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Thermal characterization of a tri-hybrid nanostructured material flow between coaxially revolving disks with activation energy and shape factor aspects

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Abstract

The current study examines the behavior of a transformer-oil-based tri-hybrid nanostructured material flow between revolving, double-stretchable disks, considering the effects of activation/dissipative energy. A better understanding of these effects helps us in the improvement of thermal management systems, efficiencies of mechanical systems, and to gain the high-performance of electrical systems. Spherical and non-spherical shapes of nanoparticles Fe_3O_4 , TiO_2 , and Cu are studied. The significance of thermal radiation, Lorentz force, and thermal source/sink is also incorporated to make the mathematical model more versatile. Through similarity transformation, the leading equations are transmuted into a highly non-linear and non-dimensional differential system. The obtained differential system depends on the physical boundary conditions and is then simulated using the bvp4c MATLAB solver. The results reveal that flow and heat/mass transport in the tri-hybrid nanostructure material are strongly influenced by revolving, magnetic, Reynolds, and thermochemical parameters. Revolving system suppresses axial motion while enhancing tangential velocity, whereas higher Reynolds numbers enhance heat/mass transport despite reducing thermal/solutal fields. The lower disk exhibits stronger transport rates, and tri-hybrid nanostructured material outperforms hybrid and simple nanostructured materials, with blade-shaped nanoparticles providing superior heat-transport performance.

Keywords: Tri-hybrid nanostructured material; shape factor; dissipative energy; activation energy; double disks; thermal characterization.

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1. Introduction

A transformer-oil-based tri-hybrid nano-structure (THNS) is a specialized heat-transfer substance created by dispersing three different types of tiny particles into traditional transformer oil. Transformer oil itself is an insulating and cooling medium used in electrical equipment such as power transformers, valued for its high dielectric strength and thermal stability. When tiny particles measured in billionths of a meter are added to such oils, the resulting mixture is called a nano-suspension, known for enhancing thermal conductivity, heat dissipation, and overall substance performance. A THNS goes a step further by combining three distinct tiny-particle materials (for example, metals, metal oxides, or carbon-based particles), allowing the substance to benefit from the complementary thermal, rheological, and stability properties of each. Together, these advanced substance is engineered to achieve superior cooling efficiency and improved energy performance in demanding engineering applications. The statistical analysis on the stream of a THNS due to a revolving disk with the entropy, thermal, and shape-factor performance is reported by Mohanty et al. [1]. Mixed convective THNS stream across a porous revolving disk with the consideration of engine oil as a base material is investigated by Paul et al. [2]. Magnetized THNS stream with the micro-rotation of nanoelements and squeezing parameter aspects is studied by Panda et al. [3]. Elastico-viscous radiative stream of a THNS over a revolving disk using the shape-factor influence of nanoelements is examined by Arif et al. [4]. Numerous solutions and stability studies for a radiative stream of a THNS in a Forchheimer porous revolving as well as shrinking disk are presented by Mandal & Pal [5]. Ali et al. [6] investigated the significance of a THNS confined between concentric disks with magnetic and squeezed effects utilizing Boger micro-rotation of nanoelements. Khan et al. [7] provided deep insights into the thermal-transport of a radiative THNS over a revolving disk with Hall current effects using an advanced neural computing scheme. The study of a THNS stream over a deformable disk using intelligent computing was completed by Umer & Ahmed [8]. Bioconvective Casson material stream with the involvement of tri-hybrid nanoelements between concentric disks is inspected by Raza et al. [9] in the attendance of entropy optimization. A complex dynamics of a THNS under magnetic field and vortex formation aspects is scrutinized by Ahmad et al. [10]. Rauf et al. [11] explored the impact of morphology on magneto THNS stream between coaxially moving disks using artificial neural technology and Levenberg–Marquardt simulation. Stream of a THNS on a spinning disk with nonlinear formulation of thermal energy is studied by Gohar Saeed et al. [12]. The vortex

formation in a THNS stream with the role of localized Lorentz force is made and simulated by Ahmad et al. [13]. Mandal [14] explored the stream of a THNS with Maxwell velocity-slip and temperature jump conditions in the provision of entropy generation over a shrinking surface. All these studies collectively demonstrate that the flow stream and thermal-transport of a THNS over revolving, shrinking, or deformable disks are significantly influenced by factors such as magnetic fields, micro-rotation, squeezing, shape and morphology of nanoelements, entropy generation, radiative effects, bioconvection, and advanced computational approaches.

Dissipative energy refers to the portion of energy in a system that is lost, usually as heat, due to friction, viscosity, or other resistive forces during a continuously deformed substance or mechanical processes. It represents the mechanisms that remove usable energy from a system, influencing temperature rise and flow behavior. Activation energy, in contrast, is the minimized amount of energy essential to initiate a chemical-reaction or a physicochemical process, such as diffusion or particle interaction within a continuously deformed substance. In material mechanics and nanomaterial studies, these concepts help describe how energy losses affect flow dynamics and how sufficient energy input is needed to trigger microscopic processes that influence substance properties. The magneto-stream of a THNS over a porous disk placed orthogonally with heat/mass transport, activation energy, and variable geometry aspects is evaluated by Wantao et al. [15]. Rehman et al. [16] studied the hybrid nano-suspension stream of a transformer oil-based fluid between deforming disks with activation energy, shape factor, and a revolving frame. Thermodynamic optimization of a magneto-stream with the significance of hybrid nanoparticles, activation energy, and radiative energy aspects is provided by Ferdows et al. [17] utilizing a neural networking approach. A ternary hybrid nanostructure study is performed by Haq et al. [18] with the influence of heat/mass transport and activation energy term. A hybrid nanomaterial formulation for Marangoni stream is provided and simulated by Manzoor et al. [19] with the role of dissipative/activation energies, entropy optimization, and melting phenomenon. Darcy-Forchheimer hybrid nanosubstance motion over a deformable surface is modeled and computationally scrutinized by Haq et al. [20] with the involvement of chemical reaction, MHD, and activation energy terms. Hybrid nano-suspension flow between revolving as well as stretching disks is analyzed by Qayyum et al. [21] with the consideration of activation energy, chemical reaction, and convective conditions. Haq et al. [22] provided the rheological-aspects of bioconvective stream of a material with the involvement of ternary nanoparticles, activation energy, and heat generation. Aspects of hybrid nanoelements and activation energy on the flow of a

water-based material over different geometries, namely, wedge, cone, and sheet are investigated by Rekha et al. [23]. Thermo-bioconvective formulation of a THNS stream over a revolving disk is scrutinized by Khouqeer et al. [24] with the significance of activation energy. Some updated works regarding the role of nanoparticles in various fluids are reported by Abbas et al. [25], Shanmugapriya et al. [26], and Emad et al. [27].

The present study uniquely investigates transformer-oil-based tri-hybrid nanostructure material flow between double-stretchable revolving disks while simultaneously considering activation/dissipative energy effects and nanoparticle shape factors. By integrating magnetic, radiative, and thermochemical influences into a versatile mathematical model, it highlights how tri-hybrid nanostructured material outperforms hybrid and single nanostructured materials in heat/mass transport, offering insights for enhanced thermal management and high-performance mechanical and electrical systems. The investigation addresses the following research questions:

- How do activation and dissipative energy effects influence the flow and heat/mass transport in a tri-hybrid nanostructured material confined between revolving disks?
- What is the impact of nanoparticle shapes (spherical vs. non-spherical) on thermal performance and transport efficiency?
- How does the rotation of the disks and variations in Reynolds number affect axial and tangential velocities and overall transport rates?
- To what extent do tri-hybrid nanofluids enhance heat/mass transport compared to hybrid and conventional nanofluids in double-disk revolving systems?

