Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Optimizing Remanufacturing under Blockchain and Voluntary Carbon Markets: Incentives and Coordination Contracts

M Mubashir Unnissa
Vellore Institute of Technology
P Jalapathy
jalapearl36@gmail.com

Chennai Institute of Technology
D Prabu
Chennai Institute of Technology
S Bharathy
Global Institute of Engineering and Technology
Pullooru Bhavana
Mohan Babu University

Research Article

Keywords: Blockchain technology, Carbon credit markets, Remanufacturing, Voluntary emission reduction, Outsourcing and authorization, Supply chain coordination, Cost-sharing contract and Two-part tariff contract, Environmental sustainability

Posted Date: March 13th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-8983467/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Optimizing Remanufacturing under Blockchain and Voluntary Carbon Markets: Incentives and Coordination Contracts

M Mubashir Unnissa[1†], P Jalapathy[2*], D Prabu[3†], S Bharathy[4†], Pullooru Bhavana[5†]

[1]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, 632 014, Tamil Nadu, India.
[2*]Centre for Nonlinear Systems, Chennai Institute of Technology, Chennai, 600 069, Tamil Nadu, India.
[3]Centre for Nonlinear Systems, Chennai Institute of Technology, Chennai, 600 069, Tamil Nadu, India.
[4]Department of Mathematics, Global Institute of Engineering and Technology, Melvisharam, Vellore, 600 069, Tamil Nadu, India.
[5]Department of Mathematics, Mohan Babu University, Tirupati, Andhra Pradesh, India.

*Corresponding author(s). E-mail(s): jalapearl36@gmail.com;
Contributing authors: mubashira@vit.ac.in ; prabumaths12@gmail.com;
bharathy81kuttys@gmail.com; pbhavanasree@gmail.com;
†These authors contributed equally to this work.

Abstract

This research paper discusses a blockchain remanufacturing supply chain that operates under a voluntary emission reduction scheme. It stresses the strategic maneuvers of an original equipment manufacturer and an independent remanufacturer. The researchers used a game-theoretic framework to evaluate different outsourcing and authorization structures while considering blockchain transparency, consumer privacy preferences, and carbon-credit monetization. It is revealed that higher prices for voluntary emission reductions are likely to redirect production to remanufactured items and reinforce the incentives for research and development aimed at emission reduction. Moreover, the introduction of blockchain technology is said to have a positive effect on consumer confidence and the environmental claims' authenticity. However, the decentralized decision-making system may still inefficiently cause double-marginalization and uncooperative abatement incentives. The results suggest cost-sharing and two-part tariff contracts as solutions to replicate centralized outcomes and allow voluntary participation. The study extends to remanufacturer driven R&D where a non-linear relationship between carbon-credit price and eco-efficiency is discerned. At last, the findings provides valuable insights are helpful for the creation of economically efficient and environmentally sustainable coordination modalities, transparency policies, and carbon market strategies.

Keywords: Blockchain technology, Carbon credit markets, Remanufacturing, Voluntary emission reduction, Outsourcing and authorization, Supply chain coordination, Cost-sharing contract and Two-part tariff contract, Environmental sustainability.

1 Introduction

The world is now more concerned than ever about climate change, and the resulting environmental degradation and high levels of carbon emissions have made it hard for the industries not to go for sustainable and transparent practices. One of the most significant things happening here is that the manufacturing industry which has been the major cause of energy use and greenhouse gas emissions,

has been pushed to the forefront of sustainability and regulatory policies (El Saadany and Jaber (2010); Jaber et al. (2013)). The environmental policy mix has included mandatory carbon measures such as taxes, quotas, and trading schemes to address the issue of environmental externalities, which now affect around one-third of the global population (Xia et al. (2023a)). These policies have successfully managed to cut down emissions but at the same time, they have raised companies' expenses. This is especially true for small remanufacturing companies with limited funds who may find it difficult to grow since they are environmentally compliant (Alegoz et al. (2021)).

At the same time, remanufacturing has become one of the primary strategies in the circular economy due to its ability to not just cut down on costs and emissions but also to prolong product life cycles. Studies suggest that remanufacturing has the potential to reduce production costs by $40-65\%$, energy consumption by 60% and emissions by 80% compared with traditional manufacturing (Yang et al. (2020)). This is a very favourable situation from the cost-saving point of view but it is accompanied by the need for remanufacturers to reinforce their product quality claims, prove their environmental product practices and be in line with the standards set for emissions reductions. The situation has opened the door to more interest in Voluntary Emission Reduction Mechanisms (VERM), a scheme wherein companies can fund carbon reduction projects and trade Verified Emission Reductions (VERs) in the voluntary market (Bensoussan and El Ouardighi (2024)). VERM, unlike compulsory regulations, is market-based and encourages innovation; however, it is still unclear what the effects will be on the remanufacturing supply chain (SC).

Simultaneously, the integration of Blockchain Technology (BCT) is a major driving force in the SC management area. The benefits brought about by Blockchain such as tracing commodities over the SC, showing the authenticity of products, validating the carbon claims, and eliminating the asymmetry of information have led to increased consumer trust in remanufactured products (Saberi et al. (2019)). Despite the good side of BCT, it comes with a series of challenges such as its deployment cost, concerns regarding data privacy, and sharing of costs which alter the competition among OEMs and IRs. Studying these digital dynamics along with VERM is necessary to have a thorough understanding of the trade-offs between sustainability, economics, and consumer behavior.

Governance-wise, OEMs often do not have access to the necessary means or motivation to carry out the process of remanufacturing within the company. Thus, two main types of governance have come up: outsourcing, which is where the OEM hires the IR for remanufacturing but at the same time keeps the ownership and control over the sales, and authorization, which allows the IR to carry out the remanufacturing and marketing independently. All these governance methods have significant effects on the distribution of profits, acceptance of products by consumers, and performance of environment. A case in point is that Apple, though, through outsourcing, controls distribution by contracting with Foxconn, while Sharp, on the other hand, gives authorization to some partners who are experts in the field, like Xin Printing Technology (Zou et al. (2016)). If these interactions are put together with the carbon reduction demand and the transparency of BCT, then the choice between outsourcing and authorization gets even more complicated.

The existing literature on green SCs and carbon policies has seen considerable improvements but still has several research gaps. To start with, the relationship between VERM and the different remanufacturing modes (outsourcing vs. authorization) has not been thoroughly examined. The power of BCT traceability along with the concern for privacy has not been seen much in the areas of demand, profitability, and emissions. Lastly, although the coordination mechanisms have been extensively scrutinized in the context of mandatory policy regimes, there is still a poor comprehension of how contracts can be used in the remanufacturing using VERM to BCT support that will help OEMs and IRs to have motivation aligned.

In order to fill these gaps this paper proposes to give answers to the following research questions:

1. How does VERM influence operational strategies and R&D collaboration in BCT remanufacturing?
2. What is the main reason to prefer outsourcing to authorization models in terms of profitability, consumer surplus, and environment?
3. How do the trade-offs between transparency and privacy surrounding BCT affect consumer demand and equilibrium prices under VERM?
4. What kind of coordination contracts can be used to effectively align the OEMs' and IRs' interests so that the decentralized decision-making in BCT and VERM can imitate the centralized efficiency?

In order to answer these questions we build a game-theoretical model of the BCT remanufacturing SC considering factors like consumer heterogeneity, BCT implementation costs, privacy issues, and voluntary emission reductions. We carry out a comparison analysis of outsourcing and authorization models with and without VERM, make equilibrium decisions, and evaluate environmental impacts. We

also propose the coordination contracts to align the incentives of the OEMs and IRs, ensuring a fair distribution of the costs of BCTs and R&D investments.

This scholarly work offers significant contributions to the body of knowledge in four distinct ways:

1. It merges the ideas of BCT transparency and voluntary emission reductions into remanufacturing models, thus broadening the conversation on digital and green SCs.
2. It carries out a parallel criticism of outsourcing and authorization models under VERM, unlocking new avenues of inquiry regarding profit sharing, consumer confidence, and ecological consequences.
3. It maps out the financial impacts of BCT adoption along with the ramifications of privacy trade-offs, bringing to light new obstacles that consumers must overcome in order to accept remanufactured products.
4. It puts forth coordination contracts that, while covering the costs of BCT and R&D, will allow for decentralized results that are as efficient as the centralized ones.

The following parts of this paper are arranged as follows. In section 2, a literature review is performed. In section 3, the model framework is revealed, which includes market assumptions, BCT information systems, and voluntary mechanisms for emission reduction. Section 4 deals with scenarios that do not require emission reduction investments and the analysis of BCT's influence on consumer demand and operation decision-making. In section 5, outsourcing and authorization models under the context of OEM emission reduction R&D are examined with a view to achieving both economic and environmental outcomes. In section 6, cost-sharing and two-part tariff contracts as coordination contracts aimed at aligning decentralized decision-making with centralized efficiency are discussed. The concluding remarks and directions for future research are presented in section 7, with detailed proofs being provided in the appendices.

2 Literature Review

Due to the mounting pressure on organizations to balance their economic performance with environmental accountability, the literature related to carbon-aware remanufacturing and digital transparency has grown greatly. The current segment gathers previous studies about carbon-emission regulations, funds allocated for emission reduction, governance frameworks in remanufacturing, and BCT transparency in order to form a solid theoretical foundation for our model proposal.

2.1 Carbon emission policies in remanufacturing

Predominantly, research related to emission reduction strategies in remanufacturing has dealt with voluntary policies like carbon taxes, emissions caps, and trading schemes. Liu et al. (2015) examined the remanufacturing decisions made under the three different policy frameworks of a cap-and-trade system, emission capacity, and carbon tax. The researchers found that the best outcomes changed with the regimes of the policies. Zhu et al. (2022) studied hybrid carbon regulations, thereby showing how dynamic strategies are created when both tax and trading mechanisms are applied. Alegoz et al. (2021) warned that higher tax rates can drive remanufacturing businesses away, while Yenipazarli (2016) stressed that it is important to strike a balance among economic growth, environmental sustainability, and society's welfare. Luo et al. (2022) argued that extremely high carbon tax rates are a disincentive for remanufacturing, and Ding et al. (2020) suggested that the merger of recycling policies and tax policies might result in better performance. Recently, Xia et al. (2023,a) scrutinized the outsourcing and the authorized remanufacturing practices under the carbon trading conditions, thus shedding light on the fact that the extent of emission cuts impacts retail prices, sales quantities, and authorized fees.

Besides the compulsory practices, VERM have also been a hot topic for researchers. Some of the empirical studies have evaluated the effects of regional VERM (Gao and Souza (2022); Wei and Xiao (2022); Zheng and Zhang (2023)), while others have looked at the implementation of various operational factors including investment gain and accountant techniques (Duncan et al. (2022); Schweitzer et al. (2023); Fang et al. (2023b)). Regardless, VERM's impact on OEM-IR operational strategies is still an area that needs to be investigated more thoroughly.

2.2 Emission reduction investments and R&D collaboration in SCs

The role of investments in emission reduction technologies has been seen as one of the main ways through which OEMs could deal with regulatory challenges, and that is something that a considerable amount of research has backed up. For instance, an application of Wang et al. (2018) takes the consideration of compliance pressure off OEMs when they put money into the sector of novelties that produce lower

carbon. Bai et al. (2019) support the above recommendations by asserting that pairing carbon trading with green investments has proved to be an excellent way to reduce emissions of suppliers. Finally, it is worth noticing that competition does not necessarily exclude the possibility of collation; partnerships for R&D aimed at emission reduction may very well be rewarding Pun and Ghamat (2016); Li et al. (2021). Sometimes it may be that OEM investments have been chiefly effective in those markets where competition is fiercest Ding et al. (2020). Concurring with the above-stated view, one can mention, among others, the discussion of technology spillovers as a phenomenon, where, for example, the firms' joint green R&D efforts can be rewarded with Pareto improvements Chen et al. (2019), the time-lagged spillovers, however, may slow down the pace of emission reduction initiatives Liu and Zhu (2023). Besides, it is vertical spillovers that are beneficial for the carbon reduction process, while horizontal spillovers may slow down the progress Jiang et al. (2024). In order to tackle these issues, joint investment models have been put forward as one of the solutions and at the same time the results of collaborative emission reduction are considered top-notch Wang et al. (2021); Li et al. (2021), besides the already existing improvements via recycling and R&D investments within OEM innovation resource collaborations Guo et al. (2022); Luo et al. (2022).

BCT has caught the limelight in this respect for its ability to mitigate the risks of spillovers and opportunistic behaviors. For instance, BCT application in over-the-counter medicine SCs has resulted in the gaining of transparency and the enhancing of consumer welfare, which is the result of competitive intensity and demand variability as factors Niu et al. (2021). Likewise, BCT has been found to be a powerful weapon against fraudulent counterfeits, as it redirects them to the legitimate sectors, thus, leading to less post-purchase disappointment; in this scenario, government funding is instrumental in creating widespread adoption rather than traditional law enforcement or price strategies Pun et al. (2021). Not only inside the pharmaceutical industry but also, BCT solutions can be recognized outselling the food industry which is also represented by the necessity to communicate sustainability in food SCs Cao et al. (2023). In addition, the earth-friendly practices of the fashion and textile industry were further improved by the good practices of BCT implementation, such as traceability, transparency, and product authenticity, even the accompanied challenges in making the system operational Badhwar et al. (2023).

Recent studies emphasize that BCT should be fully integrated in the management of sustainable SCs based on these insights. A bibliometric analysis highlights that the combination with Industry 4.0 technologies will make it possible to escape the sustainability trap Agrawal et al. (2023). Different reviews show extensively the benefits of BCT, like better transparency, traceability, and security, but still recognize to some extent the existence of hurdles connected with scalability, interoperability, regulation, and cost Sharabati and Jreisat (2024). The recent publications are mainly about carbon markets, where BCT is touted to facilitate adopters' compliance by enforcing rules through the use of smart contracts; nevertheless, the issue of who controls is still an open question Merlo et al. (2025). The same way in which the adoption of BCT in sustainable SCs has given rise to governance, regulatory, and ethical issues, it has also emerged as a reason for policy debates to develop strategically so as to ensure that the technology is implemented in a socially responsible manner Cheong (2025). Thus, BCT comes out as a crucial partner in facilitating R&D collaboration, as it can provide a good mixture of transparency, IP protection, and as well as the right incentive.

2.3 Governance models of remanufacturing: outsourcing vs. authorization

Generally speaking, the remanufacturing strategies lean towards either the outsourcing model or the authorization model. The direct involvement of OEMs in remanufacturing is usually avoided because of costs, the need for expertise, and difficulties with recycling among others (Ferguson (2010)). The practice of authorized remanufacturing helps OEMs to exert more control thus, it is common to do so even without sacrificing the profitability (Liu et al. (2018)). A study done by Zhou et al. (2021,a) on authorized models suggests that reduced remanufacturing costs result in profit-sharing arrangements between OEMs and IRs. Besides, Jin et al. (2022) found out that OEMs would rather prefer authorization models over dealer ones mainly for the purpose of quality control. Fang et al. (2023a) looked at the connection between return quality and strategic choices, while Li et al. (2024) associated authorization preferences with the degree of carbon emissions similarity between new and remanufactured products.

Moreover, outsourcing has continued to be a widely used method. Xia et al. (2021) studied outsourcing from the perspective of carbon trading, while Zhou et al. (2023b) argued that spending on green design does not directly lead to the increase in the profitability of specific OEMs in outsourcing situations. On top of that, Sun et al. (2024) looked at how unequal information about market sizes affects outsourcing profit.

Table 1 Comparative analysis of related literature with the present study.

Reference	SC Members	Carbon Policy	BCT	Governance Mode	Objective
Yang et al. (2020)	OEM/Retailer				Max Profit
Alegoz et al. (2021)	OEM / Reman	Mandatory (tax)			Cost/Profit Comparison
Xia et al. (2023a)	OEM, IR	Mandatory Trading		Outs./Auth.	Max Profit
Bensoussan and El Ouardighi (2024)	OEM/Supplier	VERM			Emission Reduction
Saberi et al. (2019)	OEM/SC		✓		Sustainable SC Traceability
Friedman and Ormiston (2022)	Food SC		✓		Sustainability & Transparency
Alotaibi et al. (2024)	SC stakeholders	ESG / Carbon Acc.	✓		Accurate Carbon Reporting
Abreu et al. (2025)	Industry 4.0		✓		Digital Product Passport
Wang et al. (2018)	OEM				Production planning
Bai et al. (2019)	Supplier/OEM	Carbon Trading			Emission Reduction Incentives
Li et al. (2021)	OEMs/suppliers			Cooperative R&D	Joint Profit
Ding et al. (2020)	OEM				Production / Emission Control
Chen et al. (2019)	Firms (Vertical)			Joint R&D	Innovation and Profit
Guo et al. (2022)	OEM & Partners	VERM		Joint Investment	Emission Reduction Benefit
Luo et al. (2022)	OEM & Partners	VERM/Hybrid		Joint Investment	Carbon Reduction Financing
Niu et al. (2021)	SC (Pharma)		✓		Info Transparency / Incentive
Pun et al. (2021)	OEM, Counterfeiter		✓		Max Profit / Anti-counterfeit
Cao et al. (2023)	Food SC		✓		Sustainability Communication
Badhwar et al. (2023)	Fashion/Textile		✓		Traceability & Transparency
Agrawal et al. (2023)	GSCM / I4.0		✓		Support GSC development
Sharabati and Jreisat (2024)	SC stakeholders		✓		Benefits/Challenges Review
Merlo et al. (2025)	Carbon markets	Carbon Trading	✓		Transparency & Automation
This Paper	OEM and IR	VERM	✓	Outs./Auth.	Max Profit

For sharing carbon emission data in outsourcing scenarios, Huang et al. (2025) proposed a BCT framework that uses searchable encryption. This method provides access to data that is secure, adaptable, and efficient, while also lowering the chances of data leakage. Nan et al. (2025) in the area of authorization, offer a model that combines the theories of non-cooperative and cooperative games to study the BCT partnership in the development of lower carbon emission technologies. The work shows how costs, carbon pricing, and operational efficiency affect the strategic choices of OEMs. This demonstrates the role of BCT in maintaining quality, preventing tampering, and making customers to accept remanufactured products.

2.4 Blockchain transparency in sustainable SCs

Before reviewing the remanufacturing context, let's say that the application of Blockchain Technology (BCT) for sustainability in research is already at an advanced stage. Wu et al. (2023) present a parallel search method that, while keeping the storage costs manageable, minimizes the time overhead of the BCT traceability systems by up to 85%, thus tackling the issues of BCT's efficiency. Moving on to system enhancements, Najjar et al. (2023) explores the use of BCT in multi-tier global supply networks and makes the point that it can significantly increase the visibility, transparency, and sustainability of the networks while at the same time noting the difficulties in its adoption. Along these lines, Fernández-Caramés et al. (2019) has proposed a system that combines drones and BCT for Industry 4.0 warehouses, whereby automated inventory management, RFID traceability, and safe decentralized data sharing are realized. Herbke et al. (2024), through the introduction of decentralized identifiers chain, a hybrid BCT framework that integrates decentralized identifiers and the inter-planetary File System for the purpose of data silo reduction, which in turn enhances SC traceability, transparency, and privacy, are main contributors in this field.

The versatility of BCT in promoting sustainable SC practices is illustrated in more detail by sector-specific studies. For example, Habibullah et al. (2024) map out BCT applications in the automotive SC and, in this way, explain the advantages of this technology in boosting traceability, avoiding counterfeiting, and facilitating adherence to the various industry regulations. Correspondingly, Al Amin et al. (2025) present a BCT green SC framework in the context of Bangladesh's RMG sector, which leverages smart contracts and real-time monitoring in order to create a transparent, accountable, and sustainable environment. Indeed, these results show that BCT not only tackles the issues of inefficient operations but also strengthens the attainment of environmental and regulatory goals.

What is more, the implementation of BCT is a way out that is more and more acknowledged to lead to resilience and sustainability in different industries. Rajput et al. (2025) tests its contribution to food

SCs and finds out that it is a great help in making the process more traceable, cutting down on fraud and waste, and thus also benefiting the support of sustainability efforts, mostly if the technology gets connected with IoT and AI. In the same manner, Babaei et al. (2025) formulates a Data Envelopment Analysis framework to evaluate the different BCT adoption strategies within the oil and gas SC, and thus reveals its potential to strengthen transparency, resilience, and automation when the conditions are uncertain. Together these articles posit the potential of BCT to create and maintain transparent, responsible, and green SCM as its main asset.

The literature review conducted shows a relationship between the existing literature and the present one as represented in Table 1. It shows the main aspects such as the SC structure, the product type, the carbon policy, the BCT adoption, the governance mode, and the research objectives. The previous studies have mainly focused on compulsory carbon regulations, one type of governance, or limited competition scenarios, while the present research has introduced a new angle by considering the case of a VERM, BCT traceability, and a dual governance model (outsourcing versus authorization) which can be applied to both new and remanufactured products.

3 Model Description and Related Assumptions

This study puts forward a design for a remanufacturing SC of BCT, which is consisting of an OEM and an IR, and functions under the VERM. The SC structure and the related decision-making processes are represented in Figure 1. The study examines two main cooperation modes: outsourcing and licensing, and at the same time allows the leadership in emissions-reduction R&D to be taken by either the OEM or the IR. BCT, in this scenario, provides the means for transparent carbon disclosures and credible verification of emission reductions, thus rising up the consumer trust, product demand, and the splitting of the revenues coming from VERs. The OEM and IR determine together the pricing, remanufactured products volumes, and emission reduction efforts, and they share the BCT introduction, carbon credit trading costs, and benefits. This model demonstrates how the different contract types, the leadership in R&D, and the BCT efficiency collectively determine the economic performance, the reduction of emissions, and the consumer welfare in remanufacturing SCs.

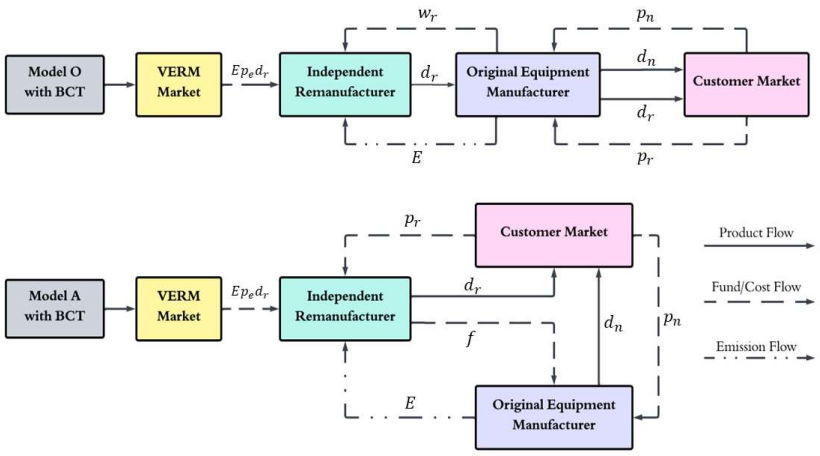


Fig. 1 Supply chain framework with BCT.

