac{\sigma^2}{2}\right]dt + \sigma dW(t).$$

Near the imitation boundary $x = 0$, the drift of the transformed process is asymptotically given by

$$(\alpha + u) - \frac{\sigma^2}{2}.$$

If

$$u > \frac{\sigma^2}{2} - \alpha,$$

this drift is strictly positive in a neighborhood of $x = 0$. Hence, trajectories starting in the interior are pushed away from the imitation boundary rather than attracted to it. The boundary $x = 0$ is therefore stochastically unstable. Consequently, the imitation equilibrium loses stability, allowing the system to admit a stable interior equilibrium characterized by persistent innovation.

Theorem 2 demonstrates that the evolutionary innovation system exhibits threshold behavior with respect to policy intervention. Define the critical intervention level

$$u_c = \frac{\sigma^2}{2} - \alpha.$$

When the policy parameter satisfies $u < u_c$, volatility continues to dominate the evolutionary dynamics and the system remains trapped in the imitation equilibrium. However, once the intervention exceeds this critical threshold, the stagnation equilibrium loses stability and the system undergoes a phase transition toward innovation-driven dynamics.

Economically, this result implies that modest policy interventions may have little effect in highly volatile environments. When firms face substantial uncertainty, incremental incentives are insufficient to offset the perceived risks of innovation. Only when policy support becomes sufficiently strong does the innovation strategy regain evolutionary viability.

This threshold structure provides a theoretical explanation for why technological transitions often appear abrupt rather than gradual. Once the effective profitability of innovation crosses the critical boundary determined by u_c, the evolutionary system rapidly shifts toward innovation-dominated dynamics.

From an operations research perspective, these findings highlight the importance of optimal policy design under uncertainty. Policymakers must determine the minimum intervention required to move the system from the stagnation region to the innovation region of the parameter space.

The phase-transition condition also induces a natural policy optimization problem. Let c_u denote the unit cost of intervention and B the available policy budget. The policymaker seeks the minimum intervention capable of destabilizing the imitation trap:

$$\min_{u \geq 0} \; c_u u$$

subject to

$$u \geq \max\left\{0, \frac{\sigma^2}{2} - \alpha\right\}, c_u u \leq B.$$

The optimal intervention is therefore the threshold level

$$u^* = \max\left\{0, \frac{\sigma^2}{2} - \alpha\right\},$$

provided that $c_u u^* \leq B$. This formulation makes explicit the resource feasibility condition for endogenous stabilization.

The next section explores these dynamics numerically by simulating the stochastic evolutionary system under different volatility and policy scenarios.

5. Numerical Experiments and Policy Phase Analysis

The analytical results derived in Sections 3 and 4 establish that macroeconomic volatility can induce an innovation trap in decentralized economic systems and that sufficiently strong policy intervention can restore the viability of innovation strategies. In this section, we complement the theoretical analysis with numerical experiments designed to illustrate the stochastic dynamics of the model and to visualize the policy thresholds derived in Theorem 2.

The numerical experiments serve two purposes. First, they demonstrate how the stochastic replicator dynamics evolve over time under different policy regimes. Second, they illustrate how the interaction between macroeconomic volatility and policy intervention determines the long-run level of innovation activity in the economy.

5.1 Simulation Design

The stochastic evolutionary system analyzed in this paper is governed by the controlled stochastic replicator equation

$$dx(t) = x(t)(1 - x(t))[(\alpha + u) - \beta x(t)]dt + \sigma x(t)(1 - x(t))dW(t),$$

where $x(t)$ represents the proportion of firms pursuing innovation strategies. The parameters α, β, and σrepresent the baseline advantage of innovation, competitive erosion among innovators, and macroeconomic volatility, respectively. The policy variable $u$captures the intensity of targeted innovation support.

