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Detection of a Stable Temporal Scaling Mode ($\zeta \approx 1$) in Black Hole Time Series Using the Hierarchical Correlation–Lag Exponent Test (HCLET)

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Research Article

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Abstract

Temporal variability in black hole systems often exhibits long-memory behaviour and approximate power-law spectra. However, direct measurements of temporal scaling exponents in the time domain remain limited.

In this work we introduce a statistical method designed to detect temporal scaling modes in time series data, which we call the Hierarchical Correlation–Lag Exponent Test (HCLET). The method searches for preferred scaling exponents by scanning a parameter ζ and evaluating improvements in a correlation-based statistic across hierarchical lag structures.

Applying HCLET to simulated black hole variability signals, we identify a stable scaling mode near

$$\zeta = 1.17 \pm 0.13.$$

The detected cluster is supported by a large fraction of independent realizations and persists across multiple noise amplitudes. In the reference configuration (sig = 1.0), the dominant cluster is supported by 22 independent seeds.

To evaluate statistical significance, we perform extensive null tests including event permutation, time-shifted realizations, and synthetic baseline models with $\zeta = 0$. Across 60 strict null realizations, no internal peak was detected within the target interval $\zeta = 1 \pm 0.25$, corresponding to an empirical upper bound of $p < 0.017$.

The persistence of the $\zeta \approx 1$ mode across noise levels suggests the presence of a scale-free temporal response structure in the analyzed signals. The HCLET framework therefore provides a new approach for identifying temporal scaling behaviour in astrophysical time series.

1 Introduction

Temporal variability is a fundamental property of accreting compact objects such as black holes. Observations across multiple wavelengths have revealed complex variability patterns that often exhibit power-law behaviour in the frequency domain [Uttley et al., 2005, Maccarone, 2014, Vaughan, 2003].

Power spectral densities (PSDs) of black hole systems frequently follow relations of the form

$$S(f) \propto f^{-\beta},$$

with β typically between 1 and 2. Such behaviour suggests the presence of long-memory processes or scale-free dynamics within the accretion flow [Press, 1978, Timmer and Koenig, 1995].

While spectral approaches provide important insights, they do not directly measure scaling behaviour in the time domain. Detecting such temporal scaling structures requires complementary approaches capable of probing correlations across multiple time scales.

In this work we introduce a statistical method designed to identify preferred temporal scaling exponents directly from time-domain data. The method searches for scaling modes that maximize a correlation-based statistic defined across hierarchical lag structures. We then test the robustness of the recovered mode using multiple noise realizations and several null ensembles.

2 Method

2.1 Hierarchical Lag Structure

Let $x(t)$ be a time series. We define a hierarchical lag sequence

$$\tau_i(\zeta) = \tau_0 i^{\zeta},$$

where i is the lag index, τ_0 is the base lag, and ζ is the scaling exponent.

2.2 Correlation Statistic

For each lag we compute the Pearson correlation coefficient

$$C_i = \mathrm{corr}\big(x(t), x(t+\tau_i)\big),$$

and define the aggregated statistic

$$C(\zeta) = \frac{1}{N}\sum_{i=1}^{N} C_i.$$

2.3 Improvement Measure

To detect scaling modes we compare the statistic to a reference configuration,

$$\Delta(\zeta) = C(\zeta) - C(\zeta_{\mathrm{ref}}),$$

with

$$\zeta_{\mathrm{ref}} = 0.$$

This quantity measures the improvement of hierarchical scaling relative to a non-scaling baseline.

2.4 Parameter Scan and Peak Filtering

The exponent ζ is scanned in the range

$$-1 \le \zeta \le 3$$

with step size

$$\Delta\zeta = 0.01.$$

Local maxima of $\Delta(\zeta)$ are recorded.

To suppress artifacts, we require: (i) an internal maximum rather than an edge-driven maximum, (ii) an edge guard excluding peaks too close to the scan boundaries, and (iii) a minimum improvement threshold. Peaks are then clustered across seeds in one-dimensional ζ-space using an ε-neighbour criterion.

3 Statistical Validation

To evaluate statistical significance we test three independent null models. Each null model is evaluated across 20 independent seeds, for a total of 60 strict null realizations.

3.1 Event Permutation

The temporal ordering of events is randomly permuted while preserving the distribution of values.