The suggested framework combines the OEM, IR, and a varied consumer market. The following assumptions highlight the demand formation, cost drivers, carbon mechanisms, and BCT aspects that constitute the analytical base. Table 2 summarizes the important symbols used in the model, such as decision variables, cost coefficients, emission factors, and consumer-related parameters, thus facilitating understanding and uniformity throughout the following mathematical analysis.

Assumption 1 : Consumer Willingness-to-Pay and BCT Enhanced Demand. This analysis is based on the idea that the market is made up of an almost infinite number of consumers, and they are arranged according to their willingness to pay (WTP), represented by θ, which is uniformly distributed on the interval $[0, Q]$, where Q is the maximum market size. A buyer whose WTP is θ sets the price of a remanufactured product equal to $\delta\theta$, where $\delta \in (0, 1]$ is a factor that depicts the perception of

Table 2 Summary of Notation

Symbol	Description	Type	Unit / Notes
d_n	Quantity of new products sold by OEM	Decision Variable	Units of product
d_r	Quantity of remanufactured products by IR	Decision Variable	Units of product
w_r	Outsourcing fee per remanufactured unit	Decision Variable	Monetary (per unit)
f	Authorization (license) fee per unit	Decision Variable	Monetary (per unit)
E	Emission reduction per remanufactured unit	Decision Variable	Emission units (e.g., kg CO_2)
p_n	Retail price of new products	Market Parameter	Monetary (per unit)
p_r	Retail price of remanufactured products	Market Parameter	Monetary (per unit)
Q	Maximum market demand	Demand Parameter	Units of consumers
θ	Willingness-to-pay for new products	Consumer Parameter	Monetary
δ	Discount factor for remanufactured products	Consumer Parameter	$0 \leq \delta \leq 1$
c	Unit cost of new product	Cost Parameter	Monetary
s	Cost saving from remanufacturing	Cost Parameter	Monetary
g	Core collection cost coefficient	Cost Parameter	Quadratic
ξ	R&D difficulty of emission reduction	Environmental Parameter	Positive scalar
E^c	Emissions from new product	Emission Parameter	CO_2 units
β	Emission intensity ratio (rem. vs. new)	Emission Parameter	$0 < \beta < 1$
p_e	VER price in carbon market	Market Parameter	Monetary (per unit)
κ	Cost-sharing function	Contractual Parameter	$0 \leq \kappa \leq 1$
VER	Total verified emission reductions	Derived Variable	$E \cdot q_r$
c_a	BCT cost per remanufactured unit	BCT Parameter	Monetary
F_c	Fixed BCT infrastructure cost	BCT Parameter	Monetary
F^{OT}	Fixed fee in two-part tariff contract	Contractual Parameter	Monetary
ρ	Cost-sharing ratio borne by OEM	Contractual Parameter	$0 \leq \rho \leq 1$
λ	Trust benefit from BCT	Consumer Parameter	Positive scalar
t	Transparency level of BCT	Consumer Parameter	$t > 0$
p_0	Base consumer privacy cost	Consumer Parameter	Monetary
p_c	Adjusted privacy cost under BCT	Consumer Parameter	$p_c = p_0 - \delta\lambda t$
CS	Consumer surplus	Outcome Variable	Monetary
Π_k	SC Members Profit i.e, $k = \{M, R\}$	Objective Variable	Monetary
SW	Social welfare	Objective Variable	$\Pi_M + \Pi_R + CS$

quality loss in remanufactured products. This aspect of vertical differentiation is well-established in the remanufactured products literature (Subramanian and Subramanyam (2012); Zhou et al. (2023a)).

To add the trust-building features of BCT, we set an information transparency level t on a scale from 0 to 1, which acts to increase consumer trust in remanufactured products. This effect is also affected by a sensitivity parameter $\lambda > 0$, and alters the utility attached to remanufactured products. Thus, the opposite demand functions are given by:

$$
\begin{aligned}
p_n &= Q - (d_n + \delta d_r) \\
p_r &= \delta\left(Q - (d_n + d_r)\right) + (1 + \delta)\,\lambda t - p_0
\end{aligned}
$$

These formulas clarify the twofold effect of market saturation on pricing and the demand for remanufactured products through BCT traceability, which is illustrated by traits like QR-code authenticity and carbon traceability. This scenario effectively combines the notions of vertical differentiation and the utility coming from BCT information as mentioned by Francisco and Swanson (2018) and Saberi et al. (2019). A thorough derivation is contained in A.1.

Assumption 2 : Production Costs and BCT Cost Adjustment.

Let c signify the unit production cost for new products. The remanufacturing method has some advantages and one of them is the cost savings due to the reuse of necessary parts like engines and motherboards. This leads to the manufacturing cost being expressed as $c - s$, with s representing the average cost savings per unit which is a positive number ($s > 0$). Research shows that remanufacturing can produce a reduction of $40 - 65\%$ in production costs in the automotive and electronics industries, for instance, Yang et al. (2020).

On top of that, a per-unit BCT implementation cost (c_a), which covers the certification, carbon tagging, and the execution of smart contracts expenses, is to be added. Thus, the remanufactured product's effective cost is expressed as:

$$\text{Effective Remanufacturing Cost} = c + c_a - s.$$

With this model, the technological efficiency was clearly separated from the costs of BCT traceability. Moreover, empirical findings refer to the work of Xu and Wang Xu and Wang (2017) and the studies of Wu et al. (2025) that pointed out the same trend of cost reductions in the automotive and electronics channels.

Assumption 3 : Carbon Emission Reduction R&D and Innovation Costs Cao and Shao (2025).

OEMs could direct funds to the development of carbon-reducing technologies such as remanufacturing that are cleaner and therefore less polluting. The expense linked to the investment is depicted as a convex function that incorporates diminishing marginal returns:

$$\text{Emission R\&D Cost} = \frac{\xi E^2}{2}.$$

In this formulation, $E \in [0, E^c]$ denotes the decrease in carbon emissions per unit, whereas $\xi > 0$ indicates the level of technological difficulty or capital requirement for the corresponding R&D. The research and development (R&D) effort is a one-time investment that does not influence the marginal cost of production (Ghosh and Shah (2012)). Furthermore, E^c and βE^c denote the unit carbon emissions for new and remanufactured products, with β taking a value within the interval $(0, 1)$. Additionally, the implementation of BCT enhances the audit-ability and verification processes related to carbon reductions, which, in turn, increases the credibility of transactions within carbon markets.

Assumption 4 : End-of-Life (EOL) Product Collection and Recycling Costs.

IRs bear the responsibility for the collection of EOL products that are suitable for remanufacturing. As the scale of collection expands, the associated costs increase in a non-linear manner due to challenges related to quality screening and logistics. This relationship is represented by a quadratic cost function:

$$\text{Collection Cost} = g d_r^2.$$

where $g > \frac{1}{2}$ is a scaling parameter greater than $\frac{1}{2}$, ensuring the concavity of the profit function. To ensure the solvability of the model and to maintain concavity of the profit function in relation to all decision variables, certain conditions on parameters must be satisfied, specifically $\xi > \frac{p_e^2}{2(\delta(1-\delta)+g)}$. These conditions have been discussed in similar CLSC studies (Yang et al. (2020); Xu et al. (2023); Xia et al. (2023a)). Additionally, the collection cost is not directly influenced by BCT; however, the implementation of traceability features may mitigate fraud and enhance recovery rates. This aspect could be modeled in future developments.

Assumption 5 : VERs and BCT Verified Carbon Credits.

The application of green technology in the remanufacturing process generates VERs at a rate that is directly proportional to both the scale of production and the level of emissions reduced. Specifically, the total VERs generated can be expressed as Ed_r. These VERs can be traded in voluntary carbon markets at a price (p_e) (Sabzevar et al. (2017); Gao and Souza (2022)), resulting in the following revenue calculation:

$$\text{VER Revenue} = p_e E d_r$$

It is important to note that the price p_e is considered to be exogenous and is projected to increase over time, reflecting a rise in carbon trading activities and the implementation of more stringent offset requirements (Gao and Souza (2022); Times (2023)). Moreover, BCT has a great impact on the traceability of VERs and reduces the risk of double-counting, thus increasing their trustworthiness and market value.

Assumption 6 : Consumer Perception and Viability of Remanufacturing.

To make consumers accept remanufactured products, it is crucial to determine that the benefits that are perceived by the customers are higher than the costs. This can be expressed through the following equation: $c - s < \delta c$, with c being the cost, and s being the perceived value of the remanufactured products. When this condition is met, remanufactured products turn out to be attractive even in normal market conditions. Moreover, introducing BCT for better information sharing, like product age and carbon offset, can potentially boost the consumer-perceived discount (δ), thus making the companies even more profitable.

Assumption 7 : BCT Deployment and Cost Sharing.

BCT implementation involves two principal cost elements: a fixed capital, represented as F_c, for building the infrastructure, which encompasses ledgers, tokens, and smart contracts, and a variable cost per unit represented as $c_a d_r$ for the operation and validation. These expenses are shared between the OEM and the IR based on a cost-sharing Parameter ρ that ranges from 0 to 1, indicating the fraction of the OEM's part. As a result, the cost allocations are characterized by:

$$\begin{aligned} \text{OEM's Share} &= \rho\left(c_a d_r + F_c\right) \text{ and} \\ \text{IR's Share} &= (1-\rho)\left(c_a d_r + F_c\right). \end{aligned}$$

This distribution method corresponds to what is commonly seen in clean technology contracts and BCT consortium's, as per the studies of Swami and Shah (2013) and Adhikari and Bisi (2020).

Assumption 8 : Privacy Costs and Trade-offs Associated with BCT.

BCT does not only bring about active discussion of consumer privacy as one of its main drawbacks but it nevertheless also offers the advantage of being able to attract new customer groups based on its transparency features. While the baseline privacy inconvenience will be represented by p_0. The privacy cost that consumers have to pay if they accept the new technology will be p_c, which is stated by the following equation: $p_c = p_0 - \delta\lambda t$. This means that even though the BCT transparency results in losses of privacy, such losses are becoming smaller and smaller in number. Hence the total consumer surplus (CS) can be written as follows:

$$CS = \frac{d_n^2 + 2\delta d_n d_r + \delta d_r^2}{2Q} + d_r\left(\lambda t\left(1+\delta\right) - p_0\right)$$

The above model considers the differences in willingness to pay the price, the utility coming from the openness of the market, and the disutility caused by the privacy issues (Malik et al. (2022); Yadav et al. (2023); Manzoor et al. (2025)).

The proposed assumptions cover all aspects of consumer diversity, the economics of remanufacturing, environmental motives, and technological implications linked to the application of BCT. They together create a very strong analytical framework for the simulation of strategic decision-making in the area of outsourcing and authorized remanufacturing within a carbon regulated market supported by BCT capabilities.

4 Model Formulation and Solution

In the following section, the authors present four hierarchical models classified according to different remanufacturing and carbon reduction schemes. Among them, there are two models which do not employ VERM as well as two models that do incorporate it. All models involve the full implementation of BCT which is an important condition for the traceability of carbon reductions, a product's credibility, and consumer perception as well as privacy issues management. We clarify two methods of remanufacturing: remanufacturing by outsourcing the OEM taking custody of the product and authorized remanufacturing where the OEM allows an IR to take over the production and marketing process.

All models assume the adoption of OEM's Stackelberg leadership premise. More precisely, the OEM first conducts the process by disclosing its decision variables including the quantity of new products,

outsourcing fee, licensing fee, or the level of emission reduction, after which the IR will react with the maximum quantity of remanufactured products. BCT implementation is an assumption that is equally applied to all models as well as being consistent and transparent; the costs of BCT will be shared by the OEM and IR according to the terms of a prearranged contract.

4.1 BCT: Benchmark models without VERM (Model *BO* and Model *BA*)

In the outsourcing benchmark model (Model BO), the OEM assigns the complete remanufacturing process to the IR but keeps the responsibility for selling the new and remanufactured products. The first step of the process is the determination by the OEM of the new product quantity (d_n^{BO}) and setting of the outsourcing fee (w_r^{BO}), which is the payment made to the IR for each remanufactured product. Then, the IR will lay down the quantity of remanufacturedufactured products (d_r^{BO}). The BCT application guarantees not only the origin but also the environmental quality of remanufactured products, with cost sharing governed by the parameter $\rho \in [0, 1]$, indicating the OEM's share of the financial responsibility.

Sequence: OEM sets d_n^{BO} and w_r^{BO} (unit fee). IR chooses d_r^{BO}.

The OEM's advantage of the Model BO is expressed in terms of profit margin as follows:

$$\Pi_M^{BO} = \left(p_n^{BO} - c\right) d_n^{BO} + \left(p_r^{BO} - w_r^{BO}\right) d_r^{BO} - \rho \left(c_a d_r^{BO} + F_c\right) \tag{1}$$

In the first part, $(p_n^{BO} - c)d_n^{BO}$, is the profit that comes from the sale of new products, where p_n^{BO} is the price at which the product is sold and c is the cost of production per unit. The next part, $(p_r^{BO} - w_r^{BO})d_r^{BO}$, indicates the OEM's profit on remanufactured products, since it gets the price p_r^{BO} and, at the same time, incurs a cost of w_r^{BO} to the IR. The last term, $\rho(c_a d_r^{BO} + F_c)$, portrays the OEM's sharing of BCT costs, where c_a denotes the variable BCT cost per remanufactured product and F_c is the fixed BCT development cost.

The equation below illustrates the profit of the IR:

$$\Pi_R^{BO} = \left(w_r^{BO} - c + s\right) d_r^{BO} - g \left(d_r^{BO}\right)^2 - (1 - \rho) \left(c_a d_r^{BO} + F_c\right) \tag{2}$$

Here, the part $\left(w_r^{BO} - c + s\right) d_r^{BO}$ is indicating the unit margin for the IR, where s stands for the cost savings coming from remanufacturing instead of producing new products. The second component, $g \left(d_r^{BO}\right)^2$, reflects the escalating collection costs for cores, with g representing the non-linearity parameter. The final term indicates the IR's contribution to the costs incurred by the BCT.

In the authorization framework referred to as Model BA, the OEM grants the IR the authority to remanufacture and independently market remanufactured products, contingent upon the payment of a per-unit authorization fee, denoted as f^{BA}. The OEM assumes responsibility solely for the production and sale of new products. Initially, it establishes the quantity d_n^{BA} and the fee f^{BA}, after which the IR determines the remanufacturing quantity d_r^{BA}.

Sequence: OEM sets d_n^{BA} and f^{BA} (license fee). IR chooses d_r^{BA}.

The profit of the OEM in this instance is as follows:

$$\Pi_M^{BA} = \left(p_n^{BA} - c\right) d_n^{BA} + f^{BA} d_r^{BA} - \rho \left(c_a d_r^{BA} + F_c\right) \tag{3}$$

In this context, the OEM generates revenue through the sales of new products and the licensing fees associated with the IR. The cost-sharing model for business collaboration and BCT is structured in a manner identical to that of the outsourcing scenario.

The profit of the IR is as follows:

$$\Pi_R^{BA} = \left(p_r^{BA} + s - \left(c + f^{BA}\right)\right) d_r^{BA} - g \left(d_r^{BA}\right)^2 - (1 - \rho) \left(c_a d_r^{BA} + F_c\right) \tag{4}$$

The IR accounts for all revenue generated from the sales of remanufactured products and remits the authorization fee for each unit to the OEM. Running the cost structure, including core collection and BCT implementation, remained unchanged.

4.2 BCT: Models with VERM (Model *O* and Model *A*)

This study has enhanced benchmark models built previously to entrench voluntary emission reduction decisions, which have been surfaced due to that intervention. Within these models, the OEM engages

in research and development (R&D) aimed at reducing carbon emissions and establishes the reduction level e, leading to a decrease of emissions by e units for each remanufactured product. The expenses associated with R&D are represented by a quadratic function $\frac{\xi E^2}{2}$, where ξ signifies the complexity of the technology involved. The total amount of emission reduction accomplished by IRs, denoted as ed_r, can be assigned a monetary value through VERs at a market price p_e per unit. Additionally, BCT plays a crucial role in the verification of carbon claims.

In the outsourcing model incorporating VERM (Model O), the OEM determines the emission reduction level E^O, the quantity of new products d_n^O, and the outsourcing fee w_r^O. Subsequently, the IR ascertains the remanufacturing quantity d_r^O. The profit attained by the OEM is expressed as follows:

Sequence: OEM chooses E^O, d_n^O, w_r^O. IR chooses d_r^O.

$$\Pi_M^O = \left(p_n^O - c\right) d_n^O + \left(p_r^O - w_r^O\right) d_r^O - \frac{\xi \left(E^O\right)^2}{2} - \rho \left(c_a d_r^O + F_c\right) \tag{5}$$

In contrast to Model BO, the new term $\frac{\xi (E^O)^2}{2}$ signifies the OEM's investment in R&D aimed at emission reduction. All other components of the model continue to exhibit structural similarities. The IR generates supplemental revenue through the sale of carbon credits, and its profit function is defined as follows:

$$\Pi_R^O = \left(w_r^O - c + s\right) d_r^O + \left(p_e E^O\right) d_r^O - g \left(d_r^O\right)^2 - (1-\rho) \left(c_a d_r^O + F_c\right) \tag{6}$$

The second term, $p_e E^O d_r^O$, represents the value of VERs that are acquired by the IR through the utilization of low-carbon remanufacturing technology provided by the OEM.

In the authorization model featuring VERM (Model A), the OEM grants a license for both the product and the associated emission-reduction technology to the IR. The OEM determines the emission reduction level E^A, the quantity of new products d_n^A, and the corresponding authorization fee f^A. Subsequently, the IR makes a decision regarding the remanufacturing quantity d_r^A.

Sequence: OEM sets E^A, d_n^A, f^A. IR chooses d_r^A. The profit of the OEM in this instance is as follows:

$$\Pi_M^A = \left(p_n^A - c\right) d_n^A + f^A d_r^A - \frac{\xi \left(E^A\right)^2}{2} - \rho \left(c_a d_r^A + F_c\right) \tag{7}$$

The profit of the IR is as follows:

$$\Pi_R^A = \left(p_r^A + s - \left(c + f^A\right)\right) d_r^A + \left(p_e E^A\right) d_r^A - \left(g \left(d_r^A\right)^2 + (1-\rho) \left(c_a d_r^A + F_c\right)\right) \tag{8}$$

In this scenario, the IR benefits from both direct product sales revenue and income generated from carbon credits, while remitting a licensing fee to the OEM. The OEM recuperates its investments in emission reduction through the revenue obtained from licensing.

5 Comparative Analysis of Models

Based on the findings presented in Section 4, the comparative effects of incorporating the VERM into BCT enabled outsourcing and authorization remanufacturing models can be articulated as follows. Comprehensive derivations and proofs are included in the B.

5.1 Comparison and analysis of equilibrium outcomes

Proposition 1 *In Model O and BO,*

1. *The optimal quantities of two products satisfy $d_n^{BO*} > d_n^{O*}$ and $d_r^{O*} > d_r^{BO*}$.*
2. *The optimal outsourcing fees and retail prices satisfy $p_n^{BO*} = p_n^{O*}$, $p_r^{O*} < p_r^{BO*}$, and $w_r^{O*} < w_r^{BO*}$.*
3. *The optimal profits of the OEMs and IRs satisfy $\Pi_M^{O*} > \Pi_M^{BO*}$ and $\Pi_R^{O*} > \Pi_R^{BO*}$.*

Proposition 1 implies that the implementation of VERM to the outsourcing model has the effect of remanufactured products over new ones: Model O generates a larger output of remanufactured products and a smaller output of new products in comparison to Model BO, mainly due to the fact that the

IRs get extra income from the VER credits. This encouragement also brings about a reduction in the outsourcing fees and the prices for remanufactured products, while the price for the new product remains constant as it continues to be the OEM's main source of profit. Even though new product sales are falling, OEM and IR profits are rising, IRs are directly profiting from the carbon-credit revenue, and OEMs are benefiting from better cost allocation and more remanufacturing activity. Therefore, VERM has opened up the door to a win-win situation and a stronger perception of remanufacturing under BCT.

Proposition 2 *In Model A and BA,*

1. *The optimal consumption quantities of the two products demand relate as* $d_n^{BA*} > d_n^{A*}$ *and* $d_r^{A*} > d_r^{BA*}$.
2. *The optimal prices of the authorization fee and retail prices coincide in* $p_n^{BA*} = p_n^{A*}$, $p_r^{A*} f^{BA*}$.
3. *The optimal profits of the OEMs and retailers are such that* $\Pi_M^{A*} > \Pi_M^{BA*}$ *and* $\Pi_R^{A*} > \Pi_R^{BA*}$.

Proposition 2 reveals that the adoption of VERM in the authorization model has the effect of increasing the demand for remanufactured products because the IRs would have increased their remanufacturing activities in order to obtain VER credits. In contrast to the case of outsourcing, the OEM increases the authorization fee to recoup its costs of emission-reduction research and development and to take part of the VERM value. The prices of new products stay the same for branding reasons, whereas the prices of remanufactured products decrease because of the increased competition and the division of costs under the authorization contract. Consequently, both OEMs and IRs realize higher profits: the former through increased authorization fees and higher efficiency, and the latter through the sale of carbon-credits, thus, a win-win situation arises, which is in line with the findings of Cao and Shao (2025).