The system was simulated using an Euler–Maruyama discretization applied to the logit-transformed state variable

$$y(t) = \log \left(\frac{x(t)}{1 - x(t)}\right),$$

which guarantees that the recovered innovation share $x(t)$ remains within the admissible interval $(0, 1)$. Simulations were conducted over a time horizon of 50 periods with time step $\Delta t = 0.01$. For each policy scenario, Monte Carlo experiments were performed using 5,000 sample paths.

Unless otherwise stated, the baseline parameter values used in the numerical experiments are

$$\alpha = 0.05, \beta = 0.15, \sigma = 0.40.$$

Under these parameters, the theoretical stabilization threshold derived in Section 4 is

$$u_c = \frac{\sigma^2}{2} - \alpha.$$

This threshold determines the minimum level of policy intervention required to destabilize the imitation equilibrium.

5.2 Evolutionary Dynamics Under Alternative Policy Regimes

We first examine the time evolution of the innovation share under three policy regimes relative to the theoretical threshold u_c. The results are shown in Figure 1, which displays Monte Carlo trajectories of the stochastic system.

Three policy scenarios are considered:

1. Baseline regime **($u = 0$)**
   In the absence of policy intervention, macroeconomic volatility dominates the evolutionary dynamics. As predicted by Theorem 1, the proportion of innovating firms declines over time and the system converges toward an imitation-dominated equilibrium. Individual trajectories exhibit stochastic fluctuations, but the population mean steadily decreases, illustrating the emergence of the innovation trap.

2. Sub-critical intervention **($u < u_c$)**
   When policy intervention is introduced but remains below the critical threshold, the innovation trap persists. Although innovation incentives improve slightly, the stochastic drift remains negative over most of the state space. As a result, the system continues to

converge toward low innovation levels. This scenario demonstrates that modest policy support is insufficient to offset the volatility penalty imposed on innovation strategies.

3. Escape regime **($u > u_c$)**

   When the intervention level exceeds the theoretical threshold, the evolutionary dynamics change qualitatively. The imitation boundary becomes unstable and the innovation share remains persistently positive. Sample trajectories fluctuate but no longer collapse toward zero. This regime corresponds to the policy-induced phase transition predicted by Theorem 2.

Together, these results confirm that the long-run dynamics of the innovation system exhibit strong nonlinear threshold behavior. Below the stabilization threshold, innovation collapses; above the threshold, innovation becomes self-sustaining.

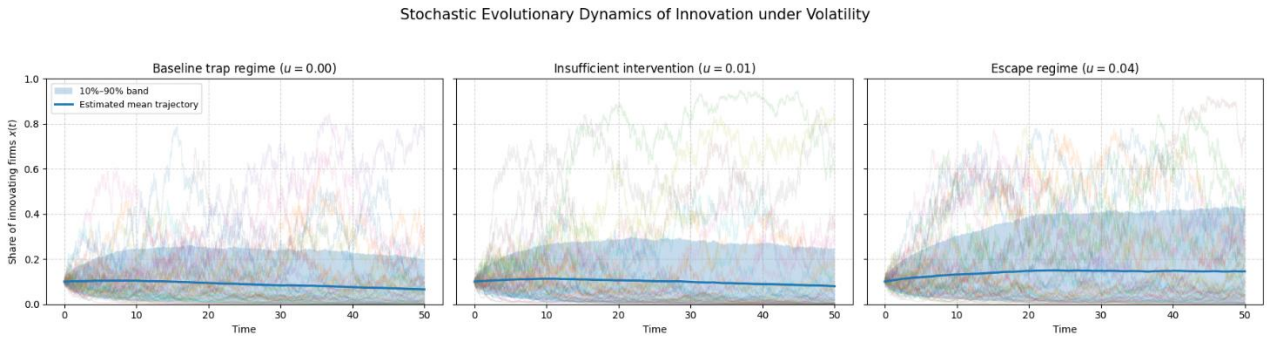


Figure 1. Monte Carlo simulation of the controlled stochastic replicator dynamics. Simulation results for the stochastic evolutionary system