2. Formulation

In this section of the study, steady and laminar flow of a tri-hybrid nanostructure (THNS) between rotatable double stretching disks is considered. The transformer oil acts as the base substance. Three nanoelements, namely, Fe_3O_4 , TiO_2 , and Cu , are uniformly dispersed into the base material to obtain THNS. The incompressibility of the nano-suspension is maintained during the flow. The aspects of spherical and non-spherical nanosolids are also investigated. Disks deform with the radial deformation rates Γ_1 and Γ_2 , as well as disks revolve with angular-velocities Ω_1 and Ω_2 . Concentration and temperature profiles are reported by C_0 and T_0 on the bottom disk, respectively, and described by C_1 and T_1 on the upper disk. Disks are separated by a constant distance d , and hence the bottom disk is at $z = 0$, and the upper disk is at $z = d$. The vector (u, v, w) represents the velocity parts in r, θ , and z directions, respectively. The flow satisfies the rotational symmetry property. An

external continuous magnetic force of a constant intensity B_0 is provided parallel to the z -axis of the configuration. The flow scenario is given in figure 1, and the leading PDEs are as follows:

$$\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r} + \frac{u}{r} = 0, \quad (1)$$

$$\rho_{thns} \left[w \frac{\partial u}{\partial z} + u \frac{\partial u}{\partial r} - \frac{v^2}{r} \right] = -\frac{\partial p}{\partial r} + \mu_{thns} \left[\frac{\partial}{\partial r} \left(\frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial z} \right) + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right] - \sigma_{thns} B_0^2 u, \quad (2)$$

$$\rho_{thns} \left[w \frac{\partial v}{\partial z} + u \frac{\partial v}{\partial r} + \frac{uv}{r} \right] = \mu_{thns} \left[\frac{\partial}{\partial r} \left(\frac{\partial v}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{\partial v}{\partial z} \right) + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right] - \sigma_{thns} B_0^2 v, \quad (3)$$

$$\rho_{thns} \left[w \frac{\partial w}{\partial z} + u \frac{\partial w}{\partial r} \right] = -\frac{\partial p}{\partial z} + \mu_{thns} \left[\frac{\partial}{\partial r} \left(\frac{\partial w}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{\partial w}{\partial z} \right) + \frac{1}{r} \frac{\partial w}{\partial r} \right], \quad (4)$$

$$(\rho c_p)_{thns} \left[w \frac{\partial T}{\partial z} + u \frac{\partial T}{\partial r} \right] = \left(k_{thns} + \frac{16\sigma^* T_1^3}{3k^*} \right) \left[\frac{\partial}{\partial r} \left(\frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{\partial T}{\partial z} \right) + \frac{1}{r} \frac{\partial T}{\partial r} \right] + Q_0(T - T_1) + \sigma_{thns} B_0^2 (u^2 + v^2), \quad (5)$$

$$\left[w \frac{\partial C}{\partial z} + u \frac{\partial C}{\partial r} \right] = D_m \left[\frac{\partial}{\partial r} \left(\frac{\partial C}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{\partial C}{\partial z} \right) + \frac{1}{r} \frac{\partial C}{\partial r} \right] - k_r^2 \left(\frac{T}{T_1} \right)^q (C - C_1) \exp \left(-\frac{E_b}{k_0 T} \right). \quad (6)$$

The BCs for the present configuration are

$$\begin{cases} C = C_0, T = T_0, u = r\Gamma_1, w = 0, v = r\Omega_1 & \text{at } z = 0, \\ C = C_1, T = T_1, u = r\Gamma_2, w = 0, v = r\Omega_2 & \text{at } z = d. \end{cases} \quad (7)$$

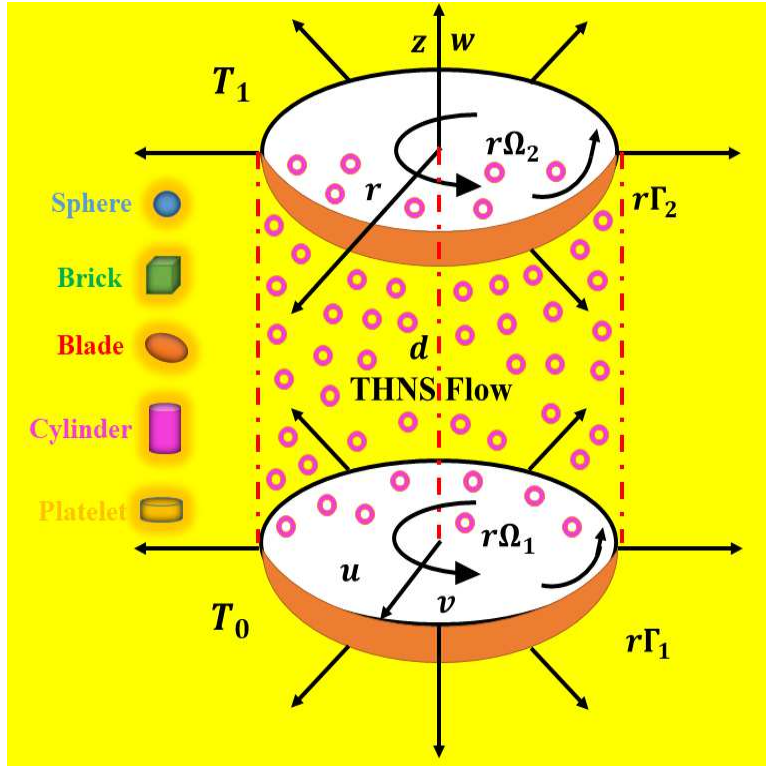


Figure 1: Flow description of a tri-hybrid nanostructure.

In the above formulation, E_b represents activation energy, k_0 is the Boltzmann constant, predicated constant rate is shown by q , k_r denotes the chemical reaction rate, D_m reports the

molecular diffusivity, the 3D thermal source constraint is denoted by Q_0 , σ^* represents the Stefan-Boltzmann constant, the mean-absorption multiplier is denoted by k^* , k symbolizes the thermal conductivity, particular thermal-capacity is represented by c_p , σ represents the electric-conductivity, μ is the dynamic-viscosity, ρ is the density, and p represents the generated pressure between coupled disks.

2.1. Similarity transformations

The following functional transforms are adopted to convert the governing PDEs into a non-dimensional system of ODEs.

$$\begin{cases} u(r, z) = r\Omega_1 F'(\chi), v(r, z) = r\Omega_1 G(\chi), \\ w(z) = -2d\Omega_1 F(\chi), p(r, z) = \rho_f \vartheta_f \Omega_1 \left(P(\chi) + \frac{r^2 \epsilon}{2d^2} \right), \\ \theta(\chi) = \frac{T-T_1}{T_0-T_1}, \phi(\chi) = \frac{C-C_1}{C_0-C_1}, \chi = \frac{z}{d}. \end{cases} \quad (8)$$

In the above transformations, χ represents the dimensionless distance between double disks, θ is the non-dimensional temperature field, followed by the non-dimensional concentration field ϕ . The function $F(\chi)$ represents the non-dimensional axial velocity, $F'(\chi)$ denotes the non-dimensional radial velocity followed by the dimensionless tangential velocity $G(\chi)$, and ϵ is the pressure parameter with non-dimensional pressure function $P(\chi)$.

2.2. Tri-hybrid nanostructure

In order to obtain the tri-hybrid nanostructure, nanoelements are uniformly dispersed into the transformer oil. The sphericity ψ of nanoelements, shape factor m , viscosity constants (A_1, A_2) , and thermophysical properties are given in Tables 1-3, respectively. The mathematical forms for the interactions of nanoelements with the transformer oil during the substance motion between the double disks are given below.

Table 1: Shape factor and sphericity of nanoelements.

Shape	Platelet	Blade	Cylinder	Brick	Sphere
ψ	0.52	0.36	0.62	0.81	1.0
$m = \frac{3}{\psi}$	5.8	8.3	4.8	3.7	3.0

Table 2: Viscosity coefficients for dissimilar shapes of nanoelements.

Shape	Platelet	Blade	Cylinder	Brick	Sphere
A_1	37.1	14.6	13.5	1.9	2.5
A_2	612.6	123.3	904.4	471.4	6.2

