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Declarations

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Author Contributions Statement

I.M.B. conducted the research, designed the methodology, implemented the methods, performed the analyses, and interpreted the results, as well as prepared the initial draft of the manuscript. F.R.L.J. contributed to the conceptualization of the study, methodological guidance, analysis and interpretation of the results, and revision and editing of the manuscript. All authors reviewed and approved the final version of the manuscript.

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Abstract

The supplier selection process is fundamental to establishing an effective supply chain. One way to address this process is through the use of multicriteria decision-making methods. Among these, Hesitant Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) methods stand out as an emerging approach capable of supporting decision-making processes in situations involving uncertainty and hesitation. However, the literature lacks studies that compare these methods in order to support the selection of the most appropriate approach for a given problem. In this context, this study performs a comparison between two Hesitant Fuzzy TOPSIS methods considering the supplier selection problem. The two methods were selected based on a mapping of different versions of the Hesitant Fuzzy TOPSIS approach. Both models were implemented and applied to an illustrative supplier selection case. The models were compared in terms of the ranking of alternatives, the consistency of the results, and the similarities and differences in the structure of the methods. The results highlight the characteristics of the methods and may assist researchers and managers in selecting an appropriate method for supplier selection and other multicriteria decision-making problems.

Keywords: Supplier selection; Multicriteria decision-making; Hesitant Fuzzy TOPSIS; Comparative study.

1. Introduction

Many decisions must be made within organizations, requiring managers to exercise discernment and make advantageous choices to achieve financial and economic objectives. In an industrial context, the selection of suppliers with whom purchasing relationships will be established is fundamental, both because of its direct impact on costs, results, and product quality, and due to the increasingly critical need to build an effective supply chain (Resende et al., 2023; Saputro et al., 2024).

Supplier selection is a complex process involving a wide range of factors to be considered for each available alternative. Therefore, to perform the selection appropriately, multiple evaluation criteria relevant to the intended relationship with each potential supplier must be considered (Saputro et al., 2022). A widely adopted approach to addressing problems involving multiple criteria is Multicriteria Decision-Making (MCDM) methods (Ghafori & Abdallah, 2024; Zhang & Wu, 2025; Zhan & Zhang, 2025).

Hundreds of MCDM methods have been used to support supplier selection, each with particular characteristics, which increases the difficulty of choosing the most appropriate technique for a given situation (Zimmer et al., 2016; Lima et al., 2023). Despite this variety, comparative studies investigating the characteristics, differences, advantages, and limitations of multiple techniques applied to the same selection problem remain scarce in the literature (Oliveira et al., 2023; Beleski & Lima, 2025). Moreover, most of these studies do not present well-defined comparison criteria; instead, they merely apply multiple techniques to the same problem and compare the numerical results from each (Yavuz, 2027).

Among the various methods used in the literature for supplier selection, those that incorporate fuzzy logic have received increasing attention due to their ability to handle uncertainty. As an extension of these methods, approaches based on Hesitant Fuzzy Sets (HFS) or their variations have been proposed as effective means of representing and handling decision-makers'

hesitations throughout the decision-making process (Rodríguez et al., 2012; Nascimento et al., 2025). Hesitation generally arises from vague, imprecise, or incomplete information, as well as from the inherent complexity of decision problems. Furthermore, experts involved in decision-making processes often cannot express their knowledge using a single linguistic term, tending instead to oscillate between different possible evaluations when making judgments (Rodríguez et al., 2012; Magalhães & Lima, 2025).

One emerging MCDM approach in the literature is the Hesitant Fuzzy TOPSIS (HF-TOPSIS) method, which presents several variations. The first version, developed by Beg and Rashid (2013), proposed the use of Hesitant Fuzzy Linguistic Term Sets (HFLTS) throughout the TOPSIS procedure, with certain adaptations. Subsequently, other authors proposed additional variations of this method (Sun et al., 2018; Wu et al., 2019; Sun et al., 2023; Song et al., 2024; Abughazalah & Khan, 2025; Cheng & Ning, 2025; Zhang & Wu, 2025; Zhan & Zhang, 2025). Despite the growing prominence of HF-TOPSIS in the academic literature, comparative studies involving these methods remain limited (Yavuz, 2017; Beleski & Lima, 2025).

Although TOPSIS and Fuzzy TOPSIS are among the most widely applied techniques in supplier selection and in comparative studies (Lima et al., 2014; Lima & Carpinetti, 2015; Saputro et al., 2022; Oliveira et al., 2023), comparative analyses between these different formulations are still scarce. To the best of our knowledge, only one study has attempted to compare different HF-TOPSIS approaches, as presented by Beleski and Lima (2025). Considering that HF-TOPSIS methods are designed to address decision-making situations involving uncertainty and hesitation, different versions of these methods may lead to distinct decision outcomes. Therefore, a systematic comparison of HF-TOPSIS approaches within the supplier selection context becomes particularly relevant to support decision makers in the identification of the most suitable method for this type of decision problem.

Accordingly, the main objective of this study is to compare two multicriteria decision-making methods based on the HF-TOPSIS approach through illustrative applications in a supplier selection context and an analysis of their characteristics against predefined comparison factors. The methods proposed by Beg and Rashid (2013) and Wu et al. (2019) were selected because of their support for group decision-making, use of linguistic expressions, and lower implementation complexity.

2. Methodological Procedures

This study can be characterized as quantitative research based on modeling and simulation. Regarding simulation, it is classified as static, since the time factor does not influence it, and as deterministic, since the model variables do not exhibit random fluctuations. The following steps were defined for conducting this study:

a) Literature review: A theoretical review of the supplier selection process was conducted to support the study. Additionally, a mapping of studies proposing variations of the HF-TOPSIS method was carried out. Comparative studies of MCDM methods were also analyzed, helping identify the comparison factors used in this research. The works considered were collected from ScienceDirect, Emerald Insight, Springer, IEEE Xplore, Scopus, and MDPI, as well as Google Scholar.

b) Computational implementation of the models: Two HF-TOPSIS methods were selected for comparison based on their ability to support group decision-making, their use of linguistic expressions, and their implementation complexity. The HF-TOPSIS methods proposed by Beg and Rashid (2013) and Wu et al. (2019) were selected. They were implemented using Microsoft Excel, chosen for its ease of use and rapid deployment.

c) Application of the models: The data used to evaluate supplier performance were based on Oliveira et al. (2023). These data were collected from an electrical services company operating within a supply chain and were originally used to compare two HF-VIKOR

(Visekriterijumska Optimizacija I Kompromisno Resenje) methods. This secondary data source was selected because it included evaluations from two decision-makers, expressed in linguistic terms and expressions. The data were used in an illustrative application. The problem was solved using both implemented methods, with identical input data in both cases to ensure comparability.

d) Analysis and comparison of the models: The results from the applications were compared, and the methods were analyzed against selected comparison factors to highlight their advantages, limitations, and characteristics, as well as similarities and differences in applications and outcomes. Computational experiments were conducted at this stage to evaluate the effect of introducing new alternatives on supplier ranking. Finally, the results were organized and discussed, emphasizing the contributions of a comparative study between MCDM techniques.

3. Theoretical Background

3.1 Decision-Making for Supplier Selection

Supplier selection is a strategic decision that directly impacts production costs, product quality, and ultimately the value delivered to the final customer, which justifies the relevance and care devoted to this process (Zimmer et al., 2016; Oliveira et al., 2023). The supplier selection process begins with identifying organizational needs, followed by formulating criteria to address them. A qualification process for available suppliers is then carried out, after which suppliers are evaluated and finally selected among the qualified candidates (Saputro et al., 2022).