$$dx = x(1-x)[(\alpha + u) - \beta x]dt + \sigma x(1-x)dW_t,$$

illustrating the effect of policy intervention on the innovation share $x(t)$. Three regimes are shown relative to the theoretical threshold $u_c = \sigma^2/2 - \alpha$. In the baseline case ($u = 0$), stochastic volatility drives the system toward the imitation boundary. Sub-critical intervention ($u = 0.01 < u_c$) remains insufficient to restore innovation incentives. When the policy level exceeds the threshold ($u = 0.04 > u_c$), the imitation boundary becomes unstable and innovation persists over time. Individual trajectories represent simulated sample paths, while the dark line and shaded region denote the Monte Carlo mean and the 10–90% quantile interval across simulations.

5.3 Drift Structure and Stability Mechanism

To better understand the mechanism underlying this transition, Figure 2 plots the deterministic drift of the log-ratio process

$$\mu(x) = (\alpha + u) - \beta x - \frac{\sigma^2}{2}.$$

The drift function determines the direction of evolutionary pressure acting on the innovation share.

When $u < u_c$, the drift remains negative over most of the state space. Consequently, the evolutionary dynamics push the system toward the imitation boundary $x = 0$, reinforcing the innovation trap.

As the intervention level increases, the drift function shifts upward. When $u > u_c$, the drift becomes positive near the imitation boundary. This change destabilizes the stagnation equilibrium and pushes the system away from the boundary, allowing innovation strategies to persist.

This analysis clarifies the economic mechanism behind the phase transition: policy intervention effectively offsets the volatility penalty imposed on innovation strategies.

