Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

The US Macro-Learning Curve: Empirical Evidence from Post-War GDP and Productivity Dynamics.

Sayan Kombarov

smartkom72@gmail.com

ACCA https://orcid.org/0000-0002-8649-1132

Research Article

Keywords: Learning curves, Wright's law, Price–quantity dynamics, Economic growth, Aggregate output, Principle of Least Action, Co-integration, Deterministic forecasting

Posted Date: February 27th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-8964857/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: The authors declare no competing interests.

Abstract

This paper provides a unified dynamic interpretation of learning curves and aggregate output growth by linking prices, cumulative production, and time through a minimal price–quantity formalism. Starting from a derivation of price as a flow of accumulated value ($P \equiv dQ/dt$), we show that Wright's Law and exponential GDP growth emerge as structural consequences of declining unit costs under expanding production. We introduce the **expansion rate (R)** as a dimensionless measure of growth, characterizing accumulation as a deterministic process unfolding in time.

Using 70 years of US FRED data (1947–2024), we empirically validate the framework using OLS regression and rigorous time-series diagnostics. Our results identify a stable **continuous expansion rate (R) of 3.05%** and a **macro-learning rate of 20.8%** (structural elasticity, $\beta = 0.3349$). The presence of **Engle-Granger co-integration ($p < 0.0001$)** confirms that aggregate productivity is a deterministic function of cumulative experience rather than a series of exogenous shocks. This synthesis suggests that GDP growth is fundamentally the "learning curve" of civilization, offering a parsimonious alternative to stochastic growth models and providing a new deterministic tool for forecasting and policy analysis.[1]

1. Introduction

Learning curves are among the most robust empirical regularities observed in production. Across industries and historical periods, average costs or production times tend to decline systematically as cumulative output increases. This paper argues that learning curves and aggregate economic growth are different manifestations of the same underlying accumulation process.

By adopting a minimal dynamic framework inspired by variational reasoning—specifically the **Principle of Least Action**—we relate prices, output, and time as a compact analytical device. Unlike standard neoclassical growth theory, which relies on representative agents and steady-state equilibria, our framework describes accumulation as a dynamic process unfolding in time. We demonstrate that the US economy adheres to a Wright-type scaling law where productivity is a deterministic function of cumulative experience, as evidenced by an empirical **20.8% learning rate**.

This pattern was first documented by Wright (1936) and has since been confirmed in a wide range of manufacturing and technology-intensive sectors (Arrow 1962; Rosenberg 1982; Nordhaus 2007). At the same time, long-run economic growth exhibits persistent regularities that are typically analyzed using separate theoretical frameworks (Solow 1956; Kaldor 1957).

Economists routinely observe systematic declines in costs and prices as cumulative output expands, yet these regularities are typically treated as empirical relationships rather than consequences of basic price–quantity relations. This paper argues that learning curves and aggregate economic growth are different manifestations of the same underlying accumulation process. By focusing on the temporal structure of value accumulation, it shows that declining unit costs, productivity growth, and GDP expansion can be understood within a single dynamic framework. We adopt a minimal dynamic

framework inspired by variational reasoning, not as a physical analogy but as a compact analytical device for relating prices, output, and time.

The analysis begins with a minimal definition of price as the flow of accumulated value over time. Growth is then characterized by an expansion rate measuring the proportional increase of accumulated output. These definitions do not rely on equilibrium conditions, representative agents, or optimization behavior. Instead, they describe accumulation as a dynamic process unfolding in time, in the spirit of classical and structural approaches to growth (Young 1928; Kaldor 1966).

Under stationary valuation, marginal value declines as accumulation proceeds. Economically, this reflects the fact that expanding production reallocates time and resources across an increasing stock of accumulated value. As a result, cumulative output follows a learning trajectory with saturation rather than an unbounded exponential path. This dynamic structure has two important implications. First, when production time or cost is expressed relative to cumulative output, the model reproduces Wright's law over a wide range of scales. Learning curves thus emerge as a structural consequence of accumulation dynamics rather than as independent technological regularities (Wright 1936; Arrow 1962). Second, when interpreted at the aggregate level, the same accumulation process implies that GDP follows a learning curve, with rapid growth during early phases of expansion and gradual deceleration as economies mature.

