



34 **Additional information**

35 **S1. Full Model Equations**

36 For each sexual activity group  $g \in \{1, 2, 3\}$  and vaccination status  $v \in \{0, 1, 2\}$  ( $v = 0$ : unvacci-  
 37 nated,  $v = 1$ : one dose,  $v = 2$ : two doses), the compartmental dynamics are:

38 **Susceptible:**

$$\begin{aligned}\frac{dS_{0,g}}{dt} &= -\lambda_g(t)S_{0,g}(t) - \alpha_1(t)S_{0,g}(t) \\ \frac{dS_{1,g}}{dt} &= \alpha_1(t)S_{0,g}(t) - \lambda_g(t)(1 - ve_1)S_{1,g}(t) - \alpha_2(t)S_{1,g}(t) \\ \frac{dS_{2,g}}{dt} &= \alpha_2(t)S_{1,g}(t) - \lambda_g(t)(1 - ve_2)S_{2,g}(t)\end{aligned}$$

39 **Exposed:**

$$\frac{dE_{v,g}}{dt} = \lambda_g(t)(1 - ve_v)S_{v,g}(t) - \epsilon_v E_{v,g}(t)$$

40 **Presymptomatic:**

$$\frac{dP_{v,g}}{dt} = \epsilon_v E_{v,g}(t) - \omega_v P_{v,g}(t)$$

41 **Asymptomatic (3-stage Gamma):**

$$\begin{aligned}\frac{dA_{v,g}^1}{dt} &= m \omega_v P_{v,g}(t) - 3\theta_v A_{v,g}^1(t) \\ \frac{dA_{v,g}^2}{dt} &= 3\theta_v A_{v,g}^1(t) - 3\theta_v A_{v,g}^2(t) \\ \frac{dA_{v,g}^3}{dt} &= 3\theta_v A_{v,g}^2(t) - 3\theta_v A_{v,g}^3(t)\end{aligned}$$

42 **Symptomatic (3-stage Gamma):**

$$\begin{aligned}\frac{dI_{v,g}^1}{dt} &= (1 - m) \omega_v P_{v,g}(t) - 3\gamma_v I_{v,g}^1(t) \\ \frac{dI_{v,g}^2}{dt} &= 3\gamma_v I_{v,g}^1(t) - 3\gamma_v I_{v,g}^2(t) \\ \frac{dI_{v,g}^3}{dt} &= 3\gamma_v I_{v,g}^2(t) - 3\gamma_v I_{v,g}^3(t)\end{aligned}$$

43 **Recovered:**

$$\frac{dR_g}{dt} = \sum_{v=0}^2 \left( 3\theta_v A_{v,g}^3(t) + 3\gamma_v I_{v,g}^3(t) \right)$$

44 **Force of Infection**

45 We considered a single transmission pathway for mpox; transmission through close contact  
 46 during sexual partnerships among gbMSM individuals. The *force of infection*,  $\lambda_g(t)$ , represents

47 the per capita rate of mpox acquisition for individuals in sexual activity group  $g$  at time  $t$ . It is  
 48 defined as:

$$\lambda_g(t) = \beta \cdot c_g \cdot \sum_{g'=1}^3 M_{g,g'}(t) \cdot \text{prev}_{g'}(t), \quad (1)$$

49 where  $\beta$  is the probability of transmission per effective contact,  $c_g$  is the average number  
 50 of sexual contacts per person per day in group  $g$ ,  $M_{g,g'}(t)$  is the time-varying sexual mixing  
 51 matrix representing the probability that a contact made by an individual in group  $g$  is with an  
 52 individual in group  $g'$ ,  $\text{prev}_{g'}(t)$  is the prevalence of infectious individuals in group  $g'$  at time  $t$ .  
 53 The transmission probability per effective contact,  $\beta$ , was estimated within the model using a  
 54 truncated normal prior:

$$\beta \sim N_{[0,1]}(\mu = 0.8, \sigma = 0.1).$$

55 This prior reflects evidence from previous modeling studies that reported relatively high  
 56 transmission probabilities for mpox among gbMSM populations during the 2022 outbreak [1].  
 57 The mean of 0.8 was chosen to represent a biologically plausible upper-bound scenario, while  
 58 the standard deviation of 0.1 allows for moderate uncertainty. Truncation ensures the parameter  
 59 remains within the valid probability interval  $[0, 1]$ .

