

Does the Elasticity of Substitution Between Capital and Energy Bolster Economic Growth? Ves Approach

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DOES THE ELASTICITY OF SUBSTITUTION BETWEEN CAPITAL AND ENERGY BOLSTER ECONOMIC GROWTH? VES APPROACH

Abstract

This paper examines the effect of the elasticity of substitution between energy and capital on sustainable economic growth by estimating the nested VES (variable elasticity of substitution) production function within the Solow growth model for 58 countries over the period 1975-2017. We employ an aggregate dataset and classify the selected countries according to their relative energy intensity levels and their accessibility to energy. In the end, we have two main groups and four subgroups. The estimating findings provide that energy and capital are complements in the final output for each country group. Complementarity indicates that a rise in energy consumption accelerates capital accumulation and long-run economic growth. Thus, any shortcomings regarding energy flow can negatively affect economic growth. If policymakers focus their attention on using energy efficiently and supplying their energy demand from renewable energy sources, not only the sustainable economic growth would be achieved, but also environmental policy goals would be reached as well.

Keyword: *Economic growth, Substitution elasticity, Energy consumption, Energy intensity, Sustainable growth, VES production function*

JEL Classification: E1, O1, O4

I. Introduction

For countries, ensuring a balanced and sustainable growth and increasing the quality of life along with the welfare level is correlated with sustainable increase in production output. In order to achieve a stable production structure, not only production factors are sought for, but also effective usage of them is substantial. And the energy, which is a component among the production factors, is highly correlated with every aspect of economic and social dimensions of sustainable growth and also a very significant parameter in formulation of domestic and foreign policy decisions. For instance, possible shortcomings in supplying energy input may cause a decrease in production level as well as welfare loss and impediment in sustainable growth. Therefore, factors such as yearly energy costs of countries, energy consumption amounts, and energy intensity in production are becoming ever more strategically important in development plans. In particular, for the countries that use energy intensely in their production process and are dependent on foreign energy sources, their energy supply security is essential as their

national security. Energy intensity is considered among the indicators of energy efficiency. Quantitatively, it can be showed as $1/\text{energy efficiency}$. Therefore, between the two countries that produce the same output during the same time interval, the one that uses more energy would have the higher energy intensity (Aydin and Esen, 2018).

Due to critical role of energy, it is very important to know the substitution elasticity between production factors regarding short- and long-term estimation of economic growth. For instance, Klump and La Grandville (2000) demonstrated in their study that higher elasticity of substitution (EoS) will lead to a higher income per capita. They also proved that higher substitution elasticity will provide higher steady state equilibrium. Lazkano and Pham (2016), on the basis of the studies of Karagiannis et al. (2005), analyzed the substitution elasticity between energy and capital. They concluded that development level of countries positively affects the substitution elasticity, while higher substitution elasticity positively affects the growth.

EoS between production factors can be estimated by the help of production functions. It is important to know which production function is utilized in this estimation. Three production functions are focused on here. First and foremost is the Cobb-Douglas Production function. The production function was developed by Charles W. Cobb and Paul Douglas in 1928 (Cobb and Douglas, 1928). While the main aim of his function was not to calculate the EoS, it is very important that EoS was set to be equal to 1, when the function was formed.

The second production function that is used in calculation of substitution elasticity is called Constant Elasticity of Substitution (CES) production function. This function was invented by Arrow et al. in 1961 in order to calculate the EoS between physical capital stock and labor. The main assumption of this production function is that EoS coefficient is constant but different from 1. Here, the EoS does not change pursuant to relative shares of factors. In different industries and countries, substitution elasticity between factors may give different results. On the other hand, when substitution elasticity parameter in the function is equal to 0, it means that the substitution elasticity is equal to 1. This result gives us the Cobb-Douglas (CD) production function. Hence, it can be argued that CES production function is the generalized version of Cobb-Douglas production function. While CES production function enables a more flexible use comparing to Cobb-Douglas function, assumption of constant EoS limits the CES production function with regard to its proximity to reality.

The third production function that is used in calculation of EoS is Variable Elasticity of Substitution (VES) production function. VES production function is a non-linear function and

was developed by Revenkar in 1971 (Revenkar, 1971a). The distinction of this production function from others is that EoS between factors changes pursuant to relative share of factors and over time. Therefore, estimated EoS coefficients via VES production function are closer to reality than CD and CES production functions.

In this paper, nested VES production function is examined within the standard Solow-Swan Growth model for 58 countries over the period 1975-2017. The main aim of this study is to analyze the effect of variable EoS between capital and energy on the economic growth. In our analysis, we firstly classify the selected countries into two main groups according to their relative energy intensity level. Later, we divide each two groups into two subgroups according to whether the country is net energy importer or net energy exporter based on the selected year. At the end, we have four subgroups to examine the EoS between energy and capital. This study is expected to contribute two significant aspects to the literature. (i) Studies focusing on VES production function in literature are limited and recent compared to Cobb-Douglas and CES production functions. Additionally, despite the fact that studies analyzing the EoS between capital and labor are ubiquitous, energy included studies are not so common. Hence, unlike the previous studies, this paper analyzes the EoS between energy and capital by estimating nested VES production function within the Solow growth model. (ii) Previous studies examining the EoS between capital and energy estimating VES production function generally have selected or have classified the sample countries according to their income levels. However, the main emphasis is needed to be given to energy. So, to the best of our knowledge, this paper is the first in classifying selected countries primarily according to their relative energy intensity levels and then according to whether the country is net energy importer or net energy exporter based on the selected year. By doing so, our paper represents complementarity between country classification and the main concern (variable elasticity of substitution between energy and capital). Consequently, we classify the countries according to their energy intensity levels and their access facilities to the energy. So, we exhibit the differences among these country groups about the effect of EoS between energy and capital to the economic growth

To that end in this paper, Section 1 discusses the importance of elasticity substitution between production factors in the relationship between energy consumption and growth. Section 2 briefly presents the theoretical and empirical literature review on EoS between production factors. Section 3 describes the model and econometric methodology which is employed in the study. While Section 4 presents data, Section 5 represents the empirical findings by estimating

elasticity substitutions between capital and energy for four country groups. Section 6 gives policy recommendations by appraising empirical findings regarding analysis.

II. Literature Review

In literature, there are many studies estimating the EoS between production factors by using three functions, mentioned in previous section. These studies can be categorized in to two groups; a) the studies using Cobb-Douglass and CES production function and b) the studies using VES production function.