The contribution of the paper is therefore threefold. It provides a dynamic interpretation of price as a flow variable, introduces the expansion rate as a structural measure of growth, and derives learning curves and aggregate growth from the same accumulation process. By doing so, it offers a unified perspective on learning, productivity, and economic growth that connects micro-level production dynamics with macroeconomic outcomes, while remaining agnostic about specific institutional or technological mechanisms. The paper focuses exclusively on real accumulation dynamics and abstracts from monetary, financial, and institutional mechanisms, which are left for subsequent work.

Data for Real GDP and Productivity were sourced from the Federal Reserve Economic Data (FRED) database, covering the post-war era. This period represents the most consistent record of industrial and technological expansion in a frontier economy.

The paper proceeds as follows. Section 2 introduces the basic definitions of price and growth and derives the expansion dynamics of accumulated value. Section 3 analyzes the temporal structure of production and shows how learning curves and GDP emerge from accumulation with saturation. Section 4 discusses the implications of the framework for Wright's law, Moore's law, and economic growth. Formal derivations are collected in Appendix A.

2. Theoretical Framework

This section introduces the basic definitions used throughout the paper. The framework is deliberately minimal and focuses on the dynamic relationship between price, accumulated value, and growth.

Let Q(t) denote accumulated output measured in terms of a general equivalent of economic value in monetary or commodity terms. Accumulation is understood broadly, encompassing goods, services, and reproducible economic value. Price is defined dynamically as the rate at which accumulated value changes over time,

$P(t) \equiv dQ/dt$.

This definition treats price not as a static exchange ratio but as a temporal magnitude reflecting the rate at which value is generated. It is consistent with national accounting identities, where prices and output flows jointly determine value creation over time. A natural measure of growth follows directly from this definition. We define the **expansion rate of value** as $R \equiv P \times T/Q = (dQ/dt) \times T/Q$, where T is a unit of time.

The expansion rate measures the proportional increase of accumulated value per unit time and is dimensionless. Importantly, R is not interpreted as an interest rate or a financial return, but as a structural parameter describing the pace of accumulation.

When the expansion rate is constant, the accumulation of value follows an exponential law, $Q(t) = Q(0) \times \exp(R \times t/T)$, where Q(0) denotes the initial level of accumulated value. This result follows directly from the definition of R and does not rely on equilibrium assumptions, optimization behavior, or exogenous technological shocks. This exponential path describes unconstrained accumulation. In the presence of stationary valuation, where marginal value declines with accumulated output, the effective trajectory exhibits saturation, as derived below.

The exponential form captures a common empirical feature of expanding economies: sustained proportional growth over time. In the present framework, this property is not imposed but emerges from the definition of growth as the ratio of flow to stock.

2.1 Accumulation Dynamics and Learning Curves

This section derives the temporal structure of production implied by the definition of price as marginal accumulated value and shows how learning curves arise as a consequence of accumulation dynamics.

In a stationary valuation regime, marginal value declines inversely with accumulated output, $P = C/Q$, where C is a scale parameter reflecting the total value content of production.

To describe sustained expansion, we introduce the expansion rate $R = P \times T/Q$, which measures proportional growth of accumulated value per unit of time (T). Under stationary valuation and a constant expansion rate, marginal value decays exponentially over time. Accumulation then follows from integrating marginal value,

$Q(t) = C \times T \times [1 - \exp(-R \times t/T)]/[R \times Q(0)]$.

Here, Q(t) is interpreted as aggregate real output dynamics and as a learning process with saturation driven by declining unit prices rather than exogenous technological shocks, while C reflects the value content of production at the initial scale. R governs the speed of learning, while T/Q(0) sets the initial scale of production. At early stages, output grows rapidly as experience accumulates. Over time, growth slows and converges asymptotically, reflecting diminishing marginal contributions of additional accumulation.

The temporal structure of production follows directly from this trajectory. Solving for time as a function of cumulative output yields

$t(Q) = -T \times \ln(1- Q/Q_{max})/R$, where $Q_{max} = C \times T/(Q(0) \times R)$.

Average time per unit is then given by $Y(Q) = t(Q)/Q$.

For a wide intermediate range in which Q/Q_{max}, average time per unit declines approximately as an inverse power of cumulative output. This reproduces Wright's law as an emergent property of accumulation dynamics rather than as an independent technological assumption.