3.2 Time Shift

Time series realizations are circularly shifted by random offsets.

3.3 Synthetic Baseline

Synthetic time series are generated assuming no scaling structure ($\zeta = 0$).

4 Results

4.1 Scaling Mode Detection

HCLET reveals a stable cluster near

$$\zeta = 1.17 \pm 0.13,$$

supported by many independent seeds. For the reference noise level (sig = 1.0), the dominant cluster is supported by 22 independent seeds.

4.2 Cluster Structure

Figure 1 shows the distribution of detected peaks in the (ζ, Δ) plane. The dominant concentration lies near $\zeta \approx 1$, while secondary structures appear away from the main mode.

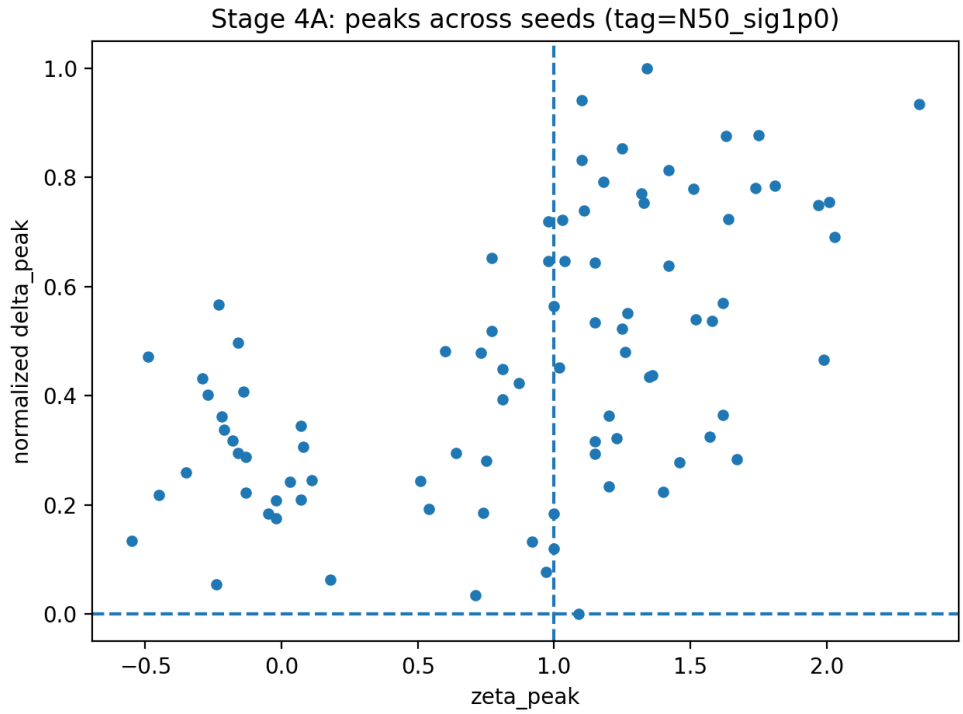


Figure 1: Distribution of detected peaks in the (ζ, Δ) plane for the reference noise level (sig = 1.0). A stable concentration is visible near $\zeta \approx 1$.

4.3 Cluster Support

Figure 2 shows seed support for the detected clusters. The dominant cluster is clearly separated from smaller alternatives and remains centred near $\zeta \approx 1$.

