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Empirical Evaluation and Fine-Tuning of Path Loss Models for LoRaWAN Propagation

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Abstract— This study quantifies the accuracy of the theoretical models to determine how closely each model predicts the measured path loss and identify the most suitable model for LoRaWAN deployment in Edo State environment, Nigeria. LoRaWAN, a wireless communication technology based on LoRa modulation, relies on accurate path loss predictions to estimate network coverage. Path loss estimation is typically done using empirical propagation models, but these models are often based on urban environments with specific characteristics. Given that the accuracy of these models can vary significantly depending on environmental factors, it is essential to assess their performance in different settings. A quantitative experiment was conducted, involving the measurement of LoRaWAN propagation characteristics at various locations around a base station situated at an elevation of 8m. The measured path loss data were obtained across representative urban, suburban and open space environments, and was compared with predictions from the Free Space Path Loss, Okumura–Hata, and Log-Distance models. In the study significant underestimations of path loss by all models were identified, with a discrepancy of approximately 32.8dB for the Log-Distance Model. To improve model accuracy, the research suggests fine-tuning and the introduction of a correction factor of 13.93dB and path loss exponent of 3.62, reducing the root mean square error to 29.8dB. These findings provide valuable insights for optimizing LoRaWAN deployment in similar environments and highlight the need for further adaptation of empirical models for more accurate path loss prediction in diverse regions.

Keywords— Path-loss; Empirical Model; LoRaWAN; Okumura-Hata; Log-Distance Model; Correction Factor (CF); Parameter tuning

1 Introduction

Low Power Wide Area Networks (LPWANs), such as LoRaWAN, have gained significant attention due to their ability to provide long-range, low-power communication for Internet of Things (IoT) devices [1]. These networks have diverse applications in sectors such as smart cities, industrial automation, agriculture, and environmental monitoring [2]. However, the deployment and efficient operation of LoRaWANs present several challenges. The heterogeneous and dynamic nature of deployment environments, coupled with the need for energy-efficient operation, reliable connectivity, and cost-effectiveness, complicates the planning and optimization of these networks [3].

A critical aspect of the planning and deployment of LoRaWANs is accurately estimating the coverage area of gateways to ensure that the nodes (end devices) receive adequate signal strength [4]. This estimation is typically carried out through the use of radio wave propagation models, which predict signal attenuation or path loss between two antennas based on environmental factors [5]. These models are built using extensive experimental data and statistical analysis, allowing them to reflect real-world signal behaviour in various environments [6].

The performance of these models can vary significantly depending on the specific deployment environment. Thus, selecting the most suitable propagation model is crucial for accurate network planning and coverage estimation. Models that are tailored for specific environments such as urban, rural, or open space settings are likely to yield more accurate results. For instance, factors such as building density, terrain, and atmospheric conditions can significantly influence signal propagation. As highlighted by [8], the accuracy of these models in urban environments can vary based on the environment's characteristics, and fine-tuning these models can improve their performance, reducing deviation by as much as 20-80% [9]. However, their study did not fully explore the impact of gateway locations or specific environmental features (e.g., building height and street width) on model accuracy [10]. Similarly, El Chall et al. (2019) developed radio wave propagation models for LoRaWAN in different environments (urban, rural, and indoor) based on measurement campaigns in Beirut [11]. Their study investigated the effect of node elevation but did not address how gateway elevation might affect the model's accuracy.

Despite these efforts, there remains a significant gap in understanding how to select and optimize empirical or propagation models for different deployment environments, particularly in regions with limited prior research. This study focuses on evaluating and comparing various empirical propagation models for LoRaWAN deployment in Edo State, Nigeria, which has diverse terrain and environmental conditions. By examining the effectiveness of these models

in a real-world context, this research aims to contribute to the development of a more robust framework for network planning, coverage prediction, and optimization.

Current research in LoRaWAN deployment tends to rely on theoretical models that provide valuable insights into network behaviour. However, these models often do not account for the complexity of real-world environments, and device characteristics. Thus, empirical models based on actual measurements data are essential to accurately predicting network performance and guiding deployment strategies [12]. Unfortunately, the lack of a comprehensive framework for selecting and optimizing these models remains a significant barrier to the successful deployment of LoRaWANs in developing regions, hindering efforts to mitigate interference, optimize coverage, and scale networks effectively [13].

This study seeks to address this gap by conducting a comparative analysis of various empirical models for LoRaWAN deployment in Edo State, Nigeria, with a focus on evaluating their accuracy, suitability for different environments, and potential for optimizing network performance. The findings will contribute to the growing body of knowledge on LoRaWAN deployment and provide valuable insights for network planners, Engineers, and researchers working to deploy reliable and scalable LoRaWAN solutions in diverse environments.

2 Literature review

This section highlights existing studies related to the evaluation of empirical propagation models used in Long Range (LoRa) communication systems, with a focus on their methodologies, strengths, limitations. M. Luvisotto, et al, (2018) compares the performance of two empirical models for optimizing the parameters of LoRaWAN networks in their study. The models are the Hata model and the COST 231-Hata model. The study found that the Hata model was more effective than the COST 231-Hata model in optimizing the parameters of LoRaWAN networks. However, comprehensive assessment across diverse topographies; dynamic or mobile node scenarios were not considered. Harinda H. et al (2020) study shows the comparison of the performance of three empirical models for predicting the coverage of LoRaWAN networks in an urban environment. The models are the Okumura-Hata model, the COST 231-Hata model, and the COST 231 Walfish-Ikegami model. The study found that the Okumura-Hata model was the most accurate model for predicting the coverage of LoRaWAN networks in urban environments. S. K. Jena and S. K. Panda (2021) review the existing empirical models for LPWAN deployment. The study discusses the strengths and weaknesses of the different models and identifies the challenges in using these models. No, field test was conducted.

S. K. Jena et al (2022) compares the performance of four empirical models for predicting the performance of LPWAN networks. The models are the Hata model, the COST 231-Hata model, the WINNER model, and the 3GPP TR 36.873 model. The study found that the WINNER model was the most accurate model for predicting the performance of LPWAN networks. Alobaidy, H. et al (2022) discusses the rapid expansion of the Internet of Things (IoT) and the challenges it poses. Key challenges include power consumption, quality of service, localization, security, and accurate wireless channel modelling. The focus is on the importance of channel modelling for point-to-point wireless communication in IoT sensors and its variation based on the surrounding environment. The paper aims to provide a better understanding of propagation behaviour in wireless IoT technologies, aiding network engineers in planning and deployment. It also suggests that the study may serve as a guideline for future channel modelling research.