## 60 **Mixing Matrix**

61 The sexual mixing matrix  $M_{g,g'}(t)$  incorporates both assortative and proportional mixing by  
 62 sexual activity groups, governed by a mixing parameter  $\kappa$ . This parameter governs the degree to  
 63 which individuals preferentially mix within their own group versus proportionally across groups.  
 64 We modeled  $\kappa$  using a Beta prior distribution

$$\kappa \sim \text{Beta}(40, 2),$$

65 which favours higher probabilities of within group mixing . The mixing matrix is defined as;

$$M_{g,g'}(t) = \kappa \cdot \delta_{g,g'} + (1 - \kappa) \cdot \frac{c_{g'} \cdot N_{g'}}{\sum_{g''} c_{g''} \cdot N_{g''}}, \quad (2)$$

66 where  $\delta_{g,g'}$  is the Kronecker delta (1 if  $g = g'$ , 0 otherwise) and  $N_g$  is the population size of  
 67 group  $g$  [1].

## 68 **Prevalence of Infectious Individuals**

69 The prevalence of infectious individuals in group  $g$  at time  $t$ ,  $\text{prev}_g(t)$ , includes individuals who  
 70 are symptomatically infectious ( $I$ ), asymptotically infectious ( $A$ ), and presymptomatically  
 71 infectious ( $P$ ) across all vaccination statuses. Contributions from  $A$  and  $P$  compartments to  
 72 the force of infection are adjusted by their relative infectiousness compared to symptomatically  
 73 infectious ( $I$ ) using the factor  $\rho=0.776$  [2]

$$\begin{aligned}
\text{prev}_g(t) = \frac{1}{N_g} & \left[ \rho \left( P_g^u(t) + P_g^{va}(t) + P_g^{vb}(t) \right) \right. \\
& + \rho \sum_{i=1}^3 \left( A_{i,g}^u(t) + A_{i,g}^{va}(t) + A_{i,g}^{vb}(t) \right) \\
& \left. + \sum_{i=1}^3 \left( I_{i,g}^u(t) + I_{i,g}^{va}(t) + I_{i,g}^{vb}(t) \right) \right]. \tag{3}
\end{aligned}$$

## 74 S2. Wastewater viral load

### 75 Deposited Viral Load in Wastewater

76 The daily concentration of MPXV DNA deposited into the wastewater was calculated based on  
77 the number of individuals actively shedding virus. All infectious individuals i.e. presymptomatic  
78 ( $P$ ), asymptomatic ( $A$ ), and symptomatic ( $I$ ) including vaccinated individuals were assumed to  
79 contribute to viral shedding. In the absence of robust empirical data quantifying differences in  
80 stage-specific viral shedding conditional on vaccination status for mpox, baseline analyses as-  
81 sumed equal shedding rates for vaccinated and unvaccinated individuals once infected. Potential  
82 post-infection effects of vaccination on viral shedding were explored through sensitivity analyses  
83 rather than imposed in the baseline model. Although individuals in the exposed compartment  
84 ( $E$ ) may begin shedding prior to becoming infectious, they were assumed not to contribute  
85 significantly to viral shedding.