After the study of Cobb and Douglas - published in 1928, Bronfenbrenner and Douglas (1939) and Gunn and Douglas (1941) analyzed the firms in the USA. In those analyses, they examined the labor and capital output elasticity by using regression models. As it known, in Cobb-Douglas production function, the EoS between production factors is equal to 1. Hence, the EoS between labor and capital did not examine in those studies. However, the results –for both two studies - show that for the sectors using capital more intensely have higher labor output elasticity. Later on, Smith (1945) analyzed the EoS between outputs and inputs for the Automobile Industry in Canada by using the Cobb-Douglas function. Smith brought novelty to the literature with his study by using non-linear regression models. Furthermore, Arrow et al. (1961) analyzed the EoS between labor and capital by using CES production function through Least Square Method (LES) for different industries belonging to 19 different countries. This analysis is known as the first study that introduces the CES production function. Hence, the results showed that the EoS between labor and capital is not equal to 1, is smaller than 1 and constant. Sato (1967) brought another novelty to the CES production function literature that he used a two-level function in his study titled “A Two-Level Constant-Elasticity-of-Substitution Production Function.” Thereby, besides analyzing the EoS between two production factors, there could be a study analyzing the EoS between three or more production factors. Later on, Kemfert (1998), and Shen and Whalley (2017) examined the EoS between production factors including energy by using nested (two-level) CES production function. In analyzes, along with microdata of industries, macro data of countries were also used. Furthermore, Klump and Grandville (2000) studied CES production function within the Solow Growth Model. According to results, higher EoS between labor and capital leads to higher GDP per capita. Hence, this study is substantial due to indicating the effect of the EoS between labor and capital on economic growth.

The literature containing VES (variable elasticity of substitution) production function begins with the study of Revenkar (1971a) which introduced VES production function for the first

time. VES production function was discussed as a special form of CES production function in that study. Revenkar (1971a) used data of two-digit United States manufacturing industries for the year 1957. Results show that for his analysis, VES production function is proper than CD production function. The same year, Revenkar (1971b) examine the substitutability between labor-capital for private non-Farm Sectors of the United States for the years 1929-1953. According to the outputs, the substitution between labor and capital differentiates upon the relative ratio of labor-capital. Bairam (1991) studied VES production function for Soviet economy with annual time series data on total economy and industry with the years of 1950-1975. According to the findings, the EoS between capital and energy is variable, less than unity and has a strong declining tendency. Bairam (1991) emphasizes that in Soviet economy over the period 1950-1975, technological progress decreased gradually. However, according to the study, the main reason behind declining in Soviet growth rate, in specified period, is not the technological degradation but low and downward EoS between capital and labor. Later on, Karagiannis et al. (2005), incorporated VES production function into Solow Growth model. In the study, the EoS between labor and capital was examined by using LES method with aggregate country-level data from 82 countries over the period 1960-1987. Lazkano and Pham (2016) studied nested VES production function within the Solow growth model based on the framework of Karagiannis et al. (2005). The paper differs from others by examining the substitution between energy and other inputs by using nested VES function. The research divided 108 countries according to their income levels, their environmental performance index and Kyoto Protocol. The results prove that the capital and energy substitutes for each other and EoS between capital and energy changes over time according to the relative ratio of capital-energy for each country groups. Additionally, Lazkano and Pham (2016) point out that the speed of EoS between capital and energy increases with a country's income level and its higher environmental performance. To conclude, the study presents that countries with high income level have higher EoS between energy and capital and reversely higher EoS between capital and energy foster economic growth. The results show that an increment in EoS between energy and capital leads to higher economic growth. Furthermore, Altunç and Yıldırım (2020) investigated the relationship between variable EoS and economic growth by analyzing the EoS between different energy sources and capital. The study used the aggregate annual data of 31 OECD countries with the years 1990-2014 and estimated the nested VES production function by Non-Linear Least Square regression method. The estimation findings present that while some energy sources (oil and primary energy consumption) are substitutes for capital, some

energy sources (hydroelectricity, natural gas and coal) are complementary to capital in the total output.

III. Model and Estimation Method

Standard VES function examines the substitution elasticity between capital and labor. In this paper, the main interest is the EoS between capital and energy. To that end, the nested (two level) VES production function is constituted and examined. For the nested VES function, preliminary functions are evaluated as follows:

$$Y = f(P, L) \quad , \quad Y = AP^{a_1 v_1} (L + b_1 \alpha_1 P)^{(1-\alpha_1) v_1} \quad (1)$$

$$P = q(K, E) \quad , \quad P = K^{\alpha_2 v_2} (E + b_2 \alpha_2 K)^{(1-\alpha_2) v_2} \quad (2)$$

In the equation (1), P shows the physical inputs and is the function of capital stock and energy as stated in the equation (2). For simplicity, total factor productivity (A) did not account twice. α_1 indicates the role of physical input in the function (1) and α_2 indicates the share of capital stock to energy in the function (2). v_1 and v_2 are the returns to the scale for the both equations. While b_1 stands for variable EoS parameter between physical inputs and labor, b_2 stands for variable EoS parameter between capital stock and energy. Thereby, the general nested VES function can be obtained as follows:

$$Y = A(K^{\alpha_2 v_2} (E + b_2 \alpha_2 K)^{(1-\alpha_2) v_2})^{\alpha_1 v_1} (L + b_1 \alpha_1 K^{\alpha_2 v_2} (E + b_2 \alpha_2 K)^{(1-\alpha_2) v_2})^{(1-\alpha_1) v_1}$$

$$MP_K = \frac{AL^{1-\alpha_1} \alpha_1 \alpha_2 (E + b_2 K) [K^{\alpha_2} (E + b_2 \alpha_2 K)^{1-\alpha_2}]^{\alpha_1}}{K(E + b_2 \alpha_2 K)} \quad (3) \text{ and } (4)$$

$$MP_E = \frac{AL^{1-\alpha_1} \alpha_1 (1-\alpha_2) [K^{\alpha_2} (E + b_2 \alpha_2 K)^{1-\alpha_2}]^{\alpha_1}}{E + b_2 \alpha_2 K} \quad (5)$$

The EoS between energy and capital in nested VES production function is described as follows by using equation (4) and equation (5) which show the marginal physical products of capital and energy:

$$\sigma(E, K) = \frac{\partial \ln(E / K)}{\partial \ln(MP_K / MP_E)} = 1 + b_2 \left(\frac{E}{K} \right) \quad (6)$$

In the equation (6), expression b_2 indicates that, EoS (σ) changes with regard to energy-capital ratio (E/K). When b_2 is smaller than zero, energy and capital are complements. Adversely, when b_2 is greater than zero, energy and capital are substitutes.