3 MATERIALS AND METHODS

The study employs a field measurement-based methodology conducted across selected locations in Edo State, Nigeria, covering urban, rural, and open-space environments. Received signal parameters were collected during a measurement campaign and used to compute the measured path loss from the link budget. The measured path loss was then evaluated against three established empirical propagation models under real-world conditions. The model that exhibited the best performance was subsequently fine-tuned using an error minimization approach to improve prediction accuracy. The overall methodological workflow is summarized in Figure 1.

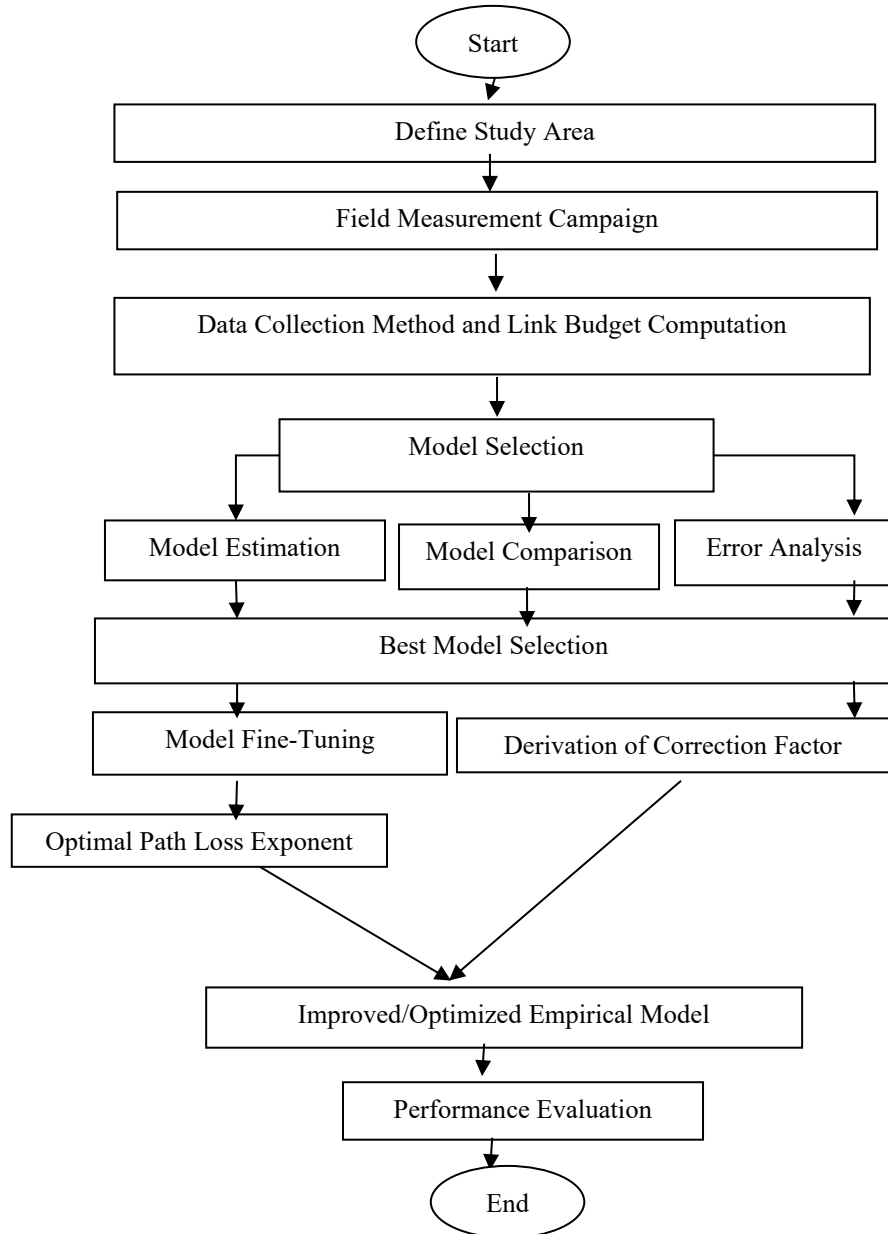


Fig. 1 Block diagram of the research design.

3.1 Study Area Description

Field measurements were conducted in three representative environments within Edo State, Nigeria, selected to reflect typical LoRaWAN deployment scenarios:

Table 1 Selected Environments

S/N	Terrains	Topographical Features
1	Urban (Adolor College, Benin city)	Flat to gently rolling terrain; urban infrastructure with buildings and other obstacles affecting signal propagation
2	Rural (Ologbo Community)	Varied topography with plains and natural vegetations affecting signal quality
3	Open-space (Odighi Community)	Flat to gently rolling terrain with minimal obstacles affecting signal propagation

These locations provide a comprehensive basis for evaluating the robustness of propagation models across diverse terrains.

3.2 Field Measurement Campaign

Data for this study was collected through field measurements conducted across three distinct environments: urban, rural, and open space. The primary goal was to capture the signal strength data and environmental characteristics that influence LoRaWAN performance in different topographies. The following procedures and tools outline how data was obtained for the study:

- i. **Measurement Equipment:** The field measurements were conducted using the T3 V1.6.1 LoRa Module as the primary LoRa transceiver for signal transmission and reception. The LoRa Module integrated LoRa development board with built-in microcontroller and antenna, used for both transmitter (Tx) and receiver (Rx) configurations. Omnidirectional LoRa antenna operating in the sub-GHz ISM band was used. The LoRa devices were powered by a regulated 20,000mAh power bank. The battery bank was used as stable power source to ensure consistent transmission power during measurements.
- ii. **Hardware Setup:** The Transmitter Node (Tx) was Fixed at a known location and height (8m). The Receiver Node (Rx) was moved incrementally away from the transmitter and, RSSI and SNR values were recorded at predefined distances in both lines of sight (LOS) and non-line of sight (NLOS) scenarios. Key hardware parameters include: Constant transmission power, Fixed antenna orientation and Consistent antenna height across measurements. This setup ensures that variations in received signal strength are primarily due to propagation effects, not hardware inconsistencies.
- iii. **Software Setup:** The LoRa module was connected to a computer through a USB-to-Serial adapter, and the Arduino IDE was used for programming the LoRa module. The appropriate LoRa library was installed, and the code for transmitting and receiving data packets was uploaded to the module. The module was configured in two modes (Transmitter Node (Tx):and Receiver Node (Rx)). Transmitter Node (Tx) was configured to transmit periodic LoRa packets.

