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Bayesian Morphometric Analysis of Recurring Esthetic Dental (RED) and Golden Proportions in the Maxillary Anterior Dentition using AutoCAD software

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TITLE PAGE

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Abstract

Objective

The purpose of the study was to approximate the population distribution of Recurring Esthetic Dental (RED) and Golden proportions in the maxillary anterior teeth of Pakistan adults by use of Bayesian statistical statistics. Precisely, the objectives of the study were the estimation of population mean of the ratio of the RED, the uncertainty of RED ratios, the ability to compare the ratios with the estimated range suggested by Ward, the prevalence of the Golden Ratio, and sexual dimorphism in the esthetic parameters.

Materials and Methods

The study involved a descriptive cross-sectional research design and was carried out on 28 young adult Pakistani respondents (13 males and 15 females; age 18-30 years) with naturally matched dentitions. Standardized frontal smile photographs were acquired and calibrated using an L-shaped scale. Mesiodistal widths and cervico-incisal heights of maxillary central incisors (CI), lateral incisors (LI), and canines (Ca) were measured digitally using AutoCAD linear dimension tool. RED ratios (LI/CI, Ca/LI) and Golden Proportion conformity (ratios within 0.618 ± 10%) were calculated. Bayesian logistic regression models were used to estimate population parameters. Informative priors based on the existing morphometric literature were used to strongly address the small sample size.

Results

The Bayesian posterior mean for the LI/CI RED ratio was 0.69 (95% Credible Interval [CrI]: 0.65–0.72), with a probability of around1.00 (translating to 100%) falling within the classic 0.60–0.80 RED range. Conversely, the Ca/LI ratio mean was 0.81 (95% CrI: 0.77–0.87), significantly exceeding the standard RED limit. The Golden Proportion was observed in 50% (95% CrI: 33–

68%) of LI/CI pairs but only 17% (95% CrI: 6–32%) of Ca/LI pairs. While females in this specific cohort exhibited larger absolute central incisor widths than males (mean difference +1.46 mm), the esthetic ratios showed no statistically significant sexual diversity.

Conclusions

The Golden Proportion is not a universal standard for the Pakistani anterior dentition. The RED proportion concept is valid on the central-to-lateral incisor transition but needs to be adapted to the lateral-to-canine transition that is generally broader in this population irrespective of gender.

Keywords

Bayesian Analysis, Sexual Dimorphism, Dental Esthetics.

Introduction

The rehabilitation of maxillary anterior dentition represents an interesting intersection of biomedical engineering and dental artistic perception. Here the goal is to create a smile that is not only functionally viable but esthetically harmonious with the patient's facial architecture. In contemporary prosthodontics and restorative dentistry, the concept of "smile design" has evolved from a purely intuitive process to one driven by universal geometric principles and morphometric data.(1) The maxillary anterior teeth, the central incisors, lateral incisors, and canines, serve as the principal determinants of perceived physical attractiveness, influencing psychosocial well-being, self-esteem, and affecting social interaction.(2) As patients demand for highly esthetic, natural-looking smile restorations, clinicians are left with little or no choice than relying on established and universal numerical ratios and visual guidelines to aid in the designing of proportionate and esthetically acceptable anterior restorations.(3)

Historically, the search for a unifying mathematical formula for beauty led to the application of the Golden Proportion (or Divine Proportion) in dentistry. Originally described by Euclid and popularized in the Renaissance, this ratio (approximately 1.618:1 or its inverse 0.618:1) describes a relationship where the ratio of the smaller part to the larger part is the same as the ratio of the larger part to the whole.(4) In 1973, Lombardi proposed the application of the Golden Proportion to dental esthetics, suggesting that in a frontal view, the width of the central incisor should be in Golden Proportion to the lateral incisor, and the lateral incisor to the canine.(5) This theory was further codified by Levin in 1978, who postulated that the "apparent" width of the maxillary anterior teeth, as viewed from the front, should diminish progressively by a factor of 0.618 as one moves distally from the midline.(6)

However, the universality of the Golden Proportion has sustained scrutiny. Anthropometric studies across diverse global populations have consistently failed to demonstrate strict adherence to the 1.618 ratio in natural, esthetically pleasing dentitions.(2,3,7) Variations in ethnic background, arch form, and skeletal class significantly influence tooth positioning and apparent width, rendering a "one-size-fits-all" ratio clinically restrictive. In response to these limitations, Ward introduced the Recurring Esthetic Dental (RED) Proportion.(8,9) Unlike the Golden Proportion, which mandates a fixed ratio of 62%, the RED concept allows for variability, stating simply that the ratio of the widths of successive teeth (LI/CI and Canine/LI) should remain *constant* within a given individual, typically falling between 60% and 80%.

Despite the theoretical appeal of the RED proportion, empirical evidence regarding its prevalence remains equivocal. Studies conducted in European, East Asian, and North American populations have reported significant deviations from constant RED ratios, often finding that the canine-to-lateral incisor ratio exceeds the lateral-to-central incisor ratio.(2,8,10) For instance, recent investigations in Kenyan populations revealed that RED proportions were not constant but increased distally, suggesting that the canine occupies a broader visual space in certain ethnic groups than the RED theory predicts.(11) Furthermore, data specific to South Asian populations—and specifically the Pakistani demographic—is remarkably sparse. Existing literature in this region is often limited by small sample sizes, frequentist statistical approaches that are prone to overfitting in small cohorts, and a lack of rigorous standardization in photographic analysis. (12,13)

To address these gaps, this study employs a Bayesian statistical framework. Unlike traditional frequentist null-hypothesis significance testing (NHST), which can be unstable and difficult to

interpret with small sample sizes (N < 30), Bayesian analysis allows for the formal integration of prior clinical knowledge (derived from systematic reviews and previous studies) with current data to generate robust posterior probability distributions.(14–16) This approach provides clinicians with intuitive answers, such as the direct probability that a patient's tooth proportions will fall within a specific esthetic range, rather than abstract p-values. (16) The primary aim of this study was to estimate the population distribution of RED and Golden proportions in the maxillary anterior teeth of young Pakistani adults using Bayesian modeling. The secondary aim of this study was to determine gender-related disparities in LI/CI and Canine/LI RED ratios.