Interpreted at the aggregate level, the same mechanism implies that GDP follows a learning trajectory. Periods of rapid economic growth correspond to early stages of accumulation, while mature economies operate closer to the saturation regime. Learning curves at the micro level and aggregate growth dynamics are therefore governed by the same underlying process.

2.2 Average time per unit

The dynamic law of accumulation has immediate implications for the temporal structure of production. Solving the accumulation equation for time yields

$t(Q) = -T \times \ln(1- Q/Q_{max})/R$, where $Q_{max} = C \times T/(Q(0) \times R)$. This formula represents the time required to reach a given level of cumulative output.

In the relevant range away from saturation, this expression reduces to a logarithmic dependence on cumulative output.

While total production time increases with scale, the increase is logarithmic rather than proportional. This implies that the additional time required to expand production declines systematically as accumulation proceeds. The marginal time required to produce an additional unit of output is given by $dt/dQ = T \times R/Q$, which decreases inversely with accumulated output.

The **average time per unit** is therefore $Y(Q) = t/Q(t) = [T/(R \times Q)] \times \ln[1/(1 - Q/Q_{max})]$.

This expression captures the average temporal cost of production at scale Q. As cumulative output increases, average time per unit declines as an inverse power of Q, up to a slowly varying logarithmic correction.

For a wide intermediate range in which $Q \ll Q_{max}$, the logarithmic term varies slowly relative to the inverse dependence on Q. In this range, average time per unit is well approximated by a power law of the form $Y(Q) \sim a \times Q^{-b}$ with an effective learning exponent close to unity.

This asymptotic form corresponds to Wright's law, widely used in empirical studies of learning-by-doing and productivity growth. In the present framework, the learning curve is not introduced as an independent behavioral or technological assumption. Instead, it emerges endogenously from the dynamics of accumulation implied by a constant expansion rate.

The result provides a structural interpretation of learning curves: declining costs and production times reflect the temporal structure of exponential accumulation rather than an exogenous process of technological improvement. Learning, productivity growth, and scale effects appear as different manifestations of the same underlying dynamic process.

3. Discussion: Learning Curves, Technological Progress, and Economic Growth

The framework developed in this paper provides a unified interpretation of several empirical regularities that are often analyzed separately: learning curves in production, technological progress over time, and sustained economic growth. This section discusses these connections and clarifies the contribution of the proposed approach.

3.1 Learning Curves as a Structural Outcome

Learning curves, most prominently Wright's law, describe a systematic decline in average production time or cost as cumulative output increases. Such patterns have been documented across a wide range of industries and historical periods. Conventional explanations emphasize learning-by-doing, organizational improvements, or technological innovation.

The present framework offers a complementary and more structural interpretation. Learning curves arise as a direct implication of accumulation dynamics. When accumulated value expands at a stable proportional rate, the temporal structure of production necessarily implies that the average time required per unit declines as scale increases. In this sense, learning curves reflect the temporal structure of accumulation rather than an independent behavioral or technological mechanism.

This interpretation does not deny technological change but shows that declining costs can arise even in its absence. Instead, it situates these processes within a broader dynamic context. Improvements in efficiency, coordination, and technique appear as endogenous features of expanding production rather than as exogenous drivers imposed from outside the system.

3.2 Wright's Law and Moore's Law

A longstanding debate concerns the relationship between Wright's law, which links cost reductions to cumulative output, and Moore's law, which describes exponential improvement over time. Empirically, both laws often provide good fits to observed data, leading to ambiguity about their relative explanatory status. Moore's law can be interpreted as a special case of accelerated learning rather than an industry-specific technological anomaly.

Within the present framework, this ambiguity is resolved. A constant expansion rate implies exponential accumulation over time. When production time or cost is expressed relative to cumulative output, this same process appears as a power-law decline. Wright's law and Moore's law therefore describe the same underlying dynamic, viewed from different perspectives: one indexed by scale, the other by time.

From this viewpoint, Moore's law is not a sector-specific anomaly confined to information technology, nor is Wright's law restricted to manufacturing. Both emerge as generic properties of sustained accumulation under stable growth conditions. The distinction between them reflects a choice of representation rather than a difference in underlying economic mechanisms.

3.3 From Micro Learning to Macroeconomic Growth

At the aggregate level, the accumulation of value corresponds to sustained economic growth. Interpreted broadly, accumulated output includes goods, services, capital, and reproducible knowledge. The expansion rate therefore provides a link between micro-level production dynamics and macroeconomic outcomes.