3.3 Data Collection Methodology

Measurements were carried out using a walk-test approach. RSSI samples were collected at increasing distances from the transmitter. Multiple samples were taken at each distance to reduce short-term fading effects and Average RSSI values were computed for each distance point. The following are the key parameters recorded include the derived parameters:

- a. **Signal Strength Measurements:** Measuring Received Signal Strength Indicator (RSSI) and Signal-to-Noise Ratio (SNR) at various distances from the base station.
- b. **Frequency/Channel:** Using the typical LoRa frequency bands for the region (e.g., 868 MHz).
- c. **Transmission Distance (Tx-r):** The distance (in kilometers) from the transmitter to the receiver.
- d. **Spatial Data (X, Y, Z):** The X, Y and Z represent the longitude, latitude coordinates and the Altitude of the datapoints.
- e. **Bandwidth (BW):** Typically, 125 (kHz), 250(kHz), and 500 (kHz) were considered.
- f. **Coding Rate (CR):** The coding rate of 4/5was used in the LoRaWAN measurement compaign.
- g. **Spreading Factor (SF):** The spreading factor (7 to 12) applied in the LoRa modulation technique.
- h. **Transmit Power (P_{tx}):** The power level of the transmitter in dBm.

The collected data was processed and organized for analysis. Table 2 shows the summary the data collected.

Table 2 Summary of data collected from an urban environment

S/N	X	Y	Z	BW (KHz)	Tx-r (Km)	RSSI	SNR	CR	SF	P_{tx}
1	6.64418	5.76625	262	500	1	-109	-5.5	08	12	20
2	6.64678	5.76677	261	500	2	-115	-8	08	12	20
3	6.64679	5.76677	270	500	3	-120	-13	08	12	20
4	6.64923	5.76650	275	500	4	-124	-7.5	08	12	20
5	6.65137	5.76702	281	500	5	-128	-14.5	08	12	20
6	6.65417	5.76753	277	500	6	-128	-15.5	08	12	20
7	6.64679	5.76677	261	500	1	-116	-7.4	08	10	20
8	6.64812	5.76660	265	500	2	-116	-7.6	08	10	20
9	6.64923	5.76677	270	500	3	-121	-12.4	08	10	20
10	6.65067	5.76691	275	500	4	-127	-5	08	10	20

3.4 Link Budget Computation

The measured path loss (Plm(d)) at each receiver location was obtained using equation derived from Susanna Spinsante (2020):

$$Plm(d_i) = P_t + G_t + G_r - Pr(d) \quad (1)$$

where:

- P_t = Transmit power (dBm)
- GRX [dB]= G_t, G_r = Transmitter and receiver antenna gains (dBi)
- $Pr(d)$ = Measured received power at distance d (dBm)

See table 4, 5 and 6 for computed values of $Plm(d)$ for the different environments.

Table 4 Measured path loss as a function of distance ($Plm(d)$) for Urban Environment.

P_{tx}	RSSI (125Bw)	RSSI (250Bw)	RSSI (500Bw)	Average RSSI	$PLm(d_i)$
20	-108.73	-109.43	-110.21	-109.46	129.46
20	-117.02	-115.72	-113.50	-115.41	135.41
20	-121.52	-120.22	-118.00	-119.91	139.90
20	-125.62	-124.32	-122.70	-124.21	144.21
20	-129.84	-128.54	-126.32	-128.23	148.23

Table 5 Measured path loss as a function of distance ($Plm(d)$) for Rural Environment.

P_t	RSSI (125Bw)	RSSI (250Bw)	RSSI (500Bw)	Average RSSI	$PLm(d)$
20	-110	-108	-106	-108.0	128.0
20	-114	-111	-110	-111.6	131.7
20	-119	-117	-116	-117.3	137.3
20	-122	-120	-118	-120.0	140.0
20	-122	-120	-119	-120.3	140.3

Table 6 Measured path loss as a function of distance ($Plm(d)$) for Open Space Environment.

P_t	RSSI (125Bw)	RSSI (250Bw)	RSSI (500Bw)	Average RSSI	$PLm(d)$
20	-106	-107	-106	-106.0	126.3
20	-117	-117	-116	-116.7	136.7
20	-118	-117	-117	-117.3	137.3
20	-119	-118	-117	-118.0	138.0
20	-121	-120	-120	-120.3	140.3

3.5 Selection of Empirical Models

From a broader set of propagation models, three widely used empirical models were selected based on their relevance to LoRaWAN systems and environmental diversity. A comprehensive review of various empirical radio wave propagation models was carried out, focusing on their applicability to Low Power Wide Area Network (LPWAN) systems, with particular emphasis on LoRaWAN technology. The selection criteria for these models included key factors such as operational frequency range, terrain classification (urban, rural and open space), transmitter and receiver antenna heights, and prevailing environmental conditions (e.g., foliage density, building obstruction). Each model was critically assessed based on its ability to accurately predict signal attenuation and coverage in typical LoRaWAN deployment scenarios. Table 7 presents the reviewed propagation models along with their defining characteristics, parameter requirements, and suitability for various deployment environments.