Methods

Study Design and Participants

A descriptive cross-sectional investigation was conducted at the Department of Prosthodontics, Islamic International Dental College and Hospital, Islamabad, Pakistan. The study protocol was approved by the Institutional Ethical Review Committee (ERC), Islamic International Dental College & Hospital, Riphah International University, Islamabad, Pakistan (Ref. No: IIDC/IRC/2024/011/006) and adhered to the Declaration of Helsinki. Written informed consent was obtained from all participants prior to their participation. A convenience sample of 28 young Pakistani adults (13 males, 15 females; age 18–30 years) was recruited. Inclusion in this study required a full complement of natural maxillary anterior teeth (right canine to left canine), clinically healthy periodontium, and well-aligned dentition. Those with a history of orthodontic treatment, anterior restorations, crowding, spacing, craniofacial anomalies, or evidence of altered passive eruption were excluded from the study.

Image Acquisition and Measurement

Standardized frontal smile photographs were acquired using a DSLR camera (Canon EOS 200D) with a macro lens (Canon EF-S 18-55mm) and ring flash. Participants were positioned with the Frankfort horizontal plane parallel to the floor. A metallic L-shaped reference scale (10 mm x 10 mm) was placed in the focal plane of the central incisors for calibration. Images were analyzed using AutoCAD software (Autodesk Inc., USA). The apparent mesiodistal width (W) and cervico-incisal height (H) of the maxillary central incisors (CI), lateral incisors (LI), and canines (Ca) were measured to the nearest 0.01 mm.

Participants were asked to produce a natural posed smile, ensuring full exposure of the maxillary anterior teeth. An L-shaped metal scale with a known length was placed in the same plane as the labial surfaces of the maxillary anterior teeth to allow subsequent calibration (Figure 1). All photographs were taken by a single trained operator under uniform lighting conditions.



Fig. 1: A standard close-up photograph of a participant showing maximum smile.

Image calibration and measurements

Digital images were exported in high-resolution JPEG format and imported into AutoCAD (Autodesk Inc., USA) for measurement. Each image was first calibrated using the L-shaped scale. The length of the scale on the digital image was set equal to its true physical dimension, thereby defining the pixel-to-millimeter conversion factor for that image.

For each participant, the following measurements were obtained for the maxillary right canine, lateral incisor, central incisor, and their left-side counterparts: Mesiodistal width (W)-the linear distance between the most mesial and distal contact points of each clinical crown as visualized in the frontal image. Cervico-incisal height (H)-the linear distance from the gingival zenith at the mid-facial aspect of the crown to the incisal edge (Figure 2).

Measurements were taken using the AutoCAD linear dimension tool with magnification as needed. All measurements were recorded at two decimal places in millimeters.

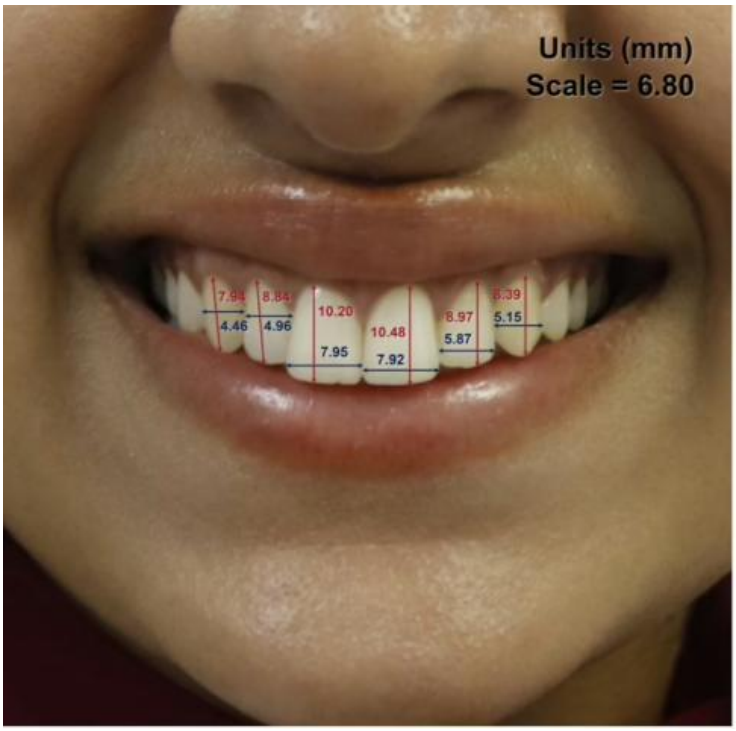


Fig. 2: Measurement of a participant's cervico-incisal height and mesio-distal width while smiling.

Bayesian Statistical Analysis

Bayesian inference was used to estimate population parameters and quantify uncertainty. This approach integrates likelihood (data) with prior distributions (previous knowledge) to produce a posterior distribution. Models were implemented using Python (PyMC v5.0) and ArviZ.(17)

Priors utilized in the study and their justifications

Informative priors were constructed based on a systematic review of anterior tooth proportions (9) and ethnic-specific data from Saudi and Asian populations. (18) The specific parameters and priors used in the analysis are detailed in Table 1.

Table 1: Parameters, Prior Distributions, and Interpretation for Bayesian Analysis

Parameter	Prior Distribution	Interpretation and Justification
μ* **-RED-LI/CI**	Normal (0.73, 0.05^2), truncated to [0, 1]	Prior Mean: 0.73. Based on Ahmed et al. (18), who reported a mean Right CI§ -to-LI‡ ratio of 72.84% (SD¶ 4.64%) in a systematic review of natural dentitions. This reflects the global tendency for the LI‡ to be roughly 70-73% of the CI§ width.

μ* -RED-Ca‖/LI	Normal (0.83, 0.06^2), truncated to [0, 1]	Prior Mean: 0.83. Based on Ahmed et al. (18), who reported a mean Right Ca‖--LI‡ ratio of 83.10% (SD¶ 6.30%). This informative prior accounts for the expectation that the distal transition is naturally broader than the RED theory suggests.
σ† -RED	Half-Normal (0, 0.10)	Represents the residual standard deviation (variability) expected in the population ratios. A weakly informative prior allowing the data to determine the spread.
μ* - Golden	Normal (0.62, 0.08^2), truncated to [0, 1]	Prior Mean: 0.62. Centered on the theoretical Golden Proportion (0.618) with moderate uncertainty to test conformity. This serves as a reference point to test if the population data pulls the posterior away from this "ideal". (15)
β-Gender-RED**	Normal (0, 0.05^2)	Gender: Weakly informative prior centered at zero (no difference), reflecting mixed evidence in morphometric literature regarding sexual dimorphism in *proportions* (as opposed to absolute dimensions). (19)

213 * Mean.
214 † Standard deviation (presented symbolically).
215 ‡ Lateral Incisor.