To evaluate alternatives within the supplier selection process, criteria must accurately describe the quantitative and qualitative aspects that enable the identification of suppliers best suited to the buyer's specifications (Ghafori & Abdallah, 2024). Economic selection criteria commonly used in the literature include cost, quality, service, flexibility, and delivery. Environmental criteria frequently adopted include materials, energy, emissions, environmental capabilities, and commitment to sustainability. From a social perspective, criteria such as wages, diversity, community involvement, and social commitment are often considered (Zimmer et al., 2016; Liang et al., 2023; Resende et al., 2023).

MCDM methods have been widely used to structure, model, and solve supplier selection problems (Ghafori & Abdallah, 2024). Different techniques rely on distinct assumptions and mathematical structures, which may influence the results and the quality of decision support (Resende et al., 2023). Therefore, it is necessary to establish criteria for comparing and evaluating such methods. A critical factor is ranking consistency when the decision problem undergoes changes, especially with the addition or removal of alternatives. Oliveira et al. (2023) highlight that some methods may exhibit structural instability, a phenomenon associated with the so-called ranking reversal effect. This aspect is particularly relevant because new suppliers may emerge and existing suppliers may cease operations (Lima et al., 2014).

Since supplier selection typically involves multiple decision-makers from areas such as procurement, quality, engineering, and sustainability, it is also essential that the selected method allows for consistent aggregation of opinions while preserving divergent opinions (Nascimento et al., 2025). Furthermore, as supply chain environments are characterized by incomplete information and imprecise judgments, methods that explicitly incorporate mechanisms to handle hesitation offer greater capability to represent decision-makers' cognitive reality, thereby increasing the reliability of results (Saputro et al., 2024; Magalhães & Lima, 2025).

The choice of a supplier selection method should also consider how its intrinsic characteristics relate to the problem context (Lima et al., 2014). Structural analysis enables understanding of the theoretical and practical implications of adopting each technique. Among

the main structural aspects to be considered are: 1) The use of weights for criteria and decision-makers; 2) The operators adopted, since these directly influence final results; 3) The format of input data, as the ability to adequately represent the informational structure of the problem increases modeling fidelity; 4) The number of alternatives and criteria, as scalability is a fundamental practical factor (Ghaleb et al., 2020; Oliveira et al., 2023).

Therefore, selecting an appropriate supplier selection approach should not be based solely on method popularity or mathematical simplicity. A systematic comparison of methods enables not only the identification of those most suitable for specific contexts but also an understanding of the managerial implications of adopting each approach in the supplier selection process.

3.2 Comparative Studies between Multicriteria Decision-Making Methods

Table 1 presents several studies comparing MCDM methods. The application of multiple techniques to the same decision problem is a common strategy for validating new methods or comparing existing ones. A central aspect of such comparative studies concerns the comparison factors adopted, which are listed in Table 1. Through these factors, it is possible to highlight the benefits and limitations of different decision techniques when applied in specific contexts.

Table 1 – Studies focusing on MCDM methods comparison

Study	Compared Techniques	Comparison Factors Considered
Lima et al. (2014)	Fuzzy AHP and Fuzzy TOPSIS	Consistency, agility in the decision-making process, computational complexity, adequacy for group decision-making, the number of suppliers and criteria, and modeling uncertainty.
Lima and Carpinetti (2015)	TOPSIS and Fuzzy TOPSIS	Number of judgements required, advantages of use, limitations of the methods, computational complexity, comparison of the algorithmic structures, and the supplied results.
Yavuz (2017)	HFLVIKOR, HFL-TODIM, and HFL-TOPSIS	Calculation methods, the relative importance of criteria, differences in terms of the determination of the criteria weights, verification of consistency, problem structure, and the final results.
Apan et al. (2018)	*Altman Z-Score* and VIKOR	Comparison of the results through the application of *Spearman* correlations.
Ghaleb et al. (2020)	AHP, TOPSIS, and VIKOR	Changes in alternatives or criteria, agility in the decision-making process, computational complexity, appropriateness for group decision-making, and modeling uncertainty.
Oliveira et al. (2023)	Extended HFL-VIKOR and Extended HFL-VIKOR with possibility distributions	Structure of the algorithms, consistency of the results, group decision making, data collection effort, computational complexity, and capacity to model uncertainty.
Pinto-DelaCadena et al. (2024)	TOPSIS, VIKOR, COPRAS, EDAS, MOORA, and WASPAS	Ranking stability, sensitivity to weight variations, robustness, correlation between methods, result dispersion, and ranking convergence.
Li et al. (2024)	TOPSIS, VIKOR, PROMETHEE, MOORA, COPRAS, EDAS, and WASPAS	Similarity between rankings, robustness, statistical correlation between methods, and stability of the best alternative.
Beleski e Lima (2025)	HF-TOPSIS e HFL-TOPSIS	Similarity between rankings, support for group decision-making, ability to model uncertainty, use of weights, and types of inputs.
Malefaki et al. (2025)	AHP, TOPSIS, VIKOR, PROMETHEE, WASPAS	Differences in the final ranking, consistency between methods, variation with changes in criteria, and stability of the dominant alternatives.
Wieckowski et al. (2025)	TOPSIS, VIKOR, PROMETHEE,	Sensitivity to weight perturbations, sensitivity to variations in evaluations, and robustness.

	ELECTRE, MOORA, COPRAS

Source: Prepared by the authors

By applying multiple methods within certain domains and analyzing them according to selected comparison factors, it becomes possible to indicate which methods may be more effectively applied, depending on the specific needs of the decision-making situation (Lima et al., 2014; Oliveira et al., 2023). Among the recurring factors are differences in alternative rankings; the number of alternatives and criteria; consistency of results when alternatives or criteria are added or removed; and conceptual similarities and differences between techniques. Other relevant aspects include result consistency, support for group decision-making, the effort required for judgment elicitation, computational complexity, and agility in the decision-making process. Consolidating these factors enables a broader and more rigorous evaluation, fostering alignment between method characteristics and the specific requirements of the decision problem.

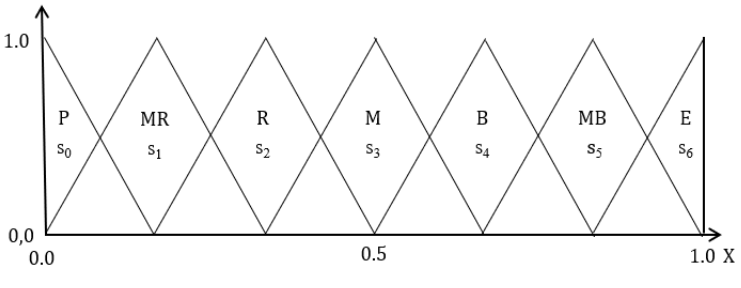
Although dozens of comparative studies involving MCDM techniques exist, only three comparative studies of methods based on HFLTS were identified: those proposed by Yavuz (2017), Oliveira et al. (2023), and Beleski and Lima (2025). The present study complements the work of Beleski and Lima (2025) by analyzing HF-TOPSIS methods that were not previously compared.

3.3 *Hesitant Fuzzy* TOPSIS

3.3.1 Fundamentals of Hesitant Fuzzy Sets

Hesitant Fuzzy Sets (HFS), developed by Torra (2010), allow modeling uncertainty arising from hesitation in the decision-making process. According to Torra (2010), the central issue addressed by this theory concerns defining the membership degree of an element in a given set, since in many real decision contexts, the decision-maker cannot express this association with a single precise value. Instead, in HFS, the evaluation is represented by a set of possible membership degrees.