86 We derived viral shedding rates using data from a previous study [3] which investigated  
87 viral dynamics in individuals with confirmed mpox infection in Spain. The study quantified  
88 viral DNA loads at multiple anatomical sites including skin lesions, rectum, saliva, semen, and  
89 blood over time using qPCR. We focused on rectal and skin lesion viral load data, given their  
90 potential relevance to fecal shedding and contact with wastewater systems. The study provided  
91 median viral DNA concentrations (in  $\log_{10}$  copies/mL) at symptom onset and estimated median  
92 clearance times for each anatomical site. To extract detailed shedding dynamics over time, we  
93 digitized the probability of detectable virus over days since symptom onset using the juicer R  
94 package [4]. For each anatomical site, we aligned the baseline probability (1.0) with the reported  
95 median viral load at day 0, and extracted relative load values for subsequent days. We computed  
96 shedding estimates corresponding to the midpoint of each of the three infectious stages. We then  
97 summed the values from skin and rectal shedding and used these as the viral shedding rates  
98 ( $\phi_i$ ) for the three symptomatic stages  $I_1$ ,  $I_2$ , and  $I_3$ ,

$$\phi_i = \{12.3, 11.5, 10.2\} \log_{10} \text{ copies/mL.}$$

99 For presymptomatic and asymptomatic individuals, we assumed reduced viral shedding rel-  
100 ative to symptomatic individuals. We used a multiplicative factor  $\rho = 0.776$  [2] representing the  
101 infectiousness and fecal shedding of presymptomatic and asymptomatic individuals relative to  
102 symptomatic individuals. The total deposited viral load  $V_{\text{shed}}(t)$  at time  $t$  was defined as:

$$V_{\text{shed}}(t) = \psi \cdot \rho \sum_{i=1}^n \phi_i P_i(t) + \sum_{i=1}^n \phi_i I_i(t) + \rho \sum_{i=1}^n \phi_i A_i(t)$$

103 ,

104 where  $\phi_i$  is the viral shedding rate associated with compartment  $i$ ,  $\rho$  is the relative reduction  
 105 in viral shedding for presymptomatic and asymptomatic individuals compared to symptomatic  
 106 individuals and  $\psi$  is a scaling parameter accounting for uncertainty in total wastewater viral load  
 107 estimates. We fitted the scaling parameter  $\psi$  using a lognormal distribution prior to account for  
 108 uncertainty in the mapping between model-predicted viral shedding and observed concentrations  
 109 in wastewater. The prior was specified as:

$$\psi \sim \text{LogNormal}(\log(3 \times 10^{-9}), 0.158).$$

110 This prior reflects prior knowledge that the scaling factor is expected to be small and positive,  
 111 while allowing for uncertainty due to variability in viral decay, sampling efficiency, and transport  
 112 dynamics within the sewer system.

### 113 **Transport and Sampled Viral Load**

114 To simulate the delay and degradation of MPXV viral DNA from the point of shedding to  
 115 the point of sampling, we employed a simplified advection-dispersion-decay framework [5]. This  
 116 model captures delay and decrease of signal of viral particles caused by hydrodynamic and envi-  
 117 ronmental processes including dilution, sedimentation, resuspension and other factors within the  
 118 sewer network. The model includes two key components; An exponential decay term with rate  
 119  $\mu$ , representing first-order degradation of MPXV DNA in wastewater due to biological activity  
 120 and environmental stressors and a plug-flow delay kernel  $g(\tau)$ , representing the distribution of  
 121 transit times  $\tau$  that viral particles experience between shedding and sampling.

122 The total sampled viral concentration  $V_s(t)$  at time  $t$  is then modeled as the convolution of  
 123 the deposited viral load  $V_{\text{shed}}(t)$  and the delay distribution  $g(\tau)$ , incorporating both temporal  
 124 dispersion and exponential decay:

$$V_s(t) = \int_0^t (V_{\text{shed}}(t - \tau) \cdot g(\tau) \cdot e^{-\mu\tau} d\tau$$

125 For computational efficiency, the plug-flow delay distribution  $g(\tau)$  was approximated using a  
 126 Gaussian distribution with mean transit time  $\bar{\tau}$  and standard deviation  $\sigma$  allowing us to capture  
 127 the average lag and its variability:

$$g(\tau) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\tau - \bar{\tau})^2}{2\sigma^2}\right).$$

