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Masuda Akter

University of Chittagong

M. M. Rizvi

`rizmm001@mymail.unisa.edu.au`

Federation University

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A Path Planning Model for Intercepting a Moving Target with Finite Obstacle Avoidance

Masuda Akter ^{*} M. M. Rizvi [†]

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Abstract

This paper investigates the problem of computing a two-dimensional curvature-constrained shortest path for an unmanned aerial vehicle (UAV) to intercept a moving target, with both the UAV (pursuer) and the target travelling at constant speeds. We formulate an optimal control problem that integrates two critical objectives: avoiding static obstacles and successfully intercepting the target. The formulation integrates curvature constraints, geometric obstacle-avoidance conditions, and time-consistent interception constraints within a constrained optimal control framework. A geometric framework is developed, along with sufficient conditions for path optimality under the imposed constraints. The problem is initially examined in the presence of a single obstacle and later extended to scenarios involving a finite number of obstacles. Numerical experiments are carried out to evaluate the performance and efficiency of the proposed model using illustrative examples. Finally, we present a realistic case study using actual geographic data, including obstacle placement, target trajectory, and heading angles, to demonstrate the practical applicability and effectiveness of the proposed method in real-world scenarios.

Keywords. Dubins path, Optimal control problem, Path planning, Mathematical modeling, Numerical methods.

AMS subject classifications: 34K35 49M37, 90C30, 90C39.

^{*}Department of Mathematics, University of Chittagong, Chittagong-4331, Bangladesh, email: masuda@cu.ac.bd.

[†]Centre for Smart Analytics (CSA), Institute of Innovation, Science and Sustainability, Federation University Australia, email: rizmm001@mymail.unisa.edu.au, (corresponding author).

1 Introduction

The development of optimal path planning techniques is crucial to improving movement efficiency, reducing operational costs, and ensuring safety and convenience in various real-world applications. Path planning is especially essential in fields such as Robotics [1], Agriculture [2], and rescue operations [3]. A critical challenge in autonomous systems and robotic navigation involves devising strategies to intercept a moving target while avoiding static obstacles positioned between the pursuer and the target. The primary objective of this study is to formulate an optimal path-planning model that takes into account the presence of static obstacles, allowing the pursuer to navigate around them while intercepting the moving target efficiently. This research seeks to contribute to the theoretical foundations and practical applications of path planning strategies in dynamic environments by employing mathematical modelling and efficient algorithms. Through the development of this model, we aim to address the complexities of intercepting a moving target in the presence of various obstacles, providing innovative, globally applicable solutions to this challenging problem in a wide range of domains.

The shortest path was first proposed by Dubins [4] for vehicles that only travel forward and make left or right unidirectional turns from predetermined starting and finishing places in the 2D plane. The author defines paths as combinations of straight (S) and curves (C) lines, where the vehicle may be straight or curved in the middle and starts and finishes with turns. $Z = \{LSL, RSL, RSR, LSR, LRL, RLR\}$ represents the six potential permutations of these pathways, where L and R stand for left and right turns, respectively. Later on, Dubins work was extended by Reeds and Shepp [5], allowing for backward adaptability in these routes. Since then, Dubins findings have been expanded upon, utilizing sophisticated theories in nonlinear optimal control [7]-[11] and integrating geometric concepts [6]. In particular, the application of Pontryagin's Maximum Principle (PMP) [12] has improved knowledge and uses of Dubins pathways. Due to these advancements, Dubins work now encompasses a broader range and is now considered a fundamental basis for the research of optimized vehicle trajectories.

Unmanned aerial vehicles (UAVs) often carry out surveillance missions when they are directed to visit specific target sites and gather the necessary data. The Dubins Touring Problem (DTP) [13] can be used to frame the problem if the order of visits to the destinations is known in advance. Many studies [14, 15] have been conducted on Dubins path problem for intercepting moving targets, looking at both relaxed optimal pathways and Dubins [16]-[19]. Looker [20] suggests a search algorithm to find the shortest route. Meyer et al. [21] study minimum-time interception, predetermined time for target movement, and the relaxed Dubins problem, focusing on finding the shortest path for a Dubin vehicle without terminal angle constraint. In [22], Yang and Kapila investigate

two-dimensional optimal path-planning problems for UAVs, accounting for both kinematic and tactical constraints. By applying vector calculus, the problem is reformulated as a parameter optimization task. An efficient numerical algorithm is developed to satisfy the optimality conditions. The proposed approach can manage multiple tactical constraints simultaneously, and its effectiveness is demonstrated through a representative numerical simulation. The optimality principles for collision-free trajectory and Dubins interpolating curves—the shortest curvature-constrained pathways in a given sequence—are the main topics of Kaya’s research [23, 24] on Dubins path reformulation. Zheng introduced a model for the Minimum-Time Intercept Problem [25], guiding a Dubins vehicle to intercept a moving target with prescribed impact angle constraints [26]. Forkan et al.[27] developed a mathematical model using arc parameterization to determine the optimal path length or time for touring a finite number of targets using an unmanned aerial vehicle. A method was designed for deploying a mobile sensor network to detect and capture moving targets in a planar environment in [28]. In order to guarantee target captureability and zero terminal guidance command, Hou [29] suggests geometry-based impact time and angle control rules that do not require time-to-go calculations.

The significance of UAV path planning for mission performance has led to the development of numerous approaches, particularly for challenges involving obstacle avoidance restrictions [30]. Recently, the shortest Dubins path between three consecutive via-points with given initial and final orientations is found using a novel technique presented in the research [31] that makes use of analytic geometry.

In this research, we formulate an optimal control problem to determine the shortest interception path for a pursuer aiming to reach a moving target while avoiding obstacles. We develop mathematical expressions to calculate the lengths of individual circular and straight line sub paths and construct a model to account for the presence of obstacles. In our model, the position of the pursuer within the obstacle’s turning circle is dynamically determined as a function of time. Unlike previous studies that considered fixed target points in path planning, our research introduces a framework in which both the touring point and the interception point are unknown. This represents a significant contribution to the field of path planning, as it allows for more realistic and adaptive interception scenarios. Furthermore, we develop an algorithm to evaluate the convergence rate of the proposed model, ensuring its stability and computational effectiveness. Several solvers are employed to solve the problem, as discussed in the numerical analysis section. Additionally, a real-world case study is presented to test the model’s capability and evaluate its effectiveness in computing an optimal interception path.

In Section 2, we define our problem statement and the motivation behind the study. The main contribution and outcome are briefly described in Section 3. The methodology of the formulation of the optimal control problem and identifying the control of the pathway

is described in Section 4. Section 5 presents the geometric structure of the engagement of pursuer, obstacle, and target. The proposed path planning model for intercepting a moving target touring with static obstacles is demonstrated in Section 6. Section 7 discusses the findings and interpretation of numerical tests for different scenarios. Later in Section 8, we present a real-life scenario of our proposed model. The conclusion of the paper is presented in the concluding part.

2 Problem Statement and Motivation

This study addresses a fundamental problem in real-world scenarios where a pursuer, moving at a constant speed, aims to intercept a moving target, also traveling at a constant speed, while navigating around a static obstacle situated between the pursuer and the target. The target is assumed to move in a straight line. This problem is particularly challenging in the contexts of autonomous systems, robotics, and unmanned vehicles, where the goal is to intercept a moving target while avoiding obstacles that may obstruct the pursuer’s path. Real-world applications, such as autonomous drone navigation, search and rescue missions, and surveillance operations, often present similar challenges. In these scenarios, the pursuer must reach the target quickly while effectively maneuvering around obstacles that may be positioned between them. While most conventional path planning models prioritize direct interception or obstacle avoidance, there has been limited research on the concept of "touring" that is, circumventing obstacles in a more strategic manner to approach the target. This gap motivates the present study, which seeks to develop a unified framework capable of simultaneously handling obstacle avoidance and moving target interception under minimum-path-length criteria.

The motivation for this work stems from the need to develop more realistic and computationally tractable models for UAV path planning. Existing approaches either simplify the obstacle environment, assume static target positions, or do not adequately capture the curvature constraints inherent to fixed-wing UAV dynamics. By integrating Dubins path theory with a dynamic interception model, this paper aims to fill these gaps and provide a model that is both theoretically sound and practically applicable.

In the numerical experiments Section 7, we demonstrate that the proposed approach achieves a more flexible and realistic solution for the interception scenario. This significantly improves the ability of autonomous systems to operate efficiently, reliably, and safely in diverse and rapidly changing environments.