$$\left\{ \begin{array}{l} \Pi_1 = \frac{\mu_{thns}}{\mu_f} = 1 + A_1(\varphi_1 + \varphi_2 + \varphi_3) + A_2(\varphi_1 + \varphi_2 + \varphi_3)^2, \\ \Pi_2 = \frac{\rho_{thns}}{\rho_f} = (1 - \varphi_2)(1 - \varphi_3) \left((1 - \varphi_1) + \varphi_1 \frac{\rho_{ns1}}{\rho_f} \right) + \varphi_3 \frac{\rho_{ns3}}{\rho_f} + \varphi_2 \frac{\rho_{ns2}}{\rho_f}, \\ \Pi_3 = \frac{(\rho c_p)_{thns}}{(\rho c_p)_f} = (1 - \varphi_2)(1 - \varphi_3) \left((1 - \varphi_1) + \varphi_1 \frac{(\rho c_p)_{ns1}}{(\rho c_p)_f} \right) + \varphi_3 \frac{(\rho c_p)_{ns3}}{(\rho c_p)_f} + \varphi_2 \frac{(\rho c_p)_{ns2}}{(\rho c_p)_f}, \\ \Pi_4 = \left[\frac{(k_{ns3} + (m-1)\Pi_5 k_f) + \varphi_3(m-1)(k_{ns3} - \Pi_5 k_f)}{(k_{ns3} + (m-1)\Pi_5 k_f) - \varphi_3(k_{ns3} - \Pi_5 k_f)} \right], \\ \Pi_5 = \left[\frac{(k_{ns2} + (m-1)\Pi_6 k_f) + \varphi_2(m-1)(k_{ns2} - \Pi_6 k_f)}{(k_{ns2} + (m-1)\Pi_6 k_f) - \varphi_2(k_{ns2} - \Pi_6 k_f)} \right], \\ \Pi_6 = \left[\frac{(k_{ns1} + (m-1)k_f) + \varphi_1(m-1)(k_{ns1} - k_f)}{(k_{ns1} + (m-1)k_f) - \varphi_1(k_{ns1} - k_f)} \right], \\ \Pi_7 = \left[\frac{(\sigma_{ns3} + (m-1)\Pi_8 \sigma_f) + \varphi_3(m-1)(\sigma_{ns3} - \Pi_8 \sigma_f)}{(\sigma_{ns3} + (m-1)\Pi_8 \sigma_f) - \varphi_3(\sigma_{ns3} - \Pi_8 \sigma_f)} \right], \\ \Pi_8 = \left[\frac{(\sigma_{ns2} + (m-1)\Pi_9 \sigma_f) + \varphi_2(m-1)(\sigma_{ns2} - \Pi_9 \sigma_f)}{(\sigma_{ns2} + (m-1)\Pi_9 \sigma_f) - \varphi_2(\sigma_{ns2} - \Pi_9 \sigma_f)} \right], \\ \Pi_9 = \left[\frac{(\sigma_{ns1} + (m-1)\sigma_f) + \varphi_1(m-1)(\sigma_{ns1} - \sigma_f)}{(\sigma_{ns1} + (m-1)\sigma_f) - \varphi_1(\sigma_{ns1} - \sigma_f)} \right]. \end{array} \right. \quad (9)$$

Table 3: Thermo-physical properties of transformer oil and nanoelements.

Element	$\sigma/(\Omega.m)$	$k/(W.m^{-1}.K^{-1})$	$c_p/(J.kg^{-1}.K^{-1})$	$\rho/(kg.m^{-3})$
Transformer oil	10^{-12}	0.126	1860	890
Cu (φ_1)	5.96×10^7	401	385	8933
TiO ₂ (φ_2)	10^{-11}	8.9538	686.2	4250
Fe ₃ O ₄ (φ_3)	2.5×10^4	6.0	670	5200

2.3. Transformed differential system

Using Eqs. (8) and (9), the continuity equation satisfies trivially. The equations (2), (3), (5), and (6) turn into the following forms with the transformed boundary conditions.

$$F'''' + \frac{\Pi_2}{\Pi_1} Re \left(2GG' + 2FF''' - \frac{\Pi_7 \Pi_8 \Pi_9}{\Pi_2} MF'' \right) = 0, \quad (10)$$

$$G'' + \frac{\Pi_2}{\Pi_1} Re \left(2FG' - 2F'G - \frac{\Pi_7 \Pi_8 \Pi_9}{\Pi_2} MG \right) = 0, \quad (11)$$

$$\left(1 + \frac{Rd}{\Pi_4 \Pi_5 \Pi_6} \right) \theta'' + 2 \frac{\Pi_3}{\Pi_4 \Pi_5 \Pi_6} Re Pr F \theta' + \frac{Re Pr S \theta}{\Pi_4 \Pi_5 \Pi_6} + \frac{\Pi_7 \Pi_8 \Pi_9}{\Pi_4 \Pi_5 \Pi_6} M Br Re (F'^2 + G^2) = 0, \quad (12)$$

$$\frac{1}{Sc} \phi'' + 2 Re F \phi' - Re \Lambda \phi (1 + \gamma \theta)^q \exp \left(\frac{-E_n}{1 + \gamma \theta} \right) = 0. \quad (13)$$

$$\begin{cases} F(0) = 0, F'(0) = \alpha_1, G(0) = 1, \theta(0) = 1, \theta(1) = 0, \\ F(1) = 0, F'(1) = \alpha_2, G(1) = \beta, \phi(0) = 1, \phi(1) = 0. \end{cases} \quad (14)$$

The names of the involved variables are rotation parameter $\beta = \frac{\Omega_2}{\Omega_1}$, stretching parameters

$(\alpha_1 = \frac{\Gamma_1}{\Omega_1}, \alpha_2 = \frac{\Gamma_2}{\Omega_1})$, activation-energy parameter $E_n = \frac{E_b}{k_0 T_1}$, temperature-difference

parameter $\gamma = \frac{T_1 - T_0}{T_1}$, Reynolds number $Re = \frac{\Omega_1 d^2}{\vartheta_f}$, Schmidt number $Sc = \frac{\vartheta_f}{D_m}$, Brinkman number $Br = \frac{\mu_f d^2 \Omega_1^2}{k_f (T_1 - T_0)}$, magnetic parameter $M = \frac{\sigma_f B_0^2}{\rho_f \Omega_1}$, thermal source variable $S = \frac{Q_0}{\Omega_1 (\rho c_p)_f}$, radiation parameter $Rd = \frac{16\sigma^* T_1^3}{3k_f k^*}$, Prandtl number $Pr = \frac{\mu_f (c_p)_f}{k_f}$, and the reaction rate variable $\Lambda = \frac{k_r^2}{\Omega_1}$. The pressure can be computed from the Eq. (4) as follows:

$$P'(\chi) = -2(2\Pi_2 Re F F' + \Pi_1 F''). \quad (15)$$

Integrate the Eq. (15) with respect to χ from $\chi = 0$ to $\chi = \chi$, we get the pressure function in the following form.

$$P(\chi) = -2 \left(\Pi_2 Re (F(\chi))^2 + \Pi_1 F'(\chi) - \Pi_1 \alpha_1 \right). \quad (16)$$

2.4. Physical quantities

To calculate the physical quantities, it is essential to find the shear stresses at both disks in the radial and tangential directions. The mathematical forms of these stresses are as follows:

$$\tau_{zr} = \mu_{thns} \frac{\partial u}{\partial z}, \tau_{z\theta} = \mu_{thns} \frac{\partial v}{\partial z}. \quad (17)$$

The total stress is then calculated using the following equation.

$$\tau_w = \sqrt{\tau_{zr}^2 + \tau_{z\theta}^2}. \quad (18)$$

The skin-friction coefficient C_{f1} at the lower disk is

$$C_{f1} = \frac{\tau_w|_{z=0}}{\rho_f (r\Omega_1)^2} = \frac{\Pi_1}{Re} \sqrt{[F''(0)]^2 + [G'(0)]^2}. \quad (19)$$

The skin-friction coefficient C_{f2} at the upper disk is

$$C_{f2} = \frac{\tau_w|_{z=d}}{\rho_f (r\Omega_1)^2} = \frac{\Pi_1}{Re} \sqrt{[F''(1)]^2 + [G'(1)]^2}. \quad (20)$$

The expressions for Nusselt Nu and Sherwood Sh numbers at the lower disk are

$$\begin{cases} Nu_1 = -\frac{d}{k_f (T_1 - T_0)} \left(k_{thns} + \frac{16\sigma^* T_1^3}{3k^*} \right) \left(\frac{\partial T}{\partial z} \right) \Big|_{z=0} = -(\Pi_4 \Pi_5 \Pi_6 + Rd) \theta'(0), \\ Sh_1 = -\frac{dD_m}{D_m (C_1 - C_0)} \left(\frac{\partial C}{\partial z} \right) \Big|_{z=0} = -\phi'(0). \end{cases} \quad (21)$$

The expressions for Nusselt Nu and Sherwood Sh numbers at the upper disk are

$$\begin{cases} Nu_2 = -\frac{d}{k_f (T_1 - T_0)} \left(k_{thns} + \frac{16\sigma^* T_1^3}{3k^*} \right) \left(\frac{\partial T}{\partial z} \right) \Big|_{z=d} = -(\Pi_4 \Pi_5 \Pi_6 + Rd) \theta'(1), \\ Sh_2 = -\frac{dD_m}{D_m (C_1 - C_0)} \left(\frac{\partial C}{\partial z} \right) \Big|_{z=d} = -\phi'(1). \end{cases} \quad (22)$$

3. Numerical simulation

The two-point BVP obtained in the set of differential equations (10)-(13), subjected to boundary values (14), is simulated computationally using the bvp4c solver in MATLAB. The

solution to the problem is provided in Table 4 with the help of the following mathematical implementations. The following functions are introduced to gain the first-order system instead of a higher-order differential system as a first step to implement the bvp4c approach.