As it is stated before, VES production function is the extension form of CES function. So in the equation (3), the values of b_1 and b_2 are important to identify the function (3) as CES or VES function. In the case of b_1 equals to 0 (zero), the substitution between physical input and labor will be constant over time. Similarly, the condition $b_2 = 0$ leads the EoS between capital and energy to the constant elasticity. Thus, $b_1 = b_2 = 0$ implies Cobb-Douglas production function, which is the special form of CES function. Additionally, the VES function (3) can also be reduced to other well-known output functions by restricting the parameters $b_{1,2}$, $\alpha_{1,2}$ and $\nu_{1,2}$. Table 6 (in Appendix) represents in detail how the VES production function is more versatile and enhanced.

In this study, we mainly examine the implications of EoS between capital and energy on long-run economic growth by using Solow-Swan growth model. Standard Solow growth model contains capital and labor as production inputs. In our paper, the standard Solow growth model is enhanced with two new elements. Initially, we take into account the energy besides labor and capital as a production factor in the production process. Secondly, nested VES production function is estimated as a production function in Solow growth model. Nested VES function allows EoS to change depending upon ratio of capital to energy over time. Thus, the two new components allow studying non-balanced economic growth.

The standard Solow-Swan growth model accounts for population growth, technological advancement and depreciation. However in our study, we build a framework of nested VES production function within the Solow-Swan growth model without population growth and technological progress for analytical simplicity. Our framework begins with the analysis of energy accumulation. Firstly, we assume exogenous flow of energy in accordance with the exogenous saving rate in Solow growth model. Let denote E_t and θ_t as an economy's energy usage parameter in production process and aggregate energy stock at the beginning of each period (t), respectively. Hence, aggregate energy stock can be evaluated as follows:

$$\theta_{t+1} = \theta_t(1 + q_t^\theta) - E_t, \quad (7)$$

q_t^θ denotes exogenous augmentation in energy resources. θ_t grows as a result of regeneration of energy and emergence of new energy reserves. Adversely, θ_t decreases through consumption of energy by the rate of E_t . The energy usage (E_t) can be represented by aggregate energy stock as

follows. Let m^θ indicate constant and exogenous fraction of the aggregate energy stock (θ_t). Thus, the total amount of energy used in production process during time t equals to $E_t = m^\theta * \theta_t$. In line with the assumptions, equation (7) can be rephrased as follows:

$$\theta_{t+1} - \theta_t = (q_t^\theta - m^\theta)\theta_t \quad (8)$$

Equation (8) infers that energy accumulation is exogenous and unaffected by the EoS between capital and energy (b_2).

Following the definition of energy accumulation, we will now describe physical capital accumulation. Let K_t denote physical capital stock at the period t without depreciation and constant exogenous fraction. Moreover, in line with no population growth and no technological progress, we suppose that final output (Y_t) is accumulated only through physical capital stock accumulation (i.e. $Y_t = K_{t+1} - K_t$). Hence, aggregate capital stock can be derived using the nested VES production function in equation (9). In equation (9), returns to scale is constant ($v_1 = v_2 = 1$) and b_1 is taken 0 (zero) which means that the EoS between labor and physical input is constant. So, capital accumulation only depends on ease with which capital and energy are substituted (b_2).

$$K_{t+1} - K_t = sA_t K_t^{\alpha_1 \alpha_2} (E_t + b_2 \alpha_2 K_t)^{\alpha_1 (1-\alpha_2)} L_t^{(1-\alpha_1)} \quad (9)$$

Equation (9) indicates that capital accumulation is based on b_2 . As is shown before, b_2 depends on the ratio of capital and energy. Thus, we investigate how the energy-capital ratio impacts the capital accumulation.

$$\frac{\partial(K_{t+1} - K_t)}{\partial b_2} = sA_t K_t^{1+\alpha_1 \alpha_2} L_t^{(1-\alpha_1)} \alpha_1 \alpha_2 (1-\alpha_2) E_t^{\alpha_1 (1-\alpha_2)-1} \left(1 + b_2 \alpha_2 \frac{1}{E_t / K_t}\right)^{\alpha_1 (1-\alpha_2)-1} \quad (10)$$

Equation (10) is the differentiation of equation (9) with respect to b_2 and represents the impact of capital-energy ratio (E_t / K_t) to the augmented capital stock. As energy and capital are substituted, $b_2 > 0$, a rise in energy efficiency contributes to a faster accumulation of capital and long-run economic growth. Otherwise, when $b_2 < 0$, capital and energy are complements, higher capital accumulation needs more capital stock and more energy together.

In case of supply of energy is reduced compare to capital for production, for example during energy crisis, energy-capital complementarity will decrease the long-run economic growth. As a consequent, we indicated in the theoretical framework of nested VES production function

within the Solow model that substitutability between energy and capital (b_2) has an impact on capital accumulation and long-run economic growth and leads to unbalanced economic growth.

In this paper, we mainly focus on the effect of the substitutability between capital and energy on long-run economic growth. While analyzing the substitutability, we construct two main hypotheses that 1) the EoS between capital and energy is variable and changes according to the energy-capital ratio ($b_2 \neq 0$); 2) in the production process, energy is an important production input ($a_2 \neq 1$). For testing the hypotheses, the log-linearizing form of the equation (3) is produced as a baseline estimating equation as follows:

$$\log Y_{it} = \log A_{it} + \alpha_1 \alpha_2 \log K_{it} + (1 - \alpha_2) \alpha_1 \log(E_{it} + b_2 \alpha_2 K_{it}) + (1 - \alpha_1) \log(L_{it} + b_1 \alpha_1 K_{it}^{\alpha_2} (E_{it} + b_2 \alpha_2 K_{it})^{(1-\alpha_2)}) + \varepsilon_{it} \quad (11)$$

In the equation (11), i, t and ε represent the country index, year index and error term, respectively. a_1 indicates the shares of physical input (capital and energy) and labor, while a_2 indicates the shares of capital and energy. In case of $a_1 = 1$, labor plays a very small role in the production process and equation (11) turns into VES production with two inputs (capital and energy). Additionally, if $a_2 = 1$, energy plays a very small role in the production process and equation (11) evolves into VES production function with two inputs (capital and labor).

In the baseline estimation equation (11), b_1 stands for the variable EoS between physical input and labor and b_2 indicates the variable EoS between capital and energy. The values of the variables ($b_{1,2}$) determine the equation (11) to become whether VES production function or CES function. Additionally, if b_1 or b_2 is smaller than zero ($b_{1,2} < 0$), there is a complementary relationship between production inputs. Adversely, if b_1 or b_2 is larger than zero ($b_{1,2} > 0$), there is a substitutability between production inputs. To make the estimating model simpler and more understandable, we assume that the baseline equation to have a constant return to scale (i.e. $v_1 = v_2 = 1$).