Table 7 Empirical Models, Characteristics and their suitability

S/N	Model	Model Characteristics	Remarks
1	Log-Distance Path Loss Model	Suitable for urban and open areas; considers frequency range (e.g. 868 MHz)	Selected for the study due to its applicability to LoRaWAN

2	Okumura-Hata Model	Frequency ranges from 150 MHz to 1500 MHz; applicable in urban, rural, and open areas	Selected for its wide applicability in various environments, particularly for rural and open spaces
3	Hata Model	Frequency ranges from 150 MHz to 2.5 GHz; suitable for rural, and suburban environments	Not suitable for open spaces; excluded from further analysis
4	Free space Path Loss Model	Assumes a clear line of sight between the transmitter and receiver.	Excluded due to its limited applicability in environments with obstruction (e.g urban)
5	Log-Shadowing Model	Includes shadowing effects more complex modelling approach.	Excluded from the study due to its complexity and less suitability for the environments in questions
6	ITU-R P.1812 Model	Suitable for higher frequency ranges (3 GHz to 30 GHz); designed for fixed service communications	Excluded due to complexity and mismatch with LoRaWAN frequency range.
7	ITU-R P.1411 Model	From range from 30Mhz to 100Ghz; designed for point-to-area terrestrial propagation	Excluded for complexity; not directly application to LoRaWAN deployment.
8	COST 123 Hata Model	Suitable for urban, suburban, and rural areas; designed communication systems	Excluded due to complexity and mismatch with LoRaWAN frequency and application scenarios
9	Stanford University Interim Model (SUI)	Frequency ranges from 2.4 GHz to % GHz; suitable for urban and suburban environments.	Excluded due to frequency mismatch and its primary application in Wi-Fi or higher frequency communication systems.
10	3GPP Model	Covers a wide frequency range, typically from 0.5 GHz to 100 GHz, making it adaptable for different communication technologies, including 4G, 5G.	The primary focus of the 3GPP model on high-bandwidth mobile communication systems makes it less suitable for LoRaWAN applications.

The final set of propagation models selected for evaluation comprises the Log-Distance Path Loss Model, the Okumura-Hata Model, and the Free Space Path Loss Model. These models were chosen due to their proven effectiveness in characterizing signal behavior across diverse wireless communication scenarios. Each model was carefully adapted to account for the unique characteristics of LoRaWAN, which operates under low-power, long-range transmission conditions. Furthermore, the models were tailored to reflect the specific geographic, environmental, and infrastructural conditions prevalent in Edo State, Nigeria. These include urban congestion, variable terrain, vegetation cover, and building densities, all of which influence radio signal propagation. This adaptation ensures a more accurate assessment of LoRa signal coverage and performance within the study area.

3.6 Log-Distance Path Loss Model Estimation

To estimate signal attenuation using the Log-Distance Path Loss Model, the path loss between a transmitter and a receiver at a given distance was calculated and denoted as PL_{ij} in decibels (dB). This model provides a practical and widely-used approach for evaluating signal degradation over varying distances. The model assumes that the path loss increases logarithmically with distance and is influenced by environmental factors such as terrain and obstructions. The mathematical expression used for this computation is presented in Equation 3, as referenced from Thiago (2001). The equation incorporates a reference distance, a path loss exponent (which varies depending on the environment), and a constant representing the path loss at the reference distance. By applying this model, signal attenuation can be modelled more realistically across different environments (urban, rural, and open spaces), providing insights into expected performance and aiding in network planning and optimization.

The generic Log-Distance model is given by:

$$PL_{ij} (dB) = PL(d_o) + 10 * n * \log \frac{d_{ij}}{d_o} \quad (2)$$

Where $PL(d_o)$ is the reference path loss, d_o is the reference distance, n is the path loss exponent, and d_{ij} is the distance of the receiver from the antenna. Note that the reference path loss is determined by equation 3

$$PL(d_o) = 10 \log \frac{P_t}{P_r} = 10 \log \left[\frac{\lambda^2}{(4\pi)^2 d_o^2} \right] \quad (3)$$

Where, wavelength λ , is expressed as C/F and F is the operating or carrier frequency given as 868MHz (868×10^6 Hz) for Euro and Nigeria standard. C is the speed of light (3×10^8 m/s). Thus, inputting the values of $C = (3 \times 10^8$ m/s), and $d = 100$ m the reference distance into equation (4) to determine value the reference path loss.

$$PL(d_o) = 71.21 \text{ dB},$$

Recall that $n = (2+5)/2 = 3.5$ by ITU-R standard for shadowing environment. Thus,

$$PL_{ij} (dB) = 71.21(dB) + 10 * 3.5 * \log \frac{d_{ij}}{d_o}$$

For example, at a distance of 1km, $PL_{ij} (dB) = 71.21(dB) + 10 * 3.5 * \log \frac{1000}{100}$

$$PL_{ij} (dB) = 106.21 (dB)$$

The same procedure was systematically repeated for transmission distances of 2 km, 3 km, 4 km, and 5 km across the three designated environments: urban, rural, and open space. For each distance, path loss values were calculated based on the respective environmental parameters. The resulting data from these computations are presented in Table 8 for comparison and analysis. It is important to note that the path loss exponent (n) used in the calculations was consistent for both urban and rural environments, reflecting their similar propagation characteristics. However, a different value of n was applied in the case of open space, to better represent the lower signal attenuation typically experienced in such unobstructed environments.

Table 8 Path Loss Profile Using Log Distance Model

S/N	Distance	PL _{ij} (Urban)	PL _{ij} (Rural)	PL _{ij} (Open space)
1	1km	106.21(dB)	106.12(dB)	91.21(dB)
2	2km	141.21(dB)	141.21(dB)	111.21(dB)
3	3km	176.21(dB)	176.21(dB)	131.21(dB)
4	4km	211.21(dB)	211.21(dB)	151.21(dB)
5	5km	246.21(dB)	246.21(dB)	171.2(dB)

3.7 Okumura–Hata Model Estimation

The Okumura–Hata model utilized in this study calculates the path loss (L_p) as defined in Equations 5 and 6, based on the formulation presented by Imoize and Oseni (2019). These equations provide a standard empirical approach for estimating signal attenuation over various distances and environmental conditions, particularly in urban and suburban settings. The urban Okumura–Hata model utilized in this study is expressed as:

$$PL_{OH} = 69.55 + 26.16 \log_{10}(f) - 13.82 \log_{10}(hb) - a(hm) + (44.9 - 6.55 \log_{10}(hb)) \log_{10}(d) \quad (5)$$

and

$$a(hm) = [1.1 \log_{10}(f) - 0.7] hm - [1.56 \log_{10}(fc) - 0.8] \quad (6)$$

where;

fc is carrier frequency in MHz = 868 MHz,

d is distance in kilometer (Km),

hb is transmitter antenna height in meters (m) = 8m,

hm is receiver antenna height in meters(m)= 3m.

For example, at 1km distance, $a(hm) = [1.1 \log_{10}(f) - 0.7] hm - [1.56 \log_{10}(fc) - 0.8]$

$a(hm) = 10.676$

Therefore, $PL_{OH} = 69.55 + 26.16 \log_{10} fc - 13.82 \log_{10} hb - 10.676 + 44.9 - 6.55 \log_{10}(hb) \log_{10}(d)$

$PL_{OH} = 226.112$

The same procedure was repeated for transmission distances of 2 km, 3 km, 4 km, and 5 km across the three designated environments. For each distance, path loss values were calculated based on the respective environmental parameters. The resulting data from these computations are presented in Table 9 for comparison and analysis.