$$\begin{aligned}
\mathcal{L}_1 &= F, \mathcal{L}_2 = F', \mathcal{L}_3 = F'', \mathcal{L}_4 = F''', \\
\mathcal{L}_5 &= G, \mathcal{L}_6 = G', \\
\mathcal{L}_7 &= \theta, \mathcal{L}_8 = \theta', \\
\mathcal{L}_9 &= \phi, \mathcal{L}_{10} = \phi'.
\end{aligned} \tag{23}$$

Using these variables, equations (10)-(13) are transformed into the following first-order system:

$$\begin{aligned}
\mathcal{L}'_1 &= \mathcal{L}_2, \mathcal{L}'_2 = \mathcal{L}_3, \mathcal{L}'_3 = \mathcal{L}_4, \\
\mathcal{L}'_4 &= -\frac{\Pi_2}{\Pi_1} Re \left(2\mathcal{L}_5\mathcal{L}_6 + 2\mathcal{L}_1\mathcal{L}_4 - \frac{\Pi_7\Pi_8\Pi_9}{\Pi_2} M\mathcal{L}_3 \right), \\
\mathcal{L}'_5 &= \mathcal{L}_6, \\
\mathcal{L}'_6 &= -\frac{\Pi_2}{\Pi_1} Re \left(2\mathcal{L}_1\mathcal{L}_6 - 2\mathcal{L}_2\mathcal{L}_5 - \frac{\Pi_7\Pi_8\Pi_9}{\Pi_2} M\mathcal{L}_5 \right) \\
\mathcal{L}'_7 &= \mathcal{L}_8, \\
\mathcal{L}'_8 &= -\frac{2\Pi_3 Re Pr \mathcal{L}_1 \mathcal{L}_8 + Re Pr S \mathcal{L}_7 + \Pi_7 \Pi_8 \Pi_9 M Br Re (\mathcal{L}_2^2 + \mathcal{L}_5^2)}{Rd + \Pi_4 \Pi_5 \Pi_6}, \\
\mathcal{L}'_9 &= \mathcal{L}_{10}, \\
\mathcal{L}'_{10} &= Sc[-2Re\mathcal{L}_1\mathcal{L}_{10} + Re\Lambda\mathcal{L}_9(1 + \gamma\mathcal{L}_7)^q \exp\left(\frac{-E_n}{1+\gamma\mathcal{L}_7}\right)].
\end{aligned} \tag{24}$$

The BCs in Eq. (14) become:

$$\begin{cases} \mathcal{L}_1(0) = 0, \mathcal{L}_2(0) = \alpha_1, \mathcal{L}_5(0) = 1, \mathcal{L}_7(0) = 1, \mathcal{L}_7(1) = 0, \\ \mathcal{L}_1(1) = 0, \mathcal{L}_2(1) = \alpha_2, \mathcal{L}_5(1) = \beta, \mathcal{L}_9(0) = 1, \mathcal{L}_9(1) = 0. \end{cases} \tag{25}$$

These ten conditions match the ten first-order equations, ensuring a well-posed BVP. An initial uniform mesh is selected on the computational domain $\chi \in [0, 1]$. The transformed system is solved using MATLAB's bvp4c solver, which employs a collocation method based on the fourth-order Lobatto IIIa formula with adaptive mesh refinement. This method is well known for its robustness, accuracy, and efficiency in solving nonlinear boundary value problems arising in computational fluid mechanics. An initial guess profile, satisfying the BCs, is supplied to initiate the iterative procedure. The solver automatically refines the mesh until the solution converges within a specified tolerance (RelTol=10⁻⁴, AbsTol=10⁻⁶). Convergence is achieved when the residuals of the differential equations and boundary conditions satisfy the prescribed tolerance.

Table 4: Numerical solution of the mathematical problem when $M = 1.5, Re = 2.0, Br = 0.3, Rd = 0.3, S = -0.1, \Lambda = 0.5, q = 1.0 = E_n, \gamma = 0.1, \alpha_1 = 0.4, \alpha_2 = 0.6, \beta = 0.4, \varphi_1 = \varphi_2 = \varphi_3 = 0.02$, and $\psi = 1.0$.

Shape	$-F''(0)$	$-G'(0)$	$-\theta'(0)$	$-\theta'(1)$	$-\phi'(0)$	$-\phi'(1)$
Sphere	2.7213	1.6553	1.1242	1.2354	1.2933	0.98029
Platelet	2.7843	0.87942	1.023	1.2867	1.2855	0.99035
Blade	2.7705	1.2904	1.0314	1.2433	1.2888	0.98592
Cylinder	2.7821	0.88754	1.0347	1.2903	1.2856	0.99015
Brick	2.7661	1.0824	1.067	1.2782	1.2877	0.98745

4. Results and discussion

In the present analysis, both the stretched disks revolve in an identical direction because β is restricted within the range $0 < \beta < 1$. The consequences of numerous parameters on different profiles, namely, axial velocity $F(\chi)$, radial velocity $F'(\chi)$, tangential velocity $G(\chi)$, temperature $\theta(\chi)$, and concentration $\phi(\chi)$. The parametric values are considered the graphical purpose of the simulation as follows: $M = 1.5, Re = Sc = 2.0, Br = 0.3, Rd = 0.3, S = -0.1, \Lambda = 0.5, q = 1.0 = E_n, \gamma = 0.1, \alpha_1 = 0.5 = \alpha_2, \beta = 0.4, \varphi_1 = \varphi_2 = \varphi_3 = 0.01$, and $\psi = 1.0$. The tri-hybrid nanofluid is moving from the top to the lower disk as $\beta > 0$.

Figure 2 elucidates the influence of M within the range $5 \leq M \leq 25$ on the velocity components of a tri-hybrid nanomaterial flowing between two rotating stretchable disks as a function of the dimensionless distance between the disks. The left panel shows the axial velocity, which initially increases from the lower disk, reaches a maximum, and then decreases, becoming negative near the upper disk. As M increases, the magnitude of the axial velocity diminishes throughout the domain, indicating that a stronger magnetic field suppresses the axial motion due to the enhanced Lorentz force opposing the flow. The middle panel presents the radial velocity, which exhibits a symmetric parabolic-type profile with a minimum near the mid-plane. An increase in M reduces the absolute value of the radial velocity, again reflecting magnetic damping of tri-hybrid nanosubstance motion. The right panel depicts the tangential velocity, which decreases sharply from the lower disk, reaches a minimum, and then slightly increases toward the upper disk. Higher values of M significantly reduce the tangential velocity across the entire gap, demonstrating that the magnetic field strongly resists rotational motion. Overall, the figure shows that increasing magnetic effects consistently suppress axial, radial, and tangential velocities of the tri-hybrid nanosubstance

between the revolving stretchable disks. Figure 3 describes the influence of Re within the range $1 \leq Re \leq 20$ on the velocity components, and almost the same trend is noticed as previously discussed for the variation of M . The physical reason behind this trend is that both the Reynolds number Re and M act as resistive mechanisms to the flow in this configuration, even though their origins are different. Therefore, the nearly identical trends observed for increasing Re and M arise because both parameters reduce the effective momentum of the nanosubstance, one through viscous resistance (Reynolds number effect) and the other through electromagnetic drag (magnetic field effect), resulting in similar qualitative behavior of the velocity profiles. Figure 4 describes the influence of α_1 within the range $0.1 \leq \alpha_1 \leq 0.9$ on the velocity components. Axial velocity upsurges, radial velocity rises near the lower disk and then declines, and the tangential velocity declines with the variation of α_1 (stretching-parameter for lower disk) within the prescribed range. Physically, increasing the α_1 enhances the axial motion of the tri-hybrid nanosubstance by injecting additional momentum into the flow, while the radial velocity intensifies near the lower disk due to stronger surface-driven stretching and then diminishes away from it because of viscous diffusion and continuity effects. Conversely, the tangential velocity decreases as the increased stretching redistributes momentum from rotational motion toward axial and radial directions, thereby weakening the azimuthal flow between the revolving stretchable disks.