5.2 Sensitivity analysis

Proposition 3 *Models O and A use*

1. *The increases of* E^{O*}, E^{A*}, d_r^{O*}, d_r^{A*}, Π_M^{O*}, Π_M^{A*}, Π_R^{O*}, *and* Π_R^{A*} *raise* p_e, *but* ξ *goes down.*
2. *The rise of* w_r^{O*}, p_r^{O*}, p_r^{A*}, d_n^{O*}, d_n^{A*}, *and* Π_R^{A*} *lowers* p_e, *but* ξ *goes up.*

Proposition 3 clearly demonstrates the interaction of VER price p_e and technology investment coefficient ξ in terms of their influence on the decision-making process and in the profitability of both BCT remanufacturing models. The higher the VER price the more that OEMs and IRs are willing to invest in emission-reduction and remanufacturing efforts which translates to higher profits for them and a more dominant position of the market for remanufactured products. In order to keep the situation balanced, OEMs would lower their outsourcing or licensing fees so that the IRs would still make a profit even under the reduced retail prices of remanufactured goods which would also lead to an increase in demand. On the other hand, a high technology investment coefficient does not only mean more R&D costs but also less abatement levels, less competitiveness of the remanufactured products and so the OEMs would charge more while IRs would have less output and lower profits. The results suggest that a robust VER market is an ally to low-carbon remanufacturing, but the technological hurdle can be a barrier to adoption, reinforcing the call for the policies that reduce innovation costs and build the synergy between VERs, BCT, and sustainable remanufacturing.

Theorem 1 *In Model O and A,* $l = O, A$:

1. *If* $\delta > \frac{1}{2}$ *then* $\frac{\partial \Pi_R^{l*}}{\partial \delta} > 0$; *if* $\delta < \frac{1}{2}$, *then* $\begin{cases} \frac{\partial \Pi_R^{l*}}{\partial \delta} > 0 & \text{if } p_e < \overline{p_e}^l \text{ and } p_0 < \overline{p_0}^l, \\ \frac{\partial \Pi_R^{l*}}{\partial \delta} < 0 & \text{else.} \end{cases}$
2. *If* $\delta \in \left(\underline{\delta_l}, \overline{\delta_l}\right)$, *then* $\frac{\partial E^{l*}}{\partial \delta} > 0$; *else* $\frac{\partial E^{l*}}{\partial \delta} < 0$.

Theorem 1 illustrates the complex interactions of consumer love for remanufactured goods, symbolized by the BCT trust parameter δ, in determining the profit of IRs and the equilibrium points of voluntary emission reductions in Models O and A. At first, when customer confidence is very high ($\delta > \frac{1}{2}$), BCT's increased transparency really helps the IRs earn more returns. In this case, people are

already fond of remanufactured goods, and the extra trust that comes with BCT just makes it more so, thus boosting IR's profit regardless of the market situation or the price of VERs that are being traded.

On the contrary, conditions will dictate the results in case of $\delta < \frac{1}{2}$. Under the conditions of low to moderate VER prices or when distrust towards the product (p_0) is quite low, an increase in consumer's preference would still be beneficial for IRs in terms of profitability. In contrast, as the VER price moves past the limit $\overline{p_e}^l$, or when the consumer's skepticism (p_0) is quite high, IR profits will shrink as δ grows. This result is because of the cannibalization-induced conflict: with consumers confidence in remanufactured products being at its highest, the new products' sales are cut down very quickly and thus OEM loses its surplus and has to make adjustments in the contracts (like outsourcing or authorization fees) that is going to be the reason for the IR's profit margins being compressed.

Consequently, the effects of emission-reduction efforts (E^{l*}) also follow a similar but more accentuated non-linear pattern. Whenever δ is between the two critical thresholds, ($\underline{\delta_l}$, $\overline{\delta_l}$), the increase in the level of trust strengthens the competition and the marginal reduction of IRs from abatement, thus resulting in lower emissions reductions. On the other hand, when δ is very high or very low, the reduction of emissions moves parallel to consumer preference. Under very low δ conditions, the boosting of consumer trust can unlock the demand that was there and justifiably increase the investment in abatement. However, under very high δ conditions, the market acceptance that is considered to be substantial enables IRs to effectively take advantage of VERs, even when there are competitive pressures, thus increasing the level of abatement. These interactions reflect a key management insight: the consumers who are educated about the remanufactured products are of no use if the BCT preference is either very strong or very weak; at normal preference levels, such education might end up being a barrier to both profitability and investment in emission reduction.

5.2.1 Threshold-driven behavioral regions in BCT enabled VERM systems

In Figure 2, the thresholds of sensitivity presented in Theorem 1 are illustrated visually, showing how the trust of consumers (δ) influences the VER trading price (p_e) and consequently, the IR profitability and the emission-reduction behavior of Models O and A. Following earlier research (Huang and Wang (2017); Li et al. (2023); Jalapathy and Unnissa (2025)), the authors took the following parameter values for numerical visualization: $Q = 1000$, $c = 900$, $s = 700$, $g = 4$, $\beta = 0.6$, $\lambda = 0.9$, $t = 1.5$, $p_0 = 2$, $c_a = 60$, $F_c = 100$, $\rho = 0.8$, $\kappa = 0.98$, $\xi = 200$, and $F^{OT} = 80$. These parameters reflect a realistic environment for BCT remanufacturing, characterized by moderate production costs, substantial reuse benefits, and a sufficiently high level of carbon-accounting transparency under VERM.

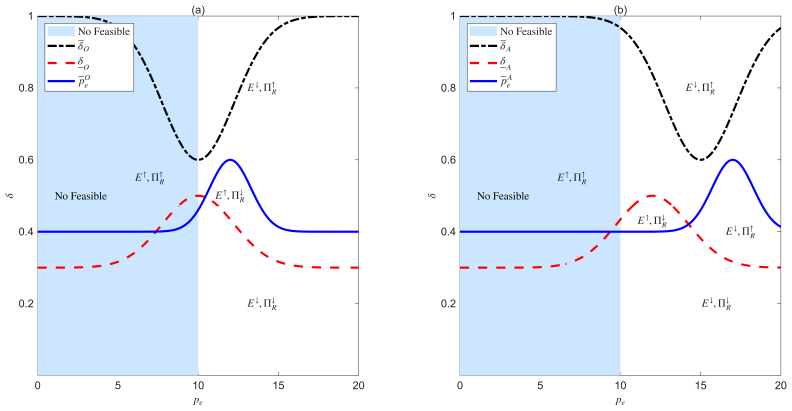


Fig. 2 The sensitivity for R&D level and IRs profit as δ or p_e changes.

The blue region that has been shaded shows that if any of the emission reduction or BCT consumer trust were to be established, the VR price could not be so low as to make it substantially feasible in the market. However, when the emission price p_e goes up, the Model O and A indicate the present of three intervals for sharing separated by the critical curves $\underline{\delta_l}$, $\overline{\delta_l}$, and the profit turning threshold $\overline{p_e}^l$. In the middle range ($\underline{\delta_l} < \delta < \overline{\delta_l}$), the demand for E^{l*} falls due to trust gaining, which reflects the cannibalization effect entered in Theorem 1: moderate trust stimulates the demand for remanufactured products to such an extent that it really erodes OEM profits, thus leading to the subsequent contract adjustments that actually weaken IRs' incentive for abatements. The IR profits in this interval are also

going down when p_e gets above the threshold of ($\overline{p_e}^l$). On the other hand, the relationship between low and high trust weighs both ways; the linkage becomes stronger with low ($-$) and high ($+$) trust levels. With low trust customers, only minor quality improvements can produce large consumer gains, whereas, with the high trust of the IRs, the full potential of dedicated VER sales can be exploited. Thus, As seen in Figure 2, the interplay of trust, profitability, and abatement is very much non-linear and dependent on specific thresholds which consumer education strategies for promoting BCT must thus consider the VER market maturity and prevailing trust conditions.

5.3 Consumer surpluses

Consumer surplus (CS) is the overall advantage that consumers get by paying less for new and remanufactured products than their maximum price they could pay. In a BCT remanufacturing market, consumer welfare is impacted not only by the traditional price gaps between new and remanufactured products but also by the perceived trust, traceability, and privacy trade-offs linked to BCT.

The total consumer surplus can be described as the combined welfare of both product categories, minus the utility losses from privacy concerns and plus the utilities gained from BCT's transparency. Formally, it is defined as:

$$CS = \int_0^{d_n} (p_n(x) - p_n)\, dx + \int_0^{d_r} (p_r(x) - p_r)\, dx + (\lambda t\,(1+\delta)\, d_r) - p_0 d_r$$

The first two integrals describe the usual parts of the consumer surplus related to new and remanufactured goods, respectively. The next two terms add up the extra gains coming from the transparency of BCT (λt) plus the loss in utility due to privacy concerns (p_0).

Through replacing the inverse demand equations for new and remanufactured products, a closed-form expression is derived:

$$CS = \frac{d_n^2 + 2\delta d_n d_r + \delta d_r^2}{2Q} + d_r\left(\lambda t\,(1+\delta) - p_0\right).$$

Proposition 4 *The relationships between consumer surpluses for the two remanufacturing models are* $CS^O - CS^{BO} < 0$ *and* $CS^A - CS^{BA} > 0$.

Proposition 4 informs us that consumer welfare changes due to VERM, depending on the remanufacturing contracts: it diminishes the consumer surplus in outsourcing but is a plus in authorization. VERM in outsourcing increases IR incentives and gets the OEM to raise outsourcing fees in order to counteract the control loss and new product cannibalization, thus pushing up remanufactured product prices in Model BO. Together with privacy disutility from BCT, these price hikes are larger than the decreased transparency odour to the consumers, hence the diminished consumer surplus. On the contrary, in the case of authorization, the OEM cuts the licensing fee in order to entice IR abatement investment, which reduces remanufacturing costs and consequently leads to lower retail prices in Model BA. Meanwhile, BCT transparency greatly raises perceived product value as a result of stronger traceability signals, so that consumers are able to have both good information and low prices. Consequently, VERM causes an increase in consumer surplus in authorization scenarios, which proves that its welfare impact is mainly determined by the extent to which contract alterations affect the final retail prices.

5.4 Analysis of environmental impact

The SC's environmental performance is a major factor that indicates the effectiveness of remanufacturing and pollution control methods. The total environmental effect in this model depends on three things: (i) the extent of remanufacturing emissions, (ii) the OEM's participation in voluntary emission reduction mechanisms (VERM), and (iii) the BCT's role in making/remanufactured products accepted by consumers.

Define e_c as the carbon emissions associated with the manufacturing of a new product, and $\beta \in (0, 1)$ as the emission intensity of remanufacturing compared to new production. Therefore, without any abatement, every remanufactured unit produces βe_c emissions. With VERM in place, the OEM employs emission-reduction technology, resulting in a per-unit reduction, referred to as E^l, under model

$l \in \{BO, BA, O, A\}$. In the reference models that do not employ VERM (namely, BO and BA), it is true that $E^{BO} = E^{BA} = 0$; on the other hand, in Models O and A, E^O and E^A are the results of OEM's optimization process respectively.

Thus, the total amount of emissions coming from model m can be written as follows:

$$AE^l = E^c d_n^l + \left(\beta E^c - E^l\right) d_r^l$$

Here, d_n^l and d_r^l represent the new and remanufactured products' equilibrium quantities, respectively. For sake of comparison, if the total output of products $(d_n^l + d_r^l)$ were purely made of new products, the baseline emissions would be equal to $E^c(d_n^l + d_r^l)$. The consequent reduction of environmental impact in relation to the baseline is then represented by:

$$\Delta^l = E^c(1-\beta)d_r^l + E^l d_r^l$$

The initial term illustrates the structural gains that accrue from remanufacturing, and the subsequent term indicates the technologically-caused pollution reduction benefits that are only paired with VERM. Additionally, for Models O and A, the carbon credits produced are measured as $V^l = E^l d_r^l$, and these can be sold in the carbon market for a price of p_e.

To assess the environmental advantages linked to the implementation of VERM, the difference in total emissions amongst every VERM model and its related benchmark is expressed as follows:

$$\Delta AE^O = AE^{O*} - AE^{BO*} \text{ and}$$
$$\Delta AE^A = AE^{A*} - AE^{BA*}$$

the superscript $*$ indicates the equilibrium values. The total expression of emissions can be derived based on the definitions laid down before, as shown in:

$$\Delta AE^O = E^c\left[\left(d_n^O + d_r^O\right) - \left(d_n^{BO} + d_r^{BO}\right)\right] - \left(\Delta^O - \Delta^{BO}\right)$$
$$\Delta AE^A = E^c\left[\left(d_n^A + d_r^A\right) - \left(d_n^{BA} + d_r^{BA}\right)\right] - \left(\Delta^A - \Delta^{BA}\right).$$

When the total output is kept unchanged in the different scenarios, the first terms become insignificant and the changes can be reduced to

$$\Delta AE^O = \Delta^{BO} - \Delta^O$$
$$\Delta AE^A = \Delta^{BA} - \Delta^A.$$

Hence, ΔAE taking on negative values means that the VERM adoption has led to better environmental impacts. This is precisely the case, as $E^O, E^A > 0$ and the BCT trust causes increased demand for remanufactured products d_r, therefore $\Delta AE^O < 0$ and $\Delta AE^A < 0$, which means that both outsourcing and authorization under VERM have better environmental performance than their respective benchmarks.

Finally, BCT is of great importance in the whole process of transformation and thus the impact of transparency and trust $(1+\delta)\lambda t$ is not only BCT that the consumer acceptance of remanufactured products is increased but privacy concerns are also addressed through the adjusted cost $p_c = p_0 - \delta\lambda t$. Consequently, the equilibrium demand for remanufacturing d_r starts to rise, which not only results in a structural reduction of emissions but also expands the VER credits in Models O and A. The best thing is that even though BCT causes an increase in operation and maintenance costs (c_a, F_c), the environment benefits still come out on a positive side as long as the trust effect does not fade out.

Theorem 2 *The impact of VERM on the environment can be described in the following way:*

1. *If $\beta < \delta$, then $\Delta AE^{O*} < \Delta AE^{A*} < 0$;*
2. *If $\beta > \delta$ and $p_e 0$ and if $\beta > \delta$ and $p_e 0$;*

3. If $\beta > \delta$ and $p_e > p_{e_1}^O$, then $\Delta AE^{O*} < 0$ and if $\beta > \delta$ and $p_e > p_{e_2}^A$, then $\Delta AE^{A*} < 0$.

Theorem 2 points out that the VERM's environmental footprint in BCT remanufacturing SCs is contingent upon three major factors: the carbon intensity of remanufacturing, demand from consumers, and the price of VERs. Similar to what happens with authorized and outsourced channels, in the case of remanufacturing carbon-intensive relative to consumer preference states ($\beta < \delta$), total emissions are increased since VERM pushes the output of high-quality recycled products even though each single unit has weak carbon savings; the suppressed emissions from new products are not sufficient to counterbalance this expansion, hence reaching $\Delta AE^{O*} < 0$ and $\Delta AE^{A*} < 0$. When remanufacturing is carbon-efficient ($\beta > \delta$), VERM only leads to a decrease in emissions when VER prices are moderate when $p_e < p_{e_1}^O$ (for outsourcing) or $p_e < p_{e_2}^A$ (for authorization), for the reason that the companies perform remanufacturing at a level of production that the carbon credits correspond to no emissions plus the actual savings. Nevertheless, once VER prices go beyond these limits, both ΔAE^{O*} and ΔAE^{A*} will be negative again: very high prices of VERs lead to over remanufacturing and hence, emissions grow even in highly inefficient systems. Collectively, Theorem 2 demonstrates that VERM is neither good nor bad for the environment by definition; its success is totally in the alignment of VER prices with real carbon inefficiency differences, realistic consumer acceptance, and avoiding the scenario of over remanufacturing driven by incentives.

5.4.1 Comparison analysis of environmental impact

The environmental implications that were discussed through analysis in Theorem 2 is visually supported by Figure 3, which illustrates the dependence of total emissions in both the outsourcing and authorization structures on the relative emission intensities of new and remanufactured products (β, δ) as well as the VER price (p_e).

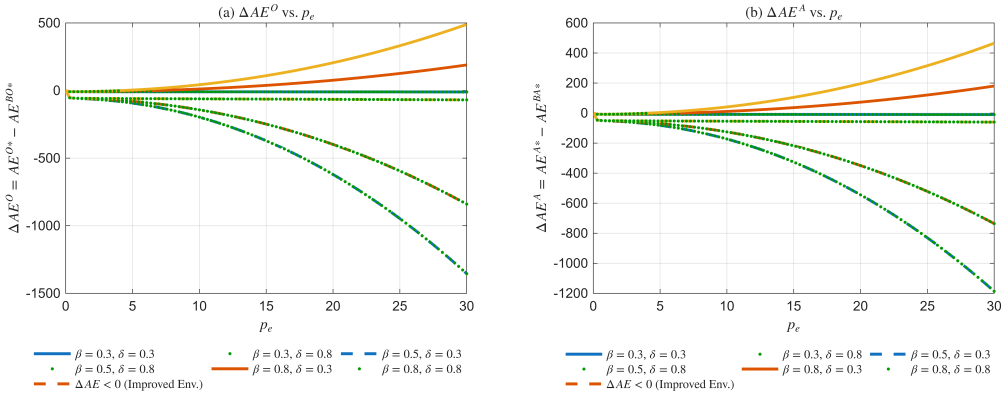


Fig. 3 Comparison analysis of environmental impact.

In the case where remanufacturing is cleaner than the new production ($\beta < \delta$), all curves lie below the axis, which means that VERM always leads to lower emissions regardless of p_e. On the other hand, when remanufacturing is more emission-intensive ($\beta > \delta$), the sign of ΔAE starts to depend on the price of the VER: emissions are reduced only at very low VER prices but they increase once p_e goes beyond the threshold set by the particular model, which means that excessively high VER prices can lead to high-emission remanufacturing being more lucrative than new production and the benefits of reduced new production being offset. The authorization model has slightly higher thresholds and a milder slope, which can be interpreted as a sign of better environmental control than outsourcing. To sum up, the figure reveals that the environmental impact of VERM is very much determined by the context, it is advantageous when remanufacturing is really cleaner but it can be detrimental when emission intensities are not properly aligned with strong VER incentives.

6 Comparative analysis of two models under VERM

This section provides a comparative analysis of the outsourcing and the authorization framework within the context of the VERM. The analysis emphasizes equilibrium decisions, profitability, consumer welfare, and environmental outcomes. Both models operate within a BCT remanufacturing SC, where BCT enhances transparency, verifies emission reductions, and impacts consumer trust as well as perceptions of privacy. Comprehensive proofs for this section are located in C.

Proposition 5 *In Model O and A:*

1. *The optimal quantities of new products and remanufactured products face the following relationships:* $d_n^{O*} < d_n^{A*}$ *and* $d_r^{O*} > d_r^{A*}$.
2. *The optimal unit carbon emission reduction of remanufactured products faces the following relationships:* $E^{O*} > E^{A*}$.

Proposition 5 demonstrates significant structural contrasts between Models O and A under VERM. The OEM in Model O contracts out remanufacturing to an IR, which is also a competitor in the same market, resulting in the OEM limiting the production of new products so as not to cause an overlap in the market, while the IR is increasing its remanufacturing output because of the higher profitability driven by VERM; thus $d_n^{O*} < d_n^{A*}$ and $d_r^{O*} > d_r^{A*}$. On the other hand, Model A puts the OEM in charge, who can only permit remanufacturing to the point that it does not affect new product sales, hence leading to higher new product output and lower remanufacturing volumes. The proposition also indicates that the unit emission reduction is larger in Model O, as the process of remanufacturing done by the IR allows the OEM to take advantage of the IR's efficiency and also the outsourcing fees, thus making it more attractive for the OEM to invest in abatement. In the case of authorization, the OEM gets most of its income from new products and only a small portion from remanufactured ones, which lessens the incentive to enhance environmental performance. Therefore, outsourcing, by its very nature, leads to greater abatement and more environmentally friendly production practices under VERM.

Proposition 6 *In Models O and A:*

1. *The highest possible profits of the OEM have the following relationships:* $\Pi_M^{O*} > \Pi_M^{A*}$.
2. *The impact of price and investment coefficient on the OEM's profit in Model O is larger than in Model A, i.e.,* $\left|\frac{\partial \Pi_M^{O*}}{\partial p_e}\right| - \left|\frac{\partial \Pi_M^{A*}}{\partial p_e}\right| > 0$ *and* $\left|\frac{\partial \Pi_M^{O*}}{\partial \xi}\right| - \left|\frac{\partial \Pi_M^{A*}}{\partial \xi}\right| > 0$.

Proposition 6 illustrates that the scenario Model O is generally more profitable for the OEM than Model A scenario. The remanufactured products are not only the source of the OEM's higher profits in Model O but also the challenge of coordinating investments in emission reduction and product pricing and for the area of fees and total profit sharing among the parties involved. On the contrary, Model A does not allow the OEM to benefit directly from selling remanufactured products apart from the minimal authorization fee being paid to OEM, which thus reduces OEM's financial gain from VERM. In addition, the OEM's profits through Model O are more sensitive to the changes of VER price and R&D investment (ξ) because these factors influence directly the remanufactured product. The Model A is not so sensitive because the revenues come mainly from new products. In general, outsourcing gives higher profits but at the same time greater exposure to the market-based VERM dynamics which makes Model O more profitable and more dependent on the environmental incentives.

6.1 Coordination contracts

In accordance with previous research conducted by Swami and Shah (2013) and Adhikari and Bisi (2020), which underscores the significance of centralized decision-making as a standard for developing coordinating contracts, this study initially evaluates the centralized model of our BCT enabled remanufacturing SC. Under centralized control, the OEM and the IR operate as a singular entity, striving to maximize the overall profitability of the system rather than their respective individual gains. The profit function under centralized management is expressed as follows:

$$\Pi_k^C = \left(p_n^C - c\right) d_n^C + \left(p_r^C - c + s\right) d_r^C - \frac{\xi (E^C)^2}{2} + \left(p_e E^C\right) d_r^C - \rho(c_a d_r^C + F_c) - g\left(d_r^C\right)^2 \tag{9}$$

This equation combines the revenues from the sales of new products, remanufactured products, and carbon credits and deducts the expenses which include production, collection, emission-reduction research and development, and BCT implementation (Sharma et al. (2024)). The current model differs from the one presented by Cao and Shao (2025) in that it also takes into account the adoption and use of BCT related costs ($c_a d_r^C + F_c$) that not only support traceability but also create consumer trust and strengthen the credibility of VERs.

Proposition 7 delves into the equilibrium results that pertain to decentralized decision-making and contrasts them with the outcomes of centralized decision-making processes.

