Supplementary Information: Navigating the Quantum Resource Landscape of Entropy Vector Space Using Machine Learning and Optimization

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I. ADDITIONAL CIRCUITS AND MAGIC EVOLUTION

Figures 1 and 2 are the circuits corresponding to Violators 2-4 in Section IVB. The Q-Learning algorithm was initialized at the state given by Eq.(36) and evolved until Ingleton violation was reached.

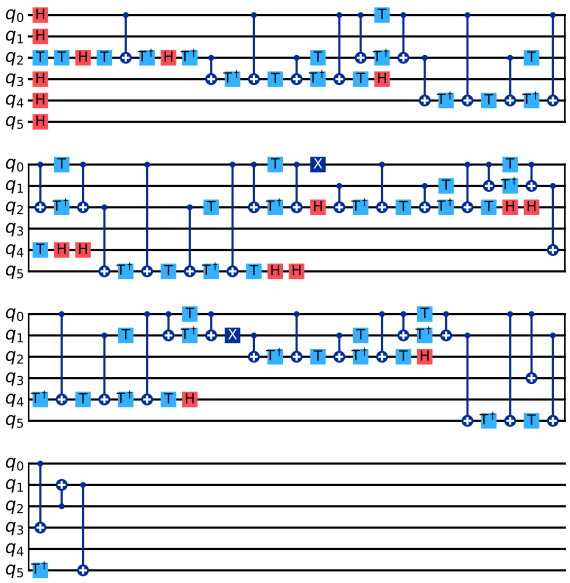
For each circuit in Appendix I that perturbs about a state on the edge of the Ingleton entropy cone, we track the overall magic of the system as the entropy vector is driven out of the Ingleton entropy cone. Figure 3 shows the evolution of $\mathcal{W}_2$, the magic witness defined in Eq.(33), across each circuit that prepares a state $|\psi\rangle_{f_i}$, beginning from the initial state $|0\rangle^{\otimes 6}$. In each circuit, the overall magic of subsystem $ABCD \subset |\psi\rangle_{f_i}$ increases throughout the evolution, as the system is driven towards Ingleton violation.

II. EXAMPLES OF INGLETON-VIOLATING STATES

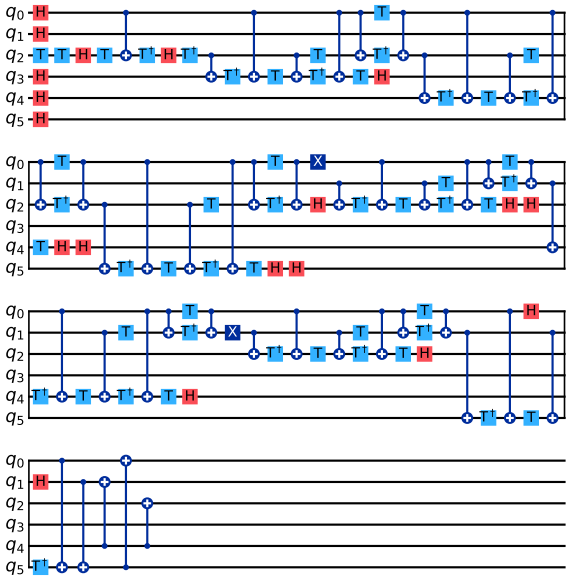
Below we include a few explicitly examples of Ingleton-violating states, with various violation amounts. A significantly larger sample list is publicly available at [1].

Please find the complete codebase, data and all other generated examples of violations at [1]. Additional functions to compute entropy vectors and evaluate entropy inequalities available at [2–4].

[1] N. Khumalo, A. Mehta, and W. Munizzi, Quantum Resource Dynamics, https://github.com/nkhumalo/william-s-magic-school.
[2] W. Munizzi, Stabilizer-States, https://github.com/WMunizzi/Stabilizer-States (2022).
[3] W. Munizzi, Cayley-Graphs, https://github.com/WMunizzi/Cayley-Graphs (2023).
[4] J. Fuentes and W. Munizzi, Mmi-failure-for-graph-states, https://github.com/jesus99222/MMI-Failure-For-Graph-States (2025).

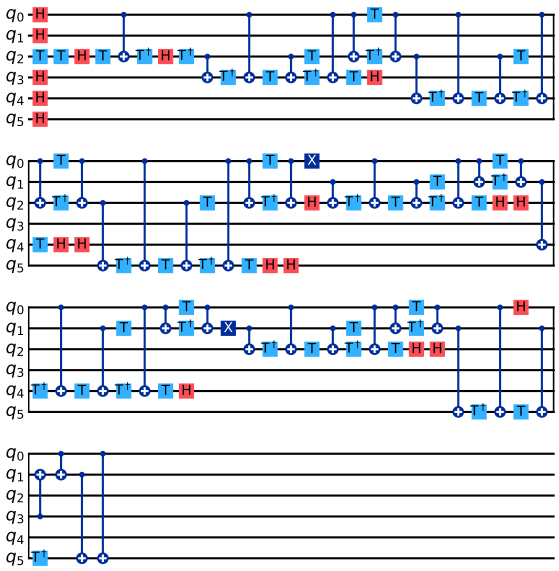


(a) Circuit to prepare violating state 2.