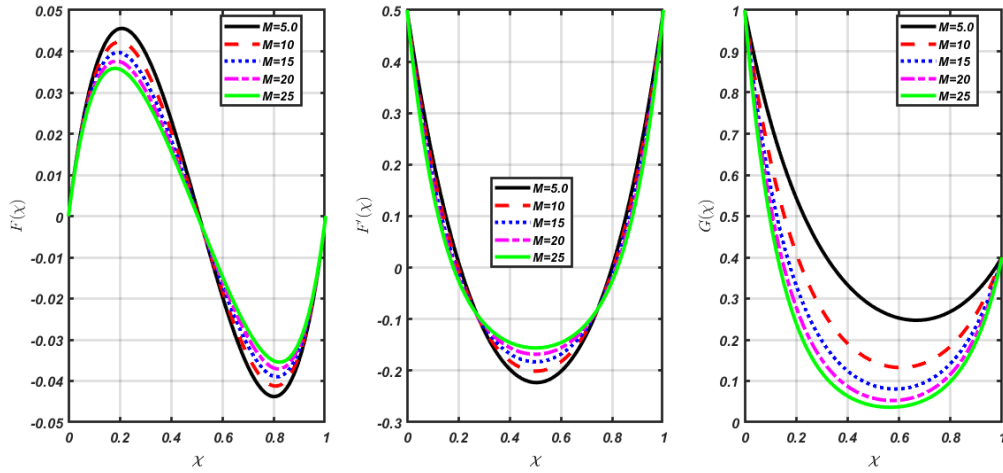


Figure 2: Curves of $F(\chi)$, $F'(\chi)$, and $G(\chi)$ for dissimilar M -values.

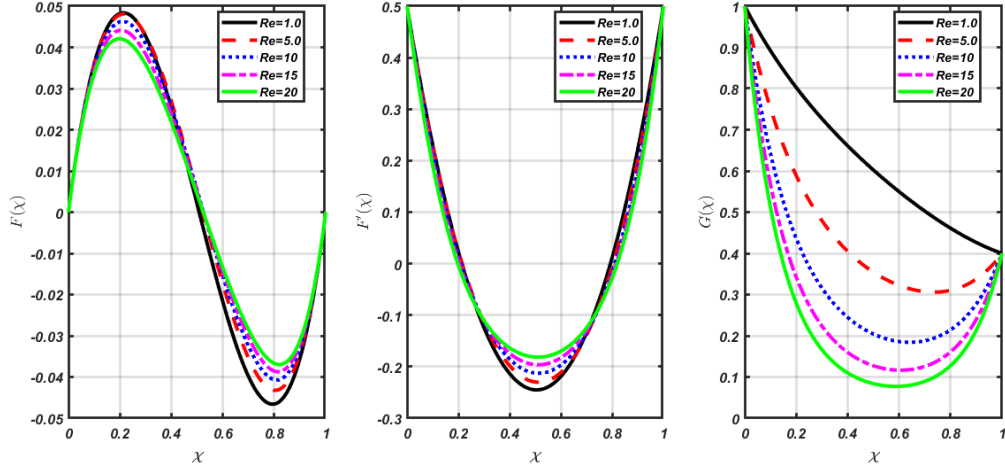


Figure 3: Curves of $F(\chi)$, $F'(\chi)$, and $G(\chi)$ for dissimilar Re -values.

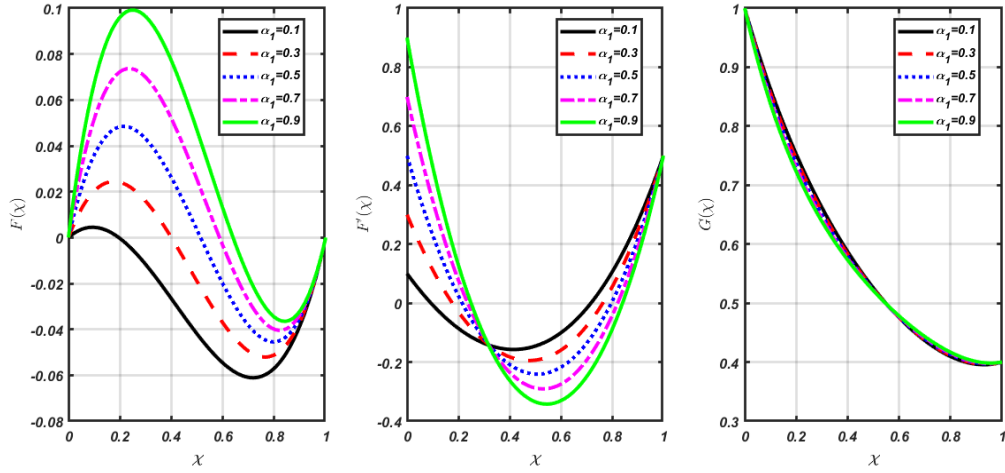


Figure 4: Curves of $F(\chi)$, $F'(\chi)$, and $G(\chi)$ for dissimilar α_1 -values.

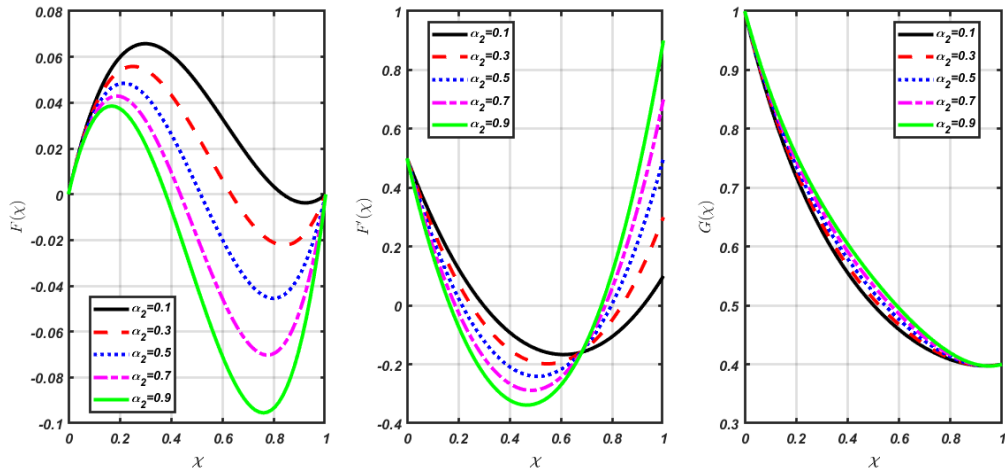


Figure 5: Curves of $F(\chi)$, $F'(\chi)$, and $G(\chi)$ for dissimilar α_2 -values.

Figure 5 explains the influence of α_2 within the range $0.1 \leq \alpha_2 \leq 0.9$ on the velocity components. Axial velocity decreases, radial velocity declines near the lower disk and then

rises, and the tangential velocity increases with the variation of α_2 (stretching parameter for upper disk) within the prescribed range. Physically, the stretching parameter α_2 represents the stretching intensity of the upper disk, and its increase significantly modifies the momentum distribution within the tri-hybrid nanosubstance. As α_2 increases, the enhanced deformation of the upper-disk draws nanosubstance toward itself, which opposes the axial momentum induced from the lower disk and thereby causes a reduction in the axial velocity across the flow domain. The radial-velocity declines near the lower-disk because the dominant influence of the upper-disk weakens the lower-disk-driven radial motion; however, as the nanosubstance approaches the upper disk, the stronger stretching there accelerates the substance radially, leading to an upsurge in radial-velocity away from the lower-disk. In contrast, the tangential velocity increases with higher α_2 since enhanced deformation at the upper-disk strengthens the coupling between rotational and surface-driven motion, allowing more azimuthal momentum to be transferred into the fluid. Thus, increasing α_2 suppresses axial and near-lower-disk radial motion while promoting radial acceleration near the upper disk and amplifying the tangential velocity. Figure 6 illustrates the influence of β within the range $0.1 \leq \beta \leq 0.9$ on the velocity components. Axial velocity decreases, radial velocity behaves like a constant parabolic function, and the tangential velocity increases with the variation of β (rotation parameter) within the prescribed range. Physically, β quantifies the relative strength of disk rotation and directly governs the balance between rotational and translational motion of the tri-hybrid nanosubstance. As β increases, stronger rotational effects enhance the centrifugal force, which redirects substance momentum from the axial direction toward azimuthal motion, leading to a decline in axial velocity. The radial-velocity maintains an almost unchanged parabolic profile because the opposing influences of centrifugal acceleration and viscous resistance balance each other, resulting in only marginal variation in its magnitude. Meanwhile, the tangential velocity increases significantly with higher β since intensified disk rotation transfers more angular momentum to the substance through viscous coupling. Consequently, increasing the rotation parameter suppresses axial flow, leaves the radial motion nearly unaffected, and amplifies the tangential velocity due to dominant rotational effects.