Based on HFS, Rodríguez et al. (2012) introduced Hesitant Fuzzy Linguistic Term Sets (HFLTS), which allow the use of multiple linguistic terms to express decision-makers' judgments. With this approach, judgments can be represented by linguistic intervals such as "between medium and good" or "at least good." Figure 1 presents an HFLTS set represented by S, where $S = \{s_0,\ldots,s_g\}$. Each decision-maker's judgment is represented by a finite ordered subset of consecutive linguistic terms from S. Each HFLTS is therefore associated with a linguistic term quantified by envelopes defined by its lower bound (s_p) and upper bound (s_q) (Rodriguez et al., 2012).



Termo linguístico	HFLTS	Envoltório [s_p, s_q]
P: Péssimo	s_0	[0, 0]
MR: Muito Ruim	s_1	[1, 1]
R: Ruim	s_2	[2, 2]
M: Médio	s_3	[3, 3]
B: Bom	s_4	[4, 4]
MB: Muito Bom	s_5	[5, 5]
E: Excelente	s_6	[6, 6]

Figure 1 - Representation of an HFLTS with seven linguistic terms. Source: Prepared by the authors

3.3.2 Mapping of Hesitant Fuzzy TOPSIS Methods

Since the introduction of Hesitant Fuzzy Set theory by Torra (2010) to support decision-making under uncertainty and hesitation, several combinations have been developed that integrate this theory or its extensions with the TOPSIS method. Table 2 summarises the

different HF-TOPSIS approaches in the literature, highlighting the form of representation of alternative scores, whether the method supports group decision-making, and whether linguistic expressions can be used. This mapping of methodological characteristics (shown in columns) was developed by the authors of this study and supported the selection of the methods compared.

Tabela 2 – Mapping of the HF-TOPSIS approaches

Authors	**Form of Representation of the Alternatives' Scores.**	**Group decision**	**Linguistic Expressions**
Beg e Rashid (2013)	HFLTS	Yes	Yes
Xu e Zhang (2013)	*Hesitant Fuzzy Sets* (HFS)	Yes	No
Liu e Rodriguez (2014)	HFLTS	No	Yes
Onar, Oztaysi e Kahraman (2014)	HFS	Yes	No
Liao e Xu (2015)	HFLTS	No	Yes
Senvar, Otay, and Boltürk (2016)	*Triangular Fuzzy* HFS	Yes	No
Wu et al. (2019)	*Possibility Distribution* HFLTS	Yes	Yes
Riaz et al. (2022)	*Cubic Hesitant Fuzzy Sets* (CHFS)	Yes	No
Jin, Mi, Li e Liang (2023)	HFS	No	No
Liang et al. (2023)	HFS	Yes	No
Sun, Luo e Wang (2023)	*Pythagorean Fuzzy Numbers* (PFN)	Yes	No
Song et al. (2024)	*Probabilistic* HFS	Yes	No
Abughazalah e Khan (2025)	*Dual* HFS	Yes	No
Cheng e Ning (2025)	*Probabilistic Picture* HFS	Yes	No
Zhang e Wu (2025)	HFLTS	No	Yes
Zhan e Zhang (2025)	*Probabilistic Hesitant Fermatean Fuzzy Linguistic Term Sets* (PHFFLTS)	Yes	Yes

To select the methods analyzed herein, the following criteria were adopted:

a) Support for group decision-making - Since supplier selection typically involves experts from different functional areas, it is desirable that the method accommodate multiple decision-makers' judgments.

b) Compatibility of input value types - To ensure consistency in comparison, the same input values (supplier evaluations) must apply to both compared methods.

c) Method complexity - This criterion considered ease of understanding, the number of operations involved, and the complexity of their implementation.

Based on these criteria, the methods proposed by Beg and Rashid (2013) and Wu et al. (2019) were selected and are detailed in the following subsections.

3.3.3 Method 1 - *Hesitant Fuzzy Linguistic* TOPSIS

The Hesitant Fuzzy Linguistic TOPSIS method developed by Beg and Rashid (2013) follows the same principle as the classical TOPSIS method, prioritizing alternatives based on their proximity to the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). The PIS is formed by the best performance values achieved by alternatives in each criterion, while the NIS is composed of the worst values. The steps of the method are summarized below based on Beg and Rashid (2013) and Magalhães and Lima (2025):

a) **Step 1:** Let $\tilde{X}^l = \left[H_{Sij}^l\right]_{mxn}$ be a fuzzy decision matrix; $E = \{e_1, e_2, \dots, e_k\}$ is the set of experts involved in the decision-making process; $A = \{A_1, A_2, \dots, A_m\}$ is the set of alternatives considered; and $C = \{C_1, C_2, \dots, C_n\}$ is the set of criteria used to evaluate the alternatives. The performance of alternative A_i with respect to criterion C_j is denoted by x_{ij}. The aggregated decision matrix $X = \left[x_{ij}\right]$, with $X_{ij} = \left[[S_{pij}, S_{qij}\right]$, i]s calculated considering the various opinions of the decision-makers $\left(\widetilde{(\tilde{X}^1}, \tilde{X}^2, \dots, \widetilde{\quad}\right)$ Xk), according to Equations 1 and 2.

$$s_{pij} = \min\left\{ \min_{l=1}^{k} \left(\max\ H_{Sij}^{l}\right), \max_{l=1}^{k} \left(\min H_{Sij}^{l}\right) \right\} \quad (1)$$

$$s_{qij} = \max\left\{ \min_{l=1}^{k} \left(\max H_{Sij}^{l}\right), \max_{l=1}^{k} \left(\min H_{Sij}^{l}\right) \right\} \quad (2)$$

b) **Step 2**: Let Ω_b be a collection of benefit criteria (i.e., the higher the performance in C_j, the higher the final score) and Ω_c be a collection of cost criteria (the lower the performance in C_j, the higher the final score). The PIS is represented as $\tilde{A}^+ = (\tilde{V}_1^+, \tilde{V}_2^+ ..., \tilde{V}_n^+)$ and the NIS is defined as $\tilde{A}^- = (\tilde{V}_1^-, \tilde{V}_2^- ..., \tilde{V}_n^-)$. Equations 3 and 4 guide the composition of the PIS and NIS for benefit and cost criteria. In these equations, $\tilde{V}_j^+ = [v_{pj}, v_{qj}]$, $(j = 1, 2, \dots, n)$ and $(\mathrm{i} = 1, 2, \dots, \mathrm{m})$.

$$\tilde{A}^{+} = \left[\left(\left(\max_{i=1}^{m}\left(H_{Sij}^{l}\right)\right) \mid j \in \Omega_b,\ \left(\min_{i=1}^{m}\left(H_{Sij}^{l}\right)\right) \mid j \in \Omega_c\right)\right] \quad (3)$$

$$\tilde{A}^{-} = \left[\left(\left(\min_{i=1}^{m}\left(H_{Sij}^{l}\right)\right) \mid j \in \Omega_b,\ \left(\max_{i=1}^{m}\left(H_{Sij}^{l}\right)\right) \mid j \in \Omega_c\right)\right] \quad (4)$$

c) **Step 3:** construct a positive ideal separation matrix (D^+) and a negative ideal separation matrix (D^-) according to equations 5 and 6. Each element of these matrices is calculated using expression 7, in which p and q are the envelope limits of set H_S^1, and p' and q' are the envelope limits of H_S^2.