§ Central Incisor.
‖ Canine.
¶ Standard Deviation (presented as an abbreviation).
**regression coefficient of sex effect on probability

Models

1. **Continuous RED Ratios:**

For each subject i, the averaged RED ratios were analyzed using a normal regression model:

$$y_{ik} \sim \text{Normal}(\mu_k + \beta_k \cdot \text{Female}_i, \sigma_k)$$

where:

- y_{ik} is the observed RED ratio for subject $i$and ratio type k(LI/CI or Canine/LI),
- μ_k is the mean RED ratio in males,
- β_k represents the mean difference between females and males,
- σ_k is the residual standard deviation, and
- Female_i is coded 1 for female and 0 for male.

2. **Golden Proportion Prevalence:**

For each definition d of Golden proportion (CI–LI, LI–canine, full sextant) and subject i , the binary indicator $G_i^{(d)}$ was analysed using a logistic regression model:

$$G_i^{(d)} \sim \text{Bernoulli}(p_i^{(d)})$$

$$\text{logit}(p_i^{(d)}) = \alpha_d + \gamma_d \cdot \text{Female}_i$$

where:

- $p_i^{(d)}$ is the probability that subject i exhibits Golden proportion under definition d ,
- α_d is the intercept (log-odds of Golden proportion in males),
- γ_d is the log-odds difference between females and males.

Model fitting, diagnostics, and sensitivity analyses

All Bayesian models were implemented in Python version 3.12 using the PyMC library. Markov chain Monte Carlo sampling with the No-U-Turn Sampler (NUTS) was used, with at least four chains, 2000 warm-up iterations, and 2000 posterior iterations per chain. Convergence was assessed using R-hat values (<1.01), effective sample sizes, and inspection of trace plots.

Posterior predictive checks were performed using ArviZ (17) to evaluate the adequacy of model fit by comparing simulated data from the posterior predictive distribution with observed data. Sensitivity analyses with slightly wider priors (for example, larger prior standard deviations for location parameters) were undertaken to assess robustness of the main findings.

Analyses were summarized as posterior medians or means with 95% CrIs. Instead of p-values, results are described in terms of posterior probabilities of clinically meaningful statements, such as the probability that the mean RED ratio lies within a predefined esthetic range.

Results

Descriptive Dimensions of the Study Population

The study sample consisted of 28 participants (15 females, 13 males). The descriptive statistics for tooth dimensions are given in **Table 2a**, while derived ratios are displayed in **Tables 2b and 2c**.

Table 2a: Tooth width comparison by gender

Tooth Type	Gender	N	Mean (mm)	SD§	Median	Min	Max	Mean Diff (F-M)
Left CI*	F	15	9.79	1.45	10.04	7.40	12.34	**+1.46**
	M	13	8.33	0.95	8.22	6.98	11.13	
Left LI †	F	15	6.53	0.90	6.53	4.84	8.84	+0.09
	M	13	6.44	0.83	6.62	4.53	7.35	
Left Ca‡	F	15	5.31	0.78	5.31	4.08	6.76	+0.31
	M	13	5.00	0.38	5.00	3.56	5.55	
Right CI*	F	15	9.60	1.14	9.98	7.76	11.18	**+1.20**
	M	13	8.40	0.90	8.23	6.97	11.08	
Right LI†	F	15	6.91	1.02	6.78	4.96	9.05	+0.71

	M	13	6.20	0.91	6.92	4.42	7.96	
Right Ca‡	F	15	5.52	0.79	5.21	4.46	6.91	+0.71
	M	13	4.81	0.75	4.62	4.17	7.34	

*Central Incisor.
† Lateral Incisor.
‡ Canine.
§ Standard Deviation (presented as an abbreviation).

Gender-Based Tooth Width Comparison (Table 2a)

Analysis of the raw tooth dimensions revealed a specific trend within this Pakistani cohort: females exhibited larger absolute tooth widths than males, particularly in the maxillary central incisors where the mean female-male difference exceeded 1.20 mm and reached up to 1.46 mm. However, despite the absolute size difference, the relative progression of widths from central to canine remained consistent across both groups, warranting further analysis of the ratios.

Table 2b: RED ratios by gender

Variable	Gender	N	Mean	SD§	Median	Min	Max	Statistical Note
Right Ca*/LI †	F	15	0.81	0.15	0.84	0.56	1.06	Similar
	M	13	0.82	0.18	0.78	0.60	1.24	
Right LI †/CI‡	F	15	0.73	0.12	0.72	0.59	1.09	Similar
	M	13	0.71	0.08	0.72	0.55	0.91	

Left Ca*/LI †	F	15	0.84	0.17	0.85	0.46	1.19	Similar
	M	13	0.83	0.16	0.81	0.48	1.11	
Left LI †/CI‡	F	15	0.68	0.13	0.67	0.46	1.05	Similar
	M	13	0.70	0.09	0.68	0.59	0.90	

*Canine.
†Lateral Incisor.
‡Central Incisor.
§ Standard Deviation (presented as an abbreviation)

RED Ratios across Genders (Table 2b)

Raw RED width ratios for lateral incisor to central incisor (LI/CI) and canine to lateral incisor (Ca/LI) were centered near the prespecified RED priors and were notably similar between genders. The averaged LI/CI ratio was 0.68–0.73 for females and 0.70–0.71 for males, aligning well with the 70% RED guideline. However, the Ca/LI ratios were systematically higher (0.81–0.84), exceeding the standard 80% limit. This indicates that while the central-lateral transition follows the RED rule, the canine is visually broader in this population than the RED concept predicts.

Table 2c: Height/Width ratios by Gender

Variable	Gender	N	Mean	SD*	Median	Min	Max	Gradient

(H/W)								Trend
Central Incisor	F	15	1.15	0.14	1.16	0.95	1.40	Lowest Ratio (Squarest)
	M	13	1.18	0.11	1.16	1.07	1.39	
Lateral Incisor	F	15	1.44	0.23	1.41	1.10	1.95	Intermediate
	M	13	1.48	0.16	1.47	1.18	1.77	
Canine	F	15	1.75	0.35	1.71	1.28	2.52	Highest Ratio (Narrowest)
	M	13	1.94	0.45	1.89	1.35	2.67	

*Standard Deviation (presented as an abbreviation)

Height–Width Ratios Across Genders (Table 2c)

Height-to-width ratios followed the expected tooth-type gradient, being largest for canines (indicating a narrower appearance) and smallest for central incisors. Mean ratios were approximately 1.7–1.9 for canines, 1.4–1.5 for lateral incisors, and ~1.15–1.18 for central incisors in both genders. This confirms that while absolute dimensions may vary (Table 2a), the relative shape characteristics (aspect ratios) are preserved across the population.