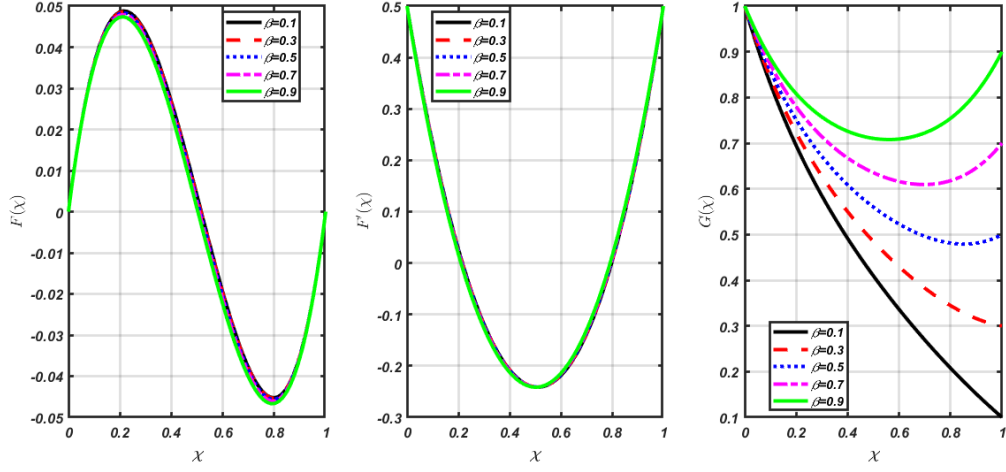


Figure 6: Curves of $F(\chi)$, $F'(\chi)$, and $G(\chi)$ for dissimilar β -values.

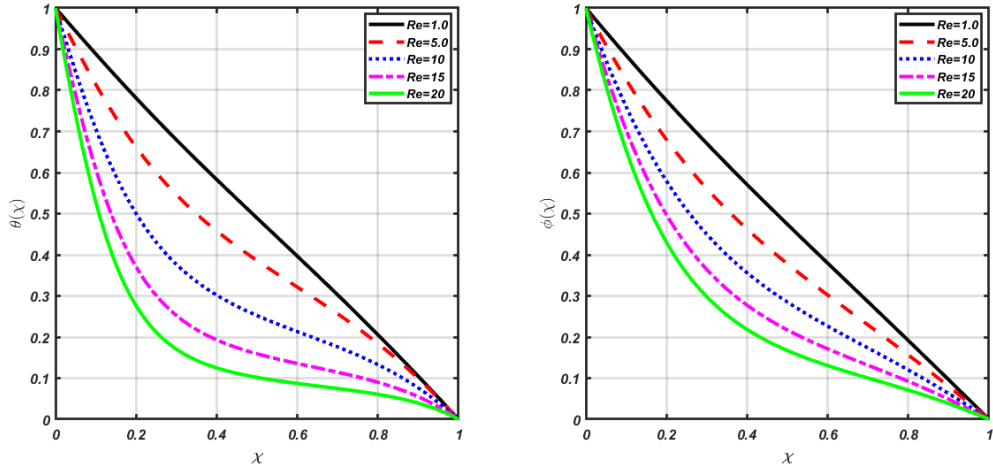


Figure 7: Curves of $\theta(\chi)$ and $\phi(\chi)$ for dissimilar Re -values.

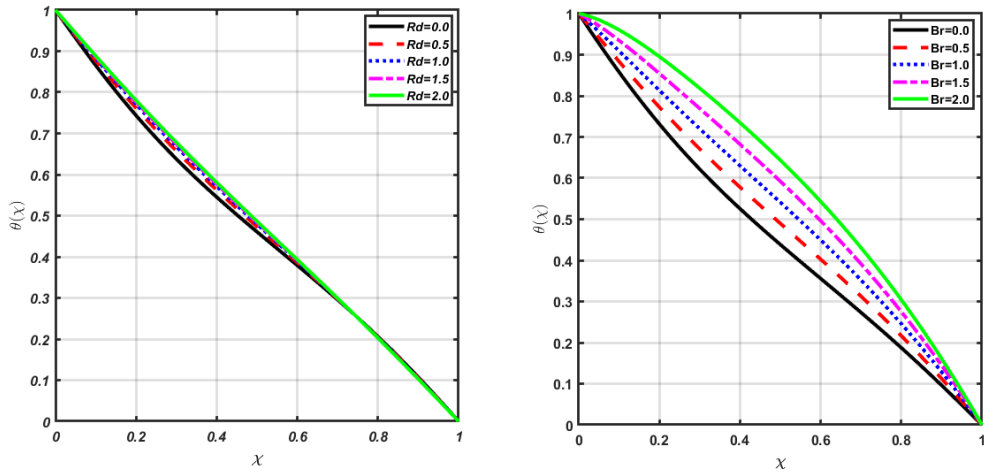


Figure 8: Profile of $\theta(\chi)$ for dissimilar Rd - and Br -values.

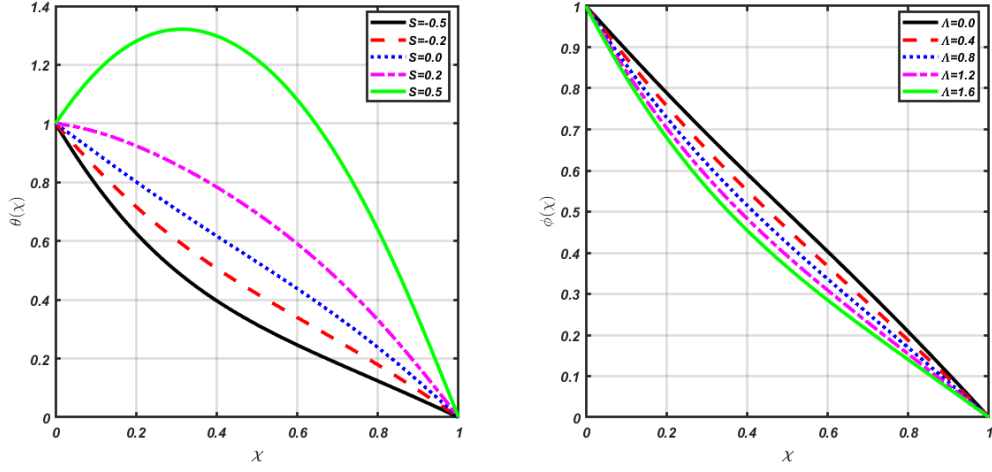


Figure 9: Profile of $\theta(\chi)$ for dissimilar S -values and profile of $\phi(\chi)$ for dissimilar Λ -values.

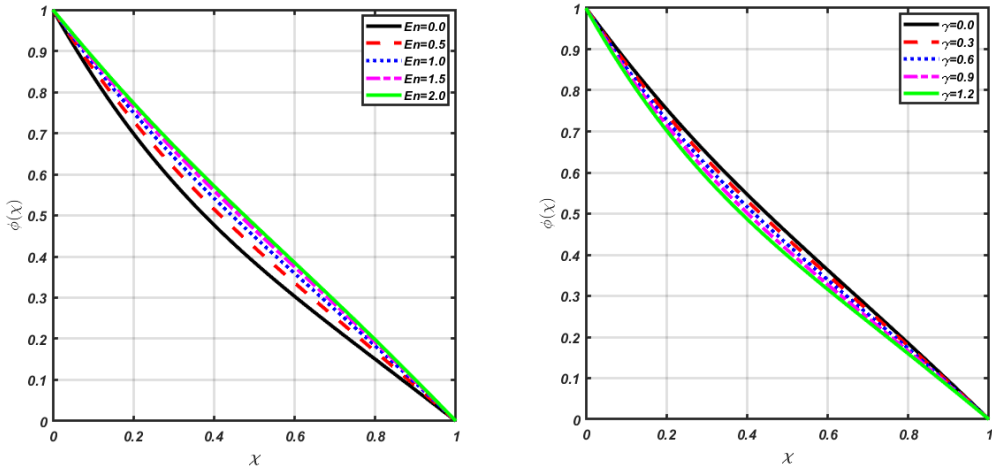


Figure 10: Profile of $\phi(\chi)$ for dissimilar En - and γ -values.