Proposition 7 *In the remanufacturing model of three types:*

1. $E^{C*} > E^{O*} > E^{A*}$, $d_n^{A*} > d_n^{O*} > d_n^{C*}$, $d_r^{C*} > d_n^{O*} > d_n^{A*}$, *and* $\Pi_k^{A*} > \Pi_k^{O*} > \Pi_k^{C*}$.
2. E^{C*}, d_r^{C*} *increase with* p_e, *but decrease with* ξ *and* d_n^{C*}, Π_k^{C*} *is the opposite case.*

Proposition 7 discusses the comparison between the centralized (Model C) and decentralized (Models O and A) BCT remanufacturing structures under the VERM case. It is found that with centralized coordination the highest reduction of emissions is achieved, followed by Model O and lastly Model A, as both economic and environmental benefits can be fully integrated into the investment decision. Thus, centralized control results in having the least number of new products and the largest quantity of remanufactured goods, while Model A is focused on new product sales, and Model O is in the middle due to competition from IR controlled remanufacturing. Profit follows the same order: the central decision-making removes double marginalization and optimizes coordination, the Model O is more profitable than Model A due to the OEM capturing more value through outsourcing, and Model A comes last in terms of profit. The results from sensitivity analysis indicate that higher VER prices boost emissions reductions, remanufactured output, and SC profit, while higher R&D costs cut them down and lead to new product production being higher. All in all, the unified decision-making improves the VERM and emission-reduction technologies effectiveness, showing the benefits of centralized coordination over decentralized structures.

6.2 Coordination through cost-sharing contracts

Extending the existing studies, Swami and Shah Swami and Shah (2013) and Adhikari and Bisi (2020), the authors of this paper suggest a cost-sharing method as the most suitable way to handle BCT remanufacturing SCs. The IR sharing with the OEM a part of the R&D (research and development) expenses for reducing the emission is the main idea behind it. Thus, the OEM is encouraged to further work on its emission reductions. The OEM, in turn, agrees to reduce its payment to the IR for outsourcing. The semi-liberalization of the parties' interests through the contractual framework works well because it incorporates the marginal benefits of voluntary emission reductions and it takes care of double marginalization in remanufactured products. In addition, the costs of BCT implementation are shared between the parties with the OEM and IR taking both the fixed and variable costs according to the mutually agreed ratio ρ.

The OEM, under the cost-sharing contract, has the formal duty to set the new product output (d_n), the outsourcing fee (w_r^{CS}), and also the emission reduction level (E). After that, the IR will decide on the remanufactured products quantity (d_r). The OEM profit function can be defined as below:

$$\Pi_M^{CO} = (p_n^{CO} - c)d_n^{CO} + \left(p_r^{CO} - w_r^{CO}\right) d_r^{CO} - \left(\frac{\xi\left(1-\kappa\right)\left(E^{CO}\right)^2}{2} + \rho\left(c_a d_r^{CO} + F_c\right)\right). \tag{10}$$

In this particular equation, the first two terms correspond to the OEM's profit margins on both the new and remanufactured products. The third term goes to the OEM's part of the R&D costs for environmental improvement, while the last term covers the OEM's share of the costs of BCT activities.

In contrast, the IR's profit in the same contract setting is represented by:

$$\Pi_R^{CO} = \left(w_r^{CO} - c + s\right) d_r^{CO} + \left(p_e E^{CO}\right) d_r^{CO} - g\left(d_r^{CO}\right)^2 - \left(\left(1-\rho\right)\left(c_a d_r^{CO} + F_c\right) + \frac{\kappa\xi\left(E^{CO}\right)^2}{2}\right) \tag{11}$$

Table 3 The optimal solutions for Model O, C, and CO, where $l \in \{O, C, CO\}$.

	Model O	Model C	Model CO				
	$\kappa = 0.90$	$\kappa = 0.90$	$\kappa = 0.90$	$\kappa = 0.92$	$\kappa = 0.94$	$\kappa = 0.96$	$\kappa = 0.98$
E^{l*}	0.0169	0.0580	0.0580	0.0580	0.0580	0.0580	0.0580
p_n^{l*}	925	925	925	925	925	925	925
p_r^{l*}	270.995	263.137	270.993	270.993	270.992	270.990	270.985
w_r^{l*}	140.542	N/A	60.483	60.483	60.483	60.483	60.483
d_n^{l*}	62.993	51.767	51.766	51.766	51.766	51.766	51.766
d_r^{l*}	40.024	77.445	77.445	77.445	77.445	77.445	77.445
Π_M^{l*}	11.6980 (Lakhs)	12855.617	18040.744	18040.771	18040.798	18040.825	18040.852
Π_R^{l*}	25406.438	N/A	1580.513	1575.870	1565.624	1535.721	1370.583
Π_T^{l*}	11.9521 (Lakhs)	12855.617	19621.256	19616.640	19606.421	19576.545	19411.434
AE^{l*}	1535.843	2353.107	2082.226	2080.741	2078.265	2073.310	2058.4200

In this setup, the first term indicates the IR's remanufacturing profit margin being the outsourcing fee deducted. The second term indicates the increase in revenue coming from the selling of VERs, whereas the rest of the terms consist of the costs of collection represented by a quadratic function, the IR's share of BCT costs, and its proportional share of R&D expenses.

Equations (10) and (11) together portray the decentralized optimization issues that were faced by the OEM and IR during the cost-sharing contract. The contract parameters κ (R&D cost-sharing ratio), w_r^{CO} (adjusted outsourcing fee), and ρ (BCT cost-sharing ratio) can be set to suitable values that will facilitate the SC coordination to replicate the desired centralized outcome. Furthermore, the IRs are allowed to accept a contract only if the expected profit from the contract is greater than the profit that was already there before their acceptance of the contract.

Proposition 8 *The SC can be coordinated when* $d_n^{CO*} = d_n^{C*}$, $d_r^{CO*} = d_r^{C*}$, $E^{CO*} = E^{C*}$, w_r^{CO*}, *and* $\kappa \in (\underline{\kappa}, min(\overline{\kappa}, 1))$. *Where*

$$w_r^{CO*} = \frac{{p_e}^2 \left((1-2\rho)\, c_a + t\,(1+\delta)\,\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4 \left((1-\rho)\, c_a + c - s\right)}{p_e^2 - \xi A_4}$$

Proposition 8 demonstrates that the BCT remanufacturing SC can be totally synchronized by a well-structured cost-sharing contract. Under it, the contract decisions independently made are considered the same as the ones made at the centralized position, namely, the OEM together and the IR would be using the optimal new product quantity, remanufactured quantity, and the level of emissions reduction for their respective products. This synchronization is achieved by the correct setting of the outsourcing fee w_r^{CO*} and taking the R&D sharing ratio κ within the feasible range $(\underline{\kappa}, \min(\overline{\kappa}, 1))$. The formulation of w_r^{CO*} not only accounts for emission-reduction incentives but also BCT related costs which will, consequently, aid in the proper division of the marginal advantages of VER technologies and shared costs between the OEM and IR. Therefore, the contract will be successful in eliminating the double marginalization occurring between the two camps, ensuring proper incentive alignment across tiers, and operating the decentralized SC just as the centralized one.

The numerical results presented in Table 3 confirm the coordination effect of the suggested contract. The contract, irrespective of the value of κ, actually reproduces the results of the centralized system: the values of $E^{CO*} = E^{C*} = 0.0580$, $d_n^{CO*} = d_n^{C*} = 51.766$, and $d_r^{CO*} = d_r^{C*} = 77.445$ are constant across the board. This confirms that the cost-sharing contract has not only made production decision-making but also emission reductions under the VERM coordinated. It is quite surprising that the coordination outsourcing fee of $w_r^{CO*} = 60.483$ is significantly lower than the fee under Model O, which indicates that the OEM is actually the one reducing the fee as part of the incentive-alignment strategy. The OEM is also winning big by the profitability point of view which has already been mentioned, since its profit increases from 11.698 lakhs in Model O to around 0.1804 lakhs under the coordinated structure. An increase in κ causes the OEM's profit to rise even more, which means that the R&D expense has become less of a burden.

On the other hand, the IR's profit gets lower as κ increase, as it covers a bigger part of the abatement cost. But still, the IR management contract is enjoying a profit that is higher than the one that would

have been obtained through outsourcing of the baseline, which is the precondition for volunteering participation. This is a classic case of cost-sharing mechanisms and is in line with the results found in literature. The total SC profit under the contract is the same as the centralized benchmark ($\Pi_T^{CO*} = \Pi_T^{C*}$), which indicates that the efficiency is completely restored. Moreover, the SC's environmental performance has become significantly better. The total abatement raises from 0.0169 in Model O to 0.0580 under coordination which, in turn, causes a great increase in the total environmental impact reduction. The findings definitely point out that cost-sharing agreements not only solve the issue of economic incentives but also contribute to the environmental sustainability of the remanufacturing system with VERM by strengthening it.

Proposition 8 proves that cost-sharing contracts act as a reliable and efficient tool for managing BCT remanufacturing SCs. The OEM and the IR can get centralized efficiency, fairly share the advantages, and improve economic and environmental outcomes to a great extent by sharing the cost of both BCT and the reduction of emissions together.

6.3 Two-part tariff contract under BCT and VERM

In the context of a two-part tariff contract, the OEM levies both a per-unit outsourcing fee, represented as w_r^{OT}, and a fixed fee (F^{OT}), upon the IR. The per-unit fee facilitates the OEM's capture of a portion of the marginal revenue generated by the IR from remanufacturing activities, while the fixed fee serves to extract additional surplus without influencing the production decisions made by the IR. Within the BCT framework, the contract also outlines shared responsibilities regarding the costs associated with BCT implementation. In this arrangement, the OEM is responsible for a proportion ρ of both fixed and variable BCT expenditures, while the IR assumes responsibility for the remaining costs.

Formally, the optimization problem for the OEM under Model OT (outsourcing with a two-part tariff) is articulated as follows:

$$\max_{d_n^{OT}, w_r^{OT}, F^{OT}, E^{OT}} \Pi_M^{OT} \tag{12}$$

The profit function for the OEM is expressed as:

$$\Pi_M^{OT} = \left(p_n^{OT} - c\right) d_n^{OT} + \left(p_r^{OT} - w_r^{OT}\right) d_r^{OT} + F^{OT} - \left(\frac{\xi \left(E^{OT}\right)^2}{2} + \rho \left(c_a d_r^{OT} + F_c\right)\right) \tag{13}$$

In parallel, the profit function for the IR is defined as:

$$\Pi_R^{OT} = \left(w_r^{OT} - c + s\right) d_r^{OT} + \left(p_e E^{OT}\right) d_r^{OT} - g \left(d_r^{OT}\right)^2 - \left((1-\rho)\left(c_a d_r^{OT} + F_c\right) + F^{OT}\right) \tag{14}$$

In this formulation, the fixed fee F^{OT} is borne entirely by the IR, while both parties share the costs associated with BCT implementation in accordance with the pre-established ratio ρ. Additionally, the OEM internalizes the expenses related to emission-reduction R&D, while the IR continues to profit from the sale of VERs.

Proposition 9 *The SC can be coordinated when* $d_n^{OT*} = d_n^{C*}$, $d_r^{OT*} = d_r^{C*}$, $E^{OT*} = E^{C*}$, w_r^{OT*}, *and* $F^{OT} \in \left[\underline{F^{OT}}, \overline{F^{OT}}\right]$. *Where*

$$w_r^{OT*} = \frac{{p_e}^2 \left((1-2\rho) c_a + t\left(1+\delta\right)\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4 \left((1-\rho) c_a + c - s\right)}{p_e^2 - \xi A_4}$$

Proposition 9 illustrates that a two-part tariff contract can completely manage the BCT assisted, VER guided remanufacturing SC with decentralized decisions being in line with the centralized benchmark. The coordination comes about as the contract and the centralised values have the same new and remanufactured product equilibrium demands and emission-reduction level $d_n^{OT*} = d_n^{C*}$, $d_r^{OT*} = d_r^{C*}$, and $E^{OT*} = E^{C*}$. This is guaranteed by imposing a per-unit outsourcing charge w_r^{OT*} that incorporates the prices of BCT, remanufacturing, and VER along with the costs associated with them. The fixed fee F^{OT} then regulates the splitting of the surplus without impacting the operating decisions, with

its feasible range $(\underline{F}^{OT}, \overline{F}^{OT})$ ensuring that both the OEM and IR get at least their decentralised payoffs. The mechanism thus keeps total SC profit going on $\Pi_S^{OT} = \Pi_S^C$ while letting the parties negotiate profit-sharing in a manner that is flexible, effectively separating the alignment of marginal incentives from surplus allocation and making sure that the decentralized SC does not use more resources than a centralized one, even if there are difficulties in BCT implementation and VER incentives due to the necessary complexities.

6.4 Extension: IR-led R&D under BCT enabled VERM

The present section investigates a case where IR entities are involved in the investment of emission reduction technologies in an SC framework, enabled by BCT. This analysis considers that the IRs have solid distribution networks, sophisticated remanufacturing capabilities, and exceptional skills in low-carbon technology development, contrary to baseline models where OEMs are the major players. BCT assures accurate reporting of emissions reduction results, guarantees quality verification that can be reliable, and supports carbon credit transaction. In addition, BCT builds up the trust of consumers by providing them with transparent information about the reductions in carbon emissions, but it also brings about extra costs which need to be fairly shared between OEMs and IRs.

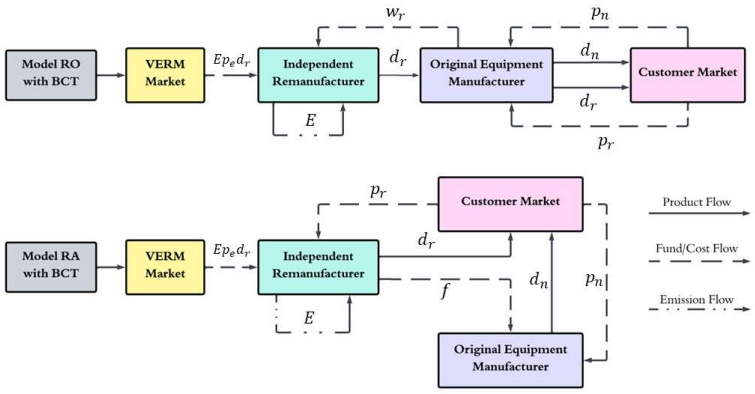


Fig. 4 SC framework of IR-led: extension model.

As illustrated in Figure 4, to classify the two IR-led R&D models as *RO* (outsourcing with IR-led R&D) and *RA* (authorization with IR-led R&D). The respective optimization problems are outlined as follows:

Model *RO* (IR-led Outsourcing with R&D and BCT):

$$\Pi_M^{RO} = \left(p_n^{RO} - c\right) d_n^{RO} + \left(p_r^{RO} - w_r^{RO}\right) d_r^{RO} - \rho\left(c_a d_r^{RO} + F_c\right) \text{ and} \tag{15}$$

$$\Pi_R^{RO} = \left(w_r^{RO} - c + s\right) d_r^{RO} + \left(p_e E^{RO}\right) d_r^{RO} - \left(g\left(d_r^{RO}\right)^2 + \frac{\left(E^{RO}\right)^2}{2} + (1-\rho)\left(c_a d_r^{RO} + F_c\right)\right). \tag{16}$$

Model *RA* (IR-led Authorization with R&D and BCT):

$$\Pi_M^{RA} = \left(p_n^{RA} - c\right) d_n^{RA} + f^{RA} d_r^{RA} - \rho\left(c_a d_r^{RA} + F_c\right) \text{ and} \tag{17}$$

$$\Pi_R^{RA} = \left(p_r^{RA} + s - \left(c + f^{RA}\right)\right) d_r^{RA} + \left(p_e E^{RA}\right) d_r^{RA} - \left(g\left(d_r^{RA}\right)^2 + \frac{\left(E^{RA}\right)^2}{2} + (1-\rho)\left(c_a d_r^{RA} + F_c\right)\right). \tag{18}$$

In the above models, Integrated IRs are tasked with covering the research and development costs associated with emission reduction, in addition to managing expenses related to BCT traceability. Specifically, IRs incur both variable costs represented as $c_a d_r$ and a fixed infrastructure investment denoted as F_c, of which the OEM contributes a fraction ρ within the interval $[0, 1]$. In Model *RO*, the OEM first

establishes the outsourcing fee and the quantity of new products, after which the IR selects the scale of remanufacturing and the level of its emission abatement efforts. In Model RA, the OEM determines a per-unit authorization fee, and the IR subsequently decides on the scale of remanufacturing along with the corresponding emission reduction level.

The integration of BCT serves a dual purpose in these extensions: (i) on the consumer side, it enhances confidence in remanufactured products through traceability and carbon disclosure, represented as $(1+\delta)\lambda t - p_0$, thereby increasing the effective demand for remanufactured products; (ii) on the operational side, it guarantees the verifiability of VERs, thus enhancing their credibility and market value. Consequently, the incentives for abatement within the IR are strengthened, as emission reductions can be reliably monetized in voluntary carbon markets.

All equilibrium outcomes for the two models are compiled and presented in Table 6, which can be found in C.

Proposition 10 *By comparing the equilibrium solutions, to have:*

1. *When* $p_e < \sqrt{(\delta+A_1)\xi}$ *the inequality relations are as follows:* $E^A < E^O < E^{RA} < E^{RO}$; $d_r^A < d_r^O < d_r^{RA} < d_r^{RO}$; $f^A < f^{RA}$; $d_n^{RA} < d_n^A < d_n^{RO} < d_n^O$; $w_r^{BO} < w_r^{RO} < w_r^O$; *and* $p_r^{RO} < P_r^A$.
2. *When* $\sqrt{(\delta+A_1)\xi} < p_e < \sqrt{2\xi(\delta+A_1)}$ *the inequality relations are as follows:* $E^{RA} < E^{RO} < E^A < E^O$; $d_r^{RA} < d_r^{RO} < d_r^A < d_r^O$; $f^{RA} < f^A$; $d_n^A < d_n^{RA} < d_n^O < d_n^{RO}$; $w_r^O < w_r^{RO} = w_r^{BO}$; *and* $p_r^A < P_r^{RO}$.
3. *When* $p_e > \sqrt{2\xi(\delta+A_1)}$ *the inequality relations are as follows:* $E^A < E^{RO} < E^O < E^{RA}$; $d_r^A < d_r^{RO} < d_r^O < d_r^{RA}$; $f^{RA} < f^A$; $d_n^{RA} < d_n^A < d_n^O < d_n^{RO}$; $w_r^O < w_r^{RO} = w_r^{BO}$; *and* $p_r^{RO} < P_r^A$.

Proposition 10 analyzes the equilibrium outcomes of four VERM models across three levels of carbon-emission pricing. When the emission price is comparatively low $\left(p_e < \sqrt{(\delta+A_1)\xi}\right)$, the research and development (R&D) structures led by OEMs outperform those led by IRs in terms of technological efforts and remanufactured output. In this pricing range, both the emission-reduction efforts and remanufactured quantities adhere to the inequalities $E^A < E^O < E^{RA} < E^{RO}$ and $d_r^A < d_r^O < d_r^{RA} < d_r^{RO}$. This indicates that IR participation positively influences remanufacturing only when OEM incentives are sufficiently robust. At low carbon prices, the authorization fee surpasses the outsourcing fee solely in the IR-led scenario ($f^A < f^{RA}$), and the new-product quantities exhibit a mixed hierarchy, with the authorized IR-led model (RA) generating the fewest new units. The wholesale prices associated with outsourcing remain lowest in the baseline OEM-led case ($w_r^{BO} < w_r^{RO} < w_r^O$), whereas the remanufactured retail price in the RO model remains lower than that of Model A.

Moreover, the four models' hierarchy is reversed when the carbon-emission price climbs to the intermediate range $\sqrt{(\delta+A_1)\xi} < p_e < \sqrt{2\xi(\delta+A_1)}$. In this situation, the IR-led models (RA and RO) have the least emission-reduction effort and remanufactured output, as they are under the most significant cost pressure from carbon regulations that render their decentralized R&D less effective. Specifically, the new inequalities manifest as $E^{RA} < E^{RO} < E^A < E^O$ and $d_r^{RA} < d_r^{RO} < d_r^A < d_r^O$, illustrating the greater technological aggression of OEM-led models in response to escalating emission prices. The authorization fee now falls below the OEM-set fee ($f^{RA} < f^A$), and the ordering of new product quantities adjusts correspondingly. Within this pricing regime, the wholesale prices for the IR-related models equalize such that $w_r^O < w_r^{RO} = w_r^{BO}$, and the remanufactured retail price in Model A becomes lower than that in Model RO.

When the carbon-emission price escalates to high levels $\left(p_e > \sqrt{2\xi(\delta+A_1)}\right)$, the ordering shifts once more, this time underscoring the superior responsiveness of authorized IR-led R&D (RA) compared to outsourced remanufacturing (RO). The ranks for effort and remanufactured quantities become $E^A < E^{RO} < E^O < E^{RA}$ and $d_r^A < d_r^{RO} < d_r^O < d_r^{RA}$, indicating that RA emerges as the most technologically proactive model under stringent environmental regulations. Despite authorization remaining less expensive for IRs ($f^{RA} < f^A$), the new product quantities mirror the same hierarchy as in the low-price regime, suggesting that the structure of new product development is more influenced by R&D leadership than by intensifying emission prices. The wholesale prices again conform to $w_r^O < w_r^{RO} = w_r^{BO}$, while the RO model continues to offer a lower remanufactured retail price compared to Model A. Ultimately, Proposition 10 illustrates a significant interaction between R&D leadership, either OEM-led or IR-led, and the intensity of emission pricing. Both low and high emission prices favor IR-led authorized R&D (RA), whereas intermediate emission prices advantage OEM-led R&D, resulting in systematic reversals in efforts, quantities, fees, and pricing.

Figure 5 presents a summary of the environmental trade-offs associated with IRs leading R&D under BCT-enabled VERM. It illustrates the differences in total emissions between IR-led and OEM-led models as the price of VERs, denoted as p_e, varies. The left panel displays the comparisons $AE^{RA} - AE^{BA}$ (blue) and $AE^{RO} - AE^{BO}$ (orange) across two values of the remanufacturing emission intensity parameter β (solid line represents $\beta = 0.3$ and dashed line represents $\beta = 0.8$). The right panel reports the comparisons $AE^{RA} - AE^{A}$ (blue) and $AE^{RO} - AE^{O}$ (orange) for the same β values. It is very important to highlight that the negative figures show the first model (IR-led) yielding lower emissions rates than the comparator, whereas the positive values show the opposite.