Bayesian Analysis of RED Ratios

The Bayesian posterior estimates provided probabilistic evidence for the RED ratios in the Pakistani population (Table 3). These estimates especially catered to:

LI/CI Ratio: The pooled posterior mean was **0.69** (95% CrI: 0.65–0.72). The probability that the population mean lies within Ward's RED range (0.60–0.80) was **1.00** (100%).

Ca/LI Ratio: The posterior mean was **0.81** (95% CrI: 0.77–0.87). The probability of this ratio falling within the RED range was only **0.2** (27.9%), indicating a high likelihood that the canine is proportionately wider than the RED theory predicts.

Table 3: Bayesian Posterior Probabilities for RED Ratios- Overall and Gender based

Overall and gender-based Ratio	Posterior Mean	Posterior Median	95% CrI§ Lower	95% CrI§ Upper	Pr‖ ()
Posterior summaries for overall RED means (pooled genders)					
LI*/CI†	0.69	0.69	0.65	0.72	1.00
Ca‡/LI*	0.81	0.82	0.77	0.87	0.28
Posterior summaries for gender effects on RED ratios					
LI*/CI†	-0.006	-0.005	-0.06	0.06	0.43
Ca‡/LI*	0.01	0.01	-0.06	0.07	0.62

*Lateral Incisor.
†Central Incisor
‡Canine
§ Credible Interval.
‖Probability.

Golden Proportion Prevalence

The analysis of the Golden Proportion prevalence, utilizing a tolerance of ±10% around the 0.618 ratio, revealed a progressive decline in adherence from the midline toward the distal aspects of the dental arch. The posterior mean prevalence for the lateral-to-central incisor (LI/CI) relationship was 50.0% (95% CrI: 33–68%). This frequency decreased significantly in the canine-to-lateral

incisor (Ca/LI) relationship, which demonstrated a prevalence of 16.7% (95% CrI: 6–32%). Furthermore, the probability of the entire anterior sextant following the Golden Proportion as a cohesive unit was found to be only 3.3% (95% CrI: 0.1–12%). (Table 4)

Table 4: Summary of findings of Golden Proportion Prevalence

Anatomical Relationship	Posterior Mean Prevalence	95%CrI §
LI*/CI†	50.0%	33% – 68%
Ca ‡/LI*	16.7%	6% – 32%
Full Anterior Sextant	3.3%	0.1% – 12%

*Lateral Incisor.
†Central Incisor.
‡Canine.
§Credible Interval.

Combined boxplots illustrating gender-based distributions of cervico-incisal heights, mesiodistal widths, width/height ratios, and RED ratios are shown in Figure 3.

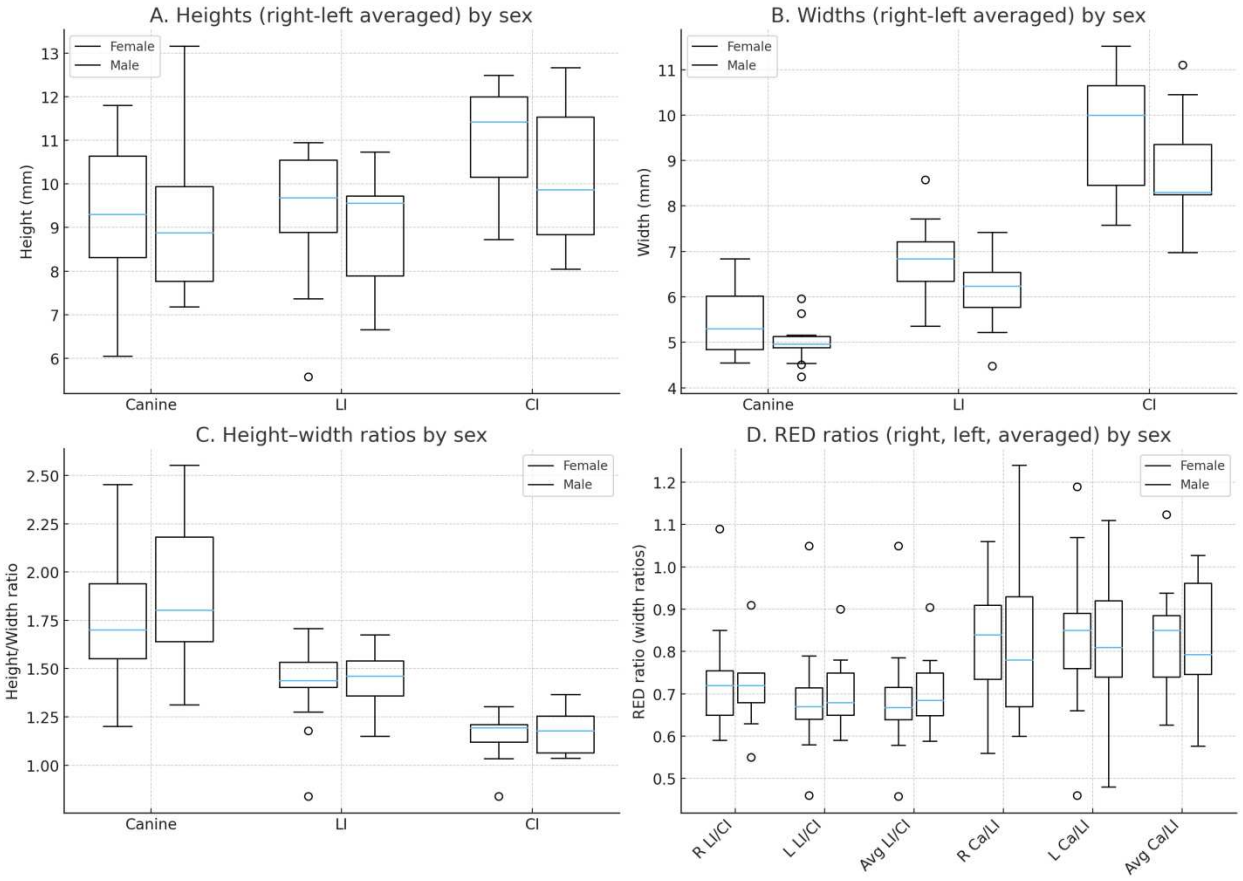


Fig. 3 Combined boxplots of morphometric data by gender. (A) Cervico-incisal heights. (B) Mesiodistal widths. (C) Width/Height ratios. (D) RED Ratios (LI/CI and Ca/LI). The central line represents the median; box edges represent the interquartile range (IQR), and whiskers extend to 1.5x IQR.