Figure 7 illustrates the significance of the Reynolds number Re on $\theta(\chi)$ and $\phi(\chi)$ curves of a tri-hybrid nanosubstance flowing between two revolving stretchable disks. Increasing Re causes both $\theta(\chi)$ and $\phi(\chi)$ profiles to decrease throughout the flow domain. Physically, a higher Reynolds number indicates stronger inertial-forces relative to viscous-forces, which enhances substance motion and convective transport. As a result, thermal energy and nanoelement species are convected away more efficiently from the disk surfaces, leading to thinner thermal and concentration boundary layers. This enhanced convection reduces the residence time of heat and nanoelements within the substance, thereby lowering the temperature/concentration levels across the gap between the disks. Consequently, increasing Re promotes heat/mass transport rates while suppressing the thermal/solutal distributions of the tri-hybrid nanosubstance. Figure 8 describes the role of the Brinkman-number Br and radiation variable Rd on $\theta(\chi)$ curves. The temperature of the tri-hybrid nanosubstance rises

with the augmentation of Br and Rd within the range $0.0 \leq Br, Rd \leq 2$. Physically, Br represents the ratio of viscous dissipation to conductive heat-transport, and its increase intensifies the conversion of mechanical energy into thermal-energy due to enhanced internal friction within the tri-hybrid nanosubstance. This additional viscous heating raises the substance temperature throughout the flow domain. Similarly, an escalation in Rd strengthens the contribution of thermal radiation to the overall heat-transport mechanism, allowing more radiative energy to be absorbed by the substance. The combined effect of stronger viscous dissipation and enhanced radiative heat flux supplies extra thermal-energy to the system, thickens the thermal-boundary-layer, and consequently elevates the temperature function between the revolving stretchable disks. Figure 9 explains the effect S (thermal source/sink parameter) on $\theta(\chi)$ and the effect of Λ (chemical reaction parameter) on $\phi(\chi)$ profiles. Increasing S causes an increment in $\theta(\chi)$ profile, while increasing Λ causes a decrement in $\phi(\chi)$ profile. Physically, S characterizes internal heat generation/absorption within the tri-hybrid nanosubstance. An increase in S corresponds to stronger internal heat generation/absorption, which supplies additional thermal energy to the substance and raises its temperature, thereby increasing $\theta(\chi)$ across the flow region. On the other hand, Λ quantifies the strength of chemical reactions that consume the nanoparticle species. As Λ increases, the reaction rate intensifies, leading to a higher consumption of solutal species and a reduction in nanoparticle concentration. This enhanced depletion weakens mass diffusion and fallouts in a decline in the $\phi(\chi)$ curve. Thus, stronger thermal sources elevate the temperature field, while intensified chemical reactions diminish the concentration distribution within the tri-hybrid nanosubstance. Figure 10 describes the effect of the activation-energy parameter En and temperature-difference parameter γ on $\phi(\chi)$ profile. The concentration of the tri-hybrid nanosubstance increases with the augmentation of En and decreases with an increment in γ . Physically, En represents the energy barrier that must be overcome for the chemical reaction to occur. As En increases, the effective reaction rate decreases because fewer nanoelement species possess sufficient energy to participate in the reaction. This weakened reaction reduces the consumption of nanoelements, allowing a larger amount of the tri-hybrid nanosubstance to remain suspended in the material, which outcomes in an upsurge in the $\phi(\chi)$ curve. Conversely, γ measures the sensitivity of mass-diffusion to temperature-variations. An increase in γ enhances thermally driven diffusion and reaction activity, which accelerates the transport and depletion of nanoelement species away from the flow domain. As an outcome, the concentration of the tri-hybrid nanosubstance declines with increasing γ .

Hence, higher activation energy suppresses chemical consumption and elevates concentration, while larger temperature differences intensify species transport and reduce the concentration distribution. Figures 11-13 present three sets of bar charts comparing the performance of nanosubstance, hybrid nanosubstance, and tri-hybrid nanosubstance at different Reynolds numbers ($Re = 5, 10, 15$) for both lower and upper disks. The skin-friction coefficient quantifies frictional resistance between the substance and disk surfaces. In both disks, the tri-hybrid nanofluid exhibits the maximum skin friction, followed by the hybrid nanofluid and the simple nanofluid, with friction increasing as Re rises, indicating that the tri-hybrid nanofluid enhances viscous effects most strongly. For the case of Nusselt number, a measure of convective heat transfer, where again the tri-hybrid nanofluid outperforms the others, with heat transfer improving as Re increases, the lower disk consistently demonstrates higher Nusselt numbers than the upper disk. The Sherwood number represents mass transfer efficiency. While the lower disk shows a slight increase in Sherwood number with Re , the upper disk experiences a slight decrease, yet the tri-hybrid nanofluid remains superior, suggesting enhanced mass transfer and some asymmetry between disks. Overall, the tri-hybrid nanofluid consistently delivers the highest performance in friction, heat, and mass transfer, followed by hybrid and then simple nanofluid, and increasing Reynolds number generally amplifies these effects, particularly on the lower disk.

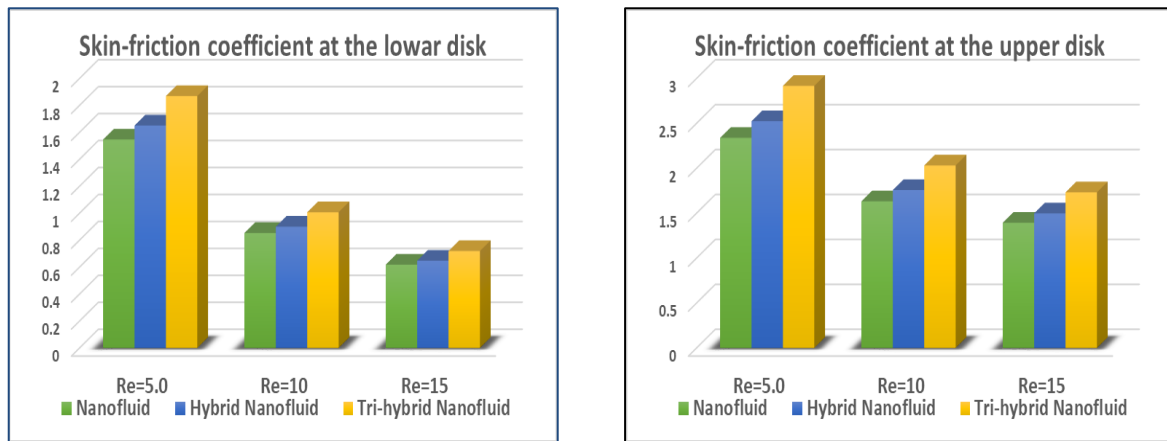


Figure 11: Skin-friction coefficients for dissimilar Re -values with $\psi = 1.0$.

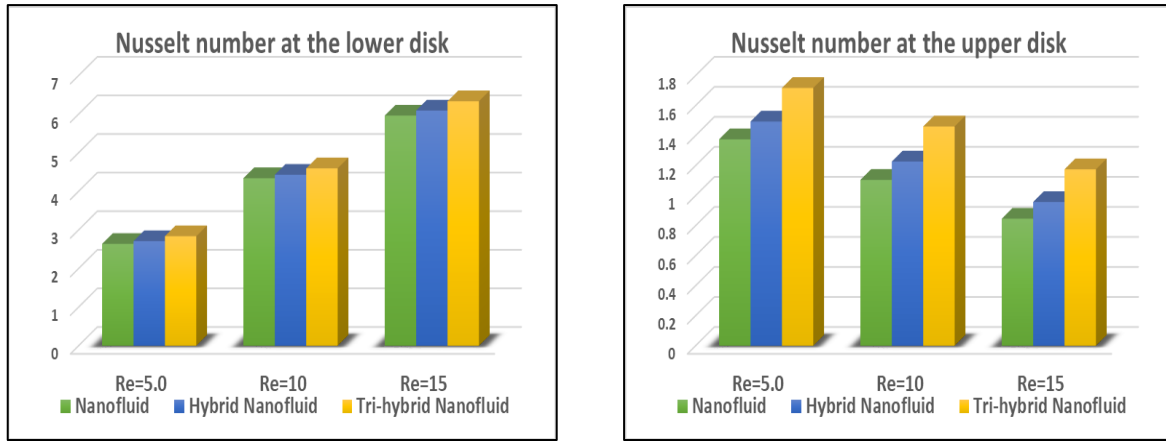


Figure 12: Nusselt number for dissimilar Re -values with $\psi = 1.0$.

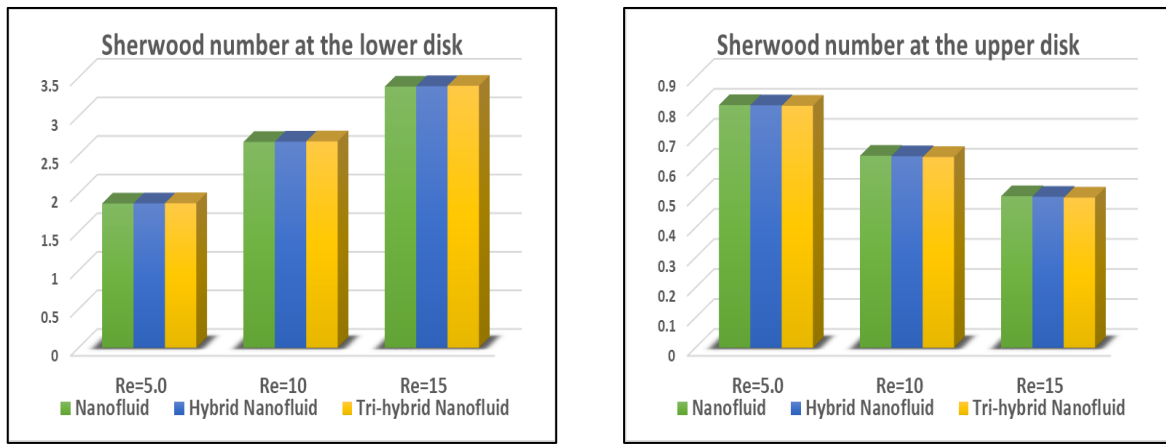


Figure 13: Sherwood number for dissimilar Re -values with $\psi = 1.0$.