128 The mean transit time  $\bar{\tau}$  and its standard deviation  $\sigma$ , which characterize the distribution  
 129 of delays between viral shedding and sampling, were estimated within the model using weakly  
 130 informative priors. These priors were chosen to reflect plausible transport dynamics for the  
 131 local sewer system, while allowing the data to guide inference. Specifically, the prior for the  
 132 mean transit time  $\bar{\tau}$  was a truncated normal distribution centered at 2.5 days, with a standard

133 deviation of approximately 0.5 days, constrained to be positive

$$\tau \sim N_{[0.1,10]}(2.5, 0.5^2).$$

134 The standard deviation  $\sigma$  of the delay distribution was assigned a truncated normal prior  
135 centered at 0.3 days, with a standard deviation of 0.1, constrained to be positive:

$$\sigma \sim N_{[0.1,1]}(0.3, 0.1^2).$$

136 These choices reflect prior expectations of moderate variability in sewer travel times, based  
137 on hydrodynamic behavior, while ensuring estimates remain within realistic bounds. In con-  
138 trast, the viral decay rate  $\mu$  was held fixed because jointly estimating decay, transit time, and  
139 dispersion led to identifiability issues. We therefore used a fixed decay rate of  $\mu = 0.143 \text{ day}^{-1}$   
140 consistent with Jacob et al [6], who measured MPXV DNA degradation in influent wastewa-  
141 ter. Using this empirically grounded value allowed us to parameterize decay realistically while  
142 avoiding over-parameterization of the wastewater transport model.

### 143 S3. Likelihood specification and model fitting

144 We fitted three versions of the model: (i) a case-data-only model, (ii) a viral-load-only model, and  
145 (iii) a combined case and viral load model. All models were implemented with a daily time step  
146 and fitted within a Bayesian framework using `rjags`, which relies on Markov Chain Monte Carlo  
147 (MCMC) sampling. JAGS primarily employs Gibbs sampling, but uses adaptive Metropolis-  
148 Hastings steps within the Gibbs sampler for parameters whose full conditional distributions do  
149 not have standard forms.

150 Each model was run using two parallel MCMC chains with 50,000 iterations per chain.  
151 The first 10,000 iterations were discarded as burn-in to allow the chains to reach the target  
152 distribution. The remaining samples were thinned by a factor of 3 to reduce autocorrelation.  
153 Convergence was assessed through visual inspection of trace plots and the Gelman–Rubin di-  
154 agnostic ( $\hat{R}$ ), ensuring that all monitored parameters had  $\hat{R} < 1.1$ .

155 To initiate the model, we estimated initial conditions for the exposed ( $E$ ), pre-symptomatic  
156 ( $P$ ), asymptomatic stage 1 ( $A_1$ ), and symptomatic stage 1 ( $I_1$ ) compartments. Priors for the  
157 total number of individuals in each compartment at  $t = 1$  were specified as:

$$\begin{aligned} E_0 &\sim \text{Poisson}(3), \\ P_0 &\sim \text{Poisson}(1), \\ A_{10} &\sim \text{Poisson}(1), \\ I_{10} &\sim \text{Poisson}(1). \end{aligned}$$

158 To reflect heterogeneity in exposure risk across the three behavioral groups, we normalized  
159 the daily sexual contact rate in each group ( $c_g$ ):

$$w_g = \frac{c_g}{\sum_{g=1}^3 c_g}$$

160 These normalized weights were used to allocate the initial state values across groups pro-  
161 portionally:

$$E_g(1) = E_0 \cdot w_g,$$

$$P_g(1) = P_0 \cdot w_g,$$

$$A_{1g}(1) = A_{10} \cdot w_g,$$

$$I_{1g}(1) = I_{10} \cdot w_g.$$

162 Initial values for other compartments were initialized to zero, with the exception of the suscep-  
163 tible compartments. Prior to fitting the model to data, we simulated the system forward for an  
164 initial 30-day period without data assimilation. This allowed the compartmental dynamics to  
165 stabilize and reduce sensitivity to initial value assumptions.