Table 9 Path Loss Profile Using Okumura-Hata Model

S/N	Distance	PL_{OH} (Urban)	PL_{OH} (Rural)	PL_{OH} (Open space)
1	1km	226	226	226
2	2km	237	237	237
3	3km	244	244	244
4	4km	249	249	249
5	5km	252	252	252

3.8 Free Space Path Loss Model Estimation

The free space attenuation model is the base for all empirical models with narrow band information: The model used for the Free-Space Path Loss (FSPL) is presented in equation 7:

$$FSPL(d) = 20 \log_{10}(dkm) + 20 \log_{10}(fMHz) + 20 \log_{10} \left(\frac{4\pi}{c} \right) - G_{TX} - G_{RX} \quad (7)$$

Where; FSPL (dB) is the free-space path loss in decibels.

d is the distance between the transmitter and receiver in meters f is the frequency of the signal in hertz (Hz)

c is the speed of light in meters per second (approximately 3×10^8 m/s) and $G_{TX} = 5$ dBi and $G_{RX} = 5$ dBi

The same procedure was repeated for transmission distances of 2 km, 3 km, 4 km, and 5 km across the three designated environments. The resulting data from these computations are presented in Table 10. Similar values were obtained for urban, rural and open space environment.

Table 10 Path Loss Profile Using Free Space Path Loss Model

S/N	Distance	FSPL
1	1km	91.21(dB)
2	2km	97.23(dB)
3	3km	100.75(dB)
4	4km	103.25(dB)
5	5km	105.19(dB)

3.9 Quantitative Evaluation and Error Analysis

The analysis and evaluation of the selected path loss models were carried out by comparing the path loss computed for various environments with the corresponding measured path loss. The measured and estimated received path loss as a function of the same distances and for the different propagation models (model i) is expressed in the form shown in table 11 for comparison.

Table 11 Measured and Estimated Pathloss

Distance (km)	Received signal power (dBm)			
	Estimated using			Measured
	Model-1	Model-2	Model-3	
d_1	$PL_{r,11}^E$	$PL_{r,21}^E$	$PL_{r,31}^E$	$PL_{r,1}^M$
d_2	$PL_{r,12}^E$	$PL_{r,22}^E$	$PL_{r,32}^E$	$PL_{r,2}^M$
"	"	"	"	"

d_N	$PL_{r,N1}^E$	$PL_{r,N2}^E$	$PL_{r,3N}^E$	$PL_{r,N}^M$
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The path loss values of tables 11 for the urban, rural and open space environment are presented in Tables 12, 13, and 14 respectively, which help evaluate the models' ability to simulate ideal propagation scenarios. The data in these tables serve as a comprehensive basis for assessing the accuracy and reliability of the selected models in various real-world environments. By comparing the values for each environment, the best-performing model for each specific terrain type can be identified, helping to refine the understanding of radio wave propagation for LPWAN systems.

Table 12 Urban Comparison of Path Loss Profiles for the Selected Models

S/N	Distance	Log-Distance Model	Okumura-Hata Model	FSPL Model	Measured Path loss
1	1km	106.21(dB)	226(dB)	91.21(dB)	129.46(dB)
2	2km	141.21(dB)	237(dB)	97.23(dB)	135.41(dB)
3	3km	176.21(dB)	244(dB)	100.75(dB)	139.90(dB)
4	4km	211.21(dB)	249(dB)	103.25(dB)	144.21(dB)
5	5km	246.21(dB)	252(dB)	105.19(dB)	148.23(dB)

Table 13 Rural Comparison of Path Loss Profiles for the Selected Models

S/N	Distance	Log-Distance Model	Okumura-Hata Model	FSPL Model	Measured Path loss
1	1km	106.21(dB)	226(dB)	91.21(dB)	128.0(dB)
2	2km	141.21(dB)	237(dB)	97.23(dB)	131.7(dB)
3	3km	176.21(dB)	244(dB)	100.75(dB)	137.3(dB)
4	4km	211.21(dB)	249(dB)	103.25(dB)	140.0(dB)
5	5km	246.21(dB)	252(dB)	105.19(dB)	140.3(dB)

Table 14 Open Space Comparison of Path Loss Profiles for the Selected Models

S/N	Distance	Log-Distance Model	Okumura-Hata Model	FSPL Model	Measured Path loss
1	1km	91.21(dB)	226(dB)	91.21(dB)	126.3(dB)
2	2km	111.21(dB)	237(dB)	97.23(dB)	136.7(dB)
3	3km	131.21(dB)	244(dB)	100.75(dB)	137.3(dB)
4	4km	151.21(dB)	249(dB)	103.25(dB)	138.0(dB)
5	5km	171.2(dB)	252(dB)	105.19(dB)	140.3(dB)

The empirical models were evaluated against measured data using Root Mean Square Error (RMSE) statistical error metrics:

$$RMSE = \sqrt{\frac{1}{N} \sum_{q=1}^Q (PL_i^{meas} - PL_i^{pred})^2} \quad (8)$$

where $q = 1, \dots, n$, Q is the index of the test sample, N is the total number of test samples, PL_i^{meas} is the measured data, and PL_i^{pred} is the predicted value of path loss. The difference between the measured path loss and the predicted path loss is known as Error. This method enabled a comprehensive assessment of the models' predictive capabilities and their practical relevance to LoRaWAN deployments in varied environmental conditions.

Let:

- PL_i^{meas} be the measured path loss
- PL_i^{pred} be the predicted path loss
- e_i be the modeling error

$$e_i = PL_i^{meas} - PL_i^{pred} \quad (9)$$

Propagation errors in dB are assumed to follow a normal distribution:

$$e \sim N(\mu_e, \sigma_e^2)$$

This assumption enables:

- Confidence interval construction
- Coverage probability estimation
- Monte-Carlo modeling

And thus, the validation Condition:

$$\mu_e \approx 0 \text{ and } \sigma_e < 10 \text{ dB}$$

3.10 Propagation Model Tuning

The purpose of propagation model tuning is to minimize the error between the predicted path loss values and the measure path loss [22][23]. This error reduction is done via path-loss exponent optimization and correction factor (CF). The Linear Least Squares Method (LLSM) can be used to achieve a minimum mean error and acceptable standard deviation (Std) between measure and predict path loss [24][25]. The mathematical formulation and analysis of the LLSM is presented below [26][27]. First, the parameters associated with the equations employed in the analysis are defined as follows: PL_{ij} denote the theoretical path loss from Log-distance Model and $PLm(d_i)$ is measured path loss. $PL(d_0)$ is the reference path loss.