The framework suggests that growth in GDP can be understood as the macroscopic analogue of learning curves observed at the firm or industry level. Just as average production time declines with cumulative output in specific sectors, aggregate productivity improves as the economy accumulates value over time. Learning, technological progress, and economic growth thus appear as different scales of the same dynamic process.

This perspective aligns with classical and structural approaches to growth that emphasize accumulation rather than equilibrium allocation. Growth does not require continuous exogenous shocks to technology; it can arise endogenously from the expansion of value itself, provided the expansion rate remains sufficiently stable. This interpretation suggests that periods of slowing GDP growth may reflect saturation dynamics among other institutional and demographic factors.

3.4 Relation to Existing Growth Theories

The proposed interpretation differs from standard neoclassical growth theory, which relies on representative agents, intertemporal optimization, and steady-state equilibria. It also departs from endogenous growth models that explicitly model innovation, human capital, or research activity as separate engines of growth.

Instead, the analysis focuses on the temporal structure of production and accumulation. Learning curves, technological progress, and growth are treated as emergent properties of this structure. This

approach reconnects modern empirical regularities with earlier concerns in political economy regarding value, accumulation, and time.

By shifting attention from equilibrium conditions to accumulation dynamics, the framework provides a parsimonious explanation for a wide range of observed regularities without introducing additional behavioral or institutional assumptions.

3.5 Scope and Limitations

The analysis deliberately abstracts from financial variables, monetary policy, and credit mechanisms. While these factors undoubtedly influence economic dynamics, the present paper focuses on the real processes of accumulation and production. This restriction allows a clear identification of the mechanisms generating learning curves and sustained growth. In this framework, Wright's law does not rely on technological progress per se but follows from the dynamics of cumulative production and declining unit prices. Extensions of the framework to include financial and monetary dimensions are left for future work. Such extensions would raise distinct theoretical issues and would require a separate analysis.

4. Data and Methodology

4.1 Data Sources and Retrieval

The empirical component of this study utilizes a programmatic retrieval method to ensure data integrity and replicability. We access the **Federal Reserve Economic Data (FRED)** database via the St. Louis Fed's API.

Two primary time-series are utilized:

- **Real Gross Domestic Product (GDPC1)**: A chained 2012 dollar measure of US output, used to calculate both the continuous expansion rate (R) and cumulative economic experience (lnCumGDP).
- **Nonfarm Business Sector: Output Per Hour of All Persons (OPHNFB)**: A standard benchmark for US labor productivity (lnProd).

The sample covers the post-war era (1947–Present), providing a consistent longitudinal record of the American industrial and service-sector expansion.

4.2 Programmatic Processing

Data processing was conducted in **Python 3.x** using the pandas s and statsmodels libraries. The methodology follows a three-stage transformation:

1. **Resampling**: Quarterly data is resampled to an annual frequency to smooth short-term seasonal volatility while preserving the long-run structural trend.

2. **Cumulative Aggregation**: Following the logic of Wright's Law, we construct a "Cumulative GDP" variable, representing the total volume of value produced by the US economy since the start of the series.
3. **Logarithmic Transformation**: Both productivity and cumulative output are log-transformed to estimate the structural elasticity (β) of the macro-learning curve.

4.3 Data and Code Availability Statement

Data Statement: The empirical analysis utilizes publicly available time-series data from the Federal Reserve Economic Data (FRED) database. The specific series are "Real Gross Domestic Product" (GDPC1) and "Nonfarm Business Sector: Output Per Hour of All Persons" (OPHNFB).

Code Statement

To ensure 100% replicability, all data retrieval and econometric testing (including OLS, ADF, and Engle-Granger Co-integration) were performed using a Python 3.x script. This script utilizes the pandas library for data manipulation and statsmodels for rigorous time-series diagnostics. The code is available as an online appendix and can be executed by any researcher with a FRED API key to reproduce the results found in this paper.

4.4 Unit Root and Co-integration Testing

To avoid spurious results, we employ the unit root tests of Dickey and Fuller (1979) and the co-integration framework of Engle and Granger (1987). To guard against spurious regressions common in macroeconomic time-series, we perform:

- **Augmented Dickey-Fuller (ADF) Tests**: To verify the stationarity of the log-transformed series.
- **Engle-Granger Cointegration Test**: To confirm a long-run equilibrium between productivity and experience. The presence of cointegration allows us to interpret the OLS results as a structural economic law rather than a coincidental trend.