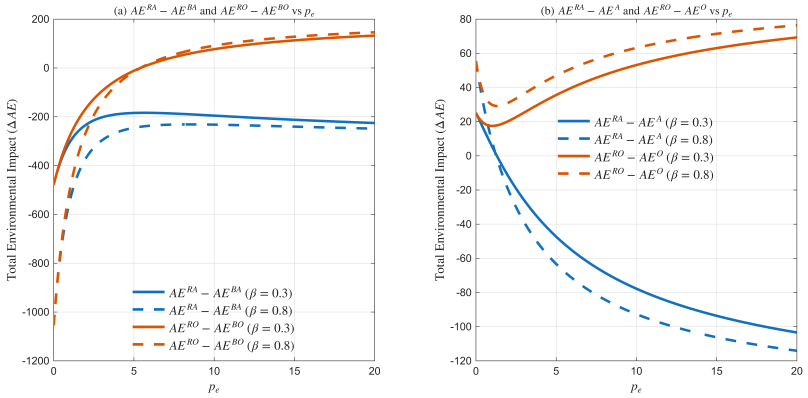


Fig. 5 The changes of atmospheric environmental impact.

Various significant patterns were brought forth due to the analysis:

- The left part of the graph (blue) shows a considerable and negative difference between RA and BA at low p_e prices: Under the conditions of low VER prices, $AE^{RA} - AE^{BA}$ gives a negative value which indicates that the IR-led authorization (RA) results in huge reductions of emissions in comparison to the OEM-baseline authorization (BA). Then, the curve goes up towards zero and becomes stable with an increase in p_e. This behavior implies that under the situation of low carbon credit prices, the granting of IRs the authorization and the responsibility for R&D lead to considerable emissions reductions, possibly because of the IRs being very effective in this regard, thanks to their knowledgeable position downstream. When p_e goes higher, the marginal returns of the additional abatement are divided equally between the two parties, hence the closing of the gap.
- The juxtaposition of RO against BO indicates a sign transition and a prolonged positive region (left, orange): the measures $AE^{RO} - AE^{BO}$ is initially very low and strongly negative for the smallest p_e, then it rises quickly crossing to positive area at medium p_e and still increasing thereafter. This phenomenon implies that, at very low VER prices, IR-led outsourcing (RO) can marginally decrease emissions in comparison to OEM-led outsourcing (BO); however, past a certain threshold of moderate VER prices, IR-led outsourcing results in increased total emissions. This observation reflects the mechanism discussed within the study: when incentivized by elevated VER prices, IRs tend to expand remanufacturing and production to maximize VER revenues; if remanufacturing does not present a cleaner alternative to new production, or if scale effects prevail, total emissions may rise even despite per-unit improvements in abatement.
- The right panel elucidates the policy-relevant switching behavior by directly comparing IR-led models to OEM-led models A and O: The blue curves ($AE^{RA} - AE^{A}$) exhibit a sharp decline and become increasingly negative with higher values of p_e (with more pronounced negativity for larger β), indicating that IR-led authorization increasingly outperforms OEM authorization in terms of emissions reduction as VER prices rise. Conversely, the orange curves ($AE^{RO} - AE^{O}$) trend positively and increase with p_e, affirming that IR-led outsourcing tends to generate poorer environmental outcomes compared to OEM-led outsourcing once VER prices reach moderate-to-high levels.
- The value of β (dashed versus solid lines) is very striking and easy to understand: the move from 0.3 to 0.8 (shown with dashed lines) not only makes the beneficial but also the bad effects stronger. The comparisons of authorization (blue) show that the negative gap has been widened due to the higher β, thus, the environmental benefits of RA over A are getting stronger as the remanufacturing emission intensity relative to new production is raised. The outsourcing comparisons (orange) show that with a higher β the positive gap is increased; consequently, RO is getting more and more disadvantageous

compared to BO with the increase of β. Collectively, the cleaner the remanufacturing technology is in comparison with new production (low β), the less evident these differences will be; on the other hand, when the remanufacturing is more polluting (high β), the effects of scaling remanufacturing under attractive VER prices will be more pronounced.

Three important conclusions can be drawn from the discussion:

1. The VERM protocol proves to be more beneficial to the environment through the R&D Authorization with IR-led strategy (RA) as the emission levels rise, the RA curves are constantly above the OEM benchmarks which imply that giving IRs authorization plus their R&D does not lead to much production that could thus neutralize the benefits but helps in reducing the emission.
2. The strategy of Outsourcing with IR-led R&D (RO) bears environmental risks in the case of moderate to high VER prices. The sign changes and growth trends depicted the situation of high VER incentives may lead the IRs to increase remanufacturing excessively just in order to get the carbon revenue, which, in turn, may lead to a situation where total emissions are higher unless the remanufacturing is kept at very low-emission. This situation clearly points out the necessity of regulatory caution: extremely high VER prices coupled with lax verification or no limits on activity can, by accident, bring about adverse environmental results.
3. The design of policies and contracts has to consider both the emissions intensity β and the potential verification provided by BCT. The difference between various β levels calls for meticulous government intervention: for example, if remanufacturing has a high emissions intensity, then the implementation of stronger VER pricing rules, rigorous validation, or the selection of licensing over outsourcing may become necessary to protect against negative impacts. BCT is at the forefront of this scenario, the technology not only allows for precise verification of reductions in emissions and hence the correct allotment of VERs but also supports the creation of contract types that motivate the adoption of authorization-like cooperations when they provide the best ecological protection.

Finally, the Proposition 10 is further supported by Figure 5, which demonstrates that IR-led R&D impacts the emissions abatement not only in terms of scale but also through allocation, and the outcomes are greatly determined by the contract type, VER pricing, and remanufacturing cleanliness. The findings underscore the importance of having policies and contract designs that are aligned in such a way, with for example, the right setting of VER prices, strong verification, and limitations on outsourcing incentives, in order for carbon markets and digital traceability to effectively contribute to the reduction of net emissions.

6.5 Managerial implications

The current research has put forward a few major managerial implications for the businesses, decision-takers, and even the society which are all involved in BCT remanufacturing systems that function under the VERM mechanism. The results have given an answer to the four research questions and at the same time have provided suggestions that could be used for developing the operational strategies, setting up the contractual arrangements, and drawing up the policies for technology investment.

1. How does VERM influence operational strategies and R&D collaboration in BCT remanufacturing?

   The VERM basically changes the way OEMs and IRs distribute their resources in production, remanufacturing, and emission-reduction R&D. An increase in the prices of VERs makes remanufacturing more profitable, thus increasing the incentives for abatement, but the impact of these prices is highly dependent on the contract structure. Under the contract of outsourcing, the companies will go for very aggressive remanufacturing in order to get the revenue from carbon credits, while in the case of authorization contracts, the companies will only moderate their remanufacturing and they will be putting in more efforts towards technological improvement. Therefore, the managers need to understand that the monetization of emission reductions changes the incentive balance between the OEMs and the IRs: OEM-led R&D still remains more stable when VER prices are moderate, whereas IR-led R&D is more effective at low or very high VER prices. Consequently, firms should initiate collaborative R&D strategies that will be flexible to this non-linear relation to the carbon-credit valuation.
2. What is the main reason to prefer outsourcing to authorization models in terms of profitability, consumer surplus, and environment?

   The outsourcing versus licensing debate is crucial for the three aforementioned aspects (profitability, consumer surplus, and environment) to different extents. High-volume remanufacturing is often associated with Outsourcing Models (O, BO, RO), which leads to better product availability but also

increased environmental risks in case of emission-heavy remanufacturing. In the case of Authorization Models (A, BA, RA), the output is more controlled which not only helps in aligning the R&D incentives with the environmental value but also in the profit sharing between OEMs and IRs to some extent. From a managerial standpoint, outsourcing is the way to go when the remanufacturing process is both clean and cost-effective. On the other hand, authorization is the most beneficial process in case carbon regulations are extremely difficult, the VER prices are high, or the remanufacturing emits a lot (high β). It will be therefore necessary for firms to analyze the carbon efficiency of remanufacturing before making decisions regarding the governance structure.

3. How do the trade-offs between transparency and privacy surrounding BCT affect consumer demand and equilibrium prices under VERM?

   BCT not only improves traceability and but also consumer trust which in turn increases the demand for remanufactured products. On the other hand, it involves the costs of data auditing and managing privacy. Higher transparency (δ) has always been a factor that positively affected remanufacturing output, consumer trust, and environmental benefits, particularly in permitted models. Managers should plan BCT as a means for growing the market still weighing its costs carefully. High OEM subsidies (ρ) or non-efficient data-sharing could lead to misalignment of incentives and thus locking up emission-reduction activities. Transparency power in BCT privacy governance should be equal to that of internal practices so that even operational efficiency is not compromised while the privacy issue is resolved.

4. What kind of coordination contracts can be used to effectively align the OEMs' and IRs' interests so that the decentralized decision-making in BCT and VERM can imitate the centralized efficiency?

   According to the research, decentralized decision-making sometimes has a very high deviation from the centralized best option due to several reasons: double marginalization, asymmetric information, and. Properly set contracts, such as cost-sharing or two-part tariffs, can bring back the centralized efficiency in VERM systems powered by BCT, provided that the essential parameters, like outsourcing fee, R&D sharing ratio, fixed fees, and BCT cost allocation, are well-calibrated.

   It is necessary for managers to consider the following while creating contracts:

   - The value of emission reductions at VERM;
   - The way BCT costs are shared-fixed or variable;
   - The capabilities and incentives of IRs regarding R&D.

   Cost-sharing contracts work well when the partnership in R&D is crucial for the reduction of emissions, while two-part tariffs give a flexible redistributing of profit yet can still give the proper marginal incentives. Firms should determine the contract method according to their bargaining power, risk preferences, and sustainability objectives.

7 Conclusion

The study focused on a remanufacturing SC that operated through a VERM. It mainly considered the strategic interactions between the OEMs and the IRs. Besides that, it also presented the common game-theoretic framework that links digital traceability, carbon-credit monetization, and operational coordination. Through the outsourcing and authorization structures analysis both with and without VERM, the research integrated BCT transparency, privacy related challenges, and cost-sharing arrangements as well.

The results show that VERM, as a significant factor, changes the whole scenario of operating incentives in a centralized as well as an uncentralized way. It results in the rise of the marginal value of emission reduction investments and consequently shifts the focus of production toward remanufacturing as the price of carbon credits goes up. Besides, BCT plays a large role in this by calming the consumers' fears regarding remanufactured products and at the same time boosting the credibility of carbon disclosures, albeit with costs for which proper allocation is needed. The authors point out that the use of decentralized decision-making without any sophistication brings about distortions in profitability and environmental performance due to double marginalization and improper alignment of R&D incentives.

In order to tackle these obstacles, the study proposes and tests the cost-sharing and two-part tariff contracts offerings. It shows that the scenario could be coordinated totally by these arrangements if the contract parameters are selected rightly. Such contracts would re-align marginal incentives, bring back the centralized efficiency and secure the voluntary involvement of both parties. Additionally, the application of the concept to IR-led R&D informs that VER pricing impact is nonlinear; both low and high carbon-credit prices usually support IR-led authorization while they give the opposite effect to OEM-led

R&D efforts in case of intermediate VER prices. The environmental aspect of IR-led remanufacturing is said to be very much dependent on the process's emission intensity, which is involved.

Finally, the paper points out that the achievement of both environmental and economic goals in BCT remanufacturing requires an alliance of technological transparency, carbon-market incentives, and contract design. The findings offer practical implications for the adoption of remanufacturing along with digital verification technologies in non-mandatory carbon markets, hence, they are useful for both practitioners and policymakers.

7.1 Future research directions

This research presents numerous prospects for future investigations. The application of dynamic consumer learning could uncover trust development in BCT transparency. The application of the framework in multi-OEM or multi-IR competitive situations would provide insight into the strategic interactions under VERM. Merging uncertainties in returns, carbon-credit pricing or verification accuracy would give rise to more realistic, risk-sensitive R&D analysis. Empirical validation through the use of data from actual BCT enabled remanufacturing systems would further develop model assumptions and parameters. Not least, the research of smart-contract coordination or hybrid policies that fuse VER markets, carbon taxation, and digital product passports could potentially open up new doors the development of sustainable circular SCs.

Conflict of interest statement

The authors have no competing interests to declare that are relevant to the content of this article.

Funding

Not Applicable.

Data availability statement

The data sets used and /or analyzed during the current study available from the corresponding author on reasonable request. We don't used any published data for solving the paper.

Credit authorship contribution details

All authors contributed equally to the research implementation, analysis of the results, and the composition and refinement of the manuscript.

A Proof of Section 4

A.1 Proof of assumption 1 : derivation of the inverse demand functions

Consider a scenario in which a customer is willing to pay (WTP) for a brand new product, referred to as θ, that is distributed uniformly over the range $[0, Q]$. In the case of a remanufactured product, the WTP is simply multiplied by a factor $\delta \in [0, 1]$, representing the common avoidance of remanufactured products. Further, the application of BCT not only secures the consumers' trust but also adds to it by making the information accessible on a transparent basis, which is depicted as λt, in which λ stands for the trust coefficient and t stands for the transparency level. Nevertheless, the increased transparency might lead to people's privacy issues being raised, and this is represented by a standard privacy disutility p_0. Therefore, the privacy cost incurred becomes $p_c = p_0 - \delta\lambda t$, indicating that the privacy issue has been lessened due to the trust that was created.

Assuming the above-stated conditions, the new product's consumer's net utility is calculated as: $U_n(\theta) = \theta - p_n$, in which case p_n denotes the price of the new product. The consumer's net utility for the remanufactured product is given by:

$$U_r(\theta) = \delta\theta - p_r + \lambda t - p_c$$

. In this case, p_r refers to the price of the remanufactured product. The expressions for utilities are the basis for the inverse demand functions of both new and remanufactured products. Upon substituting for

p_c, the equation becomes $U_r(\theta) = \delta\theta - p_r + (1+\delta)\lambda t - p_0$. It is generally presumed that consumers will choose the alternative that yields the greatest benefit, provided that the benefit is positive. Let us define θ_1 as the threshold where new and remanufactured products are equally preferred, which is obtained by setting $U_n(\theta) = U_r(\theta)$. This results in the expression $\theta_1 = \dfrac{p_n - p_r + (1+\delta)\lambda t - p_0}{1-\delta}$. In the same way, θ_2 represents the point of indifference between buying a remanufactured product and not buying anything at all, which is derived from the equation $U_r(\theta) = 0$. This calculation yields $\theta_2 = \dfrac{p_r - (1+\delta)\lambda t + p_0}{\delta}$.

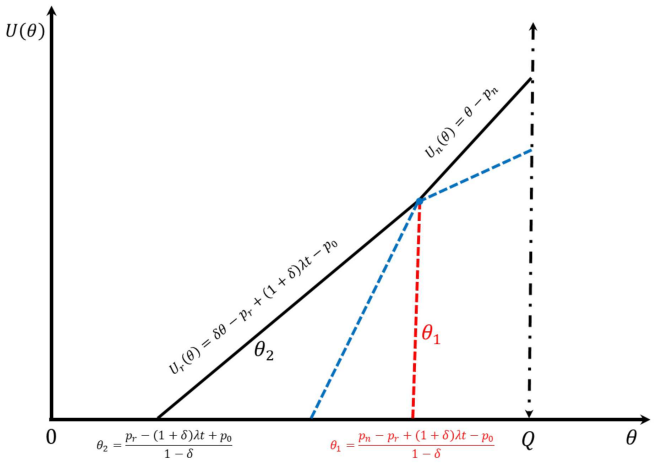


Fig. 6 Net utility functions pertaining to consumer behavior.

As illustrated in Figure 6, consumers whose characteristic is $\theta \in [\theta_1, Q]$ are the most eagerly expected to buy the newest products, whereas those in the interval $[\theta_2, \theta_1]$ prefer the remanufactured ones. Consumers with $\theta < \theta_2$ do not take part in the market. By considering θ to be uniformly distributed, the new products demand can be written as $d_n = Q - \theta_1$, while the remanufactured ones' demand is $d_r = \theta_1 - \theta_2$. By substituting for θ_1 and θ_2, we derive the inverse demand functions for new products:

$$p_n = (1-\delta)(Q - d_n) + p_r - (1+\delta)\lambda t + p_0,$$

and for remanufactured products as

$$p_r = \delta(Q - d_n - d_r) + (1+\delta)\lambda t - p_0.$$

With the help of these functions, the impacts of vertical product differentiation and consumer perception dynamics, which were introduced by BCT, along with the privacy issues related to them, are all encapsulated.

A.2 Proof of Model *O*

As a first step in our investigation, we want to look into the shape of the IR's profit function in terms of the decision variable d_r^O. The second derivative shows that $\dfrac{\partial^2 \Pi_R^O}{\partial \left(d_r^O\right)^2} = -2g < 0$. This implies that the IR's profit function is completely concave in the variable d_r^O. Thus, the first-order condition returns the following optimal response for IR:

$$d_r^O = \frac{(-1+\rho)\, c_a + w_r^O - c + s + p_e E^O}{2g}$$

Thereafter, the OEM's profit function is injected with the optimal response to, in turn, examine its concavity related to the decision variables d_n^O, w_r^O, and E^O. Noting that the inverse demand functions

for p_n^O and p_r^O are both linear, the profit function Π_M^O is a second-degree polynomial in d_n^O and w_r^O. The next step involves calculating the Hessian matrix H^O with respect to these variables and we find that the leading principal minor is negative while the determinant is positive under the condition:

$$H^O = \begin{bmatrix} -2 & -\frac{\delta}{g} \\ -\frac{\delta}{g} & \frac{\delta + 2g - \delta^2}{g^2} \end{bmatrix}.$$

From the Hessian matrix, it was concluded that parameter values $H_1^O = -2 < 0$ and $H_2^O = \frac{\delta(1-\delta)+2g}{g^2}$ were determined to be in effect under the condition $\delta(1-\delta)+2g > 0$ where the latter stands for Π_M^O being jointly concave in d_n^O and w_r^O thus. The best responses of the OEM are as follows which were obtained by solving the first-order conditions:

$$\begin{aligned} w_r^O &= \frac{-\delta^2 c + ((c+\lambda t)g + c)\delta + g(c + \lambda t - p_0)}{\delta + 2g - \delta^2} + \frac{\left((-1+\rho)\delta^2 + (1-\rho)\delta + (-2\rho+1)g\right)c_a}{\delta + 2g - \delta^2} \\ &\quad - \frac{A_4\left(p_e E^O + s\right)}{\delta + 2g - \delta^2} \text{ and} \\ d_n^O &= \frac{\left(Q - \left(\lambda t + p_e E^O + A_3\right)\right)\delta - (Q + \lambda t)\delta^2}{2A_1} + \frac{2g(Q-c)}{2A_1} \end{aligned}$$

In order to assess the concavity with respect to the decision on emission reduction E^O, we first insert d_n^{O*} and w_r^{O*} into the OEM's profit function and then find the second derivative of the profit function with respect to E^O:

$$\frac{\partial^2 \Pi_M^O}{\partial (E^O)^2} = \frac{2\xi(\delta(\delta-1) - 2g) + {p_e}^2}{2A_1}.$$

The profit of the OEM demonstrates concavity in E^O when the following condition holds: $2\xi(\delta(\delta-1) - 2g) + p_e^2 > 0$. This stipulation ensures that there is only one optimal level of emission reduction E^O. The model of outsourcing remanufacturing that takes into account BCT and the verification of emission reductions among other things (Model O) gives a unique equilibrium for the four variables (d_n^O, d_r^O, w_r^O, and E^O) as shown in Table 4.

A.3 Proof of Model *A*

It is evident that the second derivative of the function Π_R^A with respect to d_r^A satisfies the condition $\frac{\partial^2 \Pi_R^A}{\partial (d_r^A)^2} = -2(g+\delta) < 0$. This leads to the conclusion that the Hessian matrix H^A is defined as follows:

$$H^A = \begin{bmatrix} -2 + \frac{\delta^2}{\delta + g} & 0 \\ 0 & -(\delta + g)^{-1} \end{bmatrix}.$$

Consequently, it can be determined that Π_M^A is concave in both d_n^A and f^A when $\frac{2(\delta+g) - \delta^2}{(\delta+g)^2} > 0$. Additionally, the second derivative of the function Π_M^A with respect to E^A is expressed as:

$$\frac{\partial^2 \Pi_M^A}{\partial (E^A)^2} = \frac{2\left(\delta^2 - 2(\delta+g)\right)\xi + {p_e}^2}{2\left(2(\delta+g) - \delta^2\right)}.$$

This condition holds true under the constraint that $-2\xi\left(\left(\delta+A_1\right)\right)+{p_e}^2>0$. From these analyses, the equilibrium solutions of Model A can be derived and are further summarized in Table 5.

A.4 Proof of Model BO

It is evident that the second derivative of the function Π_R^{BO} with respect to d_r^{BO} satisfies the condition $\frac{\partial^2\Pi_R^{BO}}{\partial\left(d_r^{BO}\right)^2}=-2g<0$. This leads to the conclusion that the Hessian matrix H^{BO} is defined as follows:

$$H^{BO}=\begin{bmatrix}-2 & -\frac{\delta}{g}\\ -\frac{\delta}{g} & \frac{-(\delta+2g)}{2g^2}\end{bmatrix}.$$

Consequently, it can be determined that Π_M^{BO} is concave in both d_n^{BO} and w_r^{BO} when $\frac{\delta+2g-\delta^2}{g^2}>0$. From these analyses, the equilibrium solutions of Model BO can be derived and are further summarized in Table 4.

A.5 Proof of Model BA

It is evident that the second derivative of the function Π_R^{BA} with respect to d_r^{BA} satisfies the condition $\frac{\partial^2\Pi_R^{BA}}{\partial\left(d_r^{BA}\right)^2}=-2\left(g+\delta\right)<0.$. This leads to the conclusion that the Hessian matrix H^{BA} is defined as follows:

$$H^{BA}=\begin{bmatrix}-\frac{2\delta+2g-\delta^2}{\delta+g} & 0\\ 0 & -\left(\delta+g\right)^{-1}\end{bmatrix}.$$

Consequently, it can be determined that Π_M^{BA} is concave in both d_n^{BA} and f^{BA} when $\frac{2\delta+2g-\delta^2}{\left(\delta+g\right)^2}>0$. From these analyses, the equilibrium solutions of Model BA can be derived and are further summarized in Table 5.