Bayesian estimates of overall RED ratios

Using the pooled RED ratios, the Bayesian model estimated the population's mean LI/CI RED ratio to be 0.69 (posterior median 0.69; 95% CrI: 0.65–0.72). The population mean Canine/LI RED ratio was higher at 0.81 (posterior median 0.82; 95% CrI: 0.77–0.87). Posterior probabilities of the population mean lying within the 0.60–0.80 “classical” RED range were calculated from the proportion of posterior draws in this interval. For LI/CI, the probability that the population mean

lies in the 0.60–0.80 band was 1.00 (translated to a probability of 100%), indicating very strong support that the true mean LI/CI ratio conforms to the conventional RED guideline. In contrast, the corresponding probability for the Canine/LI mean was 0.28, reflecting that the posterior distribution of Canine/LI largely extends above the classical RED range (Table 3).

Posterior density curves of LI/CI and Canine/LI RED ratios

These are depicted in figure 4 in which the LI/CI distribution falls almost entirely within the Ward's classic RED range, whereas the Canine/LI distribution exceeds the 0.80 threshold of the proposed range. (9)

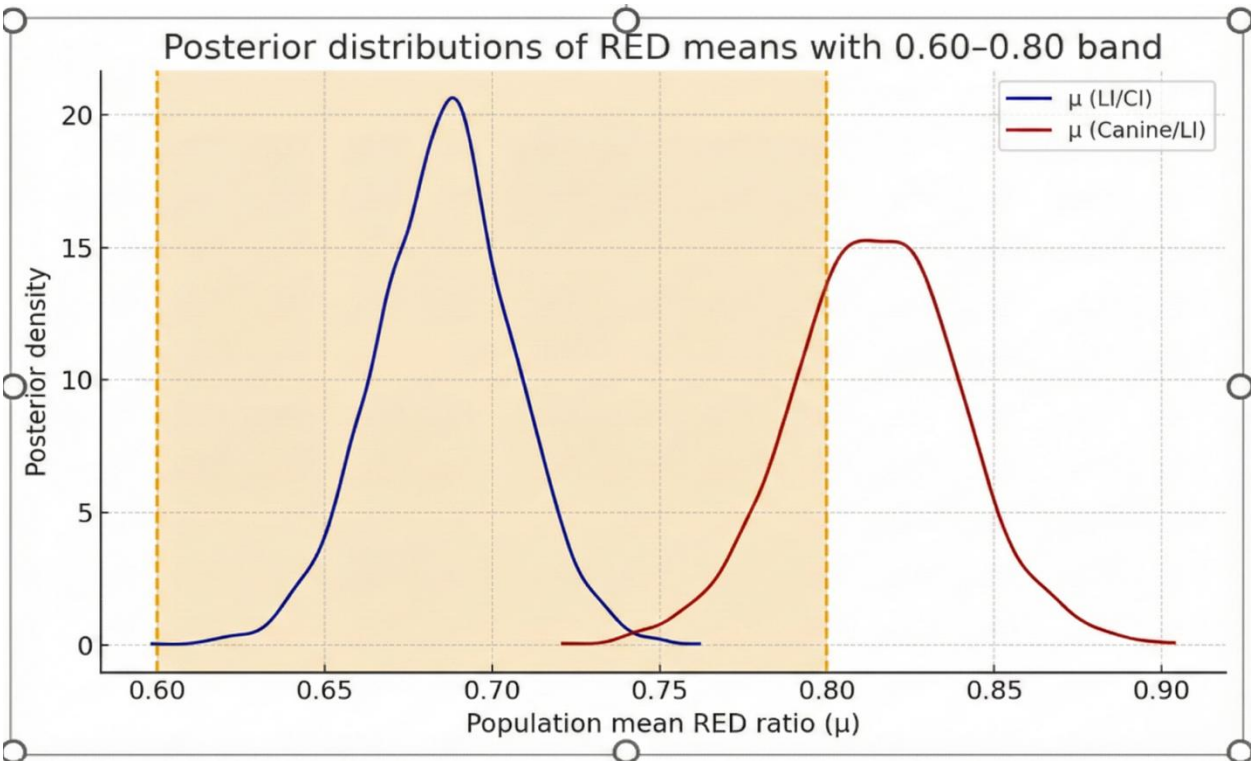


Fig. 4 Posterior density curves of LI/CI and Canine/LI RED ratios. The shaded vertical band represents Ward's classic RED range (0.60–0.80). The LI/CI distribution falls almost entirely within the range, whereas the Canine/LI distribution shifts significantly to the right,

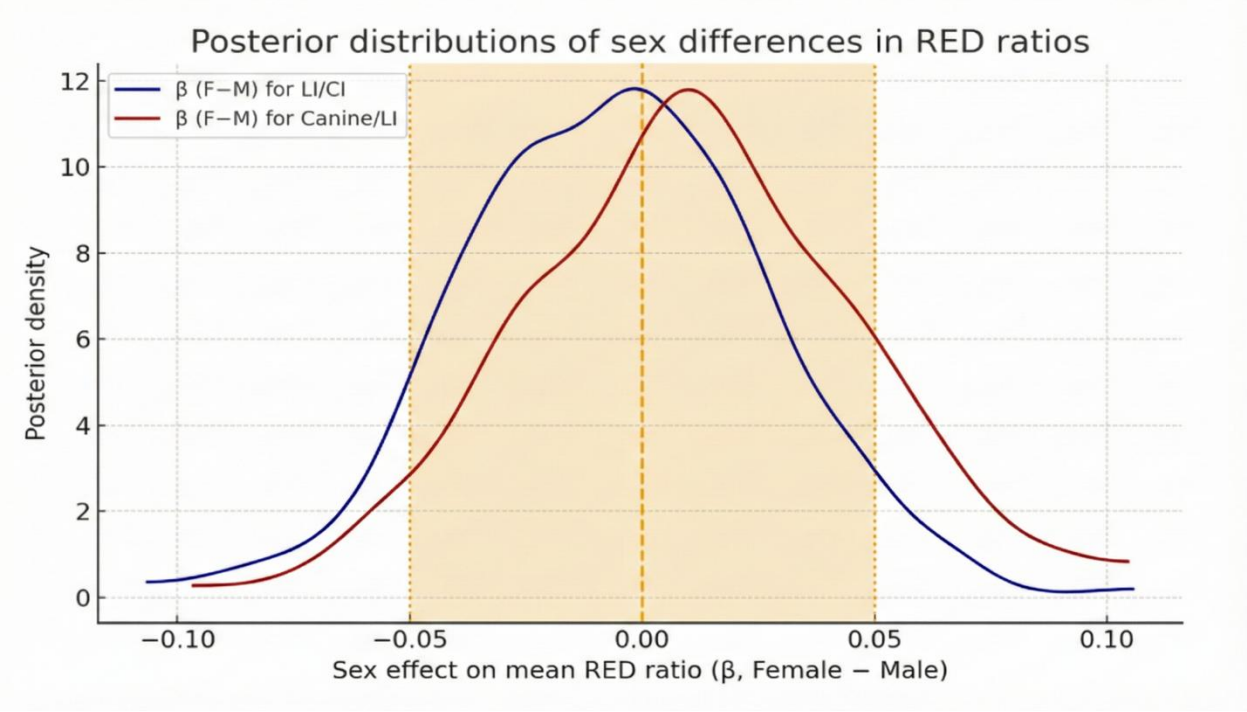
366 exceeding the 0.80 threshold.