## 166 Case data Model

167 We assumed observed case counts were *negative binomially* distributed to account for overdis-  
168 persion. Specifically, for each time point  $t$ , the number of reported cases  $C_t$  was modeled as:

$$C_t \sim \text{NegBin}(\widehat{C}_t, \eta),$$

169 where  $\widehat{C}_t$  denotes the model predicted mean number of reported cases and  $\eta$  is the dispersion  
170 parameter. We specified a *gamma prior* for the dispersion:

$$\eta \sim \text{Gamma}(2, 0.5),$$

171 which has a mean of 4 and a standard deviation of approximately 2.8 reflecting moderate  
172 overdispersion. To align model-predicted infections with observed case reporting, we incorpo-  
173 rated a reporting lag fixed at an average of 7 days. This delay was informed by the interval  
174 between symptom onset and laboratory confirmation dates in the empirical line list data. We  
175 also included a reporting fraction, rf, to account for the fact that not all symptomatic individuals  
176 are captured by the surveillance system. Underdetection, especially during the early outbreak  
177 period, has been documented in several studies. For instance, Brand et al. [2] estimated a mean  
178 weekly probability of detection of 0.45, while Xiu et al. [1] reported a detection fraction of  
179 approximately 78% (95%CrI:0.48-0.96) prior to the start of vaccination in Vancouver. Based on  
180 sensitivity analyses, we specified the prior for the reporting fraction as:

$$\text{rf} \sim \text{Beta}(10, 10),$$

181 corresponding to a mean of 0.50, with flexibility to accommodate substantial uncertainty. The  
182 proportion of infections that remain asymptomatic  $m$  was assigned a Beta prior informed by a  
183 recent meta-analysis estimating that 34% of infections are asymptomatic [7]:

$$m \sim \text{Beta}(6.8, 13.2)$$

184 We assumed that vaccine effectiveness was 66.0% (95% CI: 47.4–78.1) for individuals who

185 received two doses, and 35.8% (95% CI: 22.1–47.1) for those who received a single dose. These  
 186 estimates were based on findings from a case-control study evaluating the effectiveness of mpox  
 187 vaccine in adults [8]. Beta prior distributions were assigned to the one and two dose effectiveness  
 188 parameters, denoted  $ve_1$  and  $ve_2$ , respectively:

$$ve_1 \sim \text{Beta}(22.05, 39.56),$$

$$ve_2 \sim \text{Beta}(20.92, 10.77)$$

189 Finally, prior distributions for other model parameters governing transitions between com-  
 190 partments i.e latent and infectious periods were informed by existing literature on the biological  
 191 and epidemiological characteristics of mpox. A complete summary of all prior distributions is  
 192 provided in Table 1

### 193 **Viral Load Model**

194 We used the sampled viral load  $V_s(t)$ , previously derived from model predicted shedding and  
 195 transport dynamics, as the basis for comparison to empirical wastewater viral load data. To scale  
 196 and standardize the model predicted viral load,  $V_s(t)$  was multiplied by the daily wastewater  
 197 flow rate and divided by the population served by the sewershed, yielding daily viral load per  
 198 capita in copies/mL. We then extracted values only for time points corresponding to days with  
 199 available wastewater samples.

200 The resulting concentrations were log-transformed (base 10) to match the scale of reported  
 201 data. We refer to this final model predicted log-transformed viral load as  $V_r(t)$ , the reported  
 202 viral load. For each observed day  $t$ , the measured  $\log_{10}$  viral concentration was modeled using  
 203 a normal likelihood:

$$V_{\text{obs}}(t) \sim \text{Normal} \left( V_r(t), \frac{1}{\tau_{\text{ww}}} \right)$$

204 where  $V_{\text{obs}}(t)$  is the observed log-transformed viral load on day  $t$ ,  $V_r(t)$  is the model-predicted  
 205 reported viral load, and  $\tau_{\text{ww}}$  is the precision parameter representing observational uncertainty.  
 206 We assigned a Gamma prior to  $\tau_{\text{ww}}$ :

$$\tau_{\text{ww}} \sim \text{Gamma}(2, 1)$$

207 The Gamma distribution ensures positivity while allowing data driven estimation of observation  
 208 uncertainty.

### 209 **Combined Model**

210 For the combined model, we retained the same compartmental structure and equations used  
 211 in the individual case-data only and viral-load only models. The combined model incorporated  
 212 both data sources by jointly specifying the likelihoods for observed case counts and observed  
 213 wastewater viral load. This allowed simultaneous fitting to both clinical and environmental data  
 214 streams while sharing the same underlying transmission dynamics.