$$D^{+} = \begin{pmatrix} d\left(x_{11}, \tilde{V}_1^+\right) + d\left(x_{12}, \tilde{V}_2^+\right) + \cdots + d\left(x_{1n}, \tilde{V}_n^+\right) \\ d\left(x_{21}, \tilde{V}_1^+\right) + d\left(x_{22}, \tilde{V}_2^+\right) + \dots + d\left(x_{2n}, \tilde{V}_n^+\right) \\ \vdots \quad\quad\quad \vdots \quad\quad\quad \vdots \quad\quad\quad \vdots \\ d\left(x_{m1}, \tilde{V}_1^+\right) + d\left(x_{m2}, \tilde{V}_2^+\right) + \cdots + d\left(x_{mn}, \tilde{V}_n^+\right) \end{pmatrix} \quad (5)$$

$$D^{-} = \begin{pmatrix} d\left(x_{11}, \tilde{V}_1^-\right) + d\left(x_{12}, \tilde{V}_2^-\right) + \cdots + d\left(x_{1n}, \tilde{V}_n^-\right) \\ d\left(x_{21}, \tilde{V}_1^-\right) + d\left(x_{22}, \tilde{V}_2^-\right) + \cdots + d\left(x_{2n}, \tilde{V}_n^-\right) \\ \vdots \quad\quad\quad \vdots \quad\quad\quad \vdots \quad\quad\quad \vdots \\ d\left(x_{m1}, \tilde{V}_1^-\right) + d\left(x_{m2}, \tilde{V}_2^-\right) + \cdots + d\left(x_{mn}, \tilde{V}_n^-\right) \end{pmatrix} \quad (6)$$

$$d(H_S^1, H_S^2) = |q' - q| + |p' - p| \quad (7)$$

d) **Step 4:** calculate the relative closeness (RC) of each alternative to the ideal solution using Equation 8, in which $D_i^- = \sum_{j=1}^{n} d(x_{ij}, \tilde{V}_j^-)$ and $D_i^+ = \sum_{j=1}^{n} d(x_{ij}, \tilde{V}_j^+)$.

$$RC(A_i) = \frac{D_i^-}{D_i^+ + D_i^-} \quad (8)$$

e) **Step 5:** rank all alternatives A_i $(i = 1, 2, \dots, m)$ according to the relative closeness coefficient $RC(A_i)$. The higher the $RC(A_i)$ value, the better the final performance of the alternative A_i.

3.3.4 Method 2 – Hesitant Fuzzy Linguistic TOPSIS with Possibility Distribution

The second method analyzed in this study was proposed by Wu et al. (2019). Like Method 1, it prioritizes alternatives based on proximity to the PIS and NIS and uses HFLTS. However, it differs by incorporating possibility distributions associated with each HFLTS, explicitly modeling the relative plausibility of each linguistic term. This structure increases analytical sensitivity in distance calculation and final ranking of alternatives. The main steps of method 2 are summarized as follows (Wu *et al*., 2019; Nascimento et al., 2025):

a) Passo 1 – normalized individual decision matrices are calculated. These matrices are determined according to the assessments of the alternatives with respect to benefit and cost criteria. In this method, cost criteria are transformed by the Neg operator, defined by Equation 9, in which $S = \{s_0, s_1, \dots, s_g\}$ is the set of linguistic terms and $\vartheta = \{s_L, s_{L+1}, \dots, s_U\}$ is an HFLTS.

$$\mathrm{Neg}(\vartheta) = \{\mathrm{Neg}(s_L), \mathrm{Neg}(s_{L+1}), \dots, \mathrm{Neg}(s_U)\} = \{s_U, \dots s_{L+1}, s_L\} \quad (9)$$

b) Passo 2 – an importance value is defined for each decision-maker involved in the process, which is calculated using geodesic distances between two HFLTS. Let two HFLTS be represented by $H_s^1 = \{h_{i1}, \dots, h_{j1}\}$ and $H_s^2 = \{h_{i2}, \dots, h_{j2}\}$, then, the geodesic distance is given by Equation 10. Based on this, the weights of the decision-makers are obtained and then

normalized using Equation 11, in which k is the number of decision-makers and $\mathrm{GCI_k}$ is given by Equation 12. In Equation 12, where l and k are the corresponding numbers of decision-makers, i is the number of alternatives, and j is the number of criteria in the decision problem.

$$\mathrm{d_{geo}}(\mathrm{H_s^1}, \mathrm{H_s^2}) = \mathrm{d_{geo}}(\{\mathrm{h_{i1}}, \dots, \mathrm{h_{j1}}\}, \{\mathrm{h_{i2}}, \dots, \mathrm{h_{j2}}\}) = |\mathrm{i_1} - \mathrm{i_2}| + |\mathrm{j_1} - \mathrm{j_2}| \quad (10)$$

$$\lambda_{\mathrm{k}} = \frac{\mathrm{GCI_k}}{\sum_{\mathrm{i=1}}^{\mathrm{k}} \mathrm{GCI_l}} \quad (11)$$

$$\mathrm{GCI_k} = \sum_{\substack{\mathrm{l=1,} \\ \mathrm{l \neq k}}}^{\mathrm{t}} \sum_{\mathrm{i=1}}^{\mathrm{n}} \sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{d_{geo}}(\mathrm{H_l^1}, \mathrm{H_k^2}) \quad (12)$$

c) Passo 3 - Individual decision matrices with possibility distributions are calculated for each HFLTS, referred to as PDHFLTS matrices. The value of each element in the possibility distributions (p_l) is calculated using Equation 13. Let $S = \{s_1, s_2, \dots, s_g\}$ be the set of linguistic terms and $H_s^1 = \{s_L, s_{L+1}, \dots, s_U\}$ be an HFLTS contained in *S*; the possibility distribution corresponding to H_s^1 in S is $P = (p_0, p_1, \dots, p_l, \dots, p_g)$, where p_l denotes the possibility that the alternative has an evaluation s_l, such as $\sum_{l=L}^{U} p_l = \sum_{l=0}^{g} p_l = 1$ and $0 \leq p_l \leq 1$, with $l = 0, 1, \dots, g$. These possibility distribution values correspond to the decision-maker's level of hesitation. For instance, considering the set of terms defined in Figure 1, for a linguistic judgment defined as H_s = {s_1: Poor, s_2: Medium, s_3: Good}, the distribution will be $P = (0;\ 0.33;\ 0.33;\ 0.33; 0; 0; 0; 0)$. For H_s= {s_3: Good, s_4: Very good}, the distribution will be $\mathrm{P} = (0;\ 0;\ 0.5;\ 0.5; 0; 0; 0)$. Thus, when the decision-maker selects a larger number of linguistic terms to express a judgment, the degree of possibility (p_l) assigned to each individual term will be lower.

$$p_l = \begin{cases} 0 & l = 0, 1, \dots, L-1, \\ \dfrac{1}{U - L + 1} & l = L, L+1, \dots, U, \\ 0 & l = U+1, \dots, g. \end{cases} \quad (13)$$

d) Passo 4 - the individual matrices are aggregated using the HFLWA (Hesitant Fuzzy Linguistic Weighted Average) function, which joins the PDHFLTS matrices and normalizes them by considering the decision-makers' weights, generating a single HFLWA matrix. Let $S = \{s_1, s_2, \dots, s_g\}$ be the set of linguistic terms, $A_H = \{H_1, \dots, H_n\}$ a PDHFLTS set and $\omega = \{\omega_1, \dots, \omega_n\}$ the decision-makers' weight vector, such that $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$. Each H_k has its possibility distribution denoted as $P^k = (p_0^k, \dots, p_l^k, \dots, p_g^k)$. The HFLWA operator is defined by a possibility distribution $P = (p_0, \dots, p_l, \dots, p_g)$ in *S*, according to Equation 14. In this equation, $p_l = \sum_{k=1}^{n} \omega_k p_l^k$, where p_l is the possibility that $s_l \in S$ can be used as an evaluation value provided by the group.