3 Contribution and Outcome

This section summarizes the principal theoretical and computational contributions of the present work:

- (i) **Optimal Control Formulation:** We formulate a constrained optimal control problem for determining the minimum-length curvature-constrained path (CS path) that enables a pursuer (UAV) to intercept a moving target while avoiding one or more static circular obstacles.
- (ii) **Dynamic Touring Point:** Unlike prior studies that assume fixed waypoints, the touring point, the point at which the pursuer enters the turning circle of the obstacle, is treated as an unknown variable determined dynamically as a function of time and the relative geometry of the pursuer and target.
- (iii) **Unknown Interception Point:** The interception point is not prescribed in advance. Instead, it is computed as part of the optimization, allowing the framework to adapt to varying target speeds, headings, and initial conditions.
- (iv) **Geometric Optimality Conditions:** A set of sufficient geometric conditions is derived to characterize optimal CS paths under the imposed obstacle avoidance and interception constraints, enabling efficient computation without exhaustive enumeration of path types.
- (v) **Extension to Multiple Obstacles:** The model is systematically extended from a single-obstacle scenario to configurations involving a finite number of static obstacles, with a general algorithmic strategy for sequentially resolving each obstacle along the interception path.
- (vi) **Convergence Analysis:** An algorithm is developed to assess the convergence properties of the proposed iterative solution scheme, ensuring numerical stability across diverse initial configurations.
- (vii) **Real-World Case Study:** A realistic scenario using actual geographic data including terrain-based obstacle placement, target trajectory, and heading angles is presented to validate the proposed model’s practical applicability.

4 Methodology

In this section, we discuss the methodology of formulating the optimal control problem and how the control law has been applied to this problem. The pursuer can successfully

intercept the target only if its speed exceeds that of the target. We consider our assumption in two-dimensional space.

Consider a pursuer moving in a two-dimensional plane, with its position at time t given by $(x_p(t), y_p(t)) \in \mathbb{R}^2$. The pursuer follows an optimal trajectory $\ell(t) : [t_0, t_f] \rightarrow \mathbb{R}^2$, aiming to intercept a moving target while avoiding fixed obstacles along the way. We assume that the target moves along a straight-line path with a constant velocity, and the pursuer has complete knowledge of the target's motion, including its speed and direction.

A stationary obstacle, centered at (x_b, y_b) , is positioned along the potential interception path, requiring the pursuer to maneuver around it while maintaining an effective pursuit strategy. The pursuer's speed is denoted by $|\mathbf{V}_p|$, while the target's speed is $|\mathbf{V}_T|$. Additionally, the pursuer is subject to a minimum turning radius constraint, denoted by R , which limits its ability to make sharp turns. This constraint necessitates an optimal trajectory that balances efficient interception with obstacle avoidance, ensuring the pursuer reaches the target in the shortest possible time. Let the kinematic equations of the pursuers path be

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{y}_p(t) \\ \dot{\theta}_p(t) \end{bmatrix} = \begin{bmatrix} |\mathbf{V}_p| \cos \theta_p(t) \\ |\mathbf{V}_p| \sin \theta_p(t) \\ |\mathbf{V}_p| u(t) \end{bmatrix},$$

where the heading angle of the pursuer denotes $\theta_p(t)$ and $\theta_p(t) \in [0, 2\pi]$. This is calculated by going counterclockwise from the x -axis. Here the curvature is the main control variable $u(t)$. The control can be calculated by $u(t) = \frac{\dot{\theta}_p(t)}{|\mathbf{V}_p|}$ which can be positive or negative depending on either pursuer take left or right turn. If the pursuer moves in a straight line, $u(t)$ be zero. Consequently, the arc length functional determines the pursuer's minimal path for touring a static obstacle and intercepting a moving target

$$\int_{t_0}^{t_f} \sqrt{\dot{x}_p^2 + \dot{y}_p^2} dt = \int_{t_0}^{t_f} \|\dot{\ell}(t)\| dt = |\mathbf{V}_p|(t_f - t_0)$$

where, $\|\dot{\ell}(t)\| = |\mathbf{V}_p|$, $\dot{\ell}(t) = \frac{d\ell}{dt}$, and $\|\cdot\|$ is the Euclidean norm. When the pursuer navigates around a static obstacle, it either follows its minimum turning radius if the obstacle has a sharp corner or adopts the obstacle's turning radius. Consequently, the curvature of the pursuer's path changes, and this new curvature is denoted by $w(t)$. It is assumed that the kinematic equations describing the pursuer's motion while touring around the obstacle are given as follows.

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{y}_p(t) \\ \dot{\theta}_b(t) \end{bmatrix} = \begin{bmatrix} |\mathbf{V}_p| \cos \theta_b(t) \\ |\mathbf{V}_p| \sin \theta_b(t) \\ |\mathbf{V}_p| w(t) \end{bmatrix},$$

where $w(t) = \frac{\dot{\theta}_b(t)}{|\mathbf{V}_P|}$.

Therefore, the optimal control problem can be formulated as follows.

$$\begin{aligned}
\min \quad & f = \int_{t_0}^{t_f} |\mathbf{V}_P| dt = |\mathbf{V}_P|(t_f - t_0). \\
\text{subject to} \quad & \\
& \dot{x}_P(t) = |\mathbf{V}_P| \cos \theta_P(t), \quad x_P(t_0) = x_{P_0}, \\
& \dot{y}_P(t) = |\mathbf{V}_P| \sin \theta_P(t), \quad y_P(t_0) = y_{P_0}, \\
& \dot{\theta}_P(t) = u(t)|\mathbf{V}_P|, \quad \theta_P(t_0) = \theta_{P_0}, \\
& |u(t)| \leq a,
\end{aligned} \tag{1}$$

where $(x_P(t), y_P(t), \theta_P(t))$ represents the position along the path of pursuer. For the obstacle turning circle, the angle of direction $\theta_p(t)$ is replaced by the angle $\theta_b(t)$, and the new control input becomes $w(t)$, where $|w(t)| \leq b$.

Suppose that the target location is $(x_T(t), y_T(t), \theta_T)$ and that it moves without turns or maneuvers at a constant speed $|\mathbf{V}_T| > 0$. Within $[0, 2\pi]$, θ_T indicates the direction of the target's movement. Throughout the engagement, the value of θ_T stays constant because the target does not change its orientation. Considering that the target's starting position is (x_{T_0}, y_{T_0}) , the following equations represent its motion:

$$\begin{aligned}
\dot{x}_T(t) &= |\mathbf{V}_T| \cos \theta_T, \quad x_T(t_0) = x_{T_0}, \\
\dot{y}_T(t) &= |\mathbf{V}_T| \sin \theta_T, \quad y_T(t_0) = y_{T_0}.
\end{aligned} \tag{2}$$

To generate the shortest path for the interception of a moving target, the model is expected to first identify all feasible trajectories that the pursuer can follow. These trajectories are determined based on the pursuer's ability to turn left or right, considering both its minimum turning radius and the obstacle's turning radius. After generating this set of candidate paths, the model selects the optimal trajectory that minimizes both the total path length and the time required to intercept the moving target.

4.1 Optimality Conditions via Pontryagin's Maximum Principle

In this section, we derive necessary conditions for optimality for the interception problem formulated in (1) and (2) using Pontryagin's Maximum Principle (PMP) (see [12]). The objective is to characterize the structure of the optimal control and establish the geometric form of optimal trajectories.

Since the pursuer's speed ($|V_P|$) is constant, minimizing the function defined in (1) is equivalent to minimizing the terminal time (t_f). Therefore, the problem can be reformulated as a minimum-time optimal control problem with bounded curvature control.

Hamiltonian Formulation: Let the state vector of the pursuer be defined as

$$x(t) = (x_P(t), y_P(t), \theta_P(t))^T,$$

with dynamics given in (1), and control ($u(t) \in [-a, a]$). The Hamiltonian associated with this problem is defined as

$$H(x, \lambda, u) = 1 + \lambda_1 |V_P| \cos \theta_P + \lambda_2 |V_P| \sin \theta_P + \lambda_3 |V_P| u,$$

where $(\lambda(t) = (\lambda_1, \lambda_2, \lambda_3)^T)$ denotes the adjoint (costate) vector.

According to PMP, the costate variables satisfy

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x}(x^*(t), \lambda(t), u^*(t)).$$

Computing partial derivatives of the Hamiltonian yields the adjoint system $\dot{\lambda}_1 = 0$, $\dot{\lambda}_2 = 0$, and $\dot{\lambda}_3 = -|V_P|(-\lambda_1 \sin \theta_P + \lambda_2 \cos \theta_P)$.

Hence, λ_1 and λ_2 are constants along extremal trajectories, while λ_3 evolves according to the equation stated above. **Characterization of the Optimal Control**

The optimal control minimizes the Hamiltonian almost everywhere:

$$u^*(t) = \arg \min_{|u| \leq a} H(x^*(t), \lambda(t), u).$$

Since the Hamiltonian depends linearly on the control variable, define the switching function

$$\phi(t) = \frac{\partial H}{\partial u} = \lambda_3(t) |V_P|.$$

Since $|V_P| > 0$, the sign of $\phi(t)$ is determined by $\lambda_3(t)$. Then the optimal control satisfies

$$u^*(t) = \begin{cases} -a, & \text{if } \phi(t) > 0, \\ +a, & \text{if } \phi(t) < 0. \end{cases}$$

If $\phi(t) = 0$ on a nontrivial interval, the control is singular and requires higher-order analysis.