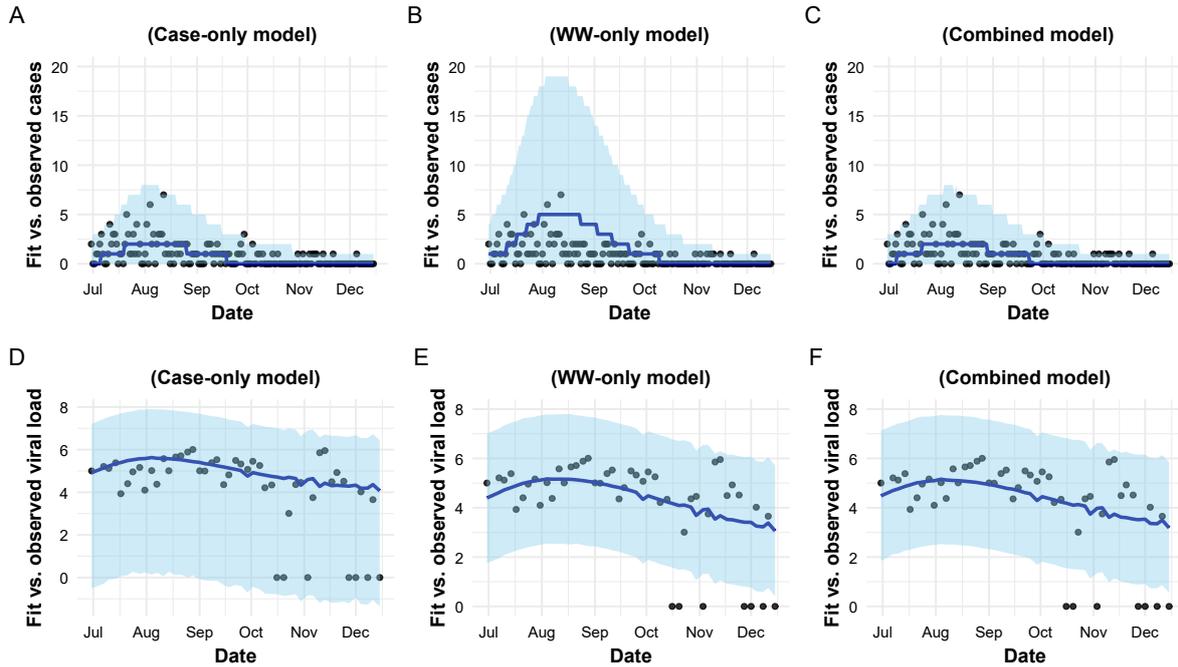


Figure S1: Posterior predictive fits of reported cases (A–C) and wastewater viral load (D–F) under the case-only, combined, and wastewater-only models. Black points represent observed data, blue lines show posterior medians, and shaded regions indicate 95% credible intervals.

Table S1: Convergence diagnostics for case-only, wastewater-only, and combined models. R-hat values (all equal to 1) indicate convergence across chains. Effective sample sizes (ESS) are  $> 200$