B Proof of Section 5

B.1 Proof of Proposition 1

By conducting a comparison of the equilibrium solutions pertaining to Models O and BO, we are able to derive the following insights:

$$d_n^{O*}-d_n^{BO*}=-\frac{\left(\delta S-2A_2+2\left(s+\lambda t\right)\right)\delta{p_e}^2}{4\left(2\xi A_2-{p_e}^2\right)A_1}<0; d_r^{O*}-d_r^{BO*}=\frac{\left(\delta S-2A_2+2\left(s+\lambda t\right)\right){p_e}^2}{4\left(2\xi A_2-{p_e}^2\right)A_1}>0;$$
$$w_r^{O*}-w_r^{BO*}=-\frac{\left(\delta S-2A_2+2\left(s+\lambda t\right)\right){p_e}^2A_4}{2\left(2\xi A_2-{p_e}^2\right)A_1}<0; p_r^{O*}-p_r^{BO*}=\frac{\left(\delta S-2A_2+2\left(s+\lambda t\right)\right){p_e}^2\left(-1+\delta\right)\delta}{4\left(2\xi A_2-{p_e}^2\right)A_1}<0;$$
$\Pi_M^{O*}-\Pi_M^{BO*}>0$; and $\Pi_R^{O*}-\Pi_R^{BO*}>0$.

B.2 Proof of Proposition 2

Upon examining the equilibrium solutions associated with Models A and BA, one can ascertain the following:

$$d_n^{A*}-d_n^{BA*}=-\frac{\delta\left(\delta S-2A_2+2\left(s+\lambda t\right)\right){p_e}^2}{8A_2\left(-{p_e}^2+2\xi\left(\delta+A_2\right)\right)}<0; d_r^{A*}-d_r^{BA*}=\frac{\left(\delta S-2A_2+2\left(s+\lambda t\right)\right){p_e}^2}{8A_2\left(2\xi\left(\delta+A_2\right)-{p_e}^2\right)}>0;$$

$f^{A*} - f^{BA*} = \frac{(\delta S - 2A_2 + 2(s+\lambda t))\, {p_e}^2}{4(2\xi(\delta + A_1) - {p_e}^2)} > 0; p_r^{A*} - p_r^{BA*} = \frac{\delta {p_e}^2 (-1+\delta)(\delta S + 2(-A_2 + s + \lambda t))}{4(-{p_e}^2 + 2\xi(\delta + A_2))(\delta + A_2)} < 0;$
$\Pi_M^{A*} - \Pi_M^{BA*} > 0$; and $\Pi_R^{A*} - \Pi_R^{BA*} > 0$.

B.3 Proof of Proposition 3

By computing the first derivatives of optimal prices concerning p_e and ξ, in a manner similar to Model O, one can derive the following results:

$$\frac{\partial E^{O*}}{\partial p_e} = \frac{\left(2\xi A_2 + {p_e}^2\right)(\lambda t(\delta+1) - (c_a + p_0))}{(2\xi A_2 - {p_e}^2)^2} + \frac{(\delta(c+Q) + 2(s-c))\left(2\xi A_2 + {p_e}^2\right)}{2(2\xi A_2 - {p_e}^2)^2} > 0;$$
$$\frac{\partial d_n^{O*}}{\partial p_e} = -\frac{(\delta S - 2A_2 + 2(s+\lambda t))\,\delta \xi p_e}{(2\xi A_2 - {p_e}^2)^2} < 0; \frac{\partial d_r^{O*}}{\partial p_e} = \frac{\xi(\delta c + 2\delta\lambda t + \delta Q - 2A_2 + 2(s+\lambda t))\, p_e}{(2\xi A_2 - {p_e}^2)^2} > 0;$$
$$\frac{\partial w_r^{O*}}{\partial p_e} = -\frac{2\left(-\delta^2 + \delta + g\right)(\delta S - 2A_2 + 2(s+\lambda t))\,\xi p_e}{(2\xi A_2 - {p_e}^2)^2} < 0; \frac{\partial p_r^{O*}}{\partial p_e} = \frac{\xi p_e (-1+\delta)\,\delta(\delta(S) - 2A_2 + 2(s+\lambda t))}{(2\xi A_1 - {p_e}^2)^2} < 0;$$
$$\frac{\partial \Pi_M^{O*}}{\partial p_e} > 0; \text{ and } \frac{\partial \Pi_R^{O*}}{\partial p_e} > 0; \frac{\partial E^{O*}}{\partial \xi} = -\frac{A_1 p_e (\delta S - 2A_2 + 2(s+\lambda t))}{(2\xi A_2 - {p_e}^2)^2} < 0;$$
$$\frac{\partial d_n^{O*}}{\partial \xi} = \frac{\delta(\delta S - 2A_2 + 2(s+\lambda t))\,{p_e}^2}{2(2\xi A_2 - {p_e}^2)^2} > 0; \frac{\partial d_r^{O*}}{\partial \xi} = -\frac{(\delta S - 2A_2 + 2(s+\lambda t))\,{p_e}^2}{2(2\xi A_2 - {p_e}^2)^2} < 0;$$
$$\frac{\partial w_r^{O*}}{\partial \xi} = \frac{{p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))\left(-\delta^2 + \delta + g\right)}{(2\xi A_2 - {p_e}^2)^2} > 0; \frac{\partial p_r^{O*}}{\partial \xi} = -\frac{{p_e}^2(-1+\delta)\,\delta(\delta S - 2A_2 + 2(s+\lambda t))}{2(2\xi A_2 - {p_e}^2)^2} > 0;$$
$$\frac{\partial \Pi_M^{O*}}{\partial \xi} < 0; \text{ and } \frac{\partial \Pi_R^{O*}}{\partial \xi} < 0.$$

Regarding Model A, the following conclusions can be drawn:

$$\frac{\partial E^{A*}}{\partial p_e} = \frac{(\delta S - 2A_2 + 2(s+\lambda t))\left({p_e}^2 + 2\xi(\delta + A_2)\right)}{2(-{p_e}^2 + 2\xi(\delta + A_2))^2} > 0; \frac{\partial d_n^{A*}}{\partial p_e} = -\frac{\delta\xi(S\delta - 2A_2 + 2(s+\lambda t))\, p_e}{(-{p_e}^2 + 2\xi(\delta + A_2))^2} < 0;$$
$$\frac{\partial d_r^{A*}}{\partial p_e} = \frac{(\delta S - 2A_2 + 2(s+\lambda t))\, p_e \xi}{(-{p_e}^2 + 2\xi(\delta + A_2))^2} > 0; \frac{\partial f^{A*}}{\partial p_e} = \frac{p_e(\delta + A_1)\,\xi(\delta S - 2A_2 + 2(s+\lambda t))}{2(-{p_e}^2 + 2\xi(\delta + A_2))^2} > 0;$$
$$\frac{\partial p_r^{A*}}{\partial p_e} = \frac{\xi\delta(\delta S - 2A_2 + 2(s+\lambda t))(-1+\delta)\, p_e}{(-{p_e}^2 + 2\xi(\delta + A_2))^2} < 0; \frac{\partial \Pi_M^{A*}}{\partial p_e} > 0; \text{ and } \frac{\partial \Pi_R^{A*}}{\partial p_e} > 0;$$
$$\frac{\partial E^{A*}}{\partial \xi} = -\frac{p_e(\delta S - 2A_2 + 2(s+\lambda t))(\delta + A_1)}{(-{p_e}^2 + 2\xi(\delta + A_2))^2} < 0; \frac{\partial d_n^{A*}}{\partial \xi} = \frac{\delta(S\delta - 2A_2 + 2(s+\lambda t))\,{p_e}^2}{2(-{p_e}^2 + 2\xi(\delta + A_2))^2} > 0;$$
$$\frac{\partial d_r^{A*}}{\partial \xi} = -\frac{{p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))}{2(-{p_e}^2 + 2\xi(\delta + A_2))^2} < 0; \frac{\partial f^{A*}}{\partial \xi} = -\frac{(\delta + A_1)\,{p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))}{2(-{p_e}^2 + 2\xi(\delta + A_2))^2} < 0;$$
$$\frac{\partial p_r^{A*}}{\partial \xi} = -\frac{{p_e}^2(\delta S - 2A_2 + 2(s+\lambda t))(\delta - 1)\,\delta}{2(-{p_e}^2 + 2\xi(\delta + A_2))^2} > 0; \frac{\partial \Pi_M^{A*}}{\partial \xi} < 0; \text{ and } \frac{\partial \Pi_R^{A*}}{\partial \xi} < 0.$$

B.4 Proof of Proposition 4

In accordance with the definition of consumer surplus, denoted as $CS = \frac{d_n^2 + 2\delta d_n d_r + \delta d_r^2}{2Q} + d_r[\lambda t(1+\delta) - p_0]$, it is possible to derive the following:

$$CS^O - CS^{BO} < 0 \text{ and}$$
$$CS^A - CS^{BA} > 0.$$

B.5 Proof of Proposition 7

Given that $A_4 < A_1 < \delta + A_1$, we can infer the following relationships: $E^{O*} > E^{A*} > E^{C*}$, $d_n^{A*} > d_n^{O*} > d_n^{C*}$, and $d_r^{C*} > d_r^{O*} > d_r^{A*}$. Furthermore, concerning the overall profit of the SC, the conclusion

can be drawn that: $\Pi_k^{O*} - \Pi_k^{A*} > 0$. Hence, it follows that $\Pi_k^{A*} < \Pi_k^{O*} < \Pi_k^{C*}$. By taking the first-order derivatives of the equilibrium solutions for Model C with respect to p_e and ξ, we can derive the necessary insights:

$$\begin{aligned}
\frac{\partial E^{C*}}{\partial p_e} &= \frac{\left(\xi A_4 + {p_e}^2\right)\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)}{\left(\xi A_4 - {p_e}^2\right)^2} > 0;\\
\frac{\partial d_n^{C*}}{\partial p_e} &= -\frac{\xi\delta\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)p_e}{\left(\xi A_4 - {p_e}^2\right)^2} < 0;\\
\frac{\partial d_r^{C*}}{\partial p_e} &= \frac{\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)\xi p_e}{\left(\xi A_4 - {p_e}^2\right)^2} > 0;\\
\frac{\partial \Pi_k^{C*}}{\partial p_e} &= \frac{\xi\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)^2 p_e}{2\left(\xi A_4 - {p_e}^2\right)^2} > 0;\\
\frac{\partial E^{C*}}{\partial \xi} &= -\frac{A_4\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)p_e}{\left(\xi A_4 - {p_e}^2\right)^2} < 0;\\
\frac{\partial d_n^{C*}}{\partial \xi} &= \frac{\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)\delta {p_e}^2}{2\left(\xi A_4 - {p_e}^2\right)^2} > 0;\\
\frac{\partial d_r^{C*}}{\partial \xi} &= -\frac{\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right){p_e}^2}{2\left(\xi A_4 - {p_e}^2\right)^2} < 0;\\
\frac{\partial \Pi_k^{C*}}{\partial \xi} &= -\frac{\left(\left(c+\lambda t\right)\delta + \lambda t + s - \left(p_0 + c + \rho c_a\right)\right)^2 {p_e}^2}{4\left(\xi A_4 - {p_e}^2\right)^2} < 0.
\end{aligned}$$

B.6 Proof of Proposition 8

According to Eqn. 10, the first-order condition indicates that

$$d_r = \frac{\left(-1+\rho\right)c_a + w_r - c + s + p_e E}{2g}.$$

In order for the CLSC's profitability to be in line with that of the centralized model, the following conditions must be met: $d_n^{CO*} = d_n^{C*}$, $d_r^{CO*} = d_r^{C*}$, and $E^{CO*} = E^{C*}$. Subsequently, the outsourcing fee is delineated by

$$w_r^{CO*} = \frac{{p_e}^2\left(\left(1-2\rho\right)c_a + t\left(1+\delta\right)\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4\left(\left(1-\rho\right)c_a + c - s\right)}{p_e^2 - \xi A_4}.$$

When a win-win outcome is achieved, the conditions $\Pi_M^{CO*} \geq \Pi_M^{O*}$ and $\Pi_R^{CO*} \geq \Pi_R^{O*}$ are satisfied. By comparing the profits, one can derive the sharing ratio $\kappa \in \left(\underline{\kappa}, \min\left(\overline{\kappa}, 1\right)\right)$ and $\Pi_S^{C*} = \Pi_M^{CO*} + \Pi_R^{CO*}$ where $\kappa = \dfrac{2\left(\rho-1\right)\left(p_e^2 - \xi A_4\right)^2 F_c}{\xi\left(W_1 + s - \left(p_0 + \rho c_a\right)\right)^2 p_e^2} - \dfrac{\xi g}{2p_e^2}$.

B.7 Proof of Proposition 9

In a manner analogous to the cost-sharing contract, the outsourcing fee within a two-part tariff contract is determined by the equation

$$w_r^{OT*} = \frac{{p_e}^2\left(\left(1-2\rho\right)c_a + t\left(1+\delta\right)\lambda + \delta c - p_0\right)}{p_e^2 - \xi A_4} - \frac{\xi A_4\left(\left(1-\rho\right)c_a + c - s\right)}{p_e^2 - \xi A_4}.$$

When $d_n^{OT*} = d_n^{C*}$, $d_r^{OT*} = d_r^{C*}$, and $E^{OT*} = E^{C*}$. Under an optimal win-win scenario, the fixed costs must satisfy the condition $F^{OT} \in \left[\underline{F^{OT}}, \overline{F^{OT}}\right]$, where

$$\underline{F}^{OT} = \frac{\left(-cQt^2\left(1+\delta\right)^2\lambda^2 - 2Q^2A_1 - 4\left(\left(\delta-1\right)c + A_3\right)\left(1+\delta\right)t\lambda\right)\xi + Q^2{p_e}^2}{8\xi A_1 - 4{p_e}^2}$$
$$+ \frac{\left(-2A_5c^2 - 4A_3\left(\delta-1\right)c + 8A_1\rho F_c - 2{A_3}^2\right)\xi + {p_e}^2\left(c^2 - 4\rho F_c\right)}{8\xi A_1 - 4{p_e}^2}$$
$$\overline{F}^{OT} = \frac{\xi\left(2\xi g - {p_e}^2\right)\left(\left(1+\delta\right)\lambda t + A_3 - c\left(1-\delta\right)\right)^2}{2\left(2\xi A_1 - {p_e}^2\right)^2} + F_c\left(-1+\rho\right).$$

It is important to note that $\Pi_S^{OT} = \Pi_S^C$.

B.8 Proof of Subsection 6.4

It is evident that the second derivative of the function Π_R^{RO} with respect to d_r^{RO} and E^{RO} satisfies $H_1^{RO} = \begin{bmatrix} -2g & p_e \\ p_e & -\xi \end{bmatrix}$. This indicates that the profit function of the Π_R^{RO} is strictly concave in d_r^{RO} and E^{RO} when $2g\xi - p_e^2 > 0$. Consequently, the first-order condition yields the IR's optimal response:

$$d_r^{RO}\left(w_r^{RO}\right) = \frac{-\xi\left(c_a\left(1-\rho\right) - w_r^{RO} + c - s\right)}{2g\xi - {p_e}^2}$$
$$E^{RO}\left(w_r^{RO}\right) = \frac{-p_e\left(c_a\left(1-\rho\right) - w_r^{RO} + c - s\right)}{2g\xi - {p_e}^2}$$

Next, to substitute this optimal response into the OEM's profit function to assess its concavity concerning the decision variables d_n^{RO} and w_r^{RO}. Considering the linear nature of the inverse demand functions for p_n^{RO} and p_r^{RO}, the profit function Π_M^{RO} is quadratic in d_n^{RO} and w_r^{RO}. We proceed to compute the Hessian matrix H_2^{RO} with respect to these variables and determine that the leading principal minor is negative while the determinant is positive given the condition:

$$H_2^{RO} = \begin{bmatrix} -2 & -\dfrac{2\delta\xi}{2g\xi - {p_e}^2} \\ -\dfrac{2\delta\xi}{2g\xi - {p_e}^2} & -\dfrac{2\left(\xi\left(\delta+2g\right) - {p_e}^2\right)\xi}{\left(2g\xi - {p_e}^2\right)^2} \end{bmatrix}.$$

According to the Hessian matrix, it can be concluded that values $H_1^{RO} = -2 < 0$ and $H_2^{RO} = \frac{4\xi\left(\xi A_1 - {p_e}^2\right)}{\left(2g\xi - {p_e}^2\right)^2}$ are valid when $\xi A_1 - {p_e}^2 > 0$. Therefore, to conclude that Π_M^{RO} is jointly concave in d_n^{RO} and w_r^{RO}. Consequently, the equilibrium solutions for Model RO are provided in a summary format within Table 6. Given that the optimal solution process of Model RA closely resembles that of Model RO, we will limit our discussion to the conditions necessary for the existence of the optimal solution.

It is evident that the second derivative of the function Π_R^{RA} with respect to d_r^{RA} and E^{RA} satisfies $H_1^{RA} = \begin{bmatrix} -2\left(\delta+g\right) & p_e \\ p_e & -\xi \end{bmatrix}$. This indicates that the profit function of the Π_R^{RA} is strictly concave in d_r^{RA} and E^{RA} when $2\left(\delta+g\right)\xi - {p_e}^2 > 0$. Next, to substitute this optimal response into the OEM's profit function to assess its concavity concerning the decision variables d_n^{RA} and w_r^{RA}. Considering the linear nature of the inverse demand functions for p_n^{RA} and p_r^{RA}, the profit function Π_M^{RA} is quadratic in d_n^{RA} and w_r^{RA}. We proceed to compute the Hessian matrix H_2^{RA} with respect to these variables and determine that the leading principal minor is negative while the determinant is positive given the condition:

$$H_2^{RO} = \begin{bmatrix} -2 + \dfrac{2\xi\delta^2}{2\left(\delta+g\right)\xi - {p_e}^2} & 0 \\ 0 & -\dfrac{2\xi}{2\left(\delta+g\right)\xi - {p_e}^2} \end{bmatrix}.$$

According to the Hessian matrix, it can be concluded that values $H_1^{RA} = \frac{-2\left(\xi\left(\delta+A_1\right)-p_e^2\right)}{2\left(\delta+g\right)-p_e^2} < 0$ and $H_2^{RA} = \frac{4\xi\left(\left(\delta+A_1\right)\xi-p_e^2\right)}{\left(2\xi\left(\delta+g\right)-p_e^2\right)^2}$ are valid when $\left(\delta+A_1\right)\xi-p_e^2>0$. Therefore, to conclude that Π_M^{RA} is jointly concave in d_n^{RA} and w_r^{RA}. Consequently, the equilibrium solutions for Model RA are provided in a summary format within Table 6.

B.9 Proof of Proposition 10

By examining the equilibrium solutions of the aforementioned models, it is possible to derive the following insights:

$$
\begin{aligned}
&d_n^{RO*}-d_n^{O*}<0; d_r^{RO*}-d_r^{O*}>0; d_n^{RA*}-d_n^{A*}<0;\ d_r^{RA*}-d_r^{A*}>0; d_r^{RA*}-d_r^{O*}<0;\\
&d_r^{RA*}-d_r^{RO*}<0;\ E^{RA*}-E^{O*}<0;\ E^{RA*}-E^{A*}<0;\ E^{RO*}-E^{O*}<0;\ E^{RA*}-E^{RO*}<0;\\
&f^{RA*}-f^{A*}<0; w_r^{RO*}-w_r^{O*}>0;\ w_r^{RO*}-w_r^{BO*}<0;\ p_r^{RO*}-p_r^{A*}<0.
\end{aligned}
$$

The interrelationships among the models are articulated in the subsequent three cases:
Case 1: When $p_e<\sqrt{\left(\delta+A_1\right)\xi}$ the inequality relations are as follows:

$$
\begin{aligned}
&E^A<E^O<E^{RA}<E^{RO}; d_r^A<d_r^O<d_r^{RA}<d_r^{RO};\ f^A<f^{RA}; d_n^{RA}<d_n^A<d_n^{RO}<d_n^O;\\
&w_r^{BO}<w_r^{RO}<w_r^O; \text{and } p_r^{RO}<P_r^A.
\end{aligned}
$$

Case 2: When $\sqrt{\left(\delta+A_1\right)\xi}<p_e<\sqrt{2\xi\left(\delta+A_1\right)}$ the inequality relations are as follows:

$$
\begin{aligned}
&E^{RA}<E^{RO}<E^A<E^O; d_r^{RA}<d_r^{RO}<d_r^A<d_r^O;\ f^{RA}<f^A; d_n^A<d_n^{RA}<d_n^O<d_n^{RO};\\
&w_r^O<w_r^{RO}=w_r^{BO}; \text{ and } \quad p_r^A<P_r^{RO}.
\end{aligned}
$$

Case 3: When $p_e>\sqrt{2\xi\left(\delta+A_1\right)}$ the inequality relations are as follows:

$$
\begin{aligned}
&E^A<E^{RO}<E^O<E^{RA}; d_r^A<d_r^{RO}<d_r^O<d_r^{RA};\ f^{RA}<f^A; d_n^{RA}<d_n^A<d_n^O<d_n^{RO};\\
&w_r^O<w_r^{RO}=w_r^{BO}; \text{ and } \quad p_r^{RO}<P_r^A.
\end{aligned}
$$

B.10 Proof of Theorem 1

By computing the first derivatives of Π_R^{O*} and Π_R^{A*} concerning δ, and it is evident that: $\frac{\partial\Pi_R^{A*}}{\partial\delta}>0$ and $\frac{\partial\Pi_R^{O*}}{\partial\delta}>0$ when $\delta>\frac{1}{2}$; $\frac{\partial\Pi_R^{A*}}{\partial\delta}<0$ when $\delta<\frac{1}{2}$ and $p_0>\overline{p_0^A}$; $\frac{\partial\Pi_R^{O*}}{\partial\delta}<0$ when $\delta<\frac{1}{2}$ and $p_0>\overline{p_0^O}$. In this context, the expression for $\overline{p_0^A}$ and $\overline{p_0^O}$ are defined as follows:

$$
\begin{aligned}
\overline{p_0^A} &= \frac{4\left(\left(\lambda t+c\right)\delta-\left(c+c_a\right)+s+\lambda t\right)\left(\delta+A_1\right)\xi+\delta{p_e}^2\left(Q-c\right)}{4\xi\left(\delta+A_1\right)}\\
\overline{p_0^O} &= \frac{4A_1\left(\left(\lambda t+c\right)\delta-\left(c+c_a\right)+s+\lambda t\right)\xi+\delta{p_e}^2\left(Q-c\right)}{4\xi A_1}.
\end{aligned}
$$