We use both raw labor and human adjusted labor separately in place of labor input as a production factor. The importance for creating human adjusted labor series comes from the studies such as Romer (1986), Lucas (1988), Tallman and Wang (1994), Duffy and Papageorgiou (2000), Lazkano and Pham (2016) and Yildirim (2018). We follow the way of the studies Lazkano and Pham (2016) and Yildirim (2018) to create human adjusted labor series. Firstly, we denote H_{it} as human capital index, where i indicates each of country at time t .

H_{it} (human capital index) is evaluated based on years of schooling and return to education by Penn World Table. So, we obtain the data from Penn World Table for each 58 countries over the period 1975-2017. Later on, we generate the human capital adjusted labor (HL) as: $HL_{it} = H_{it} * L_{it}$.

As a nonlinear function, the nested VES production function remains non-linear after taking the logarithm of the equation (3). Hence, the baseline equation (11) is estimated by Non-Linear Least Square (NLLS) regression method. Nonlinear Least Square regression method needs initial values to estimate our baseline equation (11). For the initial values of the variables, we benefit from the estimation of an OLS (Ordinary Least Square) regression.

IV. Data

The dataset is obtained from two sources: the Penn World Table (Feenstra et al., 2015) and BP Energy Statistical Review of World Energy. Penn World Table (PWT) is a large-scale dataset measuring the economic activity of a nation over time. PWT covers data on GDP, productivity, employment, population and capital stock of currently 167 countries with the period 1950-2017. BP Statistical Review of World Energy 2020 includes data of 110 countries with the period 1965-2019. However, our dataset covers 58 countries with years from 1975 to 2017 (43 years). The number of the countries and the time period are determined in order to ensure the balanced panel dataset and to increase the observation number. The variables are chosen based on the equation (11). The equation (11) needs data on variables such labor force (L), physical capital stock (K), energy (E), total output (Y) and total factor productivity (A). Table 1 summarizes the variables, unit of measurement and sources of data.

[Table 1 here]

The sample countries are divided into four groups. First classification is adopted according to their relative energy intensity levels. Energy intensity of an economy is an indicator of this economy's energy inefficiency. Simply, energy intensity is the inverse of energy efficiency and is calculated by the ratio of energy consumption over GDP. In other words, energy intensity gives the amount of energy to obtain one unit of GDP of an economy for the selected period. Quantitatively, it can be showed as $1/\text{energy efficiency}$. Therefore, between the two countries which produce the same output during the same time interval, the one that uses more energy would have the higher energy intensity (Aydin and Esen, 2018). Energy intensity level of a country is depended on several specific factors namely, structure of economic production, sizes of industries, capacity usage rates, consumption habits of society, population, technological

developments and etc. In our classification, we firstly compute energy intensity levels of 58 countries. For calculation, we use primary energy consumption data and real GDP data both in the year 2013. After determining energy intensity levels of the countries, we order the countries by ascending sort according to their energy intensity levels. Then, we take the median value, where values that are higher than median value are grouped as “high intensity” and values that are lower than median value are grouped as “low intensity”.

As for the second classification, we group the countries according to their energy import values based on the year 2013. The data on energy import is obtained from World Bank Open Data (World Bank, 2021). In the dataset, which presents the energy import values, net energy importer countries are indicated with negative values while net energy exporter countries are indicated with positive values. Due to availability of most comprehensive data, 2013 has been designated as the based year for the computing energy intensity levels and determining energy import values. The reason for the second classification is, for ‘net energy exporter’ countries, accessing to energy is easier than for ‘net energy importer’ countries. Hence, even though the countries that belong to the same upper group (‘low energy intensity’ or ‘high energy intensity’ group), in case of energy crisis net energy exporter countries can be affected differently than net energy importer countries. Thus, we have four groups namely, 1) ‘low energy intensity and net energy exporter’ countries, 2) ‘low energy intensity and net energy importer’ countries, 3) ‘high energy intensity and net energy exporter’ countries, 4) ‘high energy intensity and net energy importer’ countries that Table 7 (in Appendix) gives the country groups in detail. Descriptive statistics (mean, maximum and minimum values of variables and number of observations etc.) are represented in Table 2 for each group with the period 1975-2017.

[Table 2 here]

According to mean values on Table 2, primary energy consumption amount, real GDP and capital stock value are higher in both ‘low energy intensity - net energy importer’ countries and ‘high energy intensity – net energy importer’ countries. On the other hand, mean value of total factor productivity level is higher in both ‘low energy intensity - net energy exporter’ countries and ‘high energy intensity – net energy exporter’ countries. Additionally, mean values of human capital indexes are close to each other.

Figure 1 represents the evolution of the global primary energy consumption and Figure 2 indicates the primary energy consumption for the groups with the period 1975-2017, measured in million tons of oil equivalent. The use of primary energy has risen over time, as can be observed from the two graphs. However, the ‘high energy intensity and net importer’ countries

are experiencing the biggest increase in primary energy consumption. Additionally, the primary energy consumption pattern is more volatile in this group according to the other groups. Figure 2 also displays that the amount of primary energy consumption in ‘energy importer’ countries is higher than in ‘energy exporter’ countries over the period.

V. Empirical Findings

The Results of empirical analysis are presented in this subsection. Our main estimation builds on the baseline equation (11) that includes both physical input-labor elasticity (b_1) and capital-energy elasticity (b_2). We mainly test two hypotheses; 1) in the production process, energy is an important production input ($a_2 \neq 1$) and 2) the EoS between capital and labor is not constant ($b_2 \neq 0$) and changes according to the energy-capital ratio. The empirical findings of the nonlinear regression estimations of the model are represented by Table 3. In first two lines of Table 3, the country classifications are given where we have four groups (‘low energy intensity and net energy exporter’ countries, ‘low energy intensity and net energy importer’ countries, ‘high energy intensity and net energy exporter’ countries, ‘high energy intensity and net energy importer’ countries. Additionally, Table 3 is divided into two subsections. The first subsection gives the values of the parameters estimated for unadjusted labor (raw labor- L). The second subsection reports the values of the parameters computed for adjusted labor (human adjusted labor- HL).

[Table 3 here]

Table 3 reports that the parameter a_1 and a_2 are not equal to 1 ($a_1 \neq a_2 \neq 1$) and the elasticity parameters b_1 and b_2 are not constant ($b_1 \neq b_2 \neq 0$) for all groups. Additionally, all parameters are statistically significant with different significance levels. So, the results support our hypothesis for variable EoS between capital and energy (b_2). The constant EoS can be rejected in favor of variable EoS for all country groups. Moreover, Table 3 provides evidence for our second hypothesis that energy is an important production input in the production process. Even though our main focus is on the estimations of b_2 and a_2 , the results of b_1 and a_1 are also taken into consideration. The values of b_1 indicate that the EoS between physical input and labor is also variable ($b_1 \neq 0$) for all groups, which changes according to ratio of physical input over labor. Additionally, a_1 is not equal to one for all groups, which means that physical input is a significant production input in the production process.