367

368 **Gender differences in RED ratios**

369

370 Bayesian regression models including a gender term (Females *vs.* Males) were fitted for averaged
371 LI/CI and Canine/LI RED ratios. For LI/CI, the posterior mean gender effect was β = −0.006
372 (posterior median −0.005; 95% CrI: −0.07 to 0.06). While β representing the mean difference
373 Female − Male (Figure 5). The posterior probability that females have a higher mean LI/CI ratio
374 than males, Pr(β>0), was 0.43. For Canine/LI, the posterior mean effect was β = 0.01 (median 0.01;
375 95% CrI: −0.06 to 0.07), with Pr(β > 0) = 0.62. In both cases, the posterior distributions were
376 narrowly concentrated within approximately ±0.05 units of zero, with 95% credible intervals
377 encompassing zero.



378

Fig 5. Posterior density curves for β(LI/CI) and β(Canine/LI)

1. A vertical dashed line at 0 marks no gender difference, and the (yellow) shaded "region of practical equivalence" (ROPE) from −0.05 to +0.05 indicates small differences. Both curves lie largely within this band

2. ROPE represents differences considered small for RED ratios. LI = lateral incisor; CI = central incisor; Ca = canine; β = female − male difference in RED ratio.

Bayesian prevalence of Golden proportion

Using the continuous width ratios in the current dataset, "Golden proportion" was operationalized as ratios within ±10% of the classic value 0.618 for the averaged LI/CI and Canine/LI ratios. A "full sextant" Golden proportion required both averaged ratios to meet this criterion. For each definition, the prevalence was modeled using a Beta (1,1) prior (uniform over [0,1]), resulting in conjugate Beta posterior distributions (Figure 6).

For the Golden LI/CI definition, 14 of 28 participants met the criterion, giving a posterior mean prevalence of 0.50 with a 95% CrI 0.33–0.67. The posterior probability that prevalence exceeds 10% was essentially 1.00, and the probability that it exceeds 25% was 0.99; the probability that it exceeds 50% was 0.50.

For Canine/LI, only 4 of 28 participants met the criterion of GP, resulting in a posterior mean prevalence of 0.17 (95% CrI 0.06–0.32). The posterior probability that prevalence exceeds 10% was 0.84, but the probability of exceeding 25% fell to 0.12, and the probability of exceeding 50% was essentially 0.

The full sextant Golden proportion (both LI/CI and Canine/LI Golden in the same individual) was not observed in any participant. Under the Beta (1,1) prior, the posterior mean prevalence was 0.03

with a 95% CrI 0.001–0.12. The probability that the true prevalence exceeds 10% was only 0.05, and the probability it exceeds 25% was effectively zero. Bayesian estimates of the prevalence of the Golden proportion are summarized in Table 5.

Table 5: Bayesian prevalence of overall Golden proportion

Definition	Posterior Mean Prevalence	95% CrI‡ Lower	95% CrI‡ Upper	Pr § (p > 0.10)	Pr § (p > 0.25)	Pr § (p > 0.50)
Golden LI* /CI† (avg)	0.50	0.33	0.68	1.00	0.99	0.50
Golden Canine/LI* (avg)	0.17	0.06	0.32	0.84	0.12	0.00
Golden full sextant ‡ (both)	0.03	0.001	0.12	0.05	0.00	0.00

*Lateral Incisor.
†Central Incisor.
‡Credible Interval.
§Posterior Probability Prevalence.

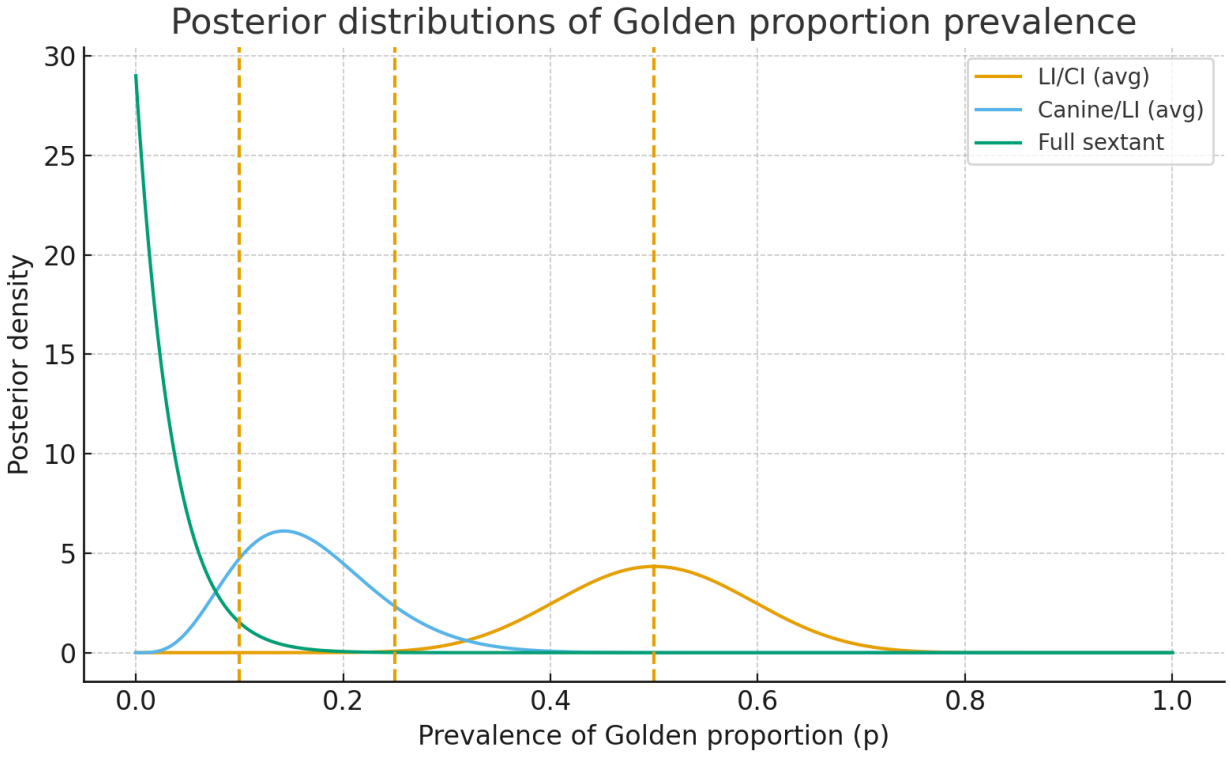


Fig 6. Posterior Beta densities of Golden proportion prevalence.