Theorem 4.1 *For the interception problem defined in (1) and (2), optimal controls are bang-bang and take values only at the bounds $u = \pm a$ or at $u = 0$ (straight-line motion).*

Proof: A singular arc would require

$$\phi(t) = 0 \quad \text{and} \quad \dot{\phi}(t) = 0,$$

over a time interval of positive measure. Since $\phi(t) = |V_P|\lambda_3(t)$, the condition $\dot{\phi}(t) = 0$ implies

$$-\lambda_1 \sin \theta_P + \lambda_2 \cos \theta_P = 0.$$

Because λ_1 and λ_2 are constants, this identity can hold on an interval only if θ_P is constant as $\theta_P = \tan^{-1} \frac{\lambda_2}{\lambda_1}$. From the state equation (1), this requires $u(t) = 0$. Hence singular arcs correspond exclusively to straight-line segments.

Therefore, optimal trajectories consist only of:

- (i). Maximum-curvature arcs ($u = \pm a$), and (ii). Straight-line segments ($u = 0$).

5 Geometric Pathway of Interception

This section presents the geometric representation of the interception path between a pursuer and a moving target. It is assumed that the initial positions and heading angles of both the pursuer $P(x_P(t_0), y_P(t_0), \theta_P(t_0))$ and the target $T(x_T(t_0), y_T(t_0), \theta_T(t_0))$ are known. In environments with static obstacles, the center and radius of each obstacle are also assumed to be known in advance. When an obstacle has a rectangular shape, a circular arc is introduced at the point where the pursuer begins a turning maneuver. The radius of this arc is set equal to the minimum turning radius of the pursuer to ensure the path remains feasible. This geometric approach provides both a visual and analytical understanding of the interception process in environments with obstacles.

According to Figure 1, the pursuer follows either path PX_1 or PX_2 to travel the circular path of the pursuer turning circle. The pursuer then follows one of the four straightline X_1X_5 , X_1X_6 , X_2X_3 , X_2X_4 to enter the obstacle turning circle. Then the pursuer tours the obstacle in one of the four feasible ways X_3X_8 , X_5X_8 , X_4X_7 , X_6X_7 . After touring the obstacle, the pursuer follows the straight way to intercept the target, either X_7X_{10} or X_8X_9 . This creates four possible feasible paths of the type $CS + CS + S_T$ which are as

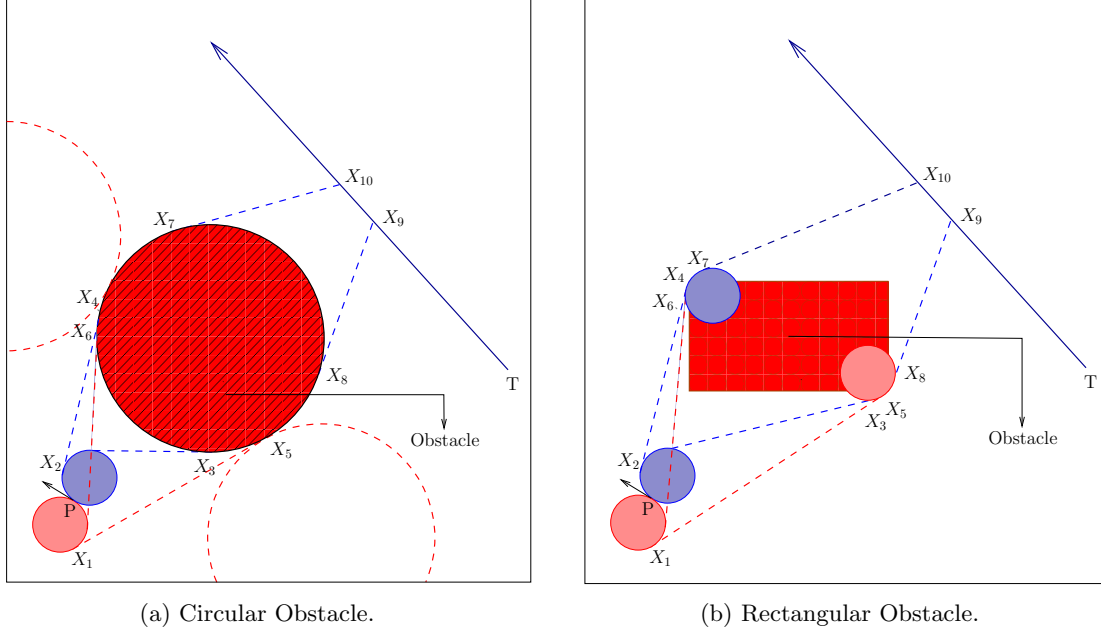


Figure 1: Paths of interception of a moving target and a pursuer touring a single obstacle.

follows:

$$\begin{aligned}
 & \left\{ \begin{aligned}
 & LS + LS + S_T \equiv \\
 & \text{arc } PX_1 + \text{st. line } X_1X_5 + \text{arc } X_5X_8 + \text{st. line } X_8X_9 + \text{st. line } TX_9, \quad \text{or,} \\
 & LS + RS + S_T \equiv \\
 & \text{arc } PX_1 + \text{st. line } X_1X_6 + \text{arc } X_6X_7 + \text{st. line } X_7X_{10} + \text{st. line } TX_{10}, \quad \text{or,} \\
 & RS + LS + S_T \equiv \\
 & \text{arc } PX_2 + \text{st. line } X_2X_3 + \text{arc } X_3X_8 + \text{st. line } X_8X_9 + \text{st. line } TX_9, \quad \text{or,} \\
 & RS + RS + S_T \equiv \\
 & \text{arc } PX_2 + \text{st. line } X_2X_4 + \text{arc } X_4X_7 + \text{st. line } X_7X_{10} + \text{st. line } TX_{10}, \quad \text{or,}
 \end{aligned} \right. \quad (3)
 \end{aligned}$$

Here, L and R represent the left and right circular arcs, respectively, while S denotes the straight-line segment of the pursuer's path. Additionally, S_T represents the straight-line path of the target.

6 Mathematical Model for Target Interception and Obstacle Avoidance

In this section, we present a mathematical model to determine the optimal path for a pursuer that must avoid a single obstacle—by touring around it—while intercepting a moving target. We develop mathematical expressions to compute the path lengths based on the geometric interpretation described in Section 5. The optimal interception path falls into one of four specific cases, as outlined in the set described in equation (3), where CS denotes a segment consisting of a curve followed by a straight line, representing the pursuer's motion, while S_T represents the straight-line path of the target.

$$\{LS + LS + S_T, LS + RS + S_T, RS + LS + S_T, RS + RS + S_T\}.$$

Consider the pursuer's starting time as $t_0 = 0$ and the terminal time as $t_f = t_6$. The length of the sub-path of the pursuer is defined as $\ell_i = |\mathbf{V_P}|(t_i - t_{i-1})$, where $|\mathbf{V_P}|$ is the speed of the pursuer. As the pursuer follows a path involving its minimum turning radius and tours around an obstacle, it generates six sub-paths. Therefore, we define ℓ_i for $i = 1, \dots, 6$. For the target, we define its path length by $\ell_7 = |\mathbf{V_T}|(t_6 - t_0)$, where $|\mathbf{V_T}|$ is the speed of the target, and ℓ_7 represents the distance the target travels until it is intercepted by the pursuer. We solve the ordinary differential equations given in (1) for $x_P(t)$, $y_P(t)$, and $\theta_P(t)$ over the time intervals $t_{i-1} \leq t \leq t_i$, for $i = 1, \dots, 6$.

$$\begin{aligned} \int_{t_{i-1}}^{t_i} \dot{x}_P(t) dt &= \int_{t_{i-1}}^{t_i} |\mathbf{V_P}| \cos \theta_P(t) dt, \\ \int_{t_{i-1}}^{t_i} \dot{y}_P(t) dt &= \int_{t_{i-1}}^{t_i} |\mathbf{V_P}| \sin \theta_P(t) dt, \\ \int_{t_{i-1}}^{t_i} \dot{\theta}_P(t) dt &= \int_{t_{i-1}}^{t_i} |\mathbf{V_P}| u(t) dt. \end{aligned} \tag{4}$$

From (4), we obtain the position $(x_P(t_i), y_P(t_i))$, $i = 1, 2$ along turning curve (C) yields

$$\begin{aligned} x_P(t_i) &= x_P(t_{i-1}) + (\sin \theta_P(t_i) - \sin \theta_P(t_{i-1})) / (\dot{\theta}_a(t) / |\mathbf{V_P}|), \\ y_P(t_i) &= y_P(t_{i-1}) - (\cos \theta_P(t_i) - \cos \theta_P(t_{i-1})) / (\dot{\theta}_a(t) / |\mathbf{V_P}|), \end{aligned} \tag{5}$$

where

$$u(t) = \frac{\dot{\theta}_a(t)}{|\mathbf{V_P}|} = \begin{cases} a & \text{if } \dot{\theta}_a(t) > 0 \\ -a & \text{if } \dot{\theta}_a(t) < 0 \\ 0 & \text{if } \dot{\theta}_a(t) = 0, \end{cases} \quad (6)$$