The estimation of the elasticity parameter between capital and energy (b_2) is negative for all country groups in case of both raw labor (L) and adjusted labor (HL). The negative value

of b_2 indicates the complementary relationship between capital and energy. Additionally, since the values b_2 is negative, the EoS for all country groups is smaller than one¹. For ‘low intensity and net energy exporter’ countries, b_2 takes negative value, which is very close to zero both in case of unadjusted labor (-6.44e-06) and adjusted labor (-6.32e-06). The negative value means that a one-unit increment in the capital-energy ratio reduces the EoS between capital and energy by approximately -6.4e-06 units in this country group. There is a similar conclusion for ‘low intensity and net energy importer country group’. For ‘high energy intensity and net energy exporter’ countries, a one-unit increment in the capital-energy ratio reduces the EoS between energy and capital by 2.5e-05 units, approximately. ‘High energy intensity and net energy importer’ countries have also negative b_2 value when both raw labor and adjusted labor are considered. If the capital-energy ratio raises one-unit, the EoS between capital and energy decreases by 1.4e-05 units, approximately. Eventually, for all country groups, the results reject constant EoS between capital-energy and prove that energy and capital are complements in the final output.

Even though the results are statistically significant, the estimations of the parameter b_1 and b_2 are very close to zero. So, we need to test the parameters b_1 and b_2 to be zero or not. We construct our null hypothesis on $b_1 = 0$ and $b_2 = 0$. The results of our null hypothesis are given in Table 4.

[Table 4 here]

Table 4 reports that our null hypothesis is rejected for all country groups both in case of unadjusted labor (L) and adjusted labor (HL). Therefore, the values of the elasticity parameters b_1 and b_2 are very small and close to zero but differ than zero. Hence, both the EoS between physical input and labor (b_1) and the EoS between capital and energy (b_2) are variable that changes over time.

After the estimation of the elasticity parameters, we compute the EoS between capital and energy for country groups over the period 1975-2107. The results are reported by Table 5.

[Table 5 here]

As it can be inferred from Table 5, the EoS takes negative value for some country groups in some years. Negative EoS is an indicator of a strong complementarity between capital and energy. However, when we consider the mean values, substitution elasticity is positive and

¹ The EoS between energy and capital for the VES production function is $\sigma(K,E) = 1 + b_2*(K/E)$.

smaller than one for each country group. The EoS being smaller than unity implies that capital is a complement to energy in the final output.

VI. Conclusion

As countries strive to improve their quality of life and welfare level, policies targeted at decreasing energy consumption are becoming more vital in maintaining long-term economic growth. The usefulness of such measures is primarily determined by the ease with which other production factors, such as capital, can be substituted for energy. Production functions are used to measure the elasticity of substitution (EoS) between production factors, and it's crucial to understand which production function is being used in this calculation. The EoS between production inputs is measured generally by estimating a) Cobb-Douglas and CES (constant elasticity of substitution) production function and b) VES (variable elasticity of substitution) production function. In comparison to Cobb-Douglas and CES production functions, there are few studies concentrating on the VES production function in the literature. VES production function differs from other functions by allowing the EoS between production inputs to change pursuant to relative share of factors and over time. Thus, VES production function appears to be closer to reality by enabling a more flexible use comparing to CES and Cobb-Douglas functions.

Our paper examines the effect of the EoS between energy and capital on sustainable economic growth by estimating VES production function within the standard Solow growth model framework. By integrating the VES production function into Solow growth model, the EoS between energy and capital directly determines the capital accumulation, which leads to an increase in total output. This paper takes a new approach by employing VES production function and classifying the countries according to their energy intensity levels and their access facilities to the energy. So, our study exhibits the differences among the country groups about the effect of EoS between energy and capital on the economic growth

The present study reports important findings; firstly, energy has a significant role in the final output and the EoS between capital and energy is not constant for all county groups. Secondly, for each group, energy is not substituted by capital that capital and energy are complements in the final production output. According to the empirical results, the magnitudes of the EoS parameter in “high energy intensity” countries (both “net energy exporter” and “net energy importer” countries) are smaller than the magnitudes in “low energy intensity” countries (both “net energy exporter” and “net energy importer” countries). Although the differences are very

low, the smaller magnitudes show that “high energy intensity” countries have a stronger complementary relationship between capital and energy than “low energy intensity” countries have. Therefore, consuming energy less efficiently reduces the elasticity of substitution between energy and capital while strengthening the complementarity relationship between energy and capital.

Moreover, “high energy intensity and net energy exporter” countries have stronger energy-capital complementarity relative to the “high energy intensity and net energy importer” countries. It is obvious that net energy exporter countries have easier access to energy than net energy importer countries. So, it can be inferred that ease of access to energy supports the complementary relationship between energy and capital. Additionally, when we consider data on TFP, we can observe that the mean value of TFP is the highest for “high energy intensity and net energy exporter” countries. Considering the TFP values, it can be expected to have the lowest complementarity in that country group. However, empirical findings suggest the opposite that “high energy intensity and net energy exporter” country group has the strongest complementary relationship between capital and energy. So, it also can be inferred that for “high energy intensity and net energy exporter” countries with highest TFP does not necessarily imply the acquisition of energy-saving technologies by these countries. In “low energy intensity” country group, the EoS between energy and capital differs for “net energy exporter countries” and “net energy importer” countries. When we consider the estimation results for unadjusted labor input, “net energy exporter” countries have a stronger complementary relationship between energy and capital than “net energy importer” countries have. The results can be explained by the inequality in countries’ accessibility to energy. Even though the countries belong to the “low energy intensity” country group, ease of access to energy strengthens the complementarity between energy and capital.

Our findings contribute a new approach to the policy discussion about minimizing energy use while sustaining economic development. Since our estimates imply that energy and capital are complements, a substantial increase in economic growth can be achieved by focusing on energy supply from renewable energy sources. Furthermore, the speed of capital and energy substitution is higher in countries which put more effort into environmental protection (Lazkano and Pham; 2016). So, supplying energy demand from renewable sources provides policymakers much flexibility when they consider both environmental protection and sustainable economic growth.