$$HFLWA(H_1, \dots, H_n) = HFLWA(P^1, \dots, P^n) \triangleq HFLWA(p_0, \dots, p_l, \dots, p_g) \quad (14)$$

From the HFLWA matrix, the mean and variance of each element are calculated. Let $S = \{s_0, s_1, \dots, s_g\}$ be the set of linguistic terms and $\vartheta \in H_s$ be an HFLTS. The possibility distribution for ϑ is $P(= \{p_0, \dots, p_l, \dots, p_g)\}$. The mean $E(\vartheta)$ for ϑ is defined by Equation 15, in which $NS(s_l)$ is the numerical scale for the linguistic term s_l. The variance $Var(\vartheta)$ for ϑ is calculated by Equation 16, in which $NS(s_l)$ is the numerical scale for the term s_l.

$$E(\vartheta) = \sum_{l=0}^{g} NS(s_l)\, p_l \quad (15)$$

$$Var(\vartheta) = \sum_{l=0}^{g} (NS(s_l) - E(\vartheta))^2 p_l \quad (16)$$

e) Step 5 - the positive ideal solution (PIS) and the negative ideal solution (NIS) are defined using Equations 17 and 18, respectively. In these equations, r_{ij}^c represents the elements of the aggregated HFLWA matrix.

$$f_j^+ = \max_i r_{ij}^c, j = 1,2,\dots,m, \quad (17)$$

$$f_j^- = \min_i r_{ij}^c, j = 1,2,\dots,m \quad (18)$$

f) Step 6 – The distances of the alternatives from the PIS and NIS are calculated using the *m-dimensional Euclidean distance operator*, by means of Equations 19 and 20, respectively. The distance between two PDHFLTS is defined by Equation 21. Let S be the set of linguistic terms and ϑ and η be two PDHFLTSs. Let $P = (p_0, \dots, p_l, \dots, p_g)$ and $Q = (q_0, \dots, q, \dots, q_g)$ be the possibility distributions for ϑ and η, respectively. Then, the distance between ϑ and η based on P and Q is calculated using Equation 21, in which $NS(s_l)$ is the numerical scale for the linguistic term s_l.

$$d_{i,Euc}^+ = \{\sum_{j=1}^{m} w_j \, (d(r_{ij}^c, f_j^+))^2\}^{\frac{1}{2}}, i = 1,2,\dots,n, \quad (19)$$

$$d_{i,Euc}^- = \{\sum_{j=1}^{m} w_j \, (d(r_{ij}^c, f_j^-))^2\}^{\frac{1}{2}}, i = 1,2,\dots,n. \quad (20)$$

$$d_{pd}(\vartheta, \eta) = \frac{1}{g}|E(\vartheta) - E(\eta)| = \frac{1}{g}|\sum_{l=0}^{g} NS(s_l)\, p_l - \sum_{l=0}^{g} NS(s_l) q_l| \quad (21)$$

f) Step 7 - calculate the relative closeness (RC) value for each alternative with respect to the ideal solutions using Equation 22, in a similar manner to the method by Beg and Rashid (2013). Using RC, all alternatives A_i $(i = 1, 2, \dots, m)$ are ranked according to the closeness coefficient $RC(A_i)$. The higher the $RC(A_i)$ value, the better the overall performance of the alternative A_i.

$$\mathrm{RC(A_i)} = \frac{\mathrm{D_i^-}}{\mathrm{D_i^+ + D_i^-}} \quad (22)$$

4. Application of the Selected HF-TOPSIS Methods

4.1 Presentation of the Decision-Making Problem

To apply the presented methods, a supplier selection problem adapted from Oliveira et al. (2023) was used as the data source. Four alternatives (A_1, A_2, A_3, and A_4), four decision criteria (C_1, C_2, C_3, and C_4), seven linguistic terms, and two decision-makers (DM_1 and DM_2). were considered. The criteria defined for the supplier selection problem were: C1 – Product quality (benefit criterion); C2 – Price (cost criterion); C3 – Delivery time (cost criterion); C4 – Delivery service (benefit criterion). The degree of importance of the criteria was considered equal for all criteria, so that the results could be adequately compared. HFLTS were used to represent the evaluations of the alternatives with respect to the criteria. Table 3 presents the terms and linguistic expressions used. The linguistic evaluations provided by decision-makers 1 and 2 (DM1 and DM2) are presented in Table 4.

Table 3 – Terms and linguistic expressions for the evaluation of alternatives

Term / linguistic expression	Abbreviation	Representation	Envelope of HFLTS [s_p, s_q]
Péssimo	P	s_0	[0]
Muito Ruim	MR	s_1	[1]
Ruim	R	s_2	[2]
Médio	M	s_3	[3]
Bom	B	s_4	[4]
Muito Bom	MB	s_5	[5]
Excelente	E	s_6	[6]
Entre Muito Bom e Excelente	Entre MB e E	s_5, s_6	[5, 6]
Entre Médio e Bom	Entre M e B	s_3, s_4	[3, 4]
Entre Bom e Muito Bom	Entre B e MB	s_4, s_5	[4, 5]
Entre Ruim e Médio	Entre R e M	s_2, s_3	[2, 3]

Table 4 – Judgments of the decision-makers regarding the performance of the suppliers

Alternatives	C_1	C_2	C_3	C_4

DM_1	**A_1**	B	E	B	M
	A_2	MB	MB	E	E
	A_3	E	B	MB	B
	A_4	B	MB	B	M
DM_2	**A_1**	Entre M e B	Entre MB e E	Entre M e B	Entre R e M
	A_2	Entre B e MB	Entre B e MB	Entre MB e E	Entre MB e E
	A_3	Entre MB e E	Entre M e B	Entre MB e E	Entre B e MB
	A_4	Entre MB e E	B	Entre M e B	MB

4.2 Application of Method 1

Initially, in applying Method 1 (Beg; Rashid, 2013), based on the judgments shown in Table 4, the aggregated judgment matrix was determined using Equations 1 and 2. The results are presented in Table 5.

Table 5 – Judgments aggregated by Method 1

Alternatives	**C_1**		**C_2**		**C_3**		**C_4**	
	p_1	**q_1**	**p_2**	**q_2**	**p_3**	**q_3**	**p_4**	**q_4**
A_1	4	4	6	6	4	4	3	3
A_2	5	5	5	5	6	6	6	6
A_3	6	6	4	4	5	5	4	4
A_4	4	5	4	5	4	4	3	5

Based on this matrix and Equations 3 and 4, the values of PIS and NIS were defined as $\tilde{A}^+ = \{[6,6],[3,4],[3,4],[6,6]\}$ and $\tilde{A}^+ = \{[3,4],[6,6],[6,6],[2,3]\}$, respectively. The distances between the aggregated judgment matrix and the ideal solutions were determined by Equations 5, 6, and 7, allowing the construction of the positive ideal separation matrix (D^+) and the negative ideal separation matrix (D^-) (Table 6). Finally, the use of Equation 8 enabled the definition of the global scores for each alternative, represented by $RC(A_i)$. The coefficients and the ranking of the alternatives are described in Table 6.