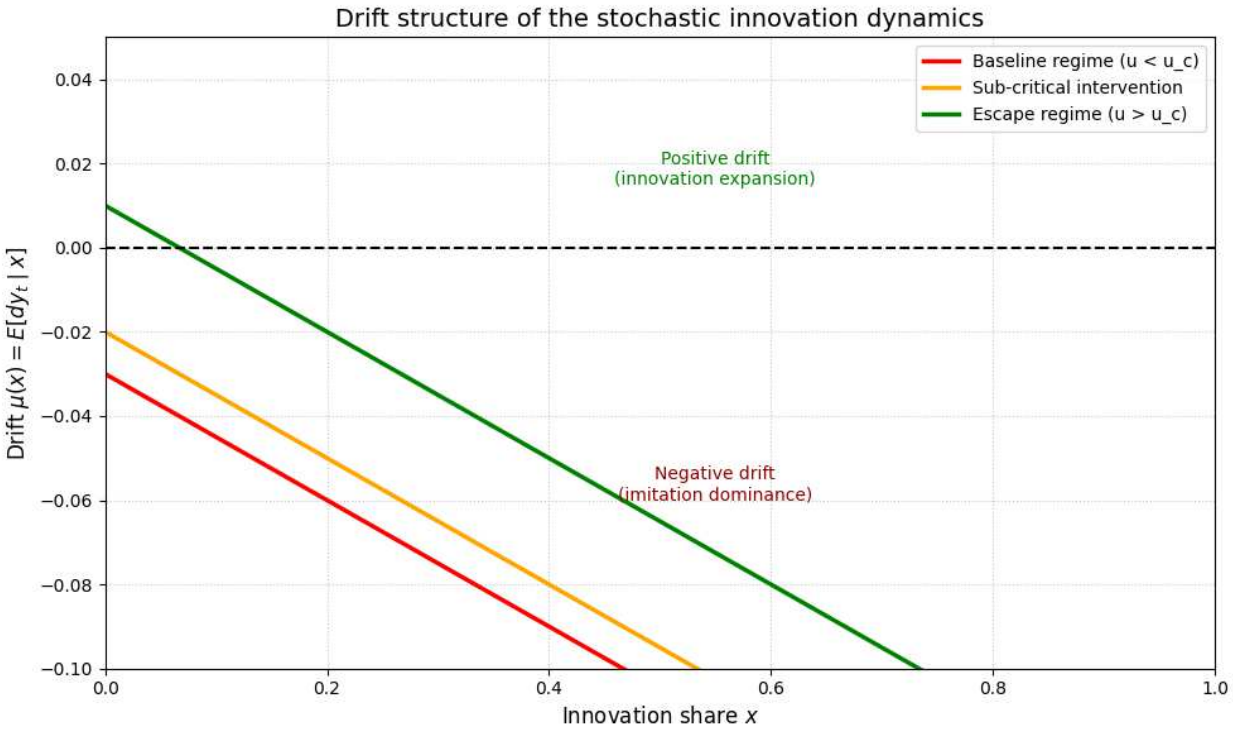


Figure 2. Drift structure of the stochastic innovation dynamics.

Deterministic drift $\mu(x) = (\alpha + u) - \beta x - \frac{\sigma^2}{2}$ of the log-ratio process governing the stochastic replicator dynamics. When $u < u_c$, the drift remains negative over most of the state space, pushing the system toward imitation-dominated outcomes. As the policy intervention increases, the drift shifts upward. When $u > u_c$, the drift becomes positive near the imitation boundary, destabilizing the stagnation equilibrium and allowing innovation to persist.

5.4 Policy Phase Space

While Figures 1 and 2 illustrate the dynamics under specific policy scenarios, it is also useful to examine how innovation outcomes vary across the entire parameter space. Figure 3 presents a phase diagram of the controlled innovation system in the (σ, u) plane.

The heatmap should be interpreted as a drift-based phase representation of the transformed dynamics rather than as a literal deterministic equilibrium surface of the full stochastic system.

The dashed curve

$$u^*(\sigma) = \max\left\{0, \frac{\sigma^2}{2} - \alpha\right\}$$

represents the critical stabilization threshold derived in Theorem 2. This curve divides the parameter space into two distinct regimes.

Below the threshold lies the imitation-dominant region, where macroeconomic volatility overwhelms innovation incentives and the economy converges toward technological stagnation. Above the threshold lies the innovation-persistent region, where policy intervention is sufficient to sustain positive levels of innovation activity.

The heatmap illustrates the drift-neutral innovation share implied by the transformed stochastic dynamics. As volatility increases, the stabilization boundary shifts upward, indicating that progressively stronger policy intervention is required to maintain innovation incentives.

This phase diagram provides a compact visual summary of the model's central insight: macroeconomic volatility and policy intervention jointly determine the long-run innovation capacity of the economy.