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Appendix

[Table 6 here]

[Table 7 here]

References

- Altunç, Ö. F., & Yildirim, A., (2020). Capital-energy substitution: Does energy sources matter for the elasticity of substitution?. An Empirical Investigation for OECD Countries, *Siyasal: Journal of Political Sciences*, 29(2), 367-378.
- Arrow, K. J., Chenery, H. B., Minhas, B. S., & Solow, R. M., (1961). Capital-labor substitution and economic efficiency. *The Review of Economics and Statistics*, 43(3), 225–250. <https://doi.org/10.2307/1927286>.
- Aydin, C., & Esen, Ö., (2018). Does the level of energy intensity matter in the effect of energy consumption on the growth of transition economies? Evidence from dynamic panel threshold analysis. *Energy Economics*, 69, 185-195.
- Bairam, E., (1989). Learning-by-doing, variable elasticity of substitution and economic growth in japan, 1878–1939. *The Journal of Development Studies*, 25(3), 344-353.
- Barro, R. J., & Lee J. W., (2013). A new data set of educational attainment in the world, 1950-2010. *Journal of Development Economics*, 104, 184–198.
- British Petroleum Statistics, (2020). Statistical review of world energy 2020 - 69th edition. <https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html>. (accessed 10 January 2021).
- Bronfenbrenner, M., & Douglas, P. H., (1939). Cross-section studies in the Cobb-Douglas function. *Journal of Political Economy*, 47(6), 761-785.
- Cobb, C. W., & Douglas, P. H., (1928). A theory of production. *The American Economic Review*, 18 (1), 139-165.
- Duffy, J., & Papageorgiou, C., (2000). A cross-country empirical investigation of the aggregate production function specification. *Journal of Economic Growth*, 5(1), 87–120. <https://doi.org/10.1023/A:1009830421147>
- Feenstra, R. C., Robert I., & Marcel, P. T., (2015). The next generation of the Penn World Table. *American Economic Review*, 105(10), 3150-3182.

502 Fisher, V. K., Jefferson, G. H., Liu, H., & Tao, Q., (2004). What is driving China's decline in
503 energy intensity?. *Resource and Energy economics*, 26(1), 77-97.

504 Gunn, G. T., & Douglas, P. H., (1941). The production function for American manufacturing
505 in 1919. *The American Economic Review*, 31(1), 67-80.

506 Inklaar R., & Timmer M. P., (2013). Capital, labor and TFP in PWT8.0. *Groningen Growth
507 and Development Centre, University of Groningen*, 1-38.

508 Karagiannis, G., Theodore P., & Chris, P., (2005). Variable elasticity of substitution and
509 economic growth: Theory and evidence. *In New Trends in Macroeconomics*, ed. Claude
510 Diebolt and Catherine Kyrtou, 21–37.

511 Kemfert, C., (1998). Estimated substitution elasticities of a nested CES production function
512 approach for Germany. *Energy Economics*, Elsevier, 20(3), 249-264.

513 Klump, R., & de La Grandville, O., (2000). Economic growth and the elasticity of substitution:
514 Two theorems and some suggestions. *American Economic Review*, 90(1), 282–291.

515 Lazkano, I., & Pham, L., (2016). Can capital-energy substitution foster economic growth?.
516 *Land Economics*, 92(3), 491-514.

517 Liao, H., Fan, Y., & Wei, Y. M., (2007). What induced China's energy intensity to fluctuate:
518 1997–2006?. *Energy Policy*, 35(9), 4640-4649.

519 Lucas, R. E., (1988). On the mechanics of economic development. *Journal of Monetary
520 Economics*, 22, 3-42.

521 Psacharopoulos, G., (1994). Returns to investment in education: A global update. *World
522 Development*, 22(9), 1325–1343.

523 Revankar, N. S., (1971a). A class of variable elasticity of substitution production functions.
524 *Econometrica*, 39(1), 61–71.

525 Revankar, N. S., (1971b). Capital-labor substitution, technological change and economic
526 growth: The U.S. experience, 1929-1953. *Metroeconomica*, 23(2), 154–176.
527 <https://doi.org/10.1111/j.1467-999X.1971.tb00168.x>.

528 Romer, P. M., (1986). Increasing returns and long-run growth. *Journal of Political Economy*,
529 94, 1002-1037.

530 Sato, K., (1967). A two-level constant-elasticity-of-substitution production function. The
531 Review of *Economic Studies*, 34(2), 201-218.

532 Shen, K., & Whalley, J., (2013). Capital-labor-energy substitution in nested CES production
533 functions for China. *The Economies of China and India*, 15-27. doi:10.3386/w19104.

534 Smith, V. E., (1945). Nonlinearity in the relation between input and output: the Canadian
535 automobile industry, 1918-1930. *Econometrica*, 13(3): 260-272.

536 World Bank, 2021. Energy imports, net (%of energy use).
537 <https://data.worldbank.org/indicator/EG.IMP.CON.S.ZS?view=chart>, (accessed 20
538 February 2021).

539 Yıldırım A., (2018). Private-public capital, elasticity of substitution and economic growth".
540 Hacettepe University Graduate School of Social Sciences Department of Economics
541 Ph.D. Dissertation, Ankara-Turkey.

Table 1. Variables and Source of Data

Variable	Unit of Measurement	Source of Data
Real GDP	Expenditure-side real GDP at chained PPPs (in millions 2011US\$)	Penn World Table (version 9.1)
Real Total Factor Productivity	Total factor productivity at constant national prices (2011=1)	Penn World Table (version 9.1)
Real Capital Stock	Capital stock at constant 2011 national prices (in millions 2011US\$)	Penn World Table (version 9.1)
Population ¹	Millions of people	Penn World Table (version 9.1)
Human Capital Index	Greater than 1	Penn World Table (version 9.1)
Primary Energy Consumption	Million tons of oil equivalent	BP Energy Statistical Review of World Energy 2020

Note: PPP indicates purchasing power parity.

¹ Population stands for the labor input (L) in VES production function. While the estimation of the model is more accurate with data on labor force for each country, this is impractical considering the vast scale of our research, which covers 58 countries with the time period 1975-2017. Thus, we employ population dataset. However, in order to obtain more appropriate estimation we also use human capital index besides data on population (unadjusted labor force). The human capital index is measured using average years of schooling approach (Barro and Lee, 2013) and the return to education rate from Mincer equation (Psacharopoulos, 1994) for each country by PWT version 9 (Inklaar and Timmer, 2013).