Table 6 – Separation matrices D^+ and D^- calculated by method 1

A_i	**Positive ideal separation matrix**					**Negative ideal separation matrix**					**Output**	
	C_1	**C_2**	**C_3**	**C_4**	**$\sum$**	**C_1**	**C_2**	**C_3**	**C_4**	**$\sum$**	**$RC(A_i)$**	***Rank***
A_1	4	5	1	6	16	1	0	4	1	6	0.273	4º
A_2	2	3	5	0	10	3	2	0	7	12	0.545	2º
A_3	0	1	3	4	8	5	4	2	3	14	0.636	1º
A_4	3	2	1	4	10	2	3	4	3	12	0.545	2º

Alternative A3 had the highest final score (0.636), indicating it is the most suitable supplier to hire. Alternatives A2 and A3 had equal scores (0.545), tying for second place. Finally, alternative A1 had the lowest score (0.273) and ranked last, making it the least suitable alternative.

4.3 Application of Method 2

For the application of Method 2 (Wu et al., 2019), the same set S and the alternative scores presented in Table 4 were used. Initially, the normalized individual decision matrices were defined, with the cost criteria (C_2 and C_3) transformed into benefit criteria using Equation 9. The decision matrices of the two decision-makers are presented in Table 7. After this step, the degree of consensus among the decision-makers was defined using Equations 10, 11, and 12. This step yielded the decision-makers' weights for each alternative, presented in Table 8.

Table 7 – Normalized decision matrix for DM_1 and DM_2

	Alternatives	**C_1**		**C_2**		**C_3**		**C_4**	
		p_1	**q_1**	**p_2**	**q_2**	**p_3**	**q_3**	**p_4**	**q_4**
DM_1	**A_1**	4	4	0	0	2	2	3	3
	A_2	5	5	1	1	0	0	6	6
	A_3	6	6	2	2	1	1	4	4
	A_4	4	4	1	1	2	2	3	3

DM_2	**A_1**	3	4	0	1	2	3	2	3
	A_2	4	5	1	2	0	1	5	6
	A_3	5	6	2	3	0	1	4	5
	A_4	5	6	2	2	2	3	5	5

Table 8 – Decision-makers' weights defined by the degree of consensus

Alternatives	**C_1**	**C_2**	**$\sum$**	**λ_1**	**λ_2**
A1	4	4	8	0.5	0.5
A2	4	4	8	0.5	0.5
A3	4	4	8	0.5	0.5
A4	10	10	20	0.5	0.5

Subsequently, the individual decision matrices with possibility distributions (PDHFLTS) were calculated using Equation 13. The resulting matrices for the decision-makers are presented in Table 9. Next, these matrices were aggregated using the HFLWA operator (Equation 14), with decision-maker weights, yielding the normalized and aggregated decision matrix shown in Table 10. The mean and the variance of each element of the aggregated and normalized matrix are calculated according to Equations 15 and 16, respectively. The obtained values are presented in Table 11.

Table 9 – Matrices with possibility distributions for DM_1 and DM_2

	A_i	**C_1**	**C_2**	**C_3**	**C_4**
DM1	**A_1**	$\{s_4(1.0)\}$	$\{s_0(1.0)\}$	$\{s_2(1.0)\}$	$\{s_3(1.0)\}$
	A_2	$\{s_5(1.0)\}$	$\{s_1(1.0)\}$	$\{s_0(1.0)\}$	$\{s_6(1.0)\}$
	A_3	$\{s_6(1.0)\}$	$\{s_2(1.0)\}$	$\{s_1(1.0)\}$	$\{s_4(1.0)\}$
	A_4	$\{s_4(1.0)\}$	$\{s_1(1.0)\}$	$\{s_2(1.0)\}$	$\{s_3(1.0)\}$
DM2	**A_1**	$\{s_3(0.5), s_4(0.5)\}$	$\{s_0(0.5), s_1(0.5)\}$	$\{s_2(0.5), s_3(0.5)\}$	$\{s_2(0.5), s_3(0.5)\}$
	A_2	$\{s_4(0.5), s_5(0.5)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.5), s_1(0.5)\}$	$\{s_5(0.5), s_6(0.5)\}$
	A_3	$\{s_5(0.5), s_6(0.5)\}$	$\{s_2(0.5), s_3(0.5)\}$	$\{s_0(0.5), s_1(0.5)\}$	$\{s_4(0.5), s_5(0.5)\}$
	A_4	$\{s_5(0.5), s_6(0.5)\}$	$\{s_2(1.0)\}$	$\{s_2(0.5), s_3(0.5)\}$	$\{s_5(1.0)\}$

Table 10 – Aggregated and normalized decision matrix

A_i	**C_1**	**C_2**	**C_3**	**C_4**
A_1	$\{s_3(0.25), s_4(0.75)\}$	$\{s_0(0.75), s_1(0.25)\}$	$\{s_2(0.75), s_3(0.25)\}$	$\{s_2(0.25), s_3(0.75)\}$
A_2	$\{s_4(0.25), s_5(0.75)\}$	$\{s_1(0.75), s_2(0.25)\}$	$\{s_0(0.75), s_1(0.25)\}$	$\{s_5(0.25), s_6(0.75)\}$
A_3	$\{s_5(0.25), s_6(0.75)\}$	$\{s_2(0.75), s_3(0.25)\}$	$\{s_0(0.25), s_1(0.75)\}$	$\{s_4(0.75), s_5(0.25)\}$
A_4	$\{s_4(0.5), s_5(0.25), s_6(0.25)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_2(0.75), s_3(0.25)\}$	$\{s_3(0.5), s_5(0.5)\}$

Table 11 – Mean and variance calculated for the elements of the aggregated and normalized matrix

Alternativas	**Média**				**Variância**			
	C_1	**C_2**	**C_3**	**C_4**	**C_1**	**C_2**	**C_3**	**C_4**
A_1	3.75	0.25	2.25	2.75	0.19	0.19	0.19	0.19
A_2	4.75	1.25	0.25	5.75	0.19	0.19	0.19	0.19
A_3	5.75	2.25	0.75	4.25	0.19	0.19	0.19	0.19
A_4	4.75	1.50	2.25	4.00	0.69	0.25	0.19	1.00

The values of PIS and NIS were determined using Equations 17 and 18. These values were selected based on the means shown in Table 11. For example, the value of PIS for C_1 $\{s_5(0.25), s_6(0.75)\}$ is equal to the score value of A_3 in Table 10, since this alternative obtained the highest mean (5.75). Alternative A_1, which obtained the lowest mean (3.75), is used to form the NIS in C_1 $\{s_3(0{,}25), s_4(0{,}75)\}$. In the case of equal means, the variance is checked, with the alternative with the lower variance chosen for the PIS and the one with the higher variance for the NIS. In this way, the PIS indicated by $f^+ = \{\{s_5(0.25), s_6(0.75)\}, \{s_2(0.75), s_3(0.25)\}, \{s_2(0.75), s_3(0.25)\}, \{s_5(0.25), s_6(0.75)\}\}$ and the NIS represented by $f^- = \{\{s_3(0.25), s_4(0.75)\}, \{s_0(0.75), s_1(0.25)\}, \{s_0(0.75), s_1(0.25)\}, \{s_2(0.25), s_3(0.75)\}\}$ were obtained.