1. Posterior Beta densities are shown for Golden LI/CI (avg), Golden Canine/LI (avg), and full sextant Golden proportion on a common 0–1 scale. Vertical dashed lines indicate prevalence thresholds at 0.10, 0.25, and 0.50.

2. The LI/CI curve peaks near 0.5. The Canine/LI curve is concentrated between ~0.05 and 0.3. The full-sextant curve is concentrated near zero and declines rapidly. LI = lateral incisor; CI = central incisor; full sextant = both LI/CI and Canine/LI meet the Golden proportion criterion.

3. LI = lateral incisor; CI = central incisor; full sextant = both LI/CI and Canine/LI meet the Golden proportion criterion. Posterior densities represent the distribution of possible prevalence values given the data and uniform Beta (1,1) prior.

Gender differences in Golden proportion prevalence

Using the same Golden-proportion definitions and a Beta (1,1) prior for prevalence in each gender, gender-specific prevalences and posterior odds ratios (ORs; Female vs Male) were estimated (Table 6). For Golden LI/CI, the posterior mean prevalence was 0.59 (95% CrI: 0.35–0.80) in females and 0.40 (95% CrI: 0.18–0.65) in males. The induced posterior distribution for the odds ratio had a median of 2.21, with a very wide 95% CrI from 0.52 to 10.06, and Pr(OR>1) = 0.86. For Golden Canine/LI, the posterior mean prevalence was 0.24 in females (95% CrI 0.07–0.46) and 0.13 in males (95% CrI 0.02–0.34). The corresponding odds ratio had a median of 2.17 (95% CrI 0.33–20.18), with Pr(OR>1) = 0.79. For the full-sextant Golden proportion, no participants met the criterion in either gender. With the Beta (1,1) prior, the posterior mean prevalence was 0.06 in females (95% CrI 0.002–0.21) and 0.07 in males (95% CrI 0.002–0.23). The odds ratio distribution was essentially centered around 1 (median 0.88, 95% CrI 0.02–34.29, Pr(OR>1) = 0.47), indicating no evidence of a gender-specific pattern when Golden proportion is defined at

the sextant level. Figure 7 shows Posterior distributions of odds ratios (Female vs Male) for Golden proportion prevalence

Table 6: Gender-specific prevalence and odds ratios for Golden proportion

Definition	Female Prevalence (Mean)	Female 95% CrI§| Lower	Female 95% CrI§ Upper	Male Prevalence (Mean)	Male 95% CrI§ Lower	Male 95% CrI§ Upper	OR‖ (F vs M) Median	OR‖ 95% CrI§ Lower	OR‖ 95% CrI§ Upper	Pr ¶ (OR‖ > 1)
Golden LI* /CI† (avg)	0.59	0.35	0.80	0.40	0.18	0.65	2.21	0.52	10.06	0.86
Golden Ca‡/LI* (avg)	0.24	0.07	0.46	0.13	0.02	0.34	2.17	0.33	20.19	0.79
Golden full sextant (both)	0.06	0.002	0.21	0.07	0.002	0.23	0.88	0.02	34.29	0.47

*Lateral Incisor.
†Central Incisor.
‡Canine.
§ Credible Interval.
‖ Odds Ratio.
¶ Posterior Probability.

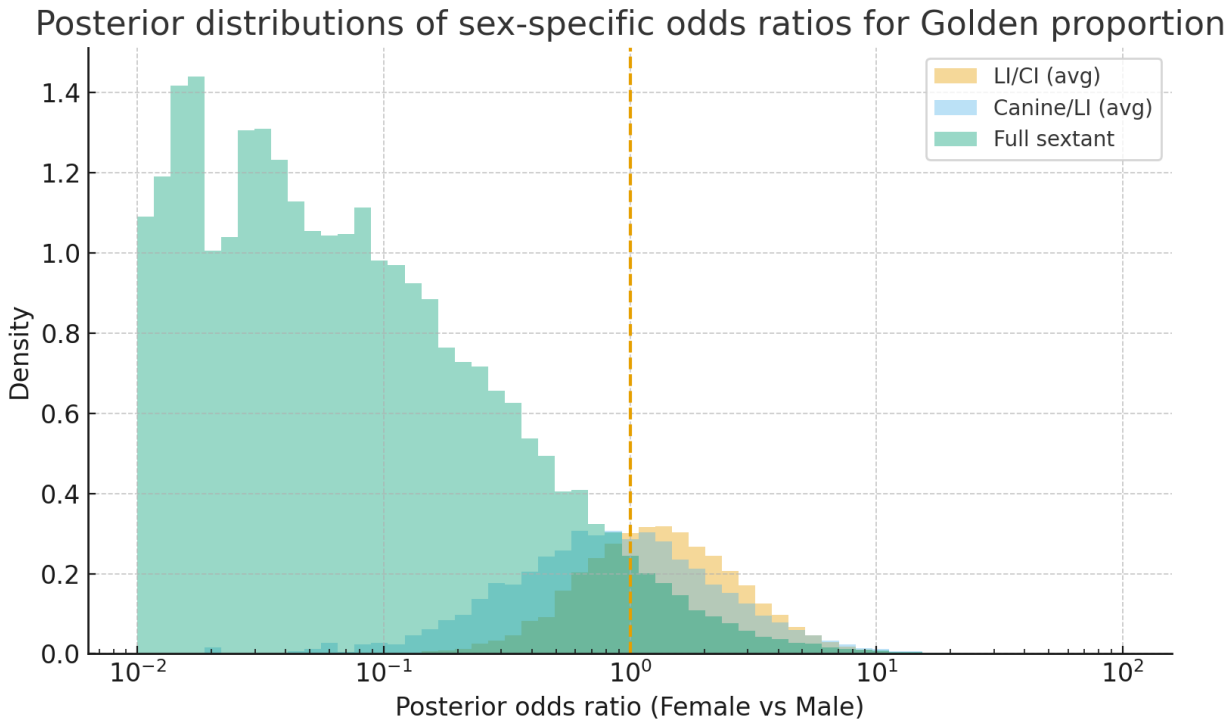


Fig 7. Posterior distributions of odds ratios (Female vs Male) for Golden proportion prevalence

1. Posterior distributions are shown on a logarithmic x-axis for Golden LI/CI (avg), Golden Canine/LI (avg), and full-sextant Golden proportion. OR = odds ratio (Female vs Male). Curves are based on joint sampling from gender-specific Beta posteriors. A vertical line at OR = 1 indicates no gender difference.