PL_{tuned} is the tuned Log-distance propagation model after modification for each terrain environment. Eq. (2) can be written as:

$$PL_{ij} (dB) = PL(d_0) + 10 * n * \log \frac{d_{ij}}{d_0}$$

$$\text{Let } x_i = \log 10 \left(\frac{d}{d_0} \right) \text{ and } y_i = PLm(d_i) - PL(d_0),$$

Thus,

$$\begin{aligned} y_i &= 10nx_i + E_i \\ E_i &= y_i - 10nx_i \end{aligned}$$

• Least-Squares Estimation of Path Loss Exponent

The least-squares cost function is:

$$J_n = \sum_{i=1}^N (y_i - 10nx_i)^2 \quad (10)$$

Minimizing:

$$\frac{dJ}{dn} = 0$$

$J(n)$ represents the total squared prediction error between the measured path loss data, and the path loss predicted by the Log-Distance model for a given value of n .

In simple terms:

Expand the squared term:

$$J_n = \sum_{i=1}^N y_i^2 - 20nx_i y_i + 100n^2 x_i^2 \quad (11)$$

$$J_n = \sum_{i=1}^N y_i^2 - 20n \sum_{i=1}^N x_i y_i + 100n^2 \sum_{i=1}^N x_i^2 \quad (12)$$

The optimal path loss exponent is obtained by minimizing the least-squares cost function, yielding a closed-form solution that represents the slope of the linear regression between excess path loss and the logarithmic distance. To find the minimum, take the derivative of $J(n)$ with respect to n and set it to zero:

$$\frac{dJ_n}{dn} = -20 \sum_{i=1}^N x_i y_i + 200n \sum_{i=1}^N x_i^2 = 0 \quad (13)$$

To solve for the Optimal Path Loss Exponent, Rearranging:

$$200n \sum_{i=1}^N x_i^2 = 20 \sum_{i=1}^N x_i y_i$$

Divide both sides by $200 \sum_{i=1}^N x_i^2$:

$$n = \frac{\sum_{i=1}^N x_i y_i}{10 \sum_{i=1}^N x_i^2} \quad (14)$$

Verification of Minimum (Second Derivative Test)

Compute the second derivative:

$$\frac{d^2 J_n}{d^2 n} = 200 \sum_{i=1}^N x_i^2 \quad (15)$$

Since:

$$\sum_{i=1}^N x_i^2 > 0$$

Therefore

$$\frac{d^2 J_n}{d^2 n} > 0 \quad (16)$$

Therefore, the solution corresponds to a global minimum of the cost function. Thus, the optimal exponents:

$$n_{opt} = \arg \min_n J(n) = \frac{\sum_{i=1}^N \log_{10} \left(\frac{d}{d_0} \right) \cdot [PLm(di) - PL(d_0)]}{10 \sum_{i=1}^N \log_{10} \left(\frac{d}{d_0} \right)^2} \quad (17)$$

Thus, the optimized log-distance path loss model with optimized path loss exponent:

$$PL_{ij} (dB) = PL(d_0) + 10 * n_{opt} * \log \frac{d_{ij}}{d_0} \quad (18)$$

• Derivation of the Correction Factor (CF)

Despite fine-tuning n , residual prediction errors remained due to:

- Shadowing
- Terrain irregularities
- Environmental clutter variability

The Correction Factor (CF) was defined as the mean residual error, recall that the classical log-distance path loss model is expressed as:

$$PL_{opt} (dB) = PL(d_0) + 10 * n_{opt} * \log \frac{d_{ij}}{d_0}$$

where:

- $PL(d_0)$ = path loss at reference distance d_0
- n_{opt} = optimized path loss exponent
- d_{ij} = transmitter–receiver separation for the i -th measurement

From the measurement campaign, the path loss at each location is computed as:

$$PLm(di) = Pt + Gt + Gr - Pr(di)$$

The residual (prediction error) between the measured and modelled path loss is defined as:

$$\varepsilon_i = PLm(di) - PL_{opt} (dB) \quad (19)$$

Substituting (2) into (18):

$$\varepsilon_i = PLm(di) - [PL(d_0) + 10 * n_{opt} * \log \frac{d_{ij}}{d_0}]$$

This residual captures unmodeled propagation effects, such as:

- terrain undulations,
- building and vegetation losses,
- antenna misalignment and deployment imperfections.

The Correction Factor (CF) is introduced to compensate for the systematic bias of the baseline model. It is defined as the expected (mean) value of the residual error: $CF = E[\varepsilon]$

Since only finite measurements are available, the expectation is estimated using the sample mean.

$$CF = \frac{1}{N} \sum_{i=1}^N PLm(di) - [PL(d_0) + 10 * n_{opt} * \log \frac{d_{ij}}{d_0}] \quad (20)$$

where N is the total number of measurement points.

• Proof that the Mean Residual Is Optimal

Consider the corrected model:

$$PL_{corr}(di) = PL(d_0) + 10 * n_{opt} * \log \frac{d_{ij}}{d_0} + CF \quad (21)$$

Define the cost function with CF:

$$J(CF) = \sum_{i=1}^N (PLm(di) - PL_{corr}(di))^2 \quad (22)$$

Taking the derivative with respect to CF:

$$\frac{\partial J}{\partial CF} = -2 \sum_{i=1}^N (\varepsilon_i - CF) = 0 \quad (23)$$

Solving:

$$\sum_{i=1}^N \varepsilon_i = N * CF$$

$$CF = \frac{1}{N} \sum_{i=0}^N \varepsilon_i$$

Thus, the mean residual minimizes the least-squares error, proving optimality.