Next, the distances of the alternatives with respect to the ideal solutions were calculated by means of Equations 19 and 20. The separation matrices presented in Table 12 show the values of the distances calculated for each alternative. Finally, the scores for each alternative ($RC(A_i)$) were calculated using Equation 21. The coefficients obtained and the ranking of the alternatives according to Method 2 are also indicated in Table 12.

Table 12 – Positive ideal separation matrix (D^+) and negative ideal separation matrix (D^-)

A_i	D^+				D^-				$RC(A_i)$	Rank
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4		
A_1	0.2857	0.2857	0	0.4286	0	0	0.2857	0	0.327	4º
A_2	0.1429	0.1429	0.2857	0	0.1429	0.1429	0	0.4286	0.575	2º
A_3	0	0	0.2143	0.2143	0.2857	0.2857	0.0714	0.2143	0.604	1º
A_4	0.1429	0.1071	0	0.25	0.1429	0.1786	0.2857	0.1786	0.570	3º

Regarding Method 2, alternative A3 presented the highest final score (0.604), while A2 and A3 achieved scores with very close values (0.570 and 0.575), occupying the second and third positions, respectively. Finally, alternative A1 obtained the lowest score (0.327) and ranked last.

5. Comparison between the HF-TOPSIS Methods

In this section, the HF-TOPSIS methods previously applied are compared with respect to comparative factors defined based on the literature. The factors chosen for this purpose are described in Table 13. These factors supported the analysis of the characteristics of the compared methods and enabled examination of their behavior across different scenarios.

Table 13 – Comparative factors utilized in this study

Comparative factors	Definition
Ranking of the alternatives (Yavuz, 2017)	The methods are compared according to the ranking results of the evaluated alternatives, identifying differences and similarities in the positions of the alternatives.
Consistency of the results (Oliveira et al., 2023)	This involves analyzing the consistency of the ranking of alternatives when new alternatives are added to the problem.
Similarities and differences between the techniques (Lima; Carpinetti, 2015; Beleski; Lima, 2025)	This analyzes the intrinsic characteristics of each method and considers various factors, including the use of weights for the criteria, the distance calculation operator, the composition of ideal solutions, the form of the input data, and the number of alternatives and criteria. It also examines the suitability for supporting group decision-making and the modeling of uncertainty and hesitation.

5.1. Ranking of the Alternatives and Consistency of the Results

Regarding the ranking of alternatives obtained in the application case of Section 4, despite specific differences in the *RC* values, caused by the different procedures and equations of each method, both pointed to A_3 as the best supplier alternative. Similarly, both models identified A_1 as the worst alternative. The only difference in the ranking concerns A_2 and A_4. In Method 1, the two alternatives obtained identical scores and jointly ranked second. In Method 2, A_2 was slightly superior to A_4. Therefore, the results of both methods were considerably close.

To evaluate the consistency of the results, simulation tests involving four scenarios were conducted. In each scenario, the method's ability to maintain ranking consistency when new alternatives are added to the decision-making process was assessed to address a known problem called ranking reversal. The ranking reversal problem refers to a change in the ranking of alternatives when a new alternative is added or removed (Oliveira et al., 2023).

In each tested scenario, an alternative named A_5 is inserted in both models to elucidate the consistency of the results. In Scenario 1, A_5 corresponds to an alternative with values equal

to the PIS. In Scenario 2, A_5 has scores equal to the NIS. In Scenario 3, A_5 yields scores equal to the best solution (A_3) in the application case, while in Scenario 4, it yields scores equal to the worst solution (A_1). The results of the simulation of the four tested scenarios are presented in Table 14.

Table 14 – Scores and ranking of the suppliers in the four tested scenarios

	A_i	Method 1		Method 2	
		$RC(A_i)$	Rank	$RC(A_i)$	Rank
Cenário 1	A_1	0.273	5º	0.327	5º
	A_2	0.545	3º	0.575	3º
	A_3	0.636	2º	0.604	2º
	A_4	0.545	3º	0.570	4º
	A_5	1.000	1º	1.000	1º
Cenário 2	A_1	0.273	4º	0.327	4º
	A_2	0.545	2º	0.575	2º
	A_3	0.636	1º	0.604	1º
	A_4	0.545	2º	0.570	3º
	A_5	0	5º	0	5º
Cenário 3	A_1	0.273	5º	0.327	5º
	A_2	0.545	3º	0.575	3º
	A_3	0.636	1º	0.604	1º
	A_4	0.545	3º	0.570	4º
	A_5	0.636	1º	0.604	1º
Cenário 4	A_1	0.273	5º	0.327	4º
	A_2	0.545	2º	0.575	2º
	A_3	0.636	1º	0.604	1º
	A_4	0.545	2º	0.570	3º
	A_5	0.273	5º	0.327	4º

In both methods, adding A_5 (with values equal to the PIS of the application case) yielded the maximum RC value, since there is no distance between two identical PISs. While A_5 occupied the first position in the ranking, the remaining alternatives did not change in either their coefficients or their rankings. Similarly, in Scenario 2, the addition of an alternative equivalent to the NIS yielded an $RC(A_5)$ value of 0 in both methods, placing it last. No changes in the ranking were observed either. Since the output range of the TOPSIS method and its variants is [0, 1], it can be concluded that both methods produced results as expected in these scenarios.

In Scenario 3, in both methods, the insertion of A_5 (with values equal to the best solution of the application case) resulted in a tie with A_3. For this reason, it assumed the first position, together with alternative A_3, which had previously been the highest ranked. The remaining alternatives did not undergo any changes. In Scenario 4, the addition of A_5 (with values equal to the worst alternative of the application case) resulted in an *RC* value identical to A_1, assuming the last position. The other alternatives did not undergo changes in the values of their respective coefficients or in their ranking positions.

Therefore, neither methods suffer from the ranking reversal problem in the four tested scenarios. These results suggest that the two methods yield consistent ranking stability, since in the supplier selection problem the inclusion of a new supplier should not alter the scores of the other alternatives.

5.2 Similarities and Differences Between the Techniques

Table 15 presents the results of the analysis of the characteristics of the compared methods.

Table 15 – Analysis of the characteristics of the compared methods

Compared characteristic	Method 1 (Beg; Rashid, 2013)	Method 2 (Wu et al., 2019)

Number of alternatives and criteria*	There is no limit to the number of alternatives and criteria used.	There is no limit to the number of alternatives and criteria used.
Use of weights**	Does not allow the use of weights for the criteria or for the decision-makers.	Allows the use of weights for the criteria and for the decision-makers.
Number of judgments required**	$J = d \times (n \times m)$.	$J = d \times (n \times m + m)$.
Form of input data*	Allows the use of terms and linguistic expressions represented by HFLTSs.	Allows the use of terms and linguistic expressions quantified by HFLTSs and subsequently converted into PDHFLTSs.
Modeling of uncertainty and hesitation*	Uses linguistic terms and expressions for modeling uncertain information and hesitant judgments.	Uses linguistic terms and expressions for modeling uncertain information and hesitant judgments. It also uses possibility distributions to prevent information loss.
Support for group decision-making**	Enables the use of judgments from multiple decision-makers, but does not assign weights to them.	Enables the use of judgments from multiple decision-makers and assigns weights to them based on the degree of consensus among the decision-makers.
Mode of aggregation of the decision-makers' opinions**	The aggregation is performed based on the maximum and minimum values of the p and q indices for the decision-makers' opinions.	The aggregation is performed based on the values and the number of responses for each alternative, with consensus among decision-makers affecting the aggregated matrix.
Mode of composition of the ideal solutions**	When the criterion is a benefit, the PIS is composed of the maximum values of the aggregated matrix, while the NIS is formed by the minimum values. The opposite is valid for cost criteria.	The PIS is composed of the maximum values of the matrix, normalized by the HFLWA operator, and the NIS is composed of the minimum values, normalized by the HFLWA operator. All criteria are treated as benefit criteria at this stage, since they have already been previously normalized using the negation operator.
Method of distance calculation**	It calculates the distance between HFLTSs using envelopes, employing the p and q indices.	It uses *m-dimensional Euclidean distance*, employing the means of the opinions.