Parameter	R-hat			ESS		
	Case-only	WW-only	Combined	Case-only	WW-only	Combined
$\log \psi$	1	1	1	40000	10031	18325
$\bar{\tau}$	1	1	1	40312	9402	17352
$\sigma$	1	1	1	40000	16057	31819
$\tau_{\text{ww}}$	1	1	1	40000	38857	77552
$\beta$	1	1	1	15845	23556	27434
$\kappa$	1	1	1	14378	13093	30214
rf	1	1	1	11027	40000	24049
$\phi_i$	1	1	1	19823	40000	43526
$\delta$	1	1	1	30084	34298	54837
$\theta$	1	1	1	19543	25799	37937
$\omega$	1	1	1	30463	33256	53995
ve <sub>1</sub>	1	1	1	33907	34550	68146
ve <sub>2</sub>	1	1	1	35439	35654	71169
$m$	1	1	1	19493	34523	41743

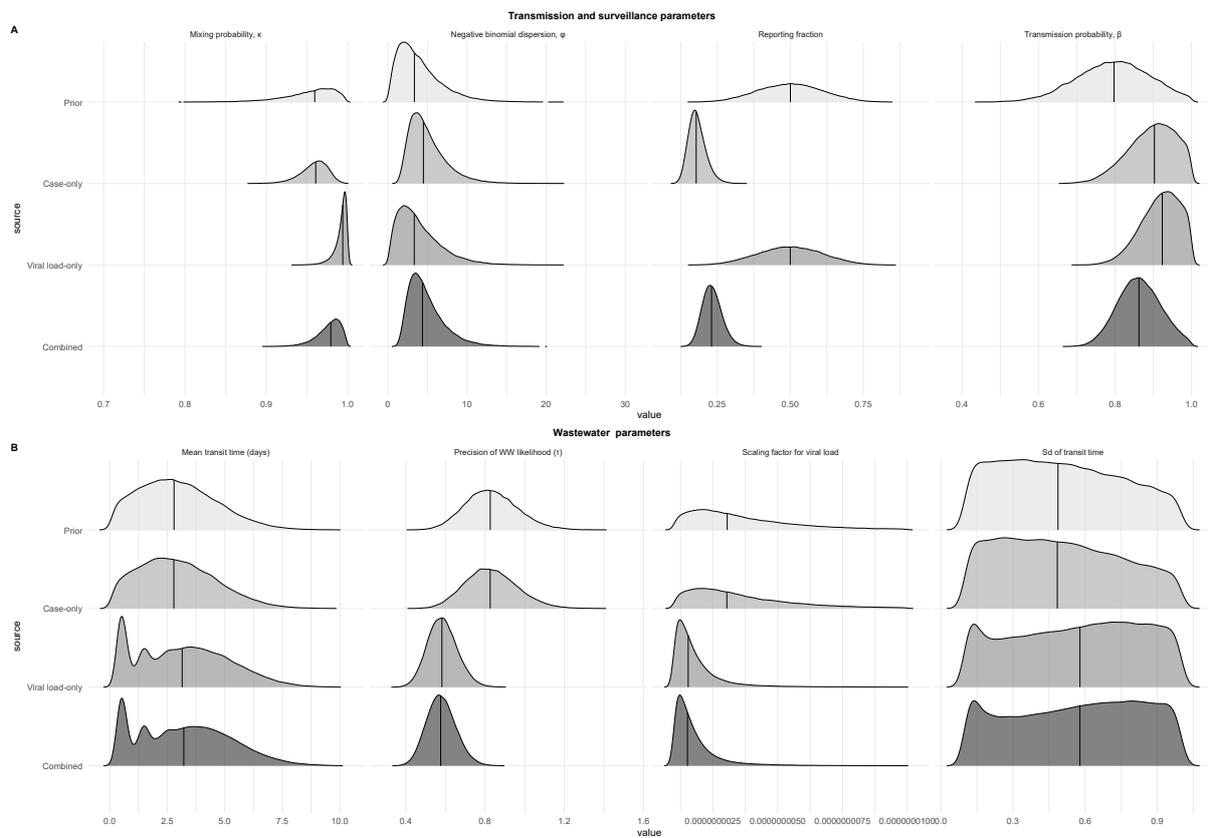


Figure S2: Posterior and prior distributions of key transmission, surveillance, and wastewater parameters across case-only, wastewater-only, and combined models.