Table 5: Influence of stretching and rotation parameters on physical quantities of engineering interest: Tri-hybrid nano-suspension in the presence of spherical nanoparticles.

Parameters			Skin-friction		Nusselt number		Sherwood number	
α_1	α_2	β	C_{f1}	C_{f2}	Nu_1	Nu_2	Sh_1	Sh_2
0.1	0.1	0.1	1.6899	3.5538	1.4047	1.9524	1.205	1.0166
0.3			2.7817	4.2737	1.6329	1.7803	1.2776	0.96972
0.5			3.9936	4.961	1.8748	1.617	1.3524	0.92426
	0.1		3.8836	3.1254	2.4972	1.0287	1.524	0.75577
	0.4		4.9513	4.9673	2.2207	1.3431	1.4525	0.84828
	0.6		5.6334	6.2851	2.0365	1.5925	1.4065	0.91273
		0.2	6.2109	7.7405	1.8922	1.8167	1.3627	0.97842
		0.4	6.2899	7.6766	1.8548	1.8728	1.3619	0.97924
		0.6	6.3908	7.5367	1.8098	1.949	1.3604	0.9807

Table 6: Nusselt and Sherwood numbers for spherical and non-spherical nanoparticles: Tri-hybrid nano-suspension.

Nanoparticles shapes		Sphere	Cylinder	Platelet	Brick	Blade
Nusselt number	Nu_1	1.8748	1.8681	1.9076	1.8437	2.0777
	Nu_2	1.617	1.8259	1.894	1.7217	2.0265
Sherwood number	Sh_1	1.3524	1.3491	1.3491	1.3501	1.3505
	Sh_2	0.92426	0.92942	0.92952	0.92793	0.92717

Table 7: Influence of activation and dissipative energies parameters on physical quantities of engineering interest: Tri-hybrid nano-suspension in the presence of spherical nanoparticles.

Parameters			Nusselt number		Sherwood number	
E_n	Br	γ	Nu_1	Nu_2	Sh_1	Sh_2
1.0	0.3	0.1	1.8748	1.617	1.3524	0.92426
1.5			1.8748	1.617	1.2523	0.97228
2.0			1.8748	1.617	1.1874	1.0039
	0.5		1.6808	1.7275	1.1878	1.0036
	1.0		1.1957	2.0037	1.1889	1.003
	1.5		0.71066	2.2799	1.19	1.0023
		0.3	0.71066	2.2799	1.242	0.98395
		0.6	0.71067	2.2799	1.3315	0.95341
		0.9	0.71067	2.2799	1.4304	0.92063

The tabulated data given in Table 5 show that increasing α_1 enhances the skin-friction coefficients at both disks, indicating higher wall shear, while simultaneously increasing heat and mass transfer at the lower-disk (Nu_1, Sh_1) and reducing them at the upper-disk (Nu_2, Sh_2). An increase in α_2 also leads to higher skin friction at both disks, but it causes a decrease in heat and mass transfer at the lower-disk and an improvement at the upper-disk, highlighting an opposite transfer behavior between the two surfaces. Moreover, increasing β outcomes in a visible rise in skin-friction coefficients, whereas both Nusselt numbers show a slight decline, implying reduced heat transfer, and the Sherwood numbers indicate a marginal decrease at the lower-disk with a slight increase at the upper-disk. Overall, the parameters significantly influence shear stress, heat-transport, and mass-transport, with distinct and asymmetric effects at the upper and lower-disks. The tabulated results given in Table 6

illustrate the effect of nanoelement shape on heat/mass transport characteristics at both disks. Among the considered shapes, the blade-shaped nanoelements yield the highest Nusselt numbers at the upper and lower-disks, indicating superior heat-transport performance compared to spherical, cylindrical, platelet, and brick shapes. Platelet-shaped nanoelements also show relatively enhanced heat-transport, whereas brick-shaped nanoparticles exhibit the lowest Nusselt numbers, reflecting weaker thermal performance. In contrast, the Sherwood numbers vary only slightly with particle shape, suggesting that nanoparticle geometry has a marginal influence on mass-transport; however, the spherical shape gives the highest mass-transport at the lower-disk, while the cylinder and platelet shapes slightly enhance mass-transport at the upper-disk. Overall, the data indicate that nanoparticle shape plays a more pronounced role in heat-transport enhancement than in mass-transport. Table 7 demonstrates the effects of the parameters E_n , Br , and γ on heat/mass transport at the upper and lower disks. As E_n increases, the Nusselt numbers at both disks remain unchanged, indicating that heat transfer is insensitive to E_n , while the Sherwood number at the lower-disk decreases and that at the upper-disk increases, reflecting a redistribution of mass-transport. Increasing the Brinkman number leads to a significant reduction in the lower disk Nusselt number and a pronounced increase in the upper disk Nusselt number, highlighting the strong influence of viscous dissipation on heat-transport, whereas the Sherwood numbers remain nearly constant. Finally, increasing γ has no effect on the Nusselt numbers but enhances mass-transport at the lower disk and reduces it at the upper disk, indicating opposite mass-transfer responses at the two disks.

5. Conclusions

This study investigated transformer-oil-based tri-hybrid nanomaterial flow between revolving stretchable disks, incorporating activation energy, dissipation, radiation, and magnetic effects. Non-spherical and spherical shapes of nanoelements are also completely analyzed. Similarity transformations converted the leading equations into a nonlinear dimensionless system, which was simulated using the bvp4c solver. The results provided useful insights for improving thermal management, mechanical efficiency, and electrical system performance. The key findings of this contribution are précised as follows:

- Nanoparticle shape strongly affects heat-transport, with blade shapes maximizing Nusselt numbers, while mass-transport varies slightly across shapes overall.
- Increasing E_n redistributes mass-transport, Br strongly alters heat-transport between disks, while γ affects only mass-transport oppositely.

- The lower disk exhibited stronger heat/mass transport, and the tri-hybrid nanofluid consistently outperformed hybrid and simple nanofluids.
- Increasing Reynolds number enhanced skin friction and heat/mass transfer rates while reducing thermal and solutal boundary-layer thicknesses.
- Variations in upper disk stretching parameter, magnetic field, and rotation significantly modified the velocity components, generally suppressing axial flow and enhancing tangential motion.

The present study has practical applications in advanced cooling systems, microfluidic devices, chemical reactors, and enhanced heat-transport technologies, where improved thermal conductivity and controlled reaction rates are essential. However, the research is limited by assumptions such as idealized disk geometry, steady-state flow, and neglect of turbulence, which may reduce real-world applicability. Future research could explore unsteady and turbulent flows, experimental validation, optimization of nanoparticle combinations, and the influence of electric fields to broaden the understanding and practical implementation of tri-hybrid nanofluids.

CRedit authorship contribution statement

“**M. Faisal:** Writing - review & editing, Writing - original draft, Investigation, Formal analysis, Conceptualization, Validation, Supervision. **Z. Mushtaq:** Writing - original draft, Visualization, Methodology. **W. Ibrahim:** Writing - review & editing, Writing - original draft, Investigation, Conceptualization, Methodology. **IA. Badruddin:** Writing - review & editing, Software, Resources, Visualization, Formal analysis. **ASAH Zedan:** Writing - review & editing, Conceptualization, Project management, Visualization.”

Data availability statement: “This manuscript does not report data generation or analysis.”

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Declarations

Conflict of interest: “The Authors declare that there are no conflicts of interest that could potentially influence the impartiality of our research or the reporting of our findings.”

Research on both animals and humans: Not applicable.

Clinical trial: Not applicable.

Consent to participate: Not applicable.

Ethics and consent to publish: Not applicable.

Ethics, consent to participate, and consent to publish: Not applicable.

Ethical statement

“The manuscript submitted to the journal is original and must not have been published or submitted elsewhere.”

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