Assume that the centre of the fixed obstacle is (x_b, y_b) . The pursuer's entry point to the obstacle is defined as the tangential point between the exit point of the pursuer's minimum turning radius circle and the point where the pursuer begins to interact with the obstacle boundary. This entry point is not fixed, as it depends primarily on the orientation angle $\theta_P(t)$ when the pursuer exits its turning circle. Thus, the entry coordinates $(x_P(t_i), y_P(t_i))$ for $(i = 3)$ to obstacle are defined as follows.

$$\begin{aligned} x_P(t_i) &= x_b - \frac{1}{b} \cos \theta_P(t_{i-1}), \\ y_P(t_i) &= y_b + \frac{1}{b} \sin \theta_P(t_{i-1}), \end{aligned} \quad (7)$$

The position $(x_P(t_i), y_P(t_i))$, with $i = 3$ at which the pursuer reaches the obstacle along a straight line (S) can be described as

$$\begin{aligned} x_P(t_i) &= x_P(t_{i-1}) + |\mathbf{V_P}| \cos \theta_P(t_{i-1})(t_i - t_{i-1}), \\ y_P(t_i) &= y_P(t_{i-1}) + |\mathbf{V_P}| \sin \theta_P(t_{i-1})(t_i - t_{i-1}). \end{aligned} \quad (8)$$

Using (4), we obtain the pursuer's position $(x_P(t_i), y_P(t_i))$ for $i = 4, 5$, as it follows the turning curve C while navigating around the obstacle.

$$\begin{aligned} x_P(t_i) &= x_P(t_{i-1}) + (\sin \theta_P(t_i) - \sin \theta_P(t_{i-1})) / (\dot{\theta}_b(t) / |\mathbf{V_P}|), \\ y_P(t_i) &= y_P(t_{i-1}) - (\cos \theta_P(t_i) - \cos \theta_P(t_{i-1})) / (\dot{\theta}_b(t) / |\mathbf{V_P}|), \end{aligned} \quad (9)$$

where

$$w(t) = \frac{\dot{\theta}_b(t)}{|\mathbf{V_P}|} = \begin{cases} b & \text{if } \dot{\theta}_b(t) > 0 \\ -b & \text{if } \dot{\theta}_b(t) < 0 \\ 0 & \text{if } \dot{\theta}_b(t) = 0, \end{cases} \quad (10)$$

And $i = 6$ at which the pursuer intercepts the moving target along a straight line (S) can be described as (8)

Since the speed of the pursuer ($|\mathbf{V_P}|$) is constant over the time duration $[t_i - t_{i-1}]$, $i = 3, 6$, so the length of straight path for pursuer can be written as

$$\ell_i = |\mathbf{V_P}|(t_i - t_{i-1}), \quad i = 3, 6. \quad (11)$$

Now using (11), we rewrite (8) as below:

$$\begin{aligned} x_P(t_i) &= x_P(t_{i-1}) + \ell_i \cos \theta_P(t_{i-1}), \\ y_P(t_i) &= y_P(t_{i-1}) + \ell_i \sin \theta_P(t_{i-1}), \end{aligned} \quad (12)$$

and heading angles of pursuer along the both curve (C) and straight line (S) can be obtained as

$$\theta_P(t_i) = \theta_P(t_{i-1}) + (\dot{\theta}_a(t)/|\mathbf{V_P}|)|\mathbf{V_P}|(t_i - t_{i-1}), \quad i = 1, 2, \quad (13)$$

where $u(t) = (\dot{\theta}_a(t)/|\mathbf{V_P}|) = a$, for left-turn circular arc, $u(t) = -a$, for right-turn circular arc and $u(t) = 0$, for straight line, and

$$\theta_P(t_i) = \theta_P(t_{i-1}) + (\dot{\theta}_b(t)/|\mathbf{V_P}|)|\mathbf{V_P}|(t_i - t_{i-1}), \quad i = 4, 5. \quad (14)$$

Similar to $u(t)$, $w(t) = (\dot{\theta}_b(t)/|\mathbf{V_P}|)$ is defined as b for a left turn, $-b$ for a right turn, and 0 for straight-line motion.

Using (5), (6), (7), and (12)(when $i = 3$) the optimum path CS of the pursuer for the time duration from t_0 to t_3 is obtained below.

$$x_P(t_0) - x_b + \frac{1}{b} \cos \theta_P(t_2) + \frac{1}{a} (-\sin \theta_P(t_0) + 2 \sin \theta_P(t_1) - \sin \theta_P(t_2)) + \ell_3 \cos \theta_P(t_2) = 0, \quad (15)$$

and

$$y_P(t_0) - y_b - \frac{1}{b} \sin \theta_P(t_2) + \frac{1}{a} (\cos \theta_P(t_0) - 2 \cos \theta_P(t_1) + \cos \theta_P(t_2)) + \ell_3 \sin \theta_P(t_2) = 0. \quad (16)$$

Since the pursuer reaches the obstacle turning circle with the same heading angle so $\theta_P(t_3) = \theta_P(t_2)$. Now we generate the optimum pathway CS of the pursuer when touring the obstacle for the time t_4 to t_6 using (7), (9), (10) and (12)(when $i = 6$).

$$x_b - \frac{1}{b} \cos \theta_P(t_2) - x_P(t_6) + \frac{1}{b} (-\sin \theta_P(t_2) + 2 \sin \theta_P(t_4) - \sin \theta_P(t_5)) + \ell_6 \cos \theta_P(t_5) = 0, \quad (17)$$

and

$$y_b + \frac{1}{b} \sin \theta_P(t_2) - y_P(t_6) + \frac{1}{b} (\cos \theta_P(t_2) - 2 \cos \theta_P(t_4) + \cos \theta_P(t_5)) + \ell_6 \sin \theta_P(t_5) = 0. \quad (18)$$

Using (2), we develop the target pathway for the time duration from t_0 to t_6 as below,

$$\int_{t_0}^{t_6} \dot{x}_T(t) dt = \int_{t_0}^{t_6} |\mathbf{V}_T| \cos \theta_T dt,$$

and

$$\int_{t_0}^{t_6} \dot{y}_T(t) dt = \int_{t_0}^{t_6} |\mathbf{V}_T| \sin \theta_T dt.$$

From the above relations and using the relation $\ell_7 = |\mathbf{V}_T|(t_6 - t_0)$, we obtain the pathway for moving target as

$$x_T(t_6) - x_T(t_0) = \ell_7 \cos \theta_T, \quad (19)$$

and

$$y_T(t_6) - y_T(t_0) = \ell_7 \sin \theta_T. \quad (20)$$

We combined the *CS* pathway from obstacle to intercept point. It is noted that their positions coincide when the pursuer intercepts a moving target. In other words, $(x_P(t_6), y_P(t_6)) = (x_T(t_6), y_T(t_6))$ at the interception moment. As a result combining (17)-(20) and thus we obtain,

$$x_b - x_T(t_0) - \frac{1}{b} \cos \theta_p(t_2) + \frac{1}{b} (-\sin \theta_P(t_2) + 2 \sin \theta_b(t_4) - \sin \theta_b(t_5)) + \ell_6 \cos \theta_b(t_5) - \ell_7 \cos \theta_T = 0, \quad (21)$$

and

$$y_b - y_T(t_0) + \frac{1}{b} \sin \theta_p(t_2) + \frac{1}{b} (\cos \theta_b(t_2) - 2 \cos \theta_b(t_4) + \cos \theta_b(t_5)) + \ell_6 \sin \theta_b(t_5) - \ell_7 \sin \theta_T = 0. \quad (22)$$

When the target is detected at time t_0 , that point is taken as the target's initial position. At the same moment, the pursuer also begins its motion, meaning both start at the same time. For a successful interception, the pursuer and the target must reach the interception point at the same time, which means they must take the same amount of time to travel their respective paths. To ensure this timing coordination, we introduce the following constraint:

$$\sum_{i=1}^6 \ell_i / |\mathbf{V}_P| = \ell_7 / |\mathbf{V}_T|. \quad (23)$$

where $|\mathbf{V}_T|$ and $|\mathbf{V}_P|$ represent the constant speeds of the target and the pursuer, respectively. The terms ℓ_1 to ℓ_6 denote the lengths of the segments comprising the pursuer's path, while ℓ_7 denotes the length of the target's path.