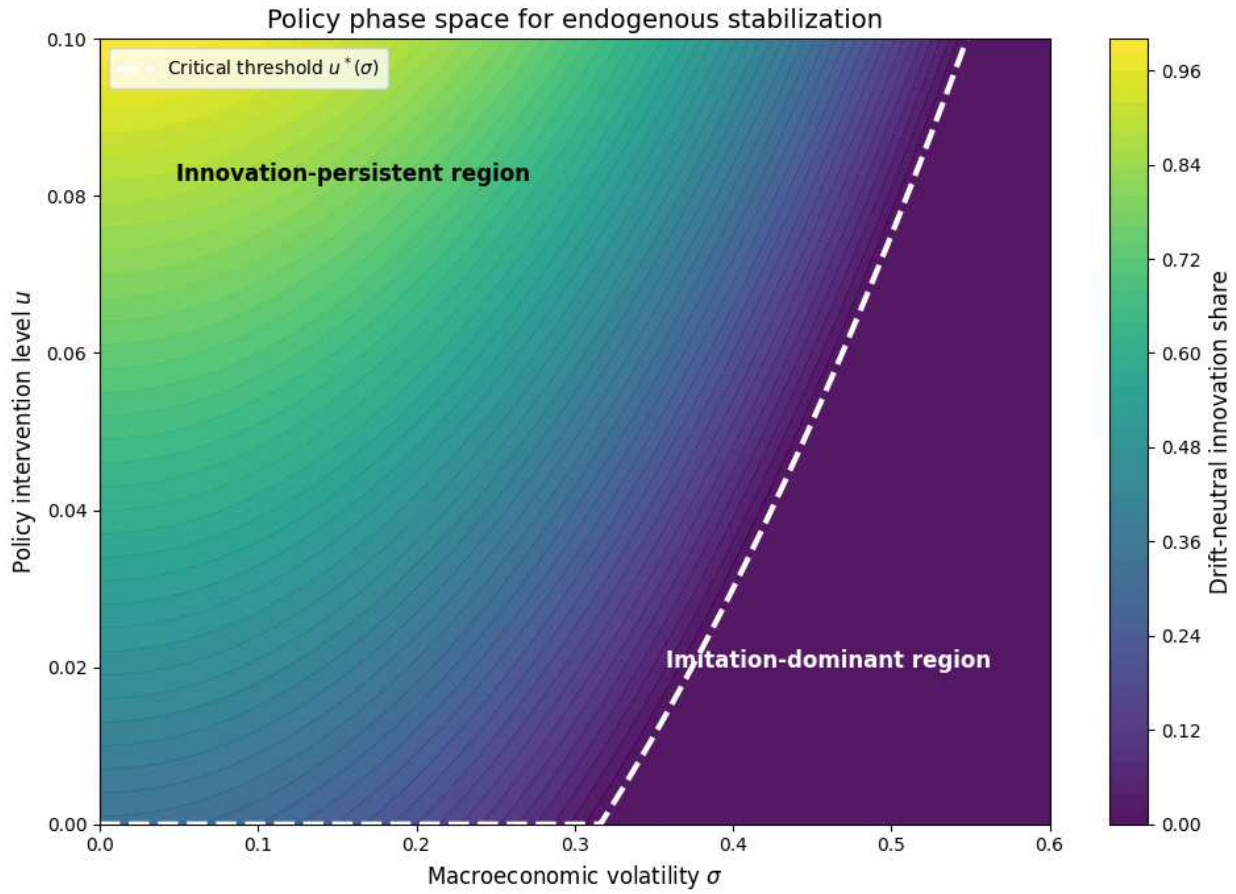


Figure 3. Policy phase space for endogenous stabilization.

Phase diagram of the controlled stochastic innovation system in the (σ, u)plane. The dashed curve marks the critical stabilization threshold

$$u^*(\sigma) = \max\left\{0, \frac{\sigma^2}{2} - \alpha\right\},$$

which separates the imitation-dominant region from the innovation-persistent region. The heatmap shows the drift-neutral innovation share implied by the transformed stochastic dynamics, truncated to the feasible interval $[0, 1]$. As macroeconomic volatility increases, progressively stronger policy intervention is required to offset the volatility penalty and sustain positive innovation activity.

5.5 Policy Implications and Summary of Numerical Findings

The numerical experiments highlight the nonlinear nature of innovation dynamics in volatile economic environments and reinforce the theoretical conclusions developed in the earlier sections of the paper. In particular, the simulations confirm that the interaction between macroeconomic volatility and policy intervention generates clear regime shifts in the long-run behavior of the innovation system. When volatility is sufficiently high and policy intervention remains below the critical threshold $u_c = \frac{\sigma^2}{2} - \alpha$, the stochastic evolutionary dynamics consistently converge toward imitation-dominated outcomes. Under these conditions, firms collectively avoid risky technological experimentation, and the economy becomes trapped in a low-innovation equilibrium.

The simulations also demonstrate that modest policy interventions may have only limited impact when volatility remains elevated. Although incremental increases in innovation support can improve expected payoffs locally, they do not fundamentally alter the stability structure of the evolutionary system unless the intervention exceeds the theoretical threshold derived in Section 4. As a result, policy efforts that fall short of this threshold tend to produce only marginal improvements in innovation activity without eliminating the underlying stagnation trap.