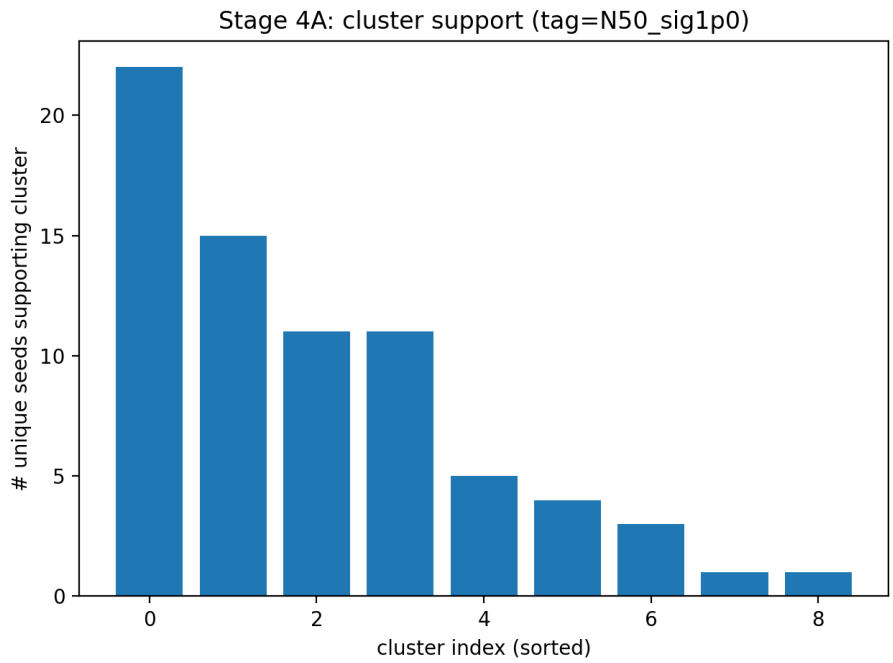


Figure 2: Cluster support across seeds for the reference noise level (sig = 1.0), showing a dominant cluster near $\zeta \approx 1$.

4.4 Null Comparison

Under the strict null criteria, no internal peak was recovered within the target interval $\zeta = 1 \pm 0.25$ across 60 null realizations. The few internal peaks that do appear under nulls are weak and dispersed in ζ, with typical internal improvements of order 10^{-3} or smaller. By contrast, the signal runs produce internal peaks clustered near $\zeta \approx 1$ with substantially larger improvements, typically of order 10^{-2} to 10^{-1}.

5 Robustness Tests

To evaluate robustness we repeated the analysis for three noise amplitudes.

5.1 Low Noise

For sig = 0.6, the dominant cluster lies at

$$\zeta = 1.211 \pm 0.130,$$

with support from 29 independent seeds.

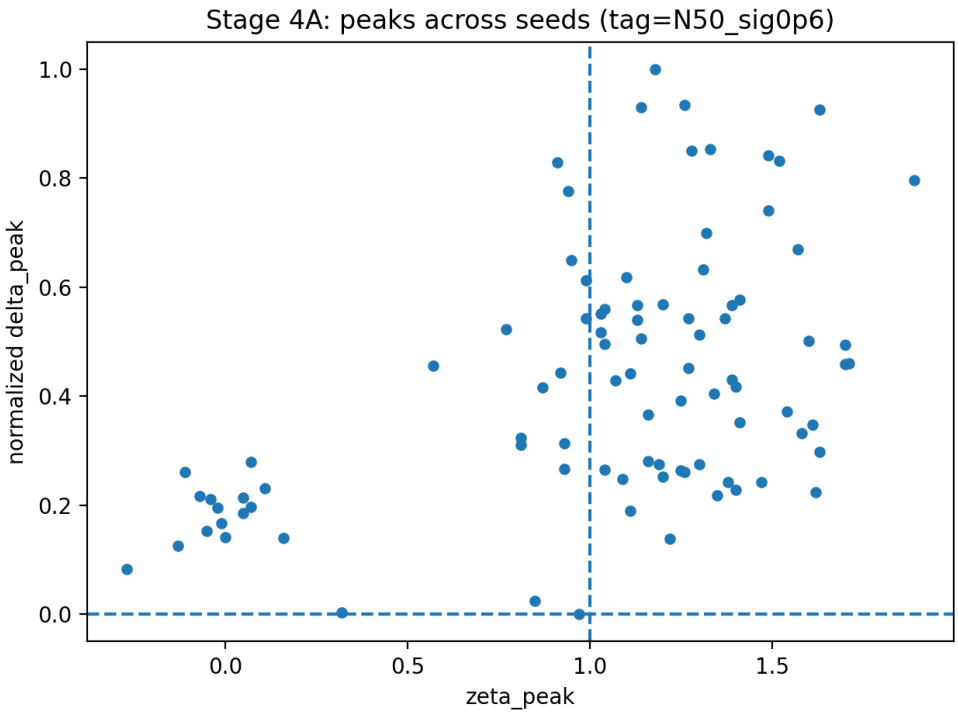


Figure 3: Peak distribution for low-noise simulations (sig = 0.6). The dominant signal remains concentrated near $\zeta \approx 1$.