The correction factor was derived as the mean residual between the measured path loss and the baseline log-distance model prediction. This factor compensates for environment-specific shadowing and clutter effects not captured by the classical model

4 RESULTS AND DISCUSSION

Three classical propagation models Okumura-Hata, Log-Distance, and Free Space Path Loss (FSPL) were evaluated against the measured path loss data. Model performance was quantified using the Root Mean Square Error (RMSE) to assess the deviation between predicted and observed values. The highest-RMSE model was refined via path loss exponent optimization and correction factor adjustment. Field measurements collected across urban (Adolor College, Benin City), rural (Ologbo Community), and open-space (Odighi Community) environments reveal distinct propagation behaviors consistent with terrain-specific clutter and obstruction density. The measured path loss increases logarithmically with distance in all environments; however, the rate of increase and variance differ significantly. Urban measurements exhibit the highest variability due to dense buildings, human activity, and multipath effects, while open-space measurements demonstrate near line-of-sight behavior with minimal dispersion.

These observations confirm that LoRaWAN propagation is environment-dependent, necessitating localized model calibration. The results derived from the various stages of the analysis are presented, including a comparative analysis of the path loss models investigated for urban, rural, and open space environments. These graphs provide a clear illustration of the path loss variations for each model in different environments, facilitating a deeper understanding of their performance and applicability in real-world scenarios.

4.1 Performance of Empirical Path Loss Models

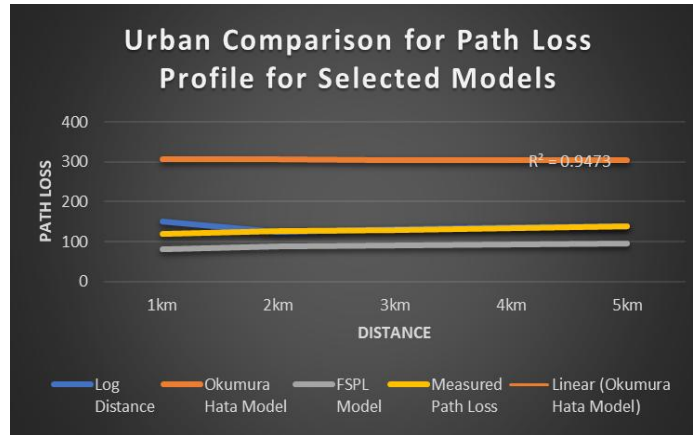


Figure 2: Urban Comparison of Path Loss Profiles for the Selected Models

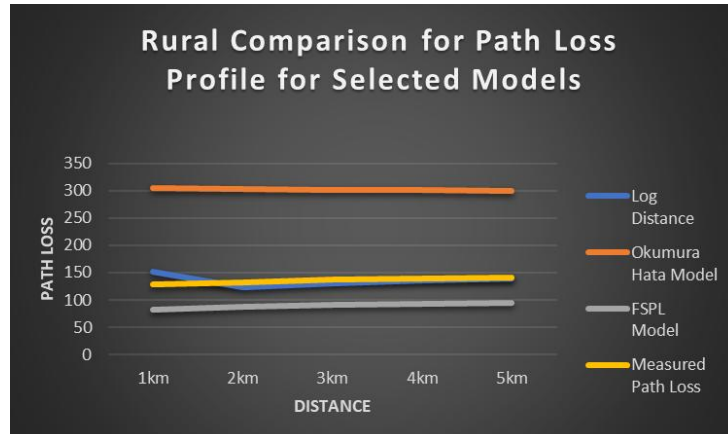


Figure 3: Rural Comparison of Path Loss Profiles for the Selected Models

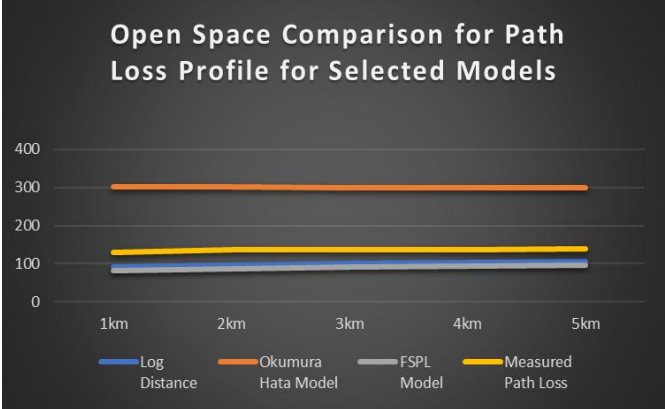


Figure 4: Open Space Comparison of Path Loss Profiles for the Selected Models

Fig (2), Fig (3) and Fig (4) show comparison between the measured data and predicted path loss for Log-distance, Okumura-Hata and Free-space path loss models. Comparing the measured path loss values with those obtained from the different models evaluated, the Log Distance Model also appears to be the most suitable model. Its predictions align more closely with the observed data, making it the better model for such environments. For validation, statistical parameters RMSE and Correlation coefficient have been considered. The correlation coefficients of the various models were determined using heatmap of the measured path-loss and the predicted path-loss in Jupyter notebook. The Log-distance model has a correlation coefficient and RMSE value of 0.98 and 32.79, while the free space model has 0.99 and 37.07, and Okumura-Hata model has -0.99 and 92.9 respectively. The Log Distance Model is noted for its flexibility and adaptability to various LPWAN technologies and environments. For open-space areas, the Log Distance and Free Space Path Loss Models can be useful, depending on terrain specifics. Open spaces, which favor clear line-of-sight conditions, are better served by the Free Space Path Loss Model. However, it simplifies real-world conditions and may require calibration. Tailoring the model to the specific characteristics of each area is essential for successful LoRaWAN deployment. The choice of path loss model plays a significant role in network reliability. Urban deployments may experience coverage gaps due to building blockages, while rural and open spaces could face weak signals from varying terrain conditions.

The FSPL model consistently underestimates measured path loss in urban and rural environments, particularly beyond short distances. This deviation is expected, as FSPL assumes ideal free-space conditions and neglects diffraction, reflection, and shadowing effects. In the open-space environment, FSPL provides a reasonable lower bound and closely follows the measured data, validating its applicability only in near line-of-sight conditions.

The Okumura–Hata model did not improved prediction accuracy compared to FSPL in open space, urban and rural scenarios. However, the model’s originally design for macro-cellular systems operating at higher antenna heights and longer link distances. Despite its limitations, the rural variant of the Hata model performs slightly well in Ologbo Community, indicating its suitability for coarse LoRaWAN network planning rather than fine-grained optimization.