Considering equations (13)-(16) and (21)-(23), we can formulate the following path planning model for target interception.

$$\begin{aligned}
& \min \quad f = \sum_{i=1}^7 \ell_i \\
& \text{subject to} \\
& \quad x_P(t_0) - x_b + \frac{1}{b} \cos \theta_P(t_2) + \frac{1}{a} (-\sin \theta_P(t_0) + 2 \sin \theta_P(t_1) - \sin \theta_P(t_2)) \\
& \quad + \ell_3 \cos \theta_P(t_2) = 0, \\
& \quad y_P(t_0) - y_b - \frac{1}{b} \sin \theta_P(t_2) + \frac{1}{a} (\cos \theta_P(t_0) - 2 \cos \theta_P(t_1) + \cos \theta_P(t_2)) \\
& \quad + \ell_3 \sin \theta_P(t_2) = 0, \\
& \quad x_b - x_T(t_0) - \frac{1}{b} \cos \theta_P(t_2) + \frac{1}{b} (-\sin \theta_P(t_2) + 2 \sin \theta_b(t_4) - \sin \theta_b(t_5)) \\
& \quad + \ell_6 \cos \theta_b(t_5) - \ell_7 \cos \theta_T = 0, \\
& \quad y_b - y_T(t_0) + \frac{1}{b} \sin \theta_P(t_2) + \frac{1}{b} (\cos \theta_b(t_2) - 2 \cos \theta_b(t_4) + \cos \theta_b(t_5)) \\
& \quad + \ell_6 \sin \theta_b(t_5) - \ell_7 \sin \theta_T = 0, \\
& \quad |\mathbf{V}_T| \sum_{i=1}^6 \ell_i - |\mathbf{V}_P| \ell_7 = 0, \\
& \quad \ell_i \geq 0, \quad i = 1, \dots, 7,
\end{aligned} \tag{24}$$

where

$$\theta_P(t_1) = \theta_P(t_0) + a\ell_1, \quad \theta_P(t_2) = \theta_P(t_1) - a\ell_2, \quad \theta_P(t_4) = \theta_P(t_2) + b\ell_4, \quad \theta_P(t_5) = \theta_P(t_4) - b\ell_5.$$

6.1 Touring n static obstacles

We extend Model (24) to the case where the pursuer must avoid n static obstacles before intercepting a moving target. The goal is to minimise the total path length, allowing the pursuer to intercept the target along the shortest possible path. The path is divided into segments made up of *CS* (curve-straight) type paths. The first *CS* path goes from the pursuer's starting point to the entry point of the first obstacle. The second *CS* path

connects the entry point of the first obstacle to the entry point of the second obstacle, and this process continues for each subsequent obstacle. In total, n CS paths are required to reach the entry point of the n th obstacle. Another path CS is added from the entry point of the last obstacle to the point of intercept with the target. This final segment requires initial target position and direction information. So, with n obstacles, the entire path includes $(n+1)$ CS paths, which are divided into $3n+4$ segments, labeled ℓ_i , where $i = 1, \dots, 3n+4$.

This extends the model for touring n obstacles and intercepting a moving target as follows.

$$\begin{aligned}
\min \quad & f = \sum_{i=1}^{3n+4} \ell_i, \quad n \in \mathbb{N} \\
\text{subject to} \quad & \begin{cases} x_P(t_{3j-3}) - x_P(t_{3j}) + \frac{1}{a_j}(-\sin \theta_P(t_{3j-3}) + 2\sin \theta_P(t_{3j-2}) - \sin \theta_P(t_{3j-1})) \\ + \ell_{3j} \cos \theta_P(t_{3j}) = 0, \\ y_P(t_{3j-3}) - y_P(t_{3j}) + \frac{1}{a_j}(\cos \theta_P(t_{3j-3}) - 2\cos \theta_P(t_{3j-2}) + \cos \theta_P(t_{3j-1})) \\ + \ell_{3j} \sin \theta_P(t_{3j}) = 0, \\ x_P(t_{3n}) - x_T(t_0) + \frac{1}{a_{n+1}}(-\sin \theta_P(t_{3n}) + 2\sin \theta_P(t_{3n+1}) - \sin \theta_P(t_{3n+2})) \\ + \ell_{3n+3} \cos \theta_P(t_{3n+3}) - \ell_{3n+4} \cos \theta_T = 0, \\ y_P(t_{3n}) - y_T(t_0) + \frac{1}{a_{n+1}}(\cos \theta_P(t_{3n}) - 2\cos \theta_P(t_{3n+1}) + \cos \theta_P(t_{3n+2})) \\ + \ell_{3n+3} \sin \theta_P(t_{3n+3}) - \ell_{3n+4} \sin \theta_T = 0, \\ |\mathbf{V}_T| \sum_{i=1}^{3n+3} \ell_i - |\mathbf{V}_P| \ell_{3n+4} = 0, \quad \ell_i \geq 0, \text{ for } i = 1, \dots, 3n+4. \end{cases}
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
x_P(t_{3j}) &= x_c^j - \frac{1}{a_{j+1}} \cos \theta_P(t_{3j}), \quad \text{and} \quad y_P(t_{3j}) = y_c^j + \frac{1}{a_{j+1}} \sin \theta_P(t_{3j}), \quad j = 1, \dots, n. \\
\theta_P(t_j) &= \theta_P(t_{j-1}) + \alpha_j \ell_j, \quad j = 1, \dots, 3n+3.
\end{aligned}$$

$$\begin{cases} \alpha_{3k-2} = a_k, \\ \alpha_{3k-1} = -a_k, \\ \alpha_{3k} = 0, \end{cases}$$

where, $k \in \{1, 2, \dots, n + 1\}$.

7 Numerical Experiments

The results of our numerical experiments are presented in this section to verify the effectiveness of our proposed model (25) in determining the optimal route that a pursuer must take to intercept a moving target interacting with static obstacles. To determine the best route for target interception, we provide the initial positions and heading angles of the pursuer and the target, as well as the obstacle's center and turning radii, taking into account their constant velocities. In each experiment, the pursuer has feasible routes, and our model determines the optimum pathway among the paths. We also evaluate our proposed model by varying the positions, velocities, and turning radii of both the pursuer and the target. This allows us to assess the efficiency of the model and examine how well it reaches the objectives of the analysis.

We perform the experiments using AMPL [32] modelling software, employing a variety of solvers, including GUROBI, SCIP, LGO, BONMIN, and IPOPT [33], all with their default settings. MATLAB is used to simulate the solutions obtained from these numerical experiments. All simulations are carried out on an HP ProBook 440 G6 laptop equipped with a 4.6 GHz Intel Core i7 processor and 8 GB of RAM.

Example 1

In this example, three experimental scenarios are carried out to evaluate the model's effectiveness in computing optimal and feasible trajectories, as shown in Table 1.

Table 1: *Test cases with a single obstacle.*

	Pursuer	Target	Obstacle (centre)	
(a)	$(0, 0, 3\pi/2)$ 6 1	$(20, 12, \pi)$ 1	$(4, 0)$ 2	Position Speed Turning Radius
(b)	$(0, 0, 7\pi/6)$ 6 0.5	$(30, 15, \pi)$ 2	$(6, 8)$ 5	Position Speed Turning Radius
(c)	$(0, 0, 7\pi/6)$ 6 1	$(6, 8, \pi/4)$ 3	$(6, 8)$ 5	Position Speed Turning Radius

In Table 1(a), we assume that the pursuer initiates its trajectory from the position $P(0, 0, 3\pi/2)$, while the target's initial position is set at $T(20, 12, \pi)$. A static obstacle

is centered at $(4, 0)$ with a turning radius of 2. In this experiment, the pursuer maintained a constant speed of 6 units, whereas the target moved along a straight path at a speed of 1 unit. This analysis demonstrated that the optimal trajectory for the pursuer followed the sequence $LS + RS + S_T$ (see Figure 2(a)). Figure 2(b) presents one of the feasible paths corresponding to the parameter settings provided in Table 1(a).

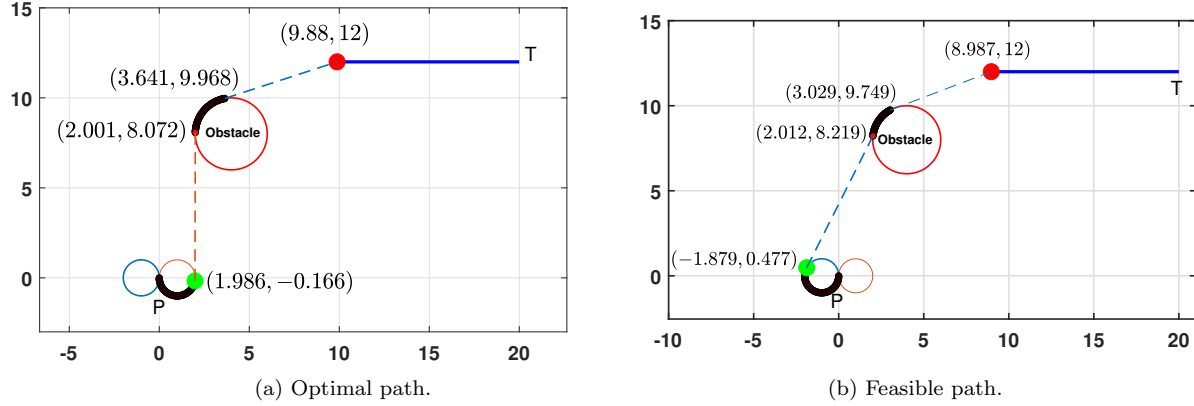


Figure 2: For Table 1 (a), optimal and feasible paths illustrating the exit points of the initial turning circles, the entry and exit points of subsequent turning circles around obstacles, and the interception points of the target for both paths.