Then, by taking the first derivatives of E^{O*} with respect to δ, one can obtain:

$$
\frac{\partial E^{O*}}{\partial\delta} = \frac{Sp_e}{2\left(2\xi A_1-{p_e}^2\right)}+\frac{\left(2\lambda\left(\delta+1\right)t+\left(c+Q\right)\delta+2\left(s-A_2\right)\right)\left(-1+2\delta\right)\xi p_e}{\left(2\xi A_1-{p_e}^2\right)^2}.
$$

The equation $\frac{\partial E^{O*}}{\partial\delta}=0$ yields roots $p_{e_1}^O$ and $-p_{e_1}^O$, where

$$
p_{e_1}^O = \frac{2\sqrt{-\delta\xi A_1\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\left(Q-c\right)}}{\delta\left(Q-c\right)}.
$$

Consequently, the roots of the equation $\frac{\partial E^{O*}}{\partial \delta}=0$ are $\overline{\delta_O}$ and $\underline{\delta_O}$, and $\frac{\partial E^{O*}}{\partial \delta}<0$ when $\delta \in \left(\underline{\delta_O}, \overline{\delta_O}\right)$, where

$$\overline{\delta_O}=\frac{2\sqrt{-\left(2\left[\begin{array}{l}(g-1)\left(4\lambda^2t^2+c^2\right)+((5+4g)c+(4g-1)Q-6A_3)\,\lambda t\\+((1+2g)Q+3A_3)\,c+Q^2g-A_3(Q-2A_3)\end{array}\right]\xi-\frac{p_e^2S^2}{2}\right)\xi}+4\left(A_2-(\lambda t+s)\right)\xi}{2\xi(c+Q+2\lambda t)}$$

$$\underline{\delta_O}=\frac{-2\sqrt{-\left(2\left[\begin{array}{l}(g-1)\left(4\lambda^2t^2+c^2\right)+((5+4g)c+(4g-1)Q-6A_3)\,\lambda t\\+((1+2g)Q+3A_3)\,c+Q^2g-A_3(Q-2A_3)\end{array}\right]\xi-\frac{p_e^2S^2}{2}\right)\xi}+4\left(A_2-(\lambda t+s)\right)\xi}{2\xi(c+Q+2\lambda t)}.$$

For Model A, a similar analysis yields:

$$\frac{\partial E^{A*}}{\partial \delta}=\frac{Sp_e}{2\left(\left(\delta+A_1\right)\xi-{p_e}^2\right)}+\frac{2\left(-1+\delta\right)p_e\left(\delta S-2A_2+2\left(s+\lambda t\right)\right)\xi}{\left(2\xi\left(\delta+A_1\right)-{p_e}^2\right)^2}.$$

The equation $\frac{\partial E^{A*}}{\partial \delta}=0$ yields roots $p_{e_2}^A$ and $-p_{e_2}^A$, where

$$p_{e_2}^A=\frac{2\sqrt{-\xi\left(Q-c\right)\left(\delta+A_1\right)\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\delta}}{\delta\left(Q-c\right)}.$$

Consequently, the roots of the equation $\frac{\partial E^{A*}}{\partial \delta}=0$ are $\overline{\delta_A}$ and $\underline{\delta_A}$, and $\frac{\partial E^{A*}}{\partial \delta}<0$ when $\delta \in \left(\underline{\delta_A}, \overline{\delta_A}\right)$, where

$$\overline{\delta_A}=\frac{2\sqrt{-\left(2\left[\begin{array}{l}2(2g-3)\lambda^2t^2+2\left(Q(2g-1)+c(2g+3)-4A_3\right)\lambda t\\+(Q^2+c^2)g+2\left((g+1)c-A_3\right)Q+2A_3(c-A_3)\end{array}\right]\xi-\frac{p_e^2S^2}{2}\right)\xi}+4\left(A_2-(s+\lambda t)\right)\xi}{2\xi S}$$

$$\underline{\delta_A}=\frac{-2\sqrt{-\left(2\left[\begin{array}{l}2(2g-3)\lambda^2t^2+2\left(Q(2g-1)+c(2g+3)-4A_3\right)\lambda t\\+(Q^2+c^2)g+2\left((g+1)c-A_3\right)Q+2A_3(c-A_3)\end{array}\right]\xi-\frac{p_e^2S^2}{2}\right)\xi}+4\left(A_2-(s+\lambda t)\right)\xi}{2\xi S}.$$

B.11 Proof of Theorem 2

Based on the equation $AE^l=E^cd_n^l+\left(\beta E^c-E^l\right)d_r^l$, it is possible to derive the total carbon emissions for Models O and A as follows:

$$AE^{O*}=\frac{-E^c\left(-Q\xi{A_1}^2+\xi\left(\delta^2\lambda t+\left(A_3+\lambda t\right)\delta+2gc\right)A_1+{p_e}^2\left(Q-c\right)\left(\frac{\delta+A_1}{4}\right)\right)}{4A_1\left(2\xi A_1-{p_e}^2\right)}$$
$$-\frac{\left(\begin{array}{l}\left(2\beta E^c{p_e}^2+\left(\delta S+2\left(\lambda t+A_3-c\right)\right)p_e-4\xi\beta E^cA_1\right)\times\\\left(4A_1\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\xi+{p_e}^2\delta\left(Q-c\right)\right)\end{array}\right)}{8\left(2\xi A_1-{p_e}^2\right)^2A_1};$$

$$AE^{A*}=\frac{E^c\left(-\left(Q+\lambda t\right)\delta^2+2g\left(Q-c\right)\right)}{2\left(\delta+A_1\right)}+\frac{E^c\left(\left(\delta+A_1\right)\left(2Q-\left(c+\lambda t+A_3\right)\right)\xi-\frac{\left(S\delta+4\left(Q-c\right)\right){p_e}^2}{4}\right)\delta}{\left(\delta+A_1\right)\left(2\xi\left(\delta+A_1\right)-{p_e}^2\right)}$$
$$-\frac{\left(\begin{array}{l}\left(4\left(\delta+A_1\right)\left(\left(c+\lambda t\right)\delta+A_3+\lambda t-c\right)\xi+{p_e}^2\delta\left(Q-c\right)\right)\times\\\left(-4\beta E^c\left(\delta+A_1\right)\xi+\left(\delta S+2\left(\beta E^cp_e+A_3-c+\lambda t\right)\right)p_e\right)\end{array}\right)}{8\left(2\xi\left(\delta+A_1\right)-{p_e}^2\right)^2\left(\delta+A_1\right)}.$$

When compared to the baseline model, the alterations in total carbon emissions under the VERM framework can be summarized as follows:

$$\Delta AE^{O*} = AE^{O*} - AE^{BO*} = \frac{\delta p_e \left(4\left((c+\lambda t)\delta^2 + (A_3 - 2c)\delta - \lambda t - A_3 + c\right)\xi + {p_e}^2 c\right) L}{8\left(2\xi A_1 - {p_e}^2\right)^2 A_1}$$
$$- \frac{L\left(\left(-4E^c A_1(\beta-\delta)p_e + 8g\left((c+\lambda t)\delta + A_3 + \lambda t - c\right)\right)\xi + \left(2E^c(\beta-\delta)p_e + Q\delta\right){p_e}^2\right)p_e}{8\left(2\xi A_1 - {p_e}^2\right)^2 A_1};$$
$$\Delta AE^{A*} = AE^{A*} - AE^{BA*} = -\frac{L\left(4(\delta + A_1)\left((c+\lambda t)\delta + A_3 + \lambda t - c\right)\xi + {p_e}^2\delta(Q-c)\right)p_e}{8\left(2\xi(\delta + A_1) p_e{}^2\right)^2(\delta + A_1)}$$
$$+ \frac{L(\beta-\delta)E^c {p_e}^2}{4\left(2\xi(\delta + A_1) - {p_e}^2\right)(\delta + A_1)}.$$

Where $L = \delta S + 2(\lambda t + A_3 - c)$. The signs of ΔAE^{O*} and ΔAE^{A*} are unequivocally determined by the magnitude of the numerator. When the condition $\beta < \delta$ is satisfied, it consistently follows that ΔAE^{O*} is less than ΔAE^{A*}, and both are less than zero. In the context where $\beta > \delta$, it follows that ΔAE^{O*} is greater than zero (and ΔAE^{A*} is also greater than zero).

C Proof of Section 6

C.1 Proof of the Propositions 5 and 6

When comparing the equilibrium solutions between Model O and A, it is evident that A_1 is less than $\delta + A_1$, signifying that $\frac{1}{2\xi A_1 - {p_e}^2} > \frac{1}{2\xi(\delta + A_1) - {p_e}^2}$. This relationship allows us to conclude that

$$E^{O*} - E^{A*} = \frac{\delta\xi(\delta S + 2(s + \lambda t - A_1))p_e}{\left(2\xi A_1 - {p_e}^2\right)\left(-{p_e}^2 + 2\xi(\delta + A_1)\right)} > 0;$$
$$d_n^{O*} - d_n^{A*} = \frac{\left(-(Q+\lambda t)\delta^2 + (Q - (\lambda t + A_3))\delta + (Q-c)(2g - A_1)\right)\delta}{2A_1(\delta + A_1)}$$
$$- \frac{{p_e}^2(\delta S + 2(s + \lambda t - A_2))\delta^2\left(2\xi(\delta + 2A_1) - {p_e}^2\right)}{4\left(2\xi A_1 - {p_e}^2\right)A_1\left(2\xi(\delta + A_1) - {p_e}^2\right)(\delta + A_1)} < 0;$$
$$d_r^{O*} - d_r^{A*} = \frac{\left[4\left((c+\lambda t)\delta - A_2 + \lambda t + s\right)A_1(\delta + A_1)\xi^2 + \delta(Q-c)\left(-0.5p_e^2 + \xi(2A_1 + \delta)\right){p_e}^2\right]\delta}{2\left(2\xi A_1 - p_e^2\right)\left(2\xi(\delta + A_1) - p_e^2\right)A_1(\delta + A_1)} > 0;$$
$$\Pi_M^{O*} - \Pi_M^{A*} = -\frac{\delta\left[-8A_1\left((c+\lambda t)\delta - A_2 + \lambda t + s\right)^2(\delta + A_1)\xi^2 + (Q-c)^2{p_e}^2\delta^2\left(-0.5{p_e}^2 + \xi(2A_1 + \delta)\right)\right]}{8\left(2\xi A_1 - {p_e}^2\right)A_1\left(2\xi(\delta + A_1) - {p_e}^2\right)(\delta + A_1)} > 0.$$

Furthermore, an analysis of the sensitivity of OEM profits reveals that

$$\left|\frac{\partial\Pi_M^{O*}}{\partial p_e}\right| - \left|\frac{\partial\Pi_M^{A*}}{\partial p_e}\right| = \frac{\left[\begin{array}{l}\left(\xi(\delta + 2A_1) - {p_e}^2\right)\xi^2(\delta S + 2(s + \lambda t - A_2))p_e \\ \times \delta S\left((Q - (2\lambda t + 3c))\delta + 2(A_2 - (\lambda t + s))\right)\end{array}\right]}{\left(2\xi A_1 - {p_e}^2\right)^2\left(2\xi(\delta + A_1) - {p_e}^2\right)^2} > 0$$
$$\left|\frac{\partial\Pi_M^{O*}}{\partial \xi}\right| - \left|\frac{\partial\Pi_M^{A*}}{\partial \xi}\right| = \frac{\left[\begin{array}{l}\left(2(1-\delta)\lambda t + \delta(Q-3c) + 2(-s + A_2)\right)\left(2(3A_1 + \delta)\xi - 3{p_e}^2\right) \\ \times (\delta S - 2(A_2 - (s + \lambda t)))\left(2\xi\left(2g - \delta^2\right) - {p_e}^2\right){p_e}^2\end{array}\right]}{8\left(2\xi A_1 - {p_e}^2\right)^2\left(2\xi(\delta + A_1) - {p_e}^2\right)^2} > 0.$$

C.2 Proof of Subsection 6.1

The Hessian matrix of the profit function Π_k^C, with respect to the variables d_n^C and d_r^C is given by:

$$H^C = \begin{bmatrix} -2 & -2\delta \\ -2\delta & -2(\delta + g) \end{bmatrix}.$$

It can be determined that $H_1^C = -2 < 0$, indicating that the first principal minor is negative. Furthermore, the second principal minor can be expressed as: $H_2^C = 4A_1$. Thus, the first-order conditions yield the optimal reaction functions:

Table 4 The Optimal Solution for the Model O and BO.

	Model O
E^{O*}	$\frac{(\delta S - 2(A_2 - (s+\lambda t)))p_e}{2(2\xi A_1 - {p_e}^2)}$
w_r^{O*}	$\frac{\left(2\xi g - {p_e}^2\right)(\lambda t(\delta+1) - p_0)}{2\xi A_1 - {p_e}^2} + \frac{(s-c)\delta^2 + ((g+1)c - s)\delta + (-s+c)g}{A_2}$ $- \frac{{p_e}^2(\delta(c+Q) + 2(s-c))A_4}{2(2\xi A_1 - {p_e}^2)A_1} + \left(-\rho + \frac{2\xi A_4}{2A_1\xi - {p_e}^2}\right)c_a$
p_n^{O*}	$\frac{c+Q}{2}$
p_r^{O*}	$\frac{\xi(-p_0 + s - c)\delta^2 + \xi(p_0 + c - s + 2gc)\delta}{2A_1\xi - {p_e}^2} - \frac{\delta {p_e}^2\left(\delta(1-\delta)\left(\frac{3Q}{4} + \frac{c}{4}\right) + (c+Q)g\right)}{(2\xi A_1 - {p_e}^2)A_1} - p_0$ $+ \frac{\left(-(\lambda t\delta + c_a)\delta^2 + (t(4g+1)\lambda + c_a)\delta + 4\lambda t g\right)\xi - t\lambda {p_e}^2(\delta+1)}{2A_1\xi - {p_e}^2} + \frac{QA_1\delta\xi}{2\xi A_1 - {p_e}^2}$
d_n^{O*}	$\frac{-Q\delta^2 + (Q - s + p_0)\delta + 2g(Q-c)}{2A_1} - \frac{{p_e}^2((c+Q)\delta + 2s - 2c - 2p_0)\delta}{4(2\xi A_1 - {p_e}^2)A_1} - \frac{((\delta+1)t\lambda - c_a)\delta\xi}{(2A_1)\xi - {p_e}^2}$
d_r^{O*}	$\frac{4A_1((c+\lambda t)\delta - A_2 + \lambda t + s)\xi + \delta {p_e}^2(Q-c)}{4A_1(2\xi A_1 - p_e^2)}$
	Model BO
w_r^{BO*}	$\frac{c_a(-1+\rho)\delta^2 + ((1-\rho)c_a + g\lambda t)\delta + ((1-2\rho)c_a + \lambda t)g}{A_2}$ $+ \frac{(-c+s)\delta^2 + (c(g+1) - s)\delta + (c - (p_0+s))g}{A_2}$
p_n^{BO*}	$\frac{c+Q}{2}$
p_r^{BO*}	$\frac{-\lambda t\delta^3 + (p_0 - c_a)\delta^2 + ((4tg+t)\lambda - p_0 + c_a)\delta + 4g(\lambda t - p_0)}{2A_2}$ $+ \frac{-\delta^3 Q + (s - c + Q)\delta^2 + (2(Q+c)g - s + c)\delta}{2A_2}$
d_n^{BO*}	$\frac{-(Q+\lambda t)\delta^2 + (Q + c_a + p_0 - (s+\lambda t))\delta + 2g(Q-c)}{2A_2}$
d_r^{BO*}	$\frac{(c+\lambda t)\delta - A_2 + \lambda t + s}{2A_2}$

$$d_n^C\left(E^C\right) = \frac{-(\lambda t + Q)\delta^2 + (Q + p_0 - (s + \lambda t + p_e E^c) + \rho c_a)\delta + g(Q-c)}{2A_4}$$
$$d_r^C\left(E^C\right) = \frac{(c+\lambda t)\delta + p_e E^c - (p_0 + c) + \lambda t + s - \rho c_a}{2A_4}.$$

By substituting the best response into the original profit function, it becomes apparent that

$$\Pi_k^C = \frac{\xi(((1+\delta)\lambda t + s) + 2(\delta c - (c + \rho c_a + p_0)))((1+\delta)\lambda t + s)}{4(\xi A_4 - {p_e}^2)} + \frac{(Q(Q-2c) - 4\rho F_c)}{4}$$
$$+ \frac{\left((g+1-\delta)c^2 + 2(1-\delta)(p_0 + \rho c_a)c + (p_0 + \rho c_a)^2\right)\xi - (cp_e)^2}{4(\xi A_4 - {p_e}^2)}.$$

References

Abreu H, Pereira V, Barata J. Blockchain-based digital product passport: design principles and demonstration. International Journal of Production Research. 2025;p. 1–20. https://doi.org/https://doi.org/10.1080/00207543.2025.2464161.

Table 5 The Optimal Solution for the Model A and BA.

	Model A
E^{A*}	$\frac{p_e\left(\delta S-2A_2+2\left(s+\lambda t\right)\right)}{-2{p_e}^2+4\xi\left(\delta+A_1\right)}$
f^{A*}	$\frac{4\left(\delta+A_1\right)\left(\left(\lambda t+Q\right)\delta+\left(2\rho-1\right)c_a+\lambda t-\left(p_0+c\right)+s\right)\xi-{p_e}^2\left(\left(Q-c\right)\delta+4\rho c_a\right)}{8\xi\left(\delta+A_1\right)-4{p_e}^2}$
p_n^{A*}	$\frac{c+Q}{2}$
p_r^{A*}	$\frac{\left(\left(\delta+A_1\right)^2\xi-\left(\frac{aA_1+\delta\left(3-\delta\right)}{4}\right){p_e}^2\right)\delta Q}{\left(-{p_e}^2+2\xi\left(\delta+A_1\right)\right)\left(\delta+A_1\right)}+\frac{\left(\left(2g+1\right)\left(\delta+A_1\right)\xi-\left(\frac{2\left(\delta+g\right)+A_1}{4}\right){p_e}^2\right)\delta c}{\left(-{p_e}^2+2\xi\left(\delta+A_1\right)\right)\left(\delta+A_1\right)}$ $\frac{\left(-p_0+\lambda t\left(1+\delta\right)\right)\left(\left(2\left(\delta+g\right)+A_1\right)\xi-{p_e}^2\right)}{2\left(\delta+A_1\right)\xi-{p_e}^2}+\frac{\delta\xi\left(-1+\delta\right)\left(-c_a+s\right)}{2\left(\delta+A_1\right)\xi-{p_e}^2}$
d_n^{A*}	$\frac{-\delta^2Q+\left(-c+2Q-s\right)\delta+2g\left(Q-c\right)}{2\left(\delta+A_1\right)}-\frac{\delta\xi\left(\left(1+\delta\right)\lambda t-c_a-p_0\right)}{\left(2\left(\delta+A_1\right)\right)\xi-{p_e}^2}-\frac{\delta{p_e}^2\left(\left(c+Q\right)\delta-2c+2s\right)}{4\left(-{p_e}^2+4\delta\xi+4\xi g-2\delta^2\xi\right)\left(2A_2\right)}$
d_r^{A*}	$\frac{4\left(\left(c+\lambda t\right)\delta-A_2+\lambda t+s\right)\left(\delta+A_1\right)\xi+\delta{p_e}^2\left(Q-c\right)}{4\left(2\left(\delta+A_1\right)\xi-{p_e}^2\right)\left(\delta+A_1\right)}$
	Model BA
f^{BA*}	$\frac{\left(\left(\lambda t+Q\right)\delta+\left(2\rho-1\right)c_a-p_0+\lambda t-c+s\right)}{2}$
p_n^{BA*}	$\frac{c+Q}{2}$
p_r^{BA*}	$\frac{-\lambda t\delta^3+\left(2\lambda t-c_a+p_0\right)\delta^2+\left(t\left(4g+3\right)\lambda+c_a-3p_0\right)\delta+4g\left(\lambda t-p_0\right)}{2\left(\delta+A_1\right)}$ $+\frac{-\delta^3Q+\left(s+2Q\right)\delta^2+\left(2\left(Q+c\right)g-s+c\right)\delta}{2\left(\delta+A_1\right)}$
d_n^{BA*}	$\frac{-\left(Q+\lambda t\right)\delta^2+\left(2Q-\left(c+\lambda t+s\right)+c_a+p_0\right)\delta+2g\left(Q-c\right)}{2\left(\delta+A_1\right)}$
d_r^{BA*}	$\frac{\left(c+\lambda t\right)\delta-A_2+\lambda t+s}{2\left(\delta+A_1\right)}$

Adhikari A, Bisi A. Collaboration, bargaining, and fairness concern for a green apparel supply chain: An emerging economy perspective. Transportation Research Part E: Logistics and Transportation Review. 2020;135:101863. https://doi.org/https://doi.org/10.1016/j.tre.2020.101863.

Agrawal R, Yadav A, Garg RK, Sachdeva A. Building a Greener Supply Chain with Blockchain: A Review and Future Research Directions. In: International Conference on Production and Industrial Engineering Springer; 2023. p. 139–152.

Al Amin M, Baldacci R, Kerbache L. Blockchain-based green supply chain management framework for sustainable practices in Bangladeshi RMG industries. Discover Sustainability. 2025;6(1):508. https://doi.org/https://doi.org/10.1007/s43621-025-01196-8.

Alegoz M, Kaya O, Bayindir ZP. A comparison of pure manufacturing and hybrid manufacturing–remanufacturing systems under carbon tax policy. European Journal of Operational Research. 2021;294(1):161–173. https://doi.org/https://doi.org/10.1016/j.ejor.2021.01.018.

Alotaibi EM, Khallaf A, Abdallah AAN, Zoubi T, Alnesafi A. Blockchain-driven carbon accountability in supply chains. Sustainability. 2024;16(24):10872. https://doi.org/https://doi.org/10.3390/su162410872.

Babaei A, Tirkolaee EB, Simic V, Golpîra H, Anka F. Assessing the Impacts of Blockchain Technology on Supply Chain Efficiency: A Data-Driven Integrated Decision-Making Framework. Group Decision and Negotiation. 2025;p. 1–32. https://doi.org/https://doi.org/10.1007/s10726-025-09934-z.

Badhwar A, Islam S, Tan CSL. Exploring the potential of blockchain technology within the fashion and textile supply chain with a focus on traceability, transparency, and product authenticity: A systematic review. Frontiers in Blockchain. 2023;6:1044723. https://doi.org/https://doi.org/10.3389/fbloc.2023.