*Similar **Different

Regarding the number of possible alternatives and criteria in the application, both methods did not show limitations in handling the inputs (4 criteria and 4 alternatives). Similar to TOPSIS and Fuzzy TOPSIS, the two analyzed methods have the capability to handle an unlimited number of inputs. Regarding the use of weights, Method 1 does not allow assigning weights to the decision criteria. Method 2, on the other hand, allows decision-makers to use linguistic judgments to weight the criteria. Neither method allows assigning weights to decision-makers' opinions, but Method 2 calculates these weights from the collected judgments.

Furthermore, in Method 1, the number of judgments required from the decision-makers is defined by the function $J = d \times (n \times m)$, where d is the number of decision-makers, n is the number of alternatives, and m is the number of criteria. In Method 2, $J = d \times (n \times m + m)$. This function shows that Method 1 requires more judgments, and that an increase in criteria or alternatives linearly increases the number of judgments required in both methods. This reduces the need for exhaustive data collection compared to methods based on pairwise comparisons, such as AHP, ANP, and their extensions.

Regarding the form of the input data, both methods allow the use of one or more linguistic terms and comparative linguistic expressions, thereby allowing decision-makers to express hesitant judgments. However, Method 1 quantifies the values of the judgments using HFLTSs, while Method 2 uses HFLTSs and subsequently converts them into PDHFLTSs.

Due to the use of HFLTSs, both methods are suitable for representing uncertain, imprecise, and/or subjective information related to supplier evaluations. However, in Method 2, the incorporation of possibility distributions associated with HFLTSs allows the explicit quantification of the degree of plausibility assigned to each linguistic term. By preserving the relative intensity of each possible evaluation during aggregation, these distributions reduce information loss, maintain the structure of the original hesitation, and increase the accuracy and discriminating power of the final results.

Both compared methods proved suitable for supporting group decision-making. In the presented case, two decision-makers participated in the process of evaluating the alternatives. In the context of supplier selection, the opinions of multiple experts can be considered in the process, as neither method limits the number of decision-makers who can participate in the problem. Both methods aggregate the decision-makers' opinions into a single matrix; however, only Method 2 allows consideration of decision-makers' weights. Nevertheless, the weights for each decision-maker are calculated based on the proximity of their judgments to those of the other decision-makers, making it impossible to assign weights based on an expert's experience, position, or knowledge of the problem.

The methods for aggregating decision-makers' opinions differ between the two. In Method 1, the aggregated judgment matrix depends on the highest and lowest values reported by any decision-maker, as determined by the indices in Equation 1. In Method 2, the aggregation is performed based on the values of the judgments. Thus, the consensus among decision-makers affects the aggregated matrix, and the repetition of judgments affects the aggregated values.

The composition of relative ideal solutions is carried out differently depending on how the models handle benefit and cost criteria. In Method 1, the PIS is formed by the maximum index values for benefit criteria and by the minimum index values of the HFLTSs for cost criteria, as explained in Equation 3. The opposite occurs in the NIS, as shown in Equation 4. In Method 2, the PIS is always formed by the maximum values of the aggregated matrix (Equation 17), since the cost criteria are previously transformed into benefit criteria by Equation 9. Similarly, the NIS is always formed from the minimum values of the aggregated matrix, as defined by Equation 18.

Regarding the procedure for calculating distances, Method 1 uses the p and q indices of each HFLTS envelope to measure the distances of the alternatives from the ideal solutions, as shown in Equation 7. Method 2, in turn, uses *the m-dimensional Euclidean distance* and the aggregated matrix means to calculate the distances of the alternatives from the ideal solutions (Equation 21).

5.3 Synthesis of the Results

The comparison carried out demonstrated the feasibility of using both applied methods to support decision-making in a supplier selection process. Table 16 shows the advantages and limitations identified for the two methods analyzed in this work.

Table 16 – Advantages and limitations of the analyzed methods

Method 1 (Beg; Rashid, 2013)	**Method 2 (Wu *et al.*, 2019)**

Advantages	- Capable of handling uncertainty and hesitation; - Supports group decision-making; - Allows the use of linguistic terms and expressions in the evaluation of alternatives; - Does not limit the number of criteria or alternatives; - Faster data collection; - Did not experience ranking reversal with the addition of alternatives; - Simpler and easier to implement.	- Capable of handling uncertainty and hesitation; - Supports group decision-making and assigns weights to decision-makers; - Allows the use of linguistic terms and expressions in the evaluation of alternatives; - Differentiates repeated linguistic terms during aggregation; - Does not limit the number of criteria or alternatives; - Allows the assignment of linguistic weights to the criteria; - Did not experience ranking reversal with the addition of new alternatives;
Limitations	- Does not allow the assignment of weights to criteria or decision-makers; - Does not differentiate repeated linguistic terms during the aggregation of judgments.	- Does not allow assigning weights to decision-makers based on experience or qualification; - Requires additional steps due to the method of calculating distances; - More complex computational implementation.

The ranking of alternatives obtained was similar in both applications, showing the same capability to solve the proposed problem in the illustrative case. Likewise, both methods yielded consistent results in the proposed supplier selection, unaffected by the addition of alternatives. Although the calculation steps of the methods differ, such as how distances are calculated and ideal solutions are determined, this did not affect performance in problem resolution, as the results remained close. Both methods handled uncertainty and decision-makers' hesitation, but Method 2 included additional measures to mitigate them.

6. Conclusion

This study compared two HF-TOPSIS methods in the context of supplier selection. Applying both methods to the same decision problem enabled a comparison of the results. It also illustrated the use of these methods in a supplier selection problem for a service company, a context still little explored with methods of this type. Through the comparison, it was possible to analyze aspects such as the similarity in the ranking of alternatives, the consistency of the results, and the structural convergences and divergences among the analyzed techniques. The analysis also considered relevant methodological characteristics that can directly influence each method's suitability for different decision-making contexts.

The identification of the usage characteristics of the compared techniques can serve as guidance for managers and researchers in choosing appropriate techniques for each application, as well as assist developers of computational solutions in the area of decision-making for supplier selection. Furthermore, another contribution of this study consists of the development of two computational models based on HF-TOPSIS to support supplier selection, which have been made available in an open-access repository.

Regarding the limitations of the work, it should be noted that the application of the methods was not exhaustive, as it comprised only a particular case of a supplier selection problem. An application involving more alternatives, more criteria, or more decision-makers could improve the understanding of the use of the described techniques. Future studies may compare HF-TOPSIS methods with other methods on additional factors, such as computational complexity and scalability in problems with a larger number of alternatives, criteria, and decision-makers. The HF-TOPSIS methods can also be compared with other techniques in the domain of supplier selection, such as HF-VIKOR and HF-QFD. Similarly, the methods addressed in this work can be applied in other contexts, such as personnel, software, or location selection problems.

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