Table 2(a) shows the optimal path taken by the pursuer to intercept the target. The path is composed of several segments, including left turns, right turns, and straight-line movements. The pursuer starts with a left turn of length $\bar{\ell}_1 = 2.97$, followed by a straight segment of $\bar{\ell}_3 = 10.28$. Next, the pursuer makes a right turn of $\bar{\ell}_5 = 2.74$ around the obstacle, then continues along a straight segment of $\bar{\ell}_6 = 10.13$ to reach the target. Meanwhile, the target moves along a straight path of $\bar{\ell}_7 = 4.35$ before being intercepted. The total length of the combined path, including both the pursuer and target segments, is $f = 30.47$, representing the shortest path that satisfies the motion constraints and interception conditions. Detailed information about the feasible path is provided in Table 2(a) under the row labelled “Feasible Path”.

Table 2: *Segment lengths of optimal and feasible paths for the setup outlined in Table 1.*

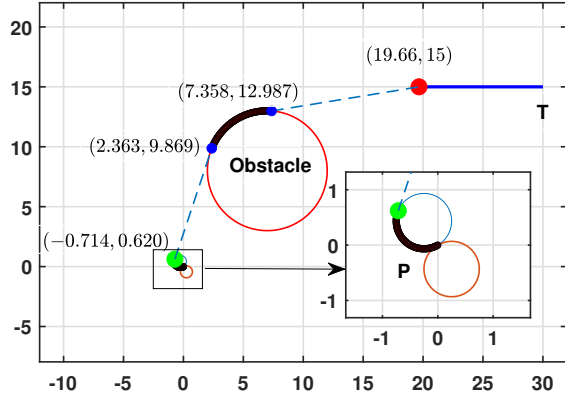
		Segment Lengths							
		Left turn $\bar{\ell}_1$	Right turn $\bar{\ell}_2$	St. line $\bar{\ell}_3$	Left turn $\bar{\ell}_4$	Right turn $\bar{\ell}_5$	St. line $\bar{\ell}_6$	Target St. line $\bar{\ell}_7$	Total length f
(a)	Optimal Path	2.97	0	10.28	0	2.74	10.13	4.35	30.47
	Feasible Path	0	3.64	10.57	0	1.93	11.04	4.53	31.71
(b)	Optimal Path	0	1.24	12.96	0	6.30	10.55	10.35	41.40
	Feasible Path	5.10	0	13.93	0	7.25	8.35	11.54	46.17
(c)	Optimal Path	0	2.59	11.56	0	2.60	10.71	13.73	41.19

Table 3 presents the time taken for each path section, including left turns, right turns, and straight-line segments, for both the optimal and feasible paths. Based on the parameters provided in Table 1(a), Table 3(a) shows that the optimal path takes in total 4.35 seconds to intercept the target. It is noted that the computational time to determine the optimal route in this experiment was approximately 0.031 seconds.

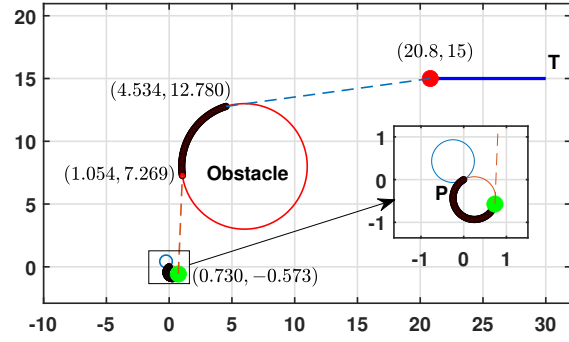
Table 3: *Path segment durations based on Table 1.*

		Time							
		Left turn $\bar{t}_1$	Right turn $\bar{t}_2$	St. line $\bar{t}_3$	Left turn $\bar{t}_4$	Right turn $\bar{t}_5$	St. line $\bar{t}_6$	Target St. line $\bar{t}_7$	Total Time t
(a)	Optimal Time	0.50	0	1.72	0	0.46	1.67	4.35	4.35
	Feasible Time	0	0.61	1.76	0	0.32	1.84	4.53	4.53
(b)	Optimal Time	0	0.21	2.16	0	1.05	1.76	5.18	5.18
	Feasible Time	0.85	0	2.32	0	1.21	1.39	5.77	5.77
(c)	Optimal Time	0	0.43	1.93	0	0.43	1.78	4.57	4.57

We now test our model (25) using the parameter settings given in Table 1 (b) and (c), where the pursuer's starting positions and heading angles are varied along with the target and obstacle locations. For the configuration in Table 1(b), the analysis shows that the optimal trajectory for the pursuer follows the sequence $RS + RS + S_T$, as illustrated in Figure 3(a), while the corresponding feasible path is shown in Figure 3(b). The total optimal path length is $f = 41.40$, as reported in Table 2(b), and the total optimal time taken is $t = 5.18$ seconds, shown in Table 3(b).



(a) Optimal path.



(b) Feasible path.

Figure 3: For Table 1 (b), optimal and feasible paths illustrating the exit points of the initial turning circles, the entry and exit points of subsequent turning circles around an obstacle, and the interception points of the target for both paths.

For the parameter settings in Table 1(c), the results are illustrated in Figure (4), with corresponding path length and time details provided in Tables 2(c) and 3(c).

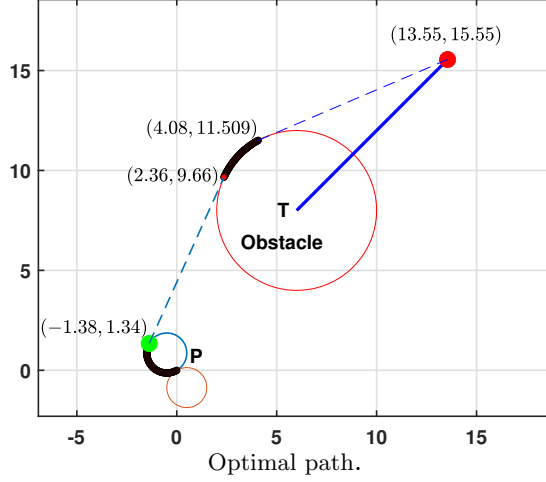


Figure 4: For Table 1 (c), optimal path illustrating the exit points of the initial turning circles, the entry and exit points of subsequent turning circles around an obstacle, and the interception points of the target for both paths.

Example 2

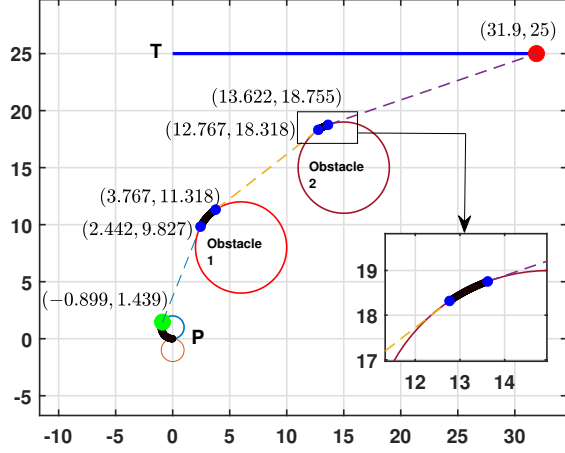
We test the proposed Model (25) to handle cases involving multiple obstacles. In this experiment, we generate optimal paths in the presence of two obstacles across four different setups, as listed in Table 4.