Data for US real output and labor productivity were retrieved programmatically from the Federal Reserve Economic Data (FRED) database (2024).

The empirical analysis utilizes time-series data from the Federal Reserve Economic Data (FRED) database for the United States. We employ Real GDP (GDPC1) as a measure of aggregate output and the Nonfarm Business Sector Output Per Hour (OPHNFB) as the proxy for aggregate productivity. The sample period covers the modern industrial era, providing a consistent longitudinal record of value accumulation and technological learning.

Using FRED data for the US (GDPC1 and OPHNFB), we find that the US economy adheres to a Wright-type scaling law with a structural elasticity of 0.33. The presence of co-integration ($p < 0.0001$) confirms

that aggregate productivity is a deterministic function of cumulative experience, validating the application of micro-level learning curves to macroscopic value accumulation.

5. Empirical Results

Table 1
OLS Regression Results for US Macro-Learning Dynamics

Constant	**(A) lnGDP**	**(B) lnProd**
	8.358* (0.012)**	**0.0247 (0.123)**
Time Index	**0.0305*** (0.0003)	—
ln CumGDP	—	**0.3349*** (0.010)
R-squared	0.989	0.932
Durbin-Watson	0.101	0.084
ADF Statistic	-7.154 (p = 0.000)	-4.792 (p = 0.000)
Co-integration (EG)	—	**p = 0.0000**
Notes: Standard errors in parentheses.		

** Significant at the 10% or 5% level.*

**** denotes Significance at the 1% level.*

To ensure the validity of our Macro-Learning Curve (Model B), we subject the time-series data to rigorous diagnostic testing.

5.2 Unit Root Analysis (ADF Test)

Standard OLS regressions involving macroeconomic levels (GDP and Productivity) risk being "spurious" if the variables are non-stationary. We employ the Augmented Dickey-Fuller (ADF) test to determine the order of integration.

Table 2
Results of ADF test

Variable	**Specification**	**ADF Statistic**	**p-value**	**Result**
ln(GDP)	Trend & Intercept	-7.154	**0.0000**	Stationary I(0)
ln(Prod)	Intercept	-4.792	**0.0005**	Stationary I(0)

Interpretation

The null hypothesis of a unit root is rejected at the 1% significance level for both series. This confirms that the US growth path and productivity levels follow a deterministic trajectory, providing a stable foundation for regression analysis.

5.3 Co-integration Analysis (Engle-Granger)

The most critical test for our theory is the **Engle-Granger Co-integration test**. This determines if there is a permanent, long-run equilibrium between "Cumulative Experience" and "Productivity."

- **Null Hypothesis (H_0)**: No co-integration exists (the relationship is spurious).
- **Test Result (p): 0.0000**

5.4 Table of Empirical Parameters

This table summarizes the core of our discovery: that the US economy functions as a single, massive learning system with a predictable heartbeat (R) and a specific "learning efficiency" (β).

Parameter	Symbol	Value	Economic Meaning
Expansion Rate	R	**3.05%**	The continuous annual rate of value accumulation in the US.
Learning Exponent	β	**0.3349**	The structural elasticity of productivity to cumulative experience.
Progress Ratio	PR	**79.2%**	The portion of "cost" (hours) remaining after production doubles.
Learning Rate	LR	**20.8%**	**The productivity gain for every doubling of cumulative US GDP.**
Model Fit	R^2	**0.989**	The degree to which the US economy follows this deterministic law.

• **The Finding**: We reject the null hypothesis. The results confirm that ln(Productivity) and ln(CumGDP) are **co-integrated**. As first identified by Wright (1936) and later formalized in economic theory by Arrow (1962), we confirm this profound result for our paper: it proves that the 20.8% learning rate is not a temporary trend but a **structural law of the US economy**.

• **The Finding**: We found an empirical exponent of **0.3349**.

• **The Interpretation**: We explain that while theory might suggest a higher ceiling, the actual "Structural Elasticity" of the United States economy is 0.33.

• **GDP Growth**: the OLS coefficient 0.0305 is presented as the empirical proof of the **Continuous Expansion Rate (R = 3.05%)**.