Table 2. Descriptive Statistics of Variables

			Low Energy Intensity		High Energy Intensity	
Variables	Unit of Measurement	Statistics	[1]	[2]	[3]	[4]
			Net Energy Exporter	Net Energy Importer	Net Energy Exporter	Net Energy Exporter
Real GDP	Millions of 2011 US Dollar	Mean	43,9275.4739	892048.7617	395983.8	1256206
		Max	2862116.75	8412113	1629508	18396068
		Min	28422.50781	30710.5781	8814.254	2996.073
		Std. Dev.	541793.5296	1176367.139	394260.5	3229540
Real TFP	Constant 2011 National Prices	Mean	1.1077	0.9607	1.1817	0.9114
		Max	3.1233	1.3164	5.2521	1.3853
		Min	0.5843	0.5648	0.5724	0.4384
		Std. Dev.	0.2953	0.1159	0.5554	0.1464
Real Capital Stock	Millions of 2011 US Dollar	Mean	1864804.656	4270330.839	1694198.562	4484390
		Max	15846835	29931072	6748094.5	94903728
		Min	90796.2188	110732.7344	42174.3515	17434.7
		Std. Dev.	2636729.213	5258859.39	1498613.715	11606617
Population	Millions of People	Mean	57.4918	83.4246	21.7307	96.9164
		Max	263.9914	1339.1802	81.1628	1409.517
		Min	4.00803	3.1841	0.1644	0.2181
		Std. Dev.	62.9127	205.4996	19.0147	278.9511
Primary Energy Consumption	Million tons of oil equivalent	Mean	46.4561	106.1902	86.4118	227.469
		Max	188.5770	748.4129	336.9992	3124.862
		Min	1.8354	1.0806	1.2950	0.7335
		Std. Dev.	46.2825	135.1136	84.9547	569.2807
Human Capital Index	Greater Than 1	Mean	2.2538	2.6896	2.4090	2.7891
		Max	3.6431	3.8071	3.70629	3.9742
		Min	1.1461	1.1224	1.1584	1.4957
		Std. Dev.	0.5524	0.6266	0.6356	0.4796
Human Capital Adjusted Labor*		Mean	123.7230	180.7671	51.8072	233.1321
		Max	611.4409	2844.1777	196.7272	3617.424
		Min	11.5756	7.9549	0.2699	0.5023
		Std. Dev.	140.2180	372.7156	44.0920	615.0471
Number of Observations			344	946	430	774
Number of			8	22	10	18

Note: GDP indicates gross domestic product. TFP indicates total factor productivity. Std. Dev. is the abbreviation of standard deviation. Max. is the maximum value. Min. is the minimum value. Obs. means the number of observation.

Table 3. Nonlinear Regression Estimation, (Country Classification)

	Low Energy Intensity		High Energy Intensity	
	[1]	[2]	[3]	[4]
Parameters	Net Energy Exporter Countries	Net Energy Importer Countries	Net Energy Exporter Countries	Net Energy Importer Countries
<i>Unadjusted Labor (L)</i>				
α_1	0.9598*** (0.0209)	0.9387*** (0.0123)	0.5470*** (0.0055)	0.9356*** (0.0118)
α_2	0.8960*** (0.0185)	0.9206*** (0.0129)	0.7233*** (0.0018)	0.9424*** (0.0132)
b_1	-0.0000111* (6.55e-06)	-7.55e-06** (3.84e-06)	-0.0000336** (1.54e-05)	-5.56e-06*** (9.76e-07)
b_2	-6.44e-06*** (1.26e-06)	-6.19e-06* (3.31e-06)	-0.0000254*** (2.96e-06)	-0.0000142*** (1.91e-07)
<i>Adjusted Labor (HL)</i>				
α_1	0.9357*** (0.0279)	0.9083*** (0.0110)	0.8330*** (0.0079)	0.9007*** (0.0144)
α_2	0.9142*** (0.236)	0.9470*** (0.0113)	0.7134*** (0.0053)	0.0733*** (0.0155)
b_1	-0.000033* (0.0000189)	-0.0000234*** 4.51e-06	-0.00011*** (2.02e-05)	-0.0000136*** (2.71e-06)
b_2	-6.32e-06*** 1.52e-07	-6.85e-06** (3.32e-06)	-0.0000252*** (1.70e-06)	-0.0000137*** (2.46e-07)
$R^2 (L)$	0.9992	0.9996	0.9986	0.9990
$R^2 (HL)$	0.9992	0.9997	0.9507	0.9991
N	344	946	430	774

Note: ***, ** and * show the significance level at 1%, 5% and 10% respectively. The terms in parentheses are standard errors. N is the number of observations.

Table 4. Test of Hypothesis

		<i>Labor</i>	$H_0: b_1 = 0 \text{ and } b_2 = 0$ $H_1: b_1 \neq 0 \text{ and } b_2 \neq 0$
Low Intensity	Net Energy Exporter	Raw Labor: L	F (2, 340) = 2797.06 Prob > F = 0.0000
		Adjusted Labor: HL	F (2, 340) = 4954.56 Prob > F = 0.0000
	Net Energy Importer	Raw Labor: L	F (2, 942) = 10.05 Prob > F = 0.0000
		Adjusted Labor: HL	F (2, 942) = 38.80 Prob > F = 0.0000
High Intensity	Net Energy Exporter	Raw Labor: L	F (2, 426) = 36.75 Prob > F = 0.0000
		Adjusted Labor: HL	F (2, 340) = 116.86 Prob > F = 0.0000
	Net Energy Importer	Raw Labor: L	F (2, 770) = 11889.71 Prob > F = 0.0000
		Adjusted Labor: HL	F (2, 770) = 18913.19 Prob > F = 0.0000

Table 5. Elasticity of Substitution by Country Groups (1975-2917)

		Low Intensity		High Intensity	
		[1] Net Energy Exporter	[2] Net Energy Importer	[3] Net Energy Exporter	[4] Net Energy Importer
Raw Labor (L)	Mean	0.7445 (0.1634)	0.7053 (0.1241)	0.4871 (0.2194)	0.6238 (0.2010)
	Max	0.9466	0.9589	0.8707	0.9673
	Min	-0.1131	0.1629	-0.3836	-0.0645
Adjusted Labor (HL)	Mean	0.7493 (0.1604)	0.6739 (0.1374)	0.4911 (0.2176)	0.6370 (0.1939)
	Max	0.9476	0.9545	0.8718	0.9684
	Min	-0.0924	0.0737	-0.3727	-0.0270
N		344	946	430	774

Note: The terms in parentheses are standard errors. N is the number of observations. . Max. is the maximum value. Min. is the minimum value.