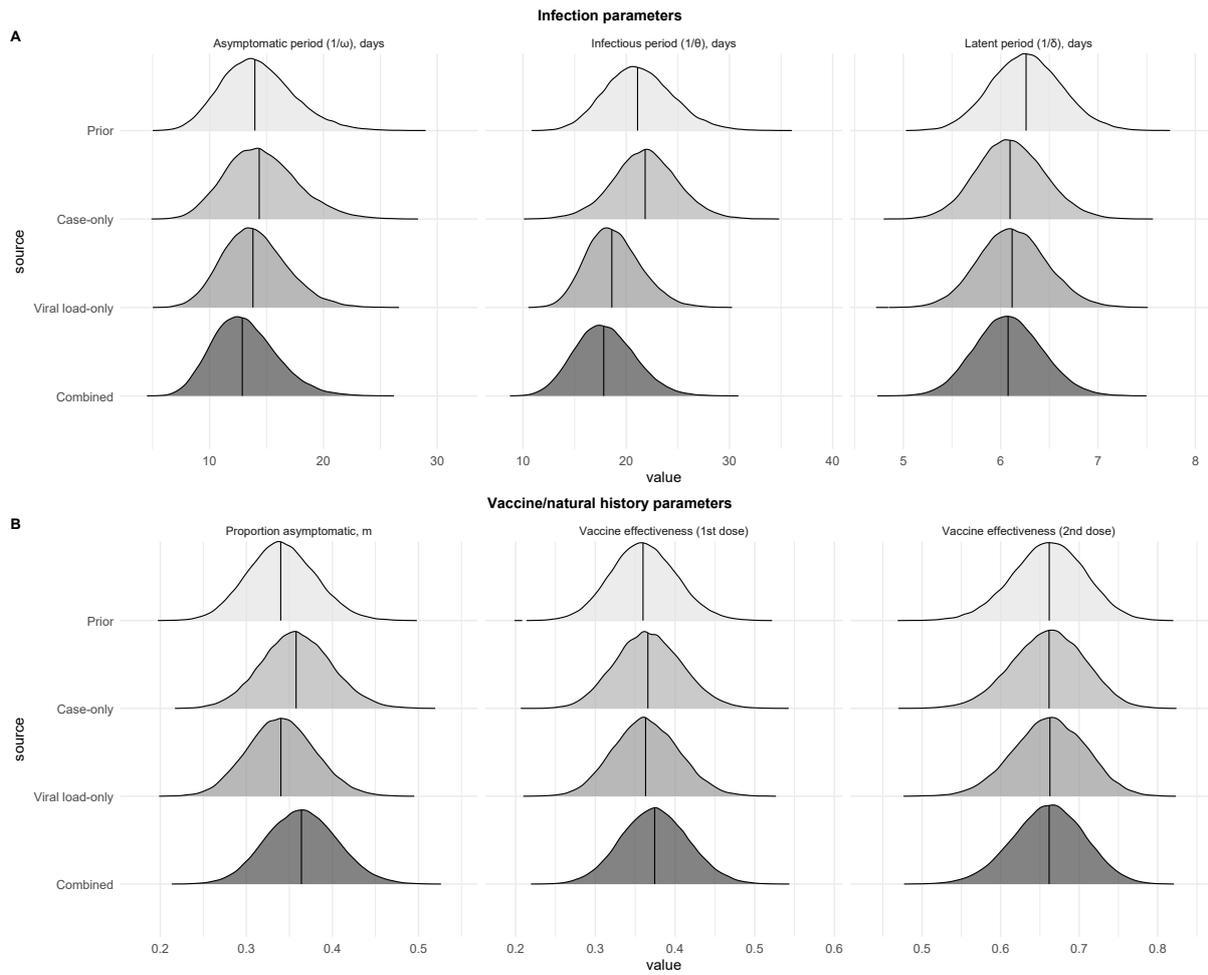


Figure S3: Posterior and prior distributions of key infection, vaccination, and natural history parameters across case-only, wastewater-only, and combined models

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