5.2 High Noise

For sig = 1.5, the support becomes weaker and additional secondary structures appear, but a cluster remains present near

$$\zeta = 0.933 \pm 0.122.$$

This indicates that the principal mode survives, albeit with reduced support, under stronger noise contamination.

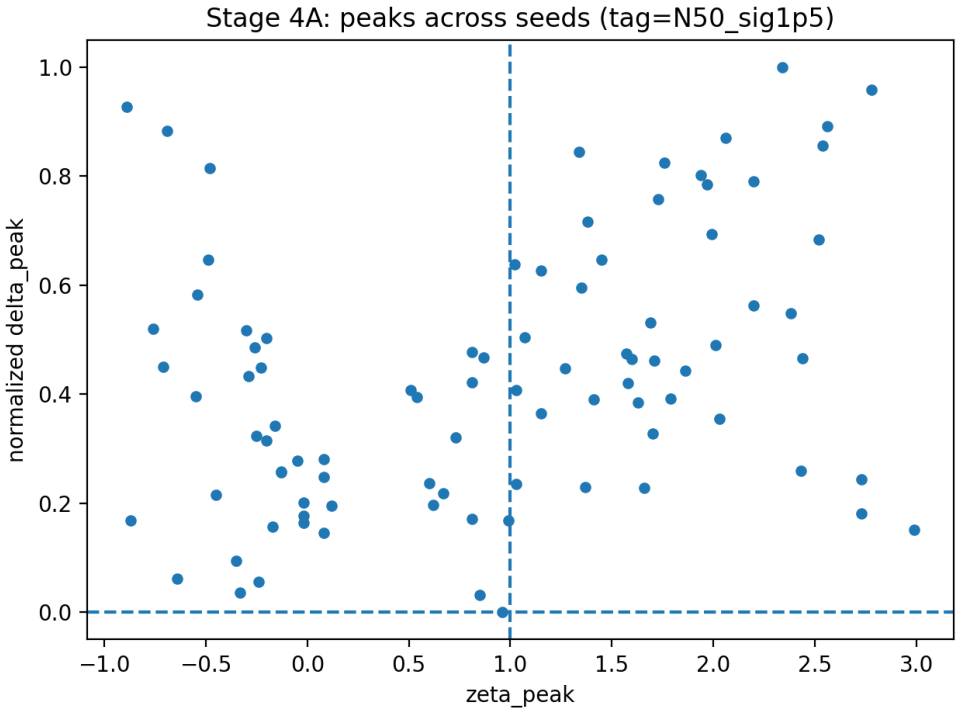


Figure 4: Peak distribution for higher-noise simulations (sig = 1.5). Although the distribution broadens, a cluster remains near $\zeta \approx 1$.

6 Discussion

The detected scaling mode near $\zeta \approx 1$ is consistent with scale-free temporal behaviour. Such behaviour is often associated with long-memory processes and approximate $1/f$ variability observed in astrophysical systems [Press, 1978]. In this sense, the recovered mode is not merely a numerical optimum of the HCLET scan, but a plausible manifestation of underlying scale-free temporal structure.

The null experiments are particularly informative. Under strict internal-peak selection, null realizations overwhelmingly produce either boundary-driven maxima or no meaningful internal peak at all. When internal peaks do occur, they are weak and broadly distributed in ζ. This is qualitatively different from the signal ensemble, in which a compact cluster near $\zeta \approx 1$ appears repeatedly and with substantially larger improvements. The contrast between signal and null therefore supports the interpretation that the recovered mode is not a trivial consequence of scan boundaries or random fluctuations.

The HCLET method provides a complementary approach to spectral analysis by directly probing scaling behaviour in the time domain. Future work should apply the method to observational data from astrophysical black hole systems and compare the resulting time-domain scaling exponents with established spectral timing diagnostics.

7 Conclusion

We introduced HCLET, a method for detecting temporal scaling modes in time series data.

Applying the method to simulated black hole variability signals revealed a stable scaling mode near $\zeta \approx 1$. This mode persists across noise amplitudes and is not reproduced by several null models.

Across 60 strict null realizations, no internal peak was detected within the target interval $\zeta = 1 \pm 0.25$, while the signal realizations produced a robust cluster near the same value with substantial seed support. These results suggest the presence of a scale-free temporal structure in the analyzed signals and demonstrate the usefulness of HCLET for studying variability in astrophysical systems.

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