Table 6. Properties of VES Production Function with Particular Constraints

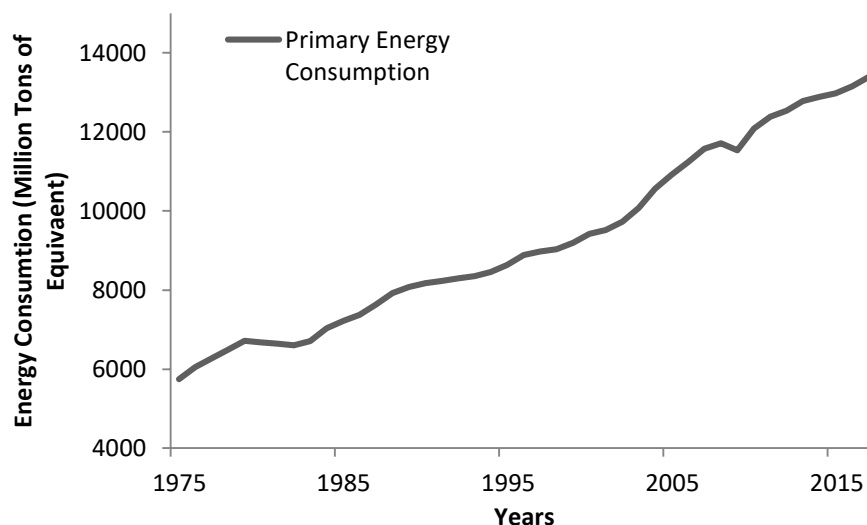
Constraints	Reduced Form of Nested VES Production Function	Property
No Restriction	$Y = A(K^{a_2 v_2} (E + b_2 \alpha_2 K)^{(1-\alpha_2) v_2})^{\alpha_1 v_1} (L + b_1 \alpha_1 K^{a_2 v_2} (E + b_2 \alpha_2 K)^{(1-\alpha_2) v_2})^{(1-\alpha_1) v_1}$	Nested VES production function
$b_1 = b_2 = 0$	$Y = A(K^{a_2 v_2} E^{(1-\alpha_2) v_2})^{\alpha_1 v_1} L^{(1-\alpha_1) v_1}$	CES Cobb-Douglas production function with labor, capital and energy
$b_1 = b_2 = 0$ and $v_1 = v_2 = 1$	$Y = A(K^{a_2} E^{(1-\alpha_2)})^{\alpha_1} L^{(1-\alpha_1)}$	CES Cobb_Douglas production function with constants return to scale
$\alpha_1 = 1$ and $v_1 = v_2 = 1$	$Y = AK^{a_2} (E + b_2 \alpha_2 K)^{(1-\alpha_2)}$	VES production function with capital and energy with CRTS
$\alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK^{\alpha_1} (L + b_1 \alpha_1 K)^{(1-\alpha_1)}$	VES production function with capital and labor with CRTS
$\alpha_1 = \alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK$	AK production function with CRTS
$b_1 = 0, \alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK^{\alpha_1} L^{1-\alpha_1}$	AK production function with capital and labor with CRTS
$b_2 = 0, \alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK^{a_2} E^{(1-\alpha_2)}$	AK production function with capital and energy with CRTS

Note: CRTS indicates constant returns to scale.

Table 7. Country Classification

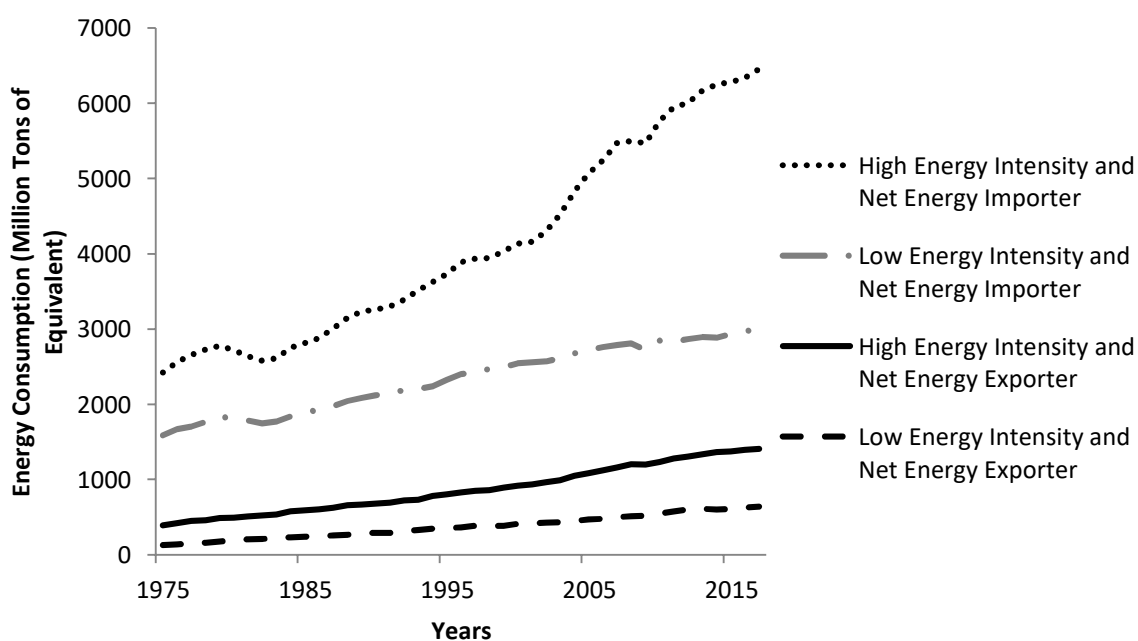
Low Energy Intensity		High Energy Intensity	
Net Energy Exporter Countries	Net Energy Importer Countries	Net Energy Exporter Countries	Net Energy Importer Countries
Colombia	Austria	Australia	Argentina
Ecuador	Brazil	Canada	Belgium
Egypt	Chile	Iran	Bulgaria
Indonesia	China, Hong Kong SAR	Kuwait	China
Iraq	Denmark	Malaysia	Cyprus
Mexico	France	Qatar	Finland
Norway	Germany	Saudi Arabia	Greece
Peru	Hungary	South Africa	Iceland
	India	Trinidad & Tobago	Luxembourg
	Ireland	Venezuela	Netherlands
	Israel		New Zealand
	Italy		Poland
	Japan		Singapore
	Morocco		South Korea
	Philippines		Sweden
	Portugal		Taiwan
	Romania		Thailand
	Spain		Unites States of America
	Sri Lanka		
	Switzerland		
	Turkey		
	United Kingdom		

Figure 1. Global Primary Energy Consumption, 1975-2017 (Million Tons of Equivalent)



Source: BP Energy Statistical Review of World Energy 2020

Figure 2. Primary Energy Consumption, by Groups 1975-2017 (Million Tons of Equivalent)



Source: BP Energy Statistical Review of World Energy 2020

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