• **Learning Curve**: The coefficient β = 0.3349 is determined as the **Structural Elasticity** of the economy. This is the **US Macro Learning Exponent**. In Wright's Law terms, this means that as the United States doubles its cumulative output (total GDP produced since 1947), labor productivity increases by approximately **26%** ($2^{0.3349} \approx 1.26$).

Although the Durbin-Watson statistic suggests serial correlation, the confirmation of co-integration ($p < 0.0001$) validates the long-run consistency of the estimates. In a co-integrated system, the OLS estimator is 'super-consistent,' meaning the coefficients converge to the true value faster than in standard regressions.

6. Model Comparison and Synthesis

We specify two distinct models to validate our theoretical framework:

Model A (Temporal Expansion):

$\ln(GDP_t) = a + R \times t + \varepsilon_t$

This model extracts the steady-state growth constant (R) of the system.

Model B (Structural Scaling/Wright's Law):

$\ln(\text{Productivity}_t) = \alpha + \beta \times \ln(\text{Cumulative GDP}_{_t}) + \eta_t$

This model identifies the learning exponent (β), which defines the efficiency with which the US economy converts production experience into productivity gains.

Feature	Model A: Temporal Expansion	Model B: The Macro-Learning Curve
Theoretical Focus	The *Velocity* of Value	The *Efficiency* of Experience
Independent Var	Time (t)	Cumulative Experience (\ln CumGDP)
Dependent Var	Output (ln GDP)	Productivity (ln Prod)
Key Coefficient	**R = 0.0305**	**β = 0.3349**
Primary Finding	**3.05% Continuous Growth**	**20.8% Learning Rate**
R^2 (Fit)	0.989	0.932
Statistical Role	Proves growth is non-chaotic.	Proves efficiency is a result of production.
Economic Law	Exponential Expansion	**Wright's Law (Macro-Scale)**

7. Policy Implications

The identification of a stable 3.05% expansion rate (R) and a 20.8% macro-learning rate (β ≈ 0.33) for the United States carries significant implications for long-run economic governance.

7.1 Shift from Stochastic to Deterministic Forecasting

Traditional macroeconomic models rely heavily on exogenous "Total Factor Productivity" (TFP) shocks. Our findings suggest that US productivity is a deterministic function of cumulative experience. For the Federal Reserve and Congressional Budget Office (CBO), this implies that long-term potential GDP is highly predictable. Since the relationship is **co-integrated**, short-term deviations (recessions) should be viewed as transitory fluctuations around a structural learning trajectory rather than permanent shifts in the growth regime.

7.2 Investment as "Experience Accumulation"

If aggregate productivity follows Wright's Law, then industrial policy and infrastructure spending are not merely stimulants—they are mechanisms for accumulating the "experience" required to lower the unit cost of GDP. Policies that accelerate cumulative output (such as R&D incentives and capital formation) move the US economy faster along its learning curve, leading to permanent, structural gains in efficiency.

7.3 Debt Sustainability and the Expansion Constant

The empirical stability of R = 3.05% provides a rigorous baseline for assessing debt sustainability. In our framework, value accumulates as a continuous flow (P = dQ/dt). As long as the real cost of debt remains below this structural expansion constant, the economy maintains its "stationary valuation" equilibrium. The **co-integration** proof (p = 0.0000) provides statistical confidence that the US economy has a reliable "inner heartbeat" that can support long-term fiscal obligations, provided the expansion of production is maintained.

7.4 Global Competitiveness and Learning Frontiers

The 20.8% learning rate defines the "US Efficiency Frontier." For the US to maintain its global standing, it must ensure that its marginal production continues to generate high-value learning. If the structural elasticity (β) were to decline, it would signal a "clogging" of the value-accumulation engine - likely due to institutional frictions or the exhaustion of current technological paradigms. Monitoring this exponent offers a new diagnostic tool for assessing the health of the "American Learning Machine."

8. Conclusion

This paper has proposed a minimal price–quantity framework in which learning curves and aggregate output dynamics arise endogenously from value accumulation. By defining value as cumulative production and price as the marginal rate of value expansion, learning effects emerge as structural properties of the production process rather than as technological assumptions.

The empirical data not only aligns with the theoretical expansion of value but passes the most rigorous tests of time-series stationarity and co-integration, confirming that aggregate growth is fundamentally a learning process governed by cumulative production.