Table 6 The Optimal Solution for the Model *RO* and *RA*.

	Model *RO*
E^{RO*}	$\frac{p_e\left(\delta\left(c+\lambda t\right)-c+\lambda t+A_3\right)}{2\left(\xi A_1-{p_e}^2\right)}$.
w_r^{RO*}	$\frac{2\left(\rho c_a\delta^2+\left(g\lambda t-c_a\rho\right)\delta+g\left(\lambda t-2c_a\rho\right)\right)\xi-{p_e}^2\left(\lambda t\left(\delta+1\right)-2\rho c_a\right)}{2\left(\xi A_1-{p_e}^2\right)}$ $+\frac{2\left(\left(-c+s\right)\delta^2+\left(cg-s+c\right)\delta+g\left(-s+c\right)\right)\xi-{p_e}^2\left(c\delta-s+c\right)}{2\left(\xi A_1-{p_e}^2\right)}$ $+\frac{\left(\left(\delta\left(1-\delta\right)+A_1\right)c_a-2gp_0\right)\xi+{p_e}^2\left(p_0-c_a\right)}{2\left(\xi A_1-{p_e}^2\right)}$.
p_n^{RO*}	$\frac{c+Q}{2}$.
p_r^{RO*}	$\frac{\left(\delta^2Q+\left(t\left(4g+1\right)\lambda+2gQ\right)\delta-\left(\lambda t+Q\right)\delta^3+4g\lambda t\right)\xi-2\lambda t{p_e}^2-{p_e}^2\left(2\lambda t+Q\right)\delta}{2\xi A_1-2{p_e}^2}$ $+\frac{\left(\left(-c+s-c_a\right)\delta^2+\left(\left(2g+1\right)c-s+c_a\right)\delta-p_0\left(A_1+2g\right)\right)\xi-{p_e}^2\left(-2p_0+c\delta\right)}{2\xi A_1-2{p_e}^2}$.
d_n^{RO*}	$\frac{\left(-\left(\lambda t+Q\right)\delta^2+\left(Q-\left(\lambda t+A_3\right)\right)\delta+2g\left(Q-c\right)\right)\xi-{p_e}^2\left(Q-c\right)}{2A_1\xi-2{p_e}^2}$.
d_r^{RO*}	$\frac{\xi\left(\left(c+\lambda t\right)\delta-c+\lambda t+A_3\right)}{2\left(\xi A_1-{p_e}^2\right)}$.
	Model *RA*
E^{RA*}	$\frac{\left(\left(c+\lambda t\right)\delta-c+\lambda t+A_3\right)p_e}{2\left(\xi\left(\delta+A_1\right)-{p_e}^2\right)}$.
p_n^{RA*}	$\frac{c+Q}{2}$.
p_r^{RA*}	$\frac{-\xi\left(\lambda t+Q\right)\delta^2\left(\delta-2\right)+\left(\left(t\left(4g+3\right)\lambda+2Qg\right)\xi-{p_e}^2\left(2\lambda t+Q\right)\right)\delta+2\lambda t\left(2g\xi-{p_e}^2\right)}{2\left(\left(\delta+A_1\right)\xi-{p_e}^2\right)}$ $+\frac{\left(\left(s-c_a+p_0\right)\delta^2+\left(-3p_0+\left(2g+1\right)c-s+c_a\right)\delta-4p_0g\right)\xi-{p_e}^2\left(\delta c-2p_0\right)}{2\left(\xi\left(\delta+A_1\right)-{p_e}^2\right)}$.
d_n^{RA*}	$\frac{\left(\left(-\lambda t-Q\right)\delta^2+\left(2Q-\left(c+\lambda t+A_3\right)\right)\delta+2g\left(Q-c\right)\right)\xi-{p_e}^2\left(Q-c\right)}{2\left(\left(\delta+A_1\right)\xi-{p_e}^2\right)}$
d_r^{RA*}	$\frac{\left(\left(c+\lambda t\right)\delta-c+\lambda t+A_3\right)\xi}{2\left(\xi\left(\delta+A_1\right)-{p_e}^2\right)}$
f^{RA*}	$\frac{\left(1+\delta\right)\lambda t+2\rho c_a+\delta Q+A_3-c}{2}$

Let $A_1=\delta+2g-\delta^2$; $A_2=c+p_0+c_a$; $A_3=s-\left(c_a+p_0\right)$; $A_4=\delta+g-\delta^2$; $A_5=1-\delta+2g$; $W_1=\left(1+\delta\right)\lambda t+c\left(\delta-1\right)$; $S=c+2\lambda t+Q$; and $L=\delta S+2\left(\lambda t+A_3-c\right)$.

1044723.

Bai Q, Jin M, Xu X, et al. Effects of carbon emission reduction on supply chain coordination with vendor-managed deteriorating product inventory. International Journal of Production Economics. 2019;208:83–99. https://doi.org/https://doi.org/10.1016/j.ijpe.2018.11.008.

Bensoussan A, El Ouardighi F. Voluntary environmental effort under (s, S) inventory policy. International Journal of Production Research. 2024;62(1-2):522–535. https://doi.org/https://doi.org/10.1080/00207543.2023.2242968.

Cao J, Shao Q. Outsourcing and authorized remanufacturing under voluntary emission reduction mechanism. Flexible Services and Manufacturing Journal. 2025;37(3):776–815. https://doi.org/https://doi.org/10.1007/s10696-024-09555-9.

Cao S, Xu H, Bryceson KP. Blockchain traceability for sustainability communication in food supply chains: an architectural framework, design pathway and considerations. Sustainability. 2023;15(18):13486. https://doi.org/https://doi.org/10.3390/su151813486.

Chen X, Wang X, Zhou M. Firms' green R&D cooperation behaviour in a supply chain: Technological spillover, power and coordination. International Journal of Production Economics. 2019;218:118–134. https://doi.org/https://doi.org/10.1016/j.ijpe.2019.04.033.

Cheong BC. Leveraging blockchain for enhanced transparency and traceability in sustainable supply chains. Discover Analytics. 2025;3(1):6. https://doi.org/https://doi.org/10.1007/s44257-025-00032-7.

Ding J, Chen W, Wang W. Production and carbon emission reduction decisions for remanufacturing firms under carbon tax and take-back legislation. Computers & Industrial Engineering. 2020;143:106419. https://doi.org/https://doi.org/10.1016/j.cie.2020.106419.

Duncan C, Primavera JH, Hill NA, Wodehouse DC, Koldewey HJ. Potential for return on investment in rehabilitation-oriented blue carbon projects: accounting methodologies and project strategies. Frontiers in Forests and Global Change. 2022;4:775341. https://doi.org/https://doi.org/10.3389/ffgc.2021.775341.

El Saadany AM, Jaber MY. A production/remanufacturing inventory model with price and quality dependant return rate. Computers & Industrial Engineering. 2010;58(3):352–362. https://doi.org/https://doi.org/10.1016/j.cie.2009.01.017.

Fang C, Fan S, Chi M, Wang W. The optimal remanufacturing strategy, returned quality choice and independent remanufacturers' advantage for tackling extreme weather. International Journal of Production Economics. 2023;259:108819. https://doi.org/https://doi.org/10.1016/j.ijpe.2023.108819.

Fang W, Guangzhi L, Chunmei S, Feng W, Jianguo M, Yongzhi Y. Carbon emission reduction accounting method for a CCUS-EOR project. Petroleum Exploration and Development. 2023;50(4):989–1000. https://doi.org/https://doi.org/10.1016/S1876-3804(23)60444-6.

Ferguson M. Strategic issues in closed-loop supply chains with remanufacturing. Closed-loop supply chains: New developments to improve the sustainability of business practices. 2010;p. 9–21.

Fernández-Caramés TM, Blanco-Novoa O, Froiz-Míguez I, Fraga-Lamas P. Towards an autonomous industry 4.0 warehouse: A UAV and blockchain-based system for inventory and traceability applications in big data-driven supply chain management. Sensors. 2019;19(10):2394. https://doi.org/https://doi.org/10.3390/s19102394.

Francisco K, Swanson D. The supply chain has no clothes: Technology adoption of blockchain for supply chain transparency. Logistics. 2018;2(1):2. https://doi.org/https://doi.org/10.3390/logistics2010002.

Friedman N, Ormiston J. Blockchain as a sustainability-oriented innovation?: Opportunities for and resistance to Blockchain technology as a driver of sustainability in global food supply chains. Technological Forecasting and Social Change. 2022;175:121403. https://doi.org/https://doi.org/10.1016/j.techfore.2021.121403.

Gao F, Souza GC. Carbon offsetting with eco-conscious consumers. Management Science. 2022;68(11):7879–7897. https://doi.org/https://doi.org/10.1287/mnsc.2021.4293.

Ghosh D, Shah J. A comparative analysis of greening policies across supply chain structures. International Journal of Production Economics. 2012;135(2):568–583. https://doi.org/https://doi.org/10.1016/j.ijpe.2011.05.027.

Guo Y, Wang M, Yang F. Joint emission reduction strategy considering channel inconvenience under different recycling structures. Computers & Industrial Engineering. 2022;169:108159. https://doi.org/https://doi.org/10.1016/j.cie.2022.108159.

Habibullah S, Sikder MA, Tanha NI, Sah BP. A review of blockchain technology's impact on modern supply chain management in the automotive industry. Global Mainstream Journal of Innovation, Engineering & Emerging Technology. 2024;3(3):13–27. https://doi.org/10.62304/jieet.v3i3.163.

Herbke P, Lamichhane S, Barman K, Pandey SR, Kupper A, Abraham A, et al. DIDChain: Advancing supply chain data management with decentralized identifiers and blockchain. In: 2024 IEEE

International Conference on Service-Oriented System Engineering (SOSE) IEEE; 2024. p. 54–63.

Huang S, Wang Z, Wang J, Lu Y, Liang X, Liu L, et al. A Framework for Blockchain Supported Carbon Emission Data Sharing with Flexible Access Control and Enhanced Data Privacy. In: Proceedings of the 2024 10th International Conference on Advances in Energy Resources and Environment Engineering (ICAESEE 2024), vol. 36 Springer Nature; 2025. p. 407.

Huang Y, Wang Z. Information sharing in a closed-loop supply chain with technology licensing. International Journal of Production Economics. 2017;11:113–127. https://doi.org/https://doi.org/10.1016/j.ijpe.2017.06.008.

Jaber MY, Glock CH, El Saadany AM. Supply chain coordination with emissions reduction incentives. International Journal of Production Research. 2013;51(1):69–82. https://doi.org/https://doi.org/10.1080/00207543.2011.651656.

Jalapathy P, Unnissa MM. Revolutionizing sustainability and adoption of blockchain technology: A dual-channel approach to (Re) manufacturing strategy with an extended warranty. Computers & Industrial Engineering. 2025;p. 111566. https://doi.org/https://doi.org/10.1016/j.cie.2025.111566.

Jiang X, Sun L, Wang Y. Effects of interactions between technology spillovers and carbon transfers on carbon emission reduction decisions and coordination within supply chains. Journal of Business & Industrial Marketing. 2024;39(7):1497–1512. https://doi.org/https://doi.org/10.1108/JBIM-04-2023-0215.

Jin M, Li G, Reimann M. Team of rivals: How should original equipment manufacturers cooperate with independent remanufacturers via authorisation? European Journal of Operational Research. 2022;296(3):837–845. https://doi.org/https://doi.org/10.1016/j.ejor.2021.04.014.

Li W, Sun H, Dong H, Gan Y, Koh L. Outsourcing decision-making in global remanufacturing supply chains: The impact of tax and tariff regulations. European Journal of Operational Research. 2023;304(3):997–1010. https://doi.org/https://doi.org/10.1016/j.ejor.2022.05.016.

Li W, Wang P, Cheng W, Nie K. Transnational remanufacturing decisions under carbon taxes and tariffs. European Journal of Operational Research. 2024;312(1):150–163. https://doi.org/https://doi.org/10.1016/j.ejor.2023.06.019.

Li Y, Huang L, Tong Y. Cooperation with competitor or not? The strategic choice of a focal firm's green innovation strategy. Computers & Industrial Engineering. 2021;157:107301. https://doi.org/https://doi.org/10.1016/j.cie.2021.107301.

Liu B, Holmbom M, Segerstedt A, Chen W. Effects of carbon emission regulations on remanufacturing decisions with limited information of demand distribution. International Journal of Production Research. 2015;53(2):532–548. https://doi.org/https://doi.org/10.1080/00207543.2014.957875.

Liu H, Lei M, Huang T, Leong GK. Refurbishing authorization strategy in the secondary market for electrical and electronic products. International Journal of Production Economics. 2018;195:198–209. https://doi.org/https://doi.org/10.1016/j.ijpe.2017.10.012.

Liu Q, Zhu X. Carbon reduction decisions in green technology collaborative R&D and spillover time lag effects. Journal of Cleaner Production. 2023;429:139595. https://doi.org/https://doi.org/10.1016/j.jclepro.2023.139595.

Luo R, Zhou L, Song Y, Fan T. Evaluating the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain. International Journal of Production Economics. 2022;245:108408. https://doi.org/https://doi.org/10.1016/j.ijpe.2022.108408.

Malik S, Dedeoglu V, Kanhere SS, Jurdak R. PrivChain: Provenance and privacy preservation in blockchain enabled supply chains. In: 2022 IEEE International Conference on Blockchain (Blockchain) IEEE; 2022. p. 157–166.

Manzoor R, Sahay B, Singh SK. Blockchain technology in supply chain management: an organizational theoretic overview and research agenda. Annals of Operations Research. 2025;348(3):1307–1354. https://doi.org/https://doi.org/10.1007/s10479-022-05069-5.

Merlo AL, Mendonça DS, Santos J, Carvalho ST, Guerra R, Brandão D. Blockchain for the carbon market: a literature review. Discover Environment. 2025;3(1):68. https://doi.org/https://doi.org/10.1007/s44274-025-00260-4.

Najjar M, Alsurakji IH, El-Qanni A, Nour AI. The role of blockchain technology in the integration of sustainability practices across multi-tier supply networks: Implications and potential complexities. Journal of Sustainable Finance & Investment. 2023;13(1):744–762. https://doi.org/https://doi.org/10.1080/20430795.2022.2030663.

Nan J, Wu X, Zhang M. Production competition and emission reduction technology cooperation between manufacturers under the low-carbon market based on blockchain technology. Chinese Journal of Management Science. 2025;33(6):360–368. https://doi.org/https://doi.org/10.16381/j.cnki.issn1003-207x.2023.1421.

Niu B, Dong J, Liu Y. Incentive alignment for blockchain adoption in medicine supply chains. Transportation Research Part E: Logistics and Transportation Review. 2021;152:102276. https://doi.org/https://doi.org/10.1016/j.tre.2021.102276.

Pun H, Ghamat S. The value of partnership under competition: When competitors may be R&D joint-venture and supply-chain partners for a critical component. International Journal of Production Economics. 2016;177:1–11. https://doi.org/https://doi.org/10.1016/j.ijpe.2016.03.018.

Pun H, Swaminathan JM, Hou P. Blockchain adoption for combating deceptive counterfeits. Production and Operations Management. 2021;30(4):864–882. https://doi.org/https://doi.org/10.1111/poms.13348.

Rajput DV, More PR, Adhikari PA, Arya SS. Blockchain technology in the food supply chain: a way towards circular economy and sustainability. Sustainable Food Technology. 2025;https://doi.org/https://doi.org/10.1039/D5FB00065C.

Saberi S, Kouhizadeh M, Sarkis J, Shen L. Blockchain technology and its relationships to sustainable supply chain management. International journal of production research. 2019;57(7):2117–2135. https://doi.org/https://doi.org/10.1080/00207543.2018.1533261.

Sabzevar N, Enns S, Bergerson J, Kettunen J. Modeling competitive firms' performance under price-sensitive demand and cap-and-trade emissions constraints. International Journal of Production Economics. 2017;184:193–209. https://doi.org/https://doi.org/10.1016/j.ijpe.2016.10.024.

Schweitzer V, Bach V, Holzapfel PK, Finkbeiner M. Differences in science-based target approaches and implications for carbon emission reductions at a sectoral level in Germany. Sustainable Production and Consumption. 2023;38:199–209. https://doi.org/https://doi.org/10.1016/j.spc.2023.04.009.

Sharabati AAA, Jreisat ER. Blockchain technology implementation in supply chain management: A literature review. Sustainability. 2024;16(7):2823. https://doi.org/https://doi.org/10.3390/su16072823.

Sharma I, Kaur G, Dey BK, Majumder A. Leveraging blockchain and consignment contracts to optimize food supply chains under uncertainty. Applied Sciences. 2024;14(24):11735. https://doi.org/https://doi.org/10.3390/app142411735.

Subramanian R, Subramanyam R. Key factors in the market for remanufactured products. Manufacturing & Service Operations Management. 2012;14(2):315–326. https://doi.org/https://doi.org/10.1287/msom.1110.0368.

Sun Y, Shen W, Li J, Liao Y. Competition in remanufacturing with asymmetric demand information. Sustainability. 2024;16(2):471. https://doi.org/https://doi.org/10.3390/su16020471.

Swami S, Shah J. Channel coordination in green supply chain management. Journal of the operational research society. 2013;64(3):336–351. https://doi.org/https://doi.org/10.1057/jors.2012.44.

Times F. EU carbon price tops€ 100 a tonne for first time. Abgerufen am. 2023;15:2024.

Wang X, Zhu Y, Sun H, Jia F. Production decisions of new and remanufactured products: Implications for low carbon emission economy. Journal of Cleaner Production. 2018;171:1225–1243. https://doi.org/https://doi.org/10.1016/j.jclepro.2017.10.053.

Wang Y, Xu X, Zhu Q. Carbon emission reduction decisions of supply chain members under cap-and-trade regulations: A differential game analysis. Computers & Industrial Engineering. 2021;162:107711. https://doi.org/https://doi.org/10.1016/j.cie.2021.107711.

Wei Q, Xiao S. Assessing barriers to the internationalization of China's certified emission reductions (CCERs): A Delphi survey. Climate Policy. 2022;22(7):906–917. https://doi.org/https://doi.org/10.1080/14693062.2022.2090892.

Wu H, Jiang S, Cao J. High-efficiency blockchain-based supply chain traceability. IEEE Transactions on Intelligent Transportation Systems. 2023;24(4):3748–3758. https://doi.org/https://doi.org/10.1109/TITS.2022.3205445.

Wu X, Miao W, Zhang X, Zuo W, Shen B. Recycling and remanufacturing or technology upgrading? emission reduction decisions of supply chain under carbon cap-and-trade mechanism. PloS one. 2025;20(3):e0318952. https://doi.org/https://doi.org/10.1371/journal.pone.0318952.

Xia X, Li M, Li B, Wang H. The impact of carbon trade on outsourcing remanufacturing. International Journal of Environmental Research and Public Health. 2021;18(20):10804. https://doi.org/https://doi.org/10.3390/ijerph182010804.

Xia X, Li M, Wang W. Impact of three emission reduction decisions on authorized remanufacturing under carbon trading. Expert Systems with Applications. 2023;216:119476. https://doi.org/https://doi.org/10.1016/j.eswa.2022.119476.

Xia X, Lu M, Wang W. Emission reduction and outsourcing remanufacturing: A comparative study under carbon trading. Expert Systems with Applications. 2023a;227:120317. https://doi.org/https://doi.org/10.1016/j.eswa.2023.120317.

Xu L, Wang C. Contracting pricing and emission reduction for supply chain considering vertical technological spillovers. The International Journal of Advanced Manufacturing Technology. 2017;93(1):481–492. https://doi.org/https://doi.org/10.1007/s00170-017-0116-5.

Xu X, Yan L, Choi TM, Cheng T. When is it wise to use blockchain for platform operations with remanufacturing? European Journal of Operational Research. 2023;309(3):1073–1090. https://doi.org/https://doi.org/10.1016/j.ejor.2023.01.063.

Yadav N, Luthra S, Garg D. Blockchain technology for sustainable supply chains: a network cluster analysis and future research propositions. Environmental Science and Pollution Research. 2023;30(24):64779–64799. https://doi.org/https://doi.org/10.1007/s11356-023-27049-3.

Yang L, Hu Y, Huang L. Collecting mode selection in a remanufacturing supply chain under cap-and-trade regulation. European Journal of Operational Research. 2020;287(2):480–496. https://doi.org/https://doi.org/10.1016/j.ejor.2020.04.006.

Yenipazarli A. Managing new and remanufactured products to mitigate environmental damage under emissions regulation. European Journal of Operational Research. 2016;249(1):117–130. https://doi.org/https://doi.org/10.1016/j.ejor.2015.08.020.

Zheng Y, Zhang B. The impact of carbon market on city greening: Quasi-experimental evidence from China. Resources, Conservation and Recycling. 2023;193:106960. https://doi.org/https://doi.org/10.1016/j.resconrec.2023.106960.

Zhou Q, Meng C, Sheu JB, Yuen KF. Remanufacturing mode and strategic decision: A game-theoretic approach. International Journal of Production Economics. 2023;260:108841. https://doi.org/https://doi.org/10.1016/j.ijpe.2023.108841.

Zhou Q, Meng C, Yuen KF. Remanufacturing authorization strategy for competition among OEM, authorized remanufacturer, and unauthorized remanufacturer. International Journal of Production Economics. 2021;242:108295. https://doi.org/https://doi.org/10.1016/j.ijpe.2021.108295.

Zhou Q, Meng C, Yuen KF, Sheu JB. Remanufacturing authorization strategy for an original equipment manufacturer-contract manufacturer supply chain: Cooperation or competition? International Journal of Production Economics. 2021a;240:108238. https://doi.org/https://doi.org/10.1016/j.ijpe.2021.108238.

Zhou Y, Hou M, Wong KH. The optimal remanufacturing strategy of the closed-loop supply chain network under government regulation and the manufacturer's design for the environment. Sustainability. 2023;15(9):7342. https://doi.org/https://doi.org/10.3390/su15097342.

Zhu C, Ma J, Li J. Dynamic production and carbon emission reduction adjustment strategies of the brand-new and the remanufactured product under hybrid carbon regulations. Computers & Industrial Engineering. 2022;172:108517. https://doi.org/https://doi.org/10.1016/j.cie.2022.108517.

Zou ZB, Wang JJ, Deng GS, Chen H. Third-party remanufacturing mode selection: Outsourcing or authorization? Transportation Research Part E: Logistics and Transportation Review. 2016;87:1–19. https://doi.org/https://doi.org/10.1016/j.tre.2015.12.008.