Once the intervention level exceeds the stabilization threshold, however, the system undergoes a qualitative transition toward innovation-driven dynamics. In this regime, the imitation boundary loses its attracting property and the share of innovating firms remains persistently positive. The

numerical experiments therefore confirm the phase-transition mechanism predicted by Theorem 2: sufficiently strong policy intervention offsets the volatility penalty imposed on innovation and restores the evolutionary viability of technological experimentation.

Taken together, these findings underscore the importance of threshold-based policy design in innovation systems subject to uncertainty. From an operations research perspective, the problem faced by policymakers can be interpreted as a resource allocation decision under stochastic dynamics. The objective is not merely to increase innovation incentives incrementally but to allocate sufficient intervention resources to push the system beyond the stabilization boundary that separates stagnation from innovation-driven growth.

6. Conclusion

This paper develops a stochastic evolutionary game-theoretic framework to explain how macroeconomic volatility can endogenously generate persistent innovation failures in decentralized economic systems. Modeling the economy as a large population of firms choosing between innovation and imitation strategies, we show that environmental uncertainty fundamentally alters the stability properties of technological competition. When volatility exceeds a critical threshold, the stochastic evolutionary dynamics converge toward an imitation-dominated equilibrium, creating an innovation trap in which firms collectively avoid risky technological experimentation despite positive aggregate returns from innovation.

The theoretical analysis identifies the mechanism driving this outcome. In the stochastic replicator dynamics governing firm behavior, volatility introduces a negative correction term that effectively penalizes innovation strategies. When this volatility penalty exceeds the baseline advantage of innovation, the imitation boundary becomes globally attracting, leading the economy toward technological stagnation. This result provides a formal explanation for why many emerging economies struggle to transition from imitation-based production toward innovation-driven growth, particularly under conditions of macroeconomic instability.

The paper also demonstrates that such stagnation traps can be overcome through targeted policy interventions. Introducing a control variable that increases the effective payoff of innovation yields a critical stabilization threshold. When policy support exceeds this threshold, the imitation equilibrium loses its stochastic stability and innovation becomes persistent. From an operations

research perspective, this result transforms the policy question into a tractable optimization problem: policymakers must determine the minimum intervention required to move the system from the stagnation region to the innovation region while respecting resource constraints.

Numerical experiments confirm the analytical results and illustrate the nonlinear nature of innovation dynamics under uncertainty. Monte Carlo simulations reveal that modest interventions have little impact when volatility remains high, whereas sufficiently strong interventions trigger a phase transition toward sustained innovation activity. A policy phase-space analysis further shows how volatility and policy intensity jointly determine the long-run innovation capacity of the economy.

Beyond its immediate application to innovation policy, the framework developed in this paper contributes more broadly to the study of strategic decision systems under uncertainty. By integrating stochastic evolutionary dynamics with policy control mechanisms, the model provides a tractable analytical tool for studying how decentralized economic systems respond to volatile environments. Future research could extend this framework by incorporating heterogeneous firms, network-based technological spillovers, or continuous innovation investment strategies, thereby providing a richer representation of real-world innovation ecosystems.

Disclosure

Declaration of Generative AI and AI-assisted technologies in the writing process.

In the course of preparing this work, the author utilized ChatGPT (OpenAI) and Gemini for language enhancement and structural editing. The author has examined and revised the content as necessary and assumes complete responsibility for the scientific accuracy and integrity of the paper.

Declarations

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Conflict of Interest The author declares that there is no conflict of interest regarding the publication of this paper.

Data Availability The simulated datasets and numerical code generated during the current study are available from the corresponding author on reasonable request. All foundational parameters required to reproduce the findings are included in the published article.

Ethical Approval This study does not involve human participants or animals and therefore does not require ethical approval.

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