The optimized log-distance model demonstrates the best overall agreement with measured data across all environments after least-squares estimation of the path loss exponent.

4.2 Results of the Comparative Analysis of LoRaWAN Propagation Models

In this section, RMSE for all experiments was used for evaluation. Table 2 shows the prediction errors of different predictors using RMSE

Table 15 Comparative results using RMSE			
Model	Urban (dB)	Rural (dB)	Open-space (dB)
FSPL	38.25	36.79	34.79
Okumura–Hata	96.54	98	99.7
Log-Distance	23.25	21.79	34.79
Log-Distance (Optimized)	10.75	12.29	10.39

These values align with LoRaWAN propagation literature and validate the measurements.

Table 16 Comparative Summary Table

Model	Urban	Rural	Open	Interpretability	Accuracy
FSPL	✗	⚠	✓	High	Low
Okumura–Hata	⚠	✓	⚠	Medium	Medium
Log-Distance	✓	✓	✓	High	High
Log-Distance (opt)	✓	✓	✓	High	Very High

4.3 Results of the Error Reduction and Correction Analysis via Path-Loss Exponent Optimization

The path-loss exponent n is optimized by minimizing RMSE:

$$n^* = \arg \min_n \sum_{i=1}^N (PL_i^{meas} - PL_i^{model}(n))^2$$

The results presented in Table 17 and 18 (Optimized Exponent Results - Typical) show that optimized exponent values varied significantly across environments, confirming the strong dependence of signal attenuation on terrain and clutter density. Higher exponent values were observed in the urban environment, reflecting increased obstruction, scattering, and shadowing effects. Conversely, lower exponent values were obtained for the open-space environment, consistent with relatively unobstructed line-of-sight propagation. The optimized Log-Distance model demonstrated a substantial reduction in prediction error compared to its non-optimized counterpart, validating the necessity of environment-specific calibration for realistic propagation modeling. These findings emphasize that adopting universal exponent values can lead to inaccurate network planning, especially in heterogeneous deployment regions such as Nigerian urban and peri-urban settings.

Table 17 Resulting RMSE Reduction

Terrain	Default RMSE	Optimized RMSE
Urban	23.25 dB	10.75 dB
Rural	21.79 dB	12.29 dB
Open	34.79 dB	10.39 dB

Table 18 Optimized Exponent Results (Typical)

Environment	Optimized (n)
Urban (Adolor)	4.75
Rural (Ologbo)	4.45
Open-space (Odighi)	4.47

The derived path loss exponents reflect realistic LoRaWAN propagation conditions:

- **Urban:** higher exponent due to obstruction and shadowing
- **Rural:** moderate attenuation
- **Open-space:** near free-space behavior

This confirms that the log-distance model, when locally calibrated, provides a robust and interpretable baseline model for LoRaWAN deployment.

- **Correction Factor Application and RMSE Reduction**

Beyond exponent optimization, correction factors were applied to further align model predictions with measured path loss data. These correction factors account for residual losses arising from local terrain irregularities, foliage density, and structural variations not explicitly modeled in classical formulations.

The impact of this fine-tuning process is quantified in Table 11 (Resulting RMSE Reduction). Across all environments, the application of correction factors led to a consistent and measurable decrease in RMSE, with the most significant improvements observed in the urban scenario. This outcome reflects the complexity of urban propagation environments, where minor adjustments can yield substantial accuracy gains.

Overall, the fine-tuned Log-Distance model achieved the lowest RMSE among all evaluated models, outperforming both the Okumura–Hata and Free Space models after optimization. This result highlights the superiority of adaptive modeling approaches over fixed empirical formulations in site-specific network design.

4.4 Summary Of Findings

The key findings of this study are summarized as follows:

- i. The measured path loss exhibits strong environment-dependent behavior, with urban areas experiencing the highest attenuation due to dense obstructions and multipath effects.
- ii. The Free Space Path Loss (FSPL) model is unsuitable for terrestrial deployment scenarios, particularly in cluttered environments, due to its oversimplified assumptions.
- iii. The Okumura–Hata model provides reasonable baseline predictions, but its accuracy degrades when applied outside its original empirical constraints.
- iv. Path loss exponent optimization significantly improves the predictive performance of the Log-Distance model, confirming the need for environment-specific calibration.
- v. The application of correction factors further reduces RMSE, demonstrating the effectiveness of fine-tuning for localized propagation modeling.
- vi. The fine-tuned Log-Distance model emerges as the most accurate and adaptable model for urban, rural, and open-space environments considered in this study.

These results provide a strong theoretical and practical foundation for site-specific radio network planning and optimization, particularly for emerging wireless deployments in heterogeneous Nigerian environments.

5 Conclusion

This case study provides a comprehensive comparative analysis and evaluation of empirical models for LoRaWAN deployment, addressing the challenges posed by the dynamic and diverse environments in which LPWANs operate. It offers valuable insights into the selection, optimization, and application of path loss models for efficient, reliable, and data-driven LoRaWAN deployment. As LoRaWAN technology evolves, this study lays the groundwork for future advancements and improvements. The findings indicate that the Log Distance Model is the most suitable path loss model for urban, rural, and open-space areas. It effectively accounts for complex urban terrain and obstacles such as buildings and streets, using calibrated path loss exponent values specific to these environments, where signal attenuation is significant. Extensive empirical validation supports its accuracy in urban scenarios, making it a trusted model for wireless communication system planning in cities. Its adaptability across various frequency bands further enhances its application in urban wireless communication. In conclusion, the Log Distance Model proves to be a reliable and accurate choice for ensuring optimal network performance in urban areas, particularly in addressing signal challenges posed by buildings and complex city layouts. Its proven track record and versatility make it an essential tool for urban network planning.

Author contributions

Dr. Augustine Obayuwana: Conceptualization of the study; design of the measurement campaign; data collection and field experiments; development and implementation of path loss model evaluation and fine-tuning methodology; data analysis and interpretation of results; drafting of the original manuscript; and revision of the manuscript for intellectual content.

Prof. Ebenezer Esenogho: Methodological support and validation; supervision and technical guidance; review and refinement of the modeling approach; critical review and editing of the manuscript; and contribution to the interpretation of results and discussion. All authors have read and approved the final version of the manuscript and agree to be accountable for all aspects of the work.

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The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

Is SDG Applicable

No.

Acknowledged lab facility

No.

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