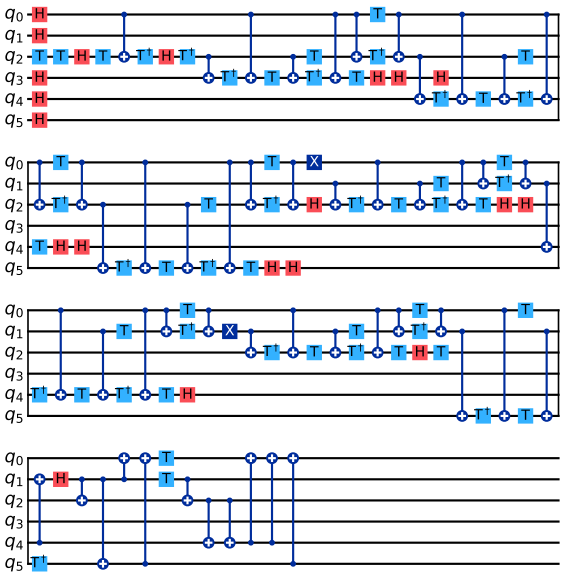


(b) Circuit to prepare violating state 3.

Figure 1: Violating states 2 and 3 found by perturbing Eq.(36) and QL paradigm in Figure (1).



(a) Circuit to prepare violating state 4.



(b) Circuit to prepare violating state 5.

Figure 2: Violating states 4 and 5 found by perturbing Eq.(36) and QL paradigm in Figure (1).

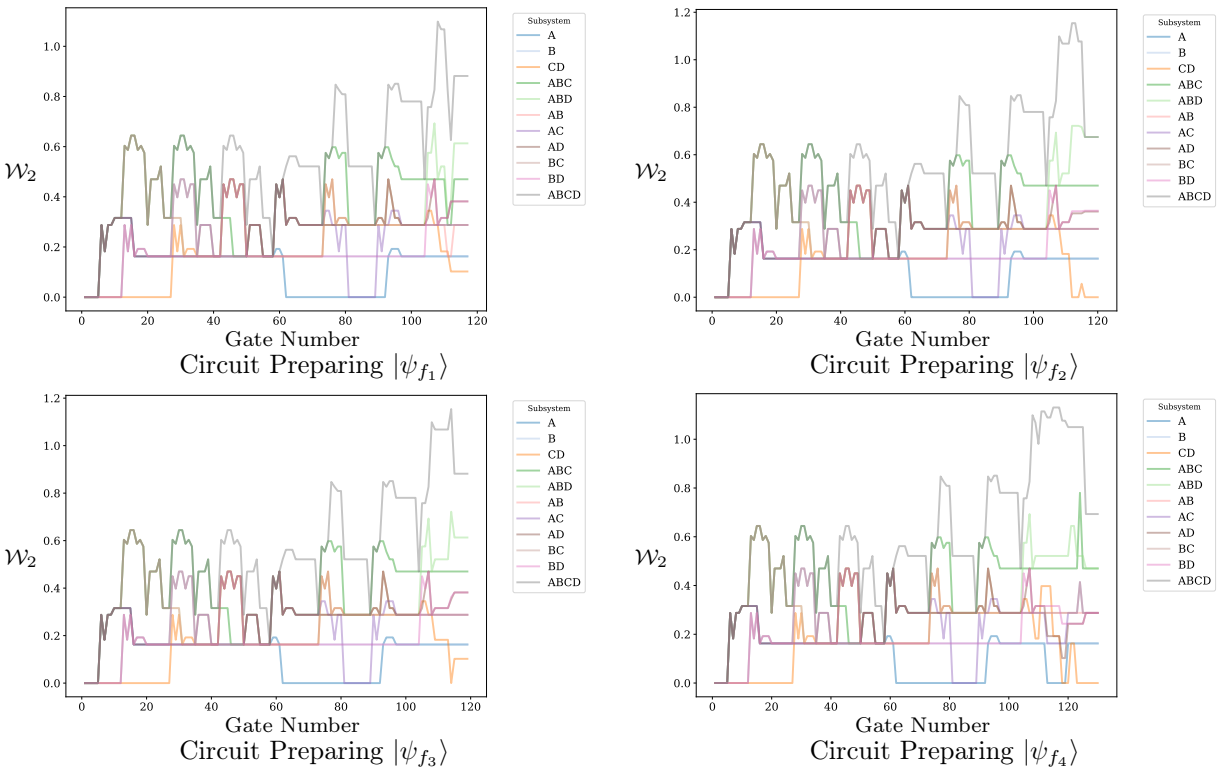


Figure 3: Evolution of the non-local magic witness for mixed states obtained from Ingleton-violating circuits identified via Q-learning–based perturbations of the known violator. In circuits 2 and 4, subsystem CD exhibits a positive $\mathcal{W}_2$ value at the point of violation, in contrast to the reference (literature) circuit where $\mathcal{W}_2(CD) = 0$.