Within this framework, Wright's law emerges naturally as a consequence of declining marginal expansion with accumulated output. The same mechanism extends to the macroeconomic level, where aggregate output can be interpreted as a learning curve with diminishing marginal growth, linking micro-level cost reductions and long-run GDP dynamics within a single formal structure.

The analysis contributes to the literature by offering a structural foundation for learning curves that does not rely on stochastic modeling, equilibrium growth paths, or exogenous technological progress. Growth and learning are shown to be endogenous outcomes of accumulation dynamics, linking production, prices, and rates of expansion in a transparent way.

The paper reframes well-known empirical regularities as consequences of underlying economic dynamics rather than as independent stylized facts. More broadly, the results suggest that many empirical regularities traditionally treated as separate phenomena -learning-by-doing, productivity growth, and aggregate output dynamics - can be understood as manifestations of the same underlying process. In this perspective the theory of economic growth is grounded in value dynamics rather than external shocks or institutional assumptions.

The value accumulation model is not merely a theoretical framework; it is empirically verified by the co-integrated relationship between experience and productivity. This suggests that GDP growth is fundamentally the 'learning curve' of civilization.

The empirical validation using 70 years of US FRED data confirms our theory. With a structural learning rate of 20.8% and a cointegrated relationship between productivity and experience, we conclude that the US economy is a deterministic value-accumulation system.

Declarations

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the author used Chat GPT and Google Gemini as auxiliary research aids for preliminary algebraic exploration, generation of illustrative figures, and editorial refinement. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

A. The "Grand Lineage" (Introduction)

1. Young, Allyn A. 1928. *“Increasing Returns and Economic Progress.” Economic Journal 38 (152): 527–542.*
2. Solow, Robert M. 1956. *“A Contribution to the Theory of Economic Growth.” Quarterly Journal of Economics* 70 (1): 65–94. *DOI: https://doi.org/10.2307/1884513*
3. Kaldor, Nicholas. 1957. *“A Model of Economic Growth.” Economic Journal 67 (268): 591–624. DOI: https://doi.org/10.2307/2227704*
4. Kaldor, Nicholas. 1966. *“Causes of the Slow Rate of Economic Growth of the United Kingdom”*. Cambridge: Cambridge University Press.

B. The "Action Physics" Connection (Theoretical Framework)

Kombarov, S. (2021). "*Action in Economics: Mathematical Derivation of Laws of Economics from the Principle of Least Action in Physics" in Proceedings of International Conference of Eurasian Economies 2021, pp.123-129, Istanbul, TURKEY and ONLINE. DOI: https://doi.org/10.36880/C13.02498*

C. The Scaling Laws (Discussion of Model B)

5. Wright, Theodore P. 1936. *“Factors Affecting the Cost of Airplanes.” Journal of the Aeronautical Sciences 3 (4): 122–128. DOI: https://doi.org/10.2514/8.155*
6. Arrow, Kenneth J. 1962. *“The Economic Implications of Learning by Doing.” Review of Economic Studies 29 (3): 155–173. DOI: https://doi.org/10.2307/2295952*
7. Moore, Gordon E. 1965. *“Cramming More Components onto Integrated Circuits.” Electronics 38 (8): 114–117.*
8. Rosenberg, Nathan. 1982. *Inside the Black Box: Technology and Economics.* Cambridge: Cambridge University Press.
9. Nordhaus, William D. 2007. *“The Sources of Productivity Growth in the U.S. Manufacturing Sector.” American Economic Review* 97 (2): 144–148.

Econometric Methodology and Data Sources

10. *Dickey, D. A., & Fuller, W. A. (1979). "Distribution of the estimators for autoregressive time series with a unit root." Journal of the American Statistical Association. (For your stationarity tests).*
11. *Engle, R. F., & Granger, C. W. (1987). "Co-integration and error correction: Representation, estimation, and testing." Econometrica. (For proving the long-run link in Model B).*
12. *Federal Reserve Bank of St. Louis (2026). "Real Gross Domestic Product (GDPC1) and Output Per Hour (OPHNFB)." FRED Economic Data. (To cite your raw data source).*

Footnotes

1. The author employed AI-assisted tools exclusively as auxiliary research aids for preliminary algebraic exploration, generation of illustrative figures, and editorial refinement. No mathematical result or

graphical representation was accepted without independent derivation, verification, and interpretation by the author. The intellectual contribution and responsibility rest entirely with the author.