Table 4: *Test cases with two obstacles.*

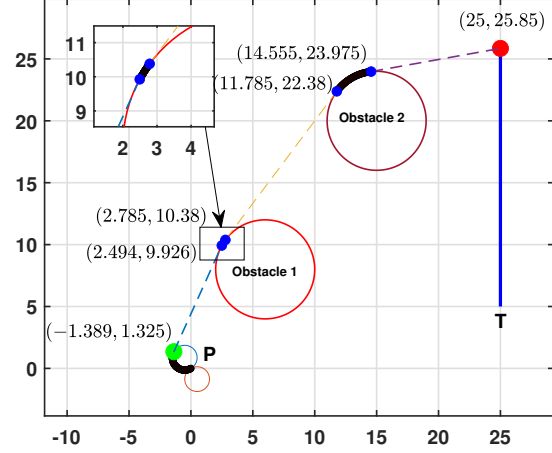
	Pursuer	Target	Obstacle 1 (centre)	Obstacle 2 (centre)	
(a)	$(0, 0, \pi)$ 7 1	$(0, 25, 0)$ 5	$(6, 8)$ 4	$(15, 15)$ 4	Position Speed Turning Radius
(b)	$(0, 0, 7\pi/6)$ 6 1	$(25, 5, \pi/2)$ 3	$(6, 8)$ 5	$(15, 20)$ 4	Position Speed Turning Radius
(c)	$(0, 0, \pi)$ 7 1	$(25, 25, 3\pi/2)$ 2	$(6, 8)$ 4	$(15, 15)$ 4	Position Speed Turning Radius
(d)	$(0, 0, 5\pi/6)$ 7 1	$(35, 5, 4\pi/6)$ 2	$(6, 8)$ 4	$(15, 15)$ 4	position Speed Turning Radius

For Table 4(a), the pursuer's trajectory originates from $P(0, 0, \pi)$, while the target begins its motion from $T(0, 25, 0)$. Two static obstacles, centered at $(6, 8)$ and $(15, 15)$, each with a turning radius of 4, are also incorporated into the analysis. The pursuer maintains a

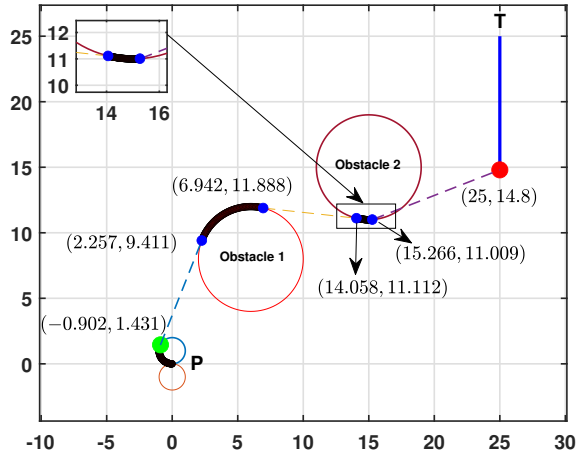
constant speed of 7 units, whereas the target moves along a straight path at a speed of 5 units. Our analysis determines that the optimal trajectory for the pursuer follows the sequence $RS + RS + RS + S_T$ demonstrated in Figure 5(a).



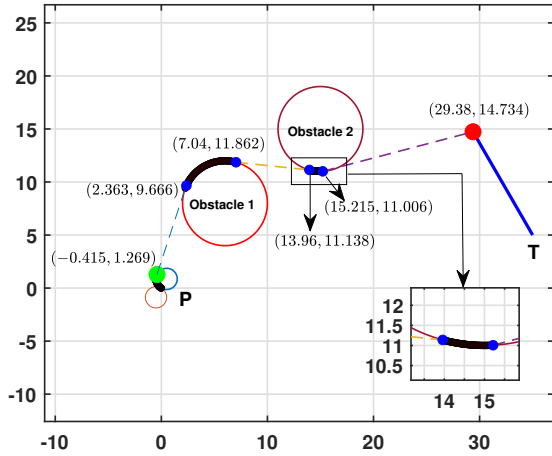
(a) Optimal path for Table 4(a).



(b) Optimal path for Table 4(b).



(c) Optimal path for Table 4(c).



(d) Optimal path for Table 4(d).

Figure 5: For Table 4(a)-(d), optimal paths illustrating the exit points of the initial turning circles, the entry and exit points of subsequent turning circles around two obstacles, and the interception points of the target for both paths.

Table 5(a) presents the optimal path taken by the pursuer to intercept the target. The path

consists of a sequence of segments—left turns, right turns, and straight-line movements similar to the structure observed in Example 1 with a single obstacle. The pursuer begins with a right turn of length $\bar{\ell}_2 = 2.05$, followed by a straight segment of $\bar{\ell}_3 = 11.37$. It then makes a right turn of $\bar{\ell}_5 = 2.02$ around the first obstacle, continues with a straight segment of $\bar{\ell}_6 = 7.45$ toward the second obstacle, and follows with a right turn and another straight segment leading to the interception point. Meanwhile, the target moves along a straight path of $\bar{\ell}_{10} = 31.95$ before being intercepted. The total combined path length, including both the pursuer and target segments, is $f = 76.45$, representing the shortest optimal path that meets the motion constraints and interception requirements.

Table 5: *Segment lengths of optimal paths for the setup outlined in Table 4.*

		Length										Total length f
		Left turn $\bar{\ell}_1$	Right turn $\bar{\ell}_2$	St. line $\bar{\ell}_3$	Left turn $\bar{\ell}_4$	Right turn $\bar{\ell}_5$	St. line $\bar{\ell}_6$	Left turn $\bar{\ell}_7$	Right turn $\bar{\ell}_8$	St. line $\bar{\ell}_9$	Target St. line $\bar{\ell}_{10}$	
(a)	Optimal Path	0	2.05	11.37	0	2.02	7.45	0	1.21	20.40	31.95	76.45
(b)	Optimal Path	0	2.60	11.61	0	0.57	13.93	0	3.29	9.74	20.88	62.62
(c)	Optimal Path	0	2.04	11.33	0	5.85	2.07	1.24	0	12.66	10.20	45.39
(d)	Optimal Path	0	1.48	11.39	0	5.96	1.94	1.27	0	17.01	11.25	50.30
	Feasible Path	4.92	0	13.67	0	5.75	1.83	1.45	0	15.98	12.60	56.20
	Feasible Path	0	1.48	11.39	0	2.23	7.07	0	3.11	14.90	11.48	51.66

Other setups are presented in Table 4(b), (c), and (d), where the pursuer and target heading angles, target positions, and the location of the second obstacle are varied. Using these configurations, we test the proposed Model (25). The corresponding optimal paths for each case are illustrated in Figures 5(b), (c), and (d), where the entry and exit points on the turning circles are indicated. Detailed sub-length information for these optimal paths is provided in Table 5(b), (c), and (d), respectively. A summary of the simulation results—including interception points, total path lengths, and time required for interception—for the cases listed in Table 4 is presented in Table 6.

Table 6: *Simulation result summary of Table 4.*

	Optimum Path	Interception Point	Optimum Length	Optimum Time
(a)	$RS + RS + RS + S_T$	(31.9, 25)	$f = 76.45$	$t = 6.39$
(b)	$RS + RS + RS + S_T$	(25, 25.85)	$f = 62.62$	$t = 6.96$
(c)	$RS + RS + LS + S_T$	(25, 14.8)	$f = 45.39$	$t = 5.10$
(d)	$RS + RS + LS + S_T$	(29.38, 14.73)	$f = 50.30$	$t = 5.58$

Two feasible paths are presented in Figure 6 for the parameter setting in Table 4(d), alongside the optimal path shown in Figure 5(d). These paths are included to illustrate that while the optimal path minimizes total length, alternative trajectories still satisfy the model’s constraints. Showing feasible paths helps demonstrate the model’s flexibility and the impact of different trajectory choices.

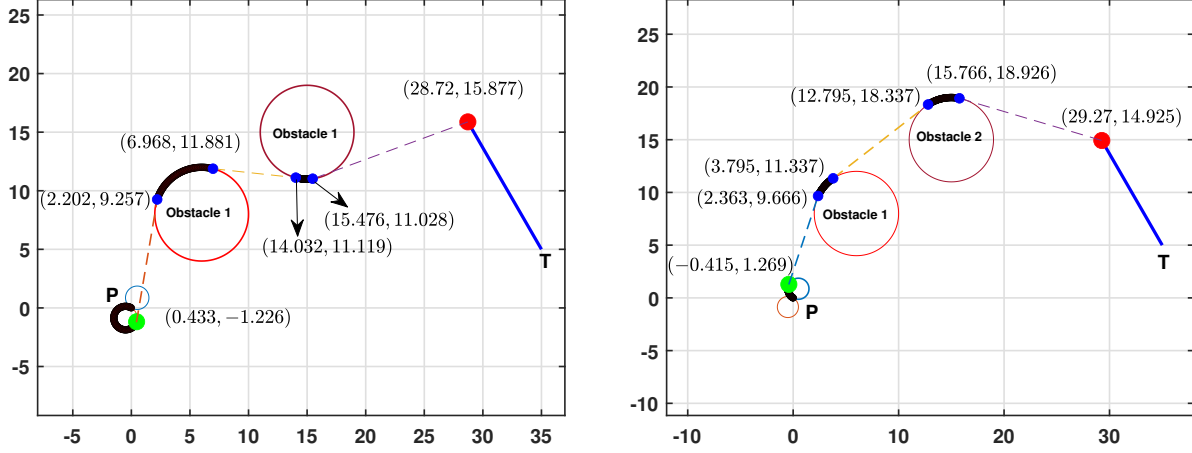


Figure 6: For Table 4(d), feasible paths illustrating the exit points of the initial turning circles, the entry and exit points of subsequent turning circles around two obstacles, and the interception points of the target for both paths.

Examples 1 and 2, along with supporting experiments, demonstrate the model’s ability to compute optimal paths under varying initial conditions while successfully avoiding obstacles and intercepting the target. The model effectively determines the shortest interception time and also provides feasible alternative trajectories. .

8 Real-World Test Scenario

In this test example, we utilise location data sourced from Google Maps, including the locations and heading angles of both the pursuer and the target, as well as the positions and radii (sizes) of obstacles that the pursuer must avoid. The experiment is set in the western region of Bangladesh using Google Maps. After defining the locations of all relevant elements, we collect the necessary data to carry out the simulation. The UAV (pursuer) starts from Satkhira City to intercept a moving target that begins travelling in a straight line from Feni City. Two restricted airspaces (or circular obstacles) are defined over Narail and Narayanganj, which the pursuer must strictly avoid. The latitude and

longitude coordinates for all relevant locations are collected using Google Maps. Initial parameters are summarized in Table 7. The autonomous pursuer is modelled with a minimum turning radius of 10 km. We used a relatively large minimum turning radius because the speed was set to 800 km/hour. However, we also tested the algorithm with smaller turning radii to evaluate its performance under tighter manoeuvring conditions. Each restricted airspace (obstacle) is represented with a turning radius of 20 km, defining the area that the pursuer must avoid.

Table 7: *Locations, coordinates, and motion parameters*

Name of City	Position in Earth	Position in 2D	Speed km/hour	Turning Radius
Satkhira	22°44'15.66"N, 89°3'25.65"E	(9133.10, 2528.32)	800	10
Narail	23°11'44.15"N, 89°29'49.47"E	(9147.19, 2579.23)		20
Narayanganj	23°38'35.46"N, 90°28'55.86"E	(9216.63, 2629.00)		20
Feni	23°1'34.54"N, 91°22'43.76"E	(9351.30, 2560.40)	400	

In our analysis, the pursuer successfully intercepts the target at geographic coordinates 23°57'38.90" N, 91°26'18.68" E, which correspond to 2D coordinates (9291.3, 2664.32). A detailed breakdown of the optimal interception path length is provided in Table 8. The pursuer covers approximately 240 km to reach the target, while the target travels about 120 km before being intercepted. The total interception distance, $f = 360$ km, is the sum of both trajectories. The simulation scenario is illustrated in Figure 7, which also depicts the corresponding geographical setting. .

Table 8: *Optimized path generation under varying model performance.*

$ \mathbf{V} $ & R	Length										
$ \mathbf{V_P} = 15$ $ \mathbf{V_T} = 7$ $R = 10$	Left turn $\bar{\ell}_1$	Right turn $\bar{\ell}_2$	St. line $\bar{\ell}_3$	Left turn $\bar{\ell}_4$	Right turn $\bar{\ell}_5$	St. line $\bar{\ell}_6$	Left turn $\bar{\ell}_7$	Right turn $\bar{\ell}_8$	St. line $\bar{\ell}_9$	Target St. line $\bar{\ell}_{10}$	Total length f
Optimal Path	0	12.82	61.05	0	16.13	55.03	0	5.36	89.98	120.18	360.55

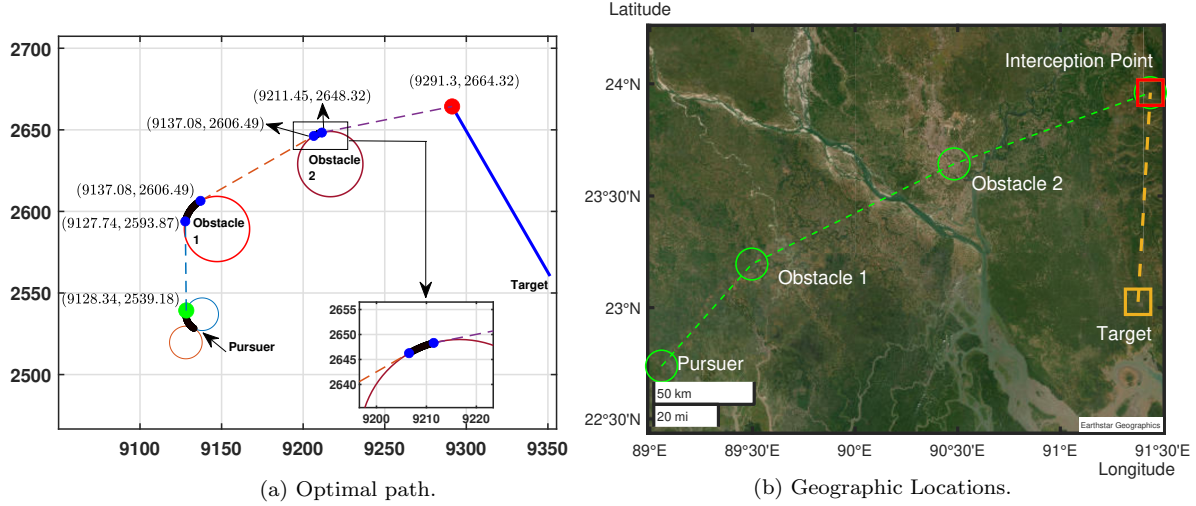


Figure 7: (a) Optimized trajectory of the automated drone from $P(9133.10, 2528.32)$ with heading $\theta_P = 5\pi/6$, intercepting the target that moved from $T(9351.30, 2560.40)$ while avoiding an obstacles centered at $(9147.19, 2579.23)$ and $(9216.63, 2629.00)$ as the target drone moves in direction $\theta_T = 4\pi/6$. (b) Geographic Locations on Map.

9 Conclusion

We have presented an optimal path planning approach for intercepting a moving target while avoiding obstacles. The study focuses on a 2D optimal control problem for a pursuer, considering both kinematic and tactical constraints. A geometric configuration is introduced to illustrate how the sub paths are determined. The pursuer's path is subject to a constraint that ensures the turning radius remains greater than or equal to its minimum turning capability. The mathematical formulation begins with the case of a single obstacle and is later extended to accommodate a finite number of obstacles. We derived necessary optimality conditions, and the model is evaluated through extensive numerical experiments. Several illustrative simulations are presented, demonstrating the interception path for various combinations of initial positions and heading angles of both the pursuer and the target. Additionally, a real-world case study is included to validate that the proposed optimal path planning method is capable of handling interception tasks in environments with obstacles.

For future research, the model can be extended to 3D space, which is particularly relevant for aerial and underwater vehicles operating in dynamic and constrained environments. Incorporating moving obstacles and time-dependent constraints would enhance the model's robustness and make it more suitable for real-time applications. Furthermore, refining

the optimization techniques could help reduce computational time, allowing for quicker responses and more efficient real-time interception.

10 Declarations

Ethics approval and consent to participate: Not applicable.

Consent for publication: Not applicable.

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A Algorithm for the Proposed Optimal Path

We now present the algorithm developed for our proposed Model (25), which determines the optimal interception path for a pursuer targeting a moving object in the presence of a single obstacle.

Algorithm A.1 Step 1 (*Input*)

Define the starting locations for a pursuer and a target as $P(x_P(t_0), y_P(t_0), \theta_P(t_0))$ and $T(x_T(t_0), y_T(t_0), \theta_T(t_0))$, respectively. Assign turning radius R_a , curvature $a = \frac{1}{R_a}$, and speed $|\mathbf{V_P}|$ and $|\mathbf{V_T}|$. Set ℓ_i , $i = 1, \dots, 7$, as variables. Set the center of the obstacle (x_b, y_b) and the curvature of the obstacle $b = \frac{1}{R_b}$, turning radius R_b .

Step 2 (*Find the distance between the pursuer's starting point and the intercept location.*)

Consider the following: left turn angle $\theta_P(t_1) = \theta_P(t_0) + a\ell_1$; right turn angle $\theta_P(t_2) = \theta_P(t_1) - a\ell_2$. Also in obstacles turning circle, $\theta_P(t_4) = \theta_P(t_2) + b\ell_4$ for left turn, $\theta_P(t_5) = \theta_P(t_4) - b\ell_5$ for right turn.

To solve Model (25), find $\ell := (\bar{\ell}_1, \dots, \bar{\ell}_7)$.

Set $\bar{F} := [\bar{\ell}_1, \dots, \bar{\ell}_7]$. Set $\bar{T} := [\bar{t}_1, \dots, \bar{t}_7]$, where $\bar{t}_i = \bar{\ell}_i/|\mathbf{V_P}|$, $i = 1, \dots, 6$, $\bar{t}_7 = \bar{\ell}_7/|\mathbf{V_T}|$ and $f = \sum_{i=1}^7 \bar{\ell}_i$.

Step 3 (*Simulation*)

Same as [34, Step 3 in Algorithm 5.1].

Step 4 (*Output*)

Calculate $\cup_{i=1}^7 \bar{X}_i$.