2. LI = lateral incisor; CI = central incisor; full sextant = both LI/CI and Canine/LI meet the Golden proportion criterion.

Posterior predictive checks and sensitivity analyses

Posterior predictive checks indicated that the fitted Bayesian models reproduced the observed data well for all key outcomes. For the continuous RED ratios, simulated values from the posterior predictive distributions closely matched the empirical distributions of the averaged LI/CI and Canine/LI ratios, both overall and stratified by gender; overlayed histograms and kernel density

plots showed no systematic misfit in central tendency or spread, and the observed summaries fell comfortably within the simulated reference intervals.

For the binary Golden-proportion indicators (LI/CI, Canine/LI, and full sextant), observed and replicated prevalences showed good agreement, with no evidence of systematic over- or underestimation.

Sensitivity analyses, including modestly broader priors on location parameters and alternative Beta priors for Golden-proportion prevalences, yielded posterior means, 95% credible intervals, and posterior probabilities very similar to the primary analyses.

Detailed posterior predictive plots and sensitivity summaries are provided in the Supplementary Figures (S1) and Tables (S2)

Discussion

This Bayesian morphometric analysis of young adult Pakistani participants offers a nuanced perspective on the applicability of standardized aesthetic tooth proportions. These findings challenge the strict application of the Golden Proportion (62%), particularly within the distal segment of the anterior arch, and suggest that the RED Proportion concept requires modification to be clinically valid for this specific population.

The Golden Proportion: A Failed Standard?

Lombardi and Levin championed the 62% ratio as the hallmark of dental harmony (5,6). However, the findings of the current study align with a growing body of global evidence rejecting the Golden Proportion as a universal biological standard. In this study, while the central-to-lateral relationship

approximated the Golden ratio in 50% of subjects, the lateral-to-canine relationship was significantly broader with a mean ratio of approximately 0.81. This deviation is a clinically critical finding; implying that strict adherence to 62% rule to this population would result in canine designs that would appear unnaturally narrow or "hidden" in the frontal view, potentially collapsing the buccal corridor and resulting in a less dominant, and arguably less esthetic smile. These results however mirror findings of a study conducted by Aldegheishem et al. in the Saudi population and Murthy et al. in India, both of which found the Golden Proportion to be an exception rather than the rule (20,21). Specifically, Aldegheishem et al. noted that the Golden Proportion was not found to exist between perceived widths of maxillary anterior teeth in a Saudi population, a demographic geographically and possibly phenotypically proximal to the study sample of the current study (20). Similarly, but not as significantly, another research in the UK by Kalia demonstrated that slightly modified Golden Percentage values were more representative than the strict Golden Proportion. (10)

The RED Proportion: Valid but Variable

Ward’s RED concept allows for a comparatively flexible, yet constant ratio of anterior teeth, falling between 60% and 80% (7). The current data supports the RED concept partially. The mean lateral-to-central incisor (LI/CI) ratio of 0.69, reported in the current study, sits comfortably within the stated Ward's "normal" range (8,9). However, the hypothesis of "constant" Ward’s ratio breaks down in the distal segment. The Ca/LI ratio of 0.81 observed in the current study dataset, however trivial, is not "recurring" or consistent with the Ward’s LI/CI ratio. This implies that in current study population, the progression of tooth widths does not follow a linear geometric degradation

as proposed by Wards. Instead, the arch form turns more sharply at the canine, or the canine itself is morphologically broader, occupying more visual weight.

This finding is remarkably similar to that of Mosomi et al. in a Kenyan population, where the RED proportion was found to increase distally rather than remain constant.(11) Clinically, this suggests that "Recurring" is a misnomer for these populations and smile designs should perhaps utilize a "Variable Proportion" approach, utilizing approximately 70% for the LI/CI interface and around 80% for the Ca/LI interface. This "70/80" rule appears to be a more biologically faithful and plausible guideline for this specific demographic rather than a single constant universal ratio.

Bayesian Robustness and Methodological strengths

A key strength of this study is its statistical methodology. Dental research often suffers from small sample sizes due to the difficulty of recruiting participants with "perfect" natural dentition. Conventional Frequentist analyses on small sample such as that in the current study (n=28) often yield p-values that are fragile and confidence intervals that are unhelpfully wide. By virtue of the Bayesian methods application, the current study was allowed incorporate prior knowledge robustly credibly and effectively, "borrowing strength" from previous credible evidence to generate robust and stable estimates. In Bayesian language, a finding that there is a 100% posterior probability of the LI/CI ratio falling within the RED range offers a stronger, more clinically actionable conclusion than a traditional p-value < 0.05. It provides the clinician with high certainty that the RED range is applicable to the lateral incisor segment in this population.(14,16,17)

Sexual Dimorphism: Dimensions vs. Proportions

Regarding sexual dimorphism, findings of the current study presented a distinct dichotomy. While absolute tooth dimensions exhibited significant differences, the esthetic proportions remained statistically indistinguishable between genders. (2,13) Contrary to the predominant odontometric pattern where male dimensions exceeded those of females, the current study cohort exhibited reverse sexual dimorphism, with female dental dimensions being larger. This phenomenon has also been documented in other populations and is commonly attributed to population-specific genetic and environmental variation. Sexual dimorphism in tooth size reflects complex interactions between genetic regulation, hormonal influences during odontogenesis, and environmental factors such as nutrition and growth. Differential expression of sex-linked genes, particularly X-chromosome–associated genes involved in enamel and dentin formation, may in some populations contribute to relatively larger tooth dimensions in females, resulting in reverse sexual dimorphism. (22–24) Crucially, despite the variation in the "canvas" size (tooth width), the "composition" (ratios) that postulates that a harmonious smile remained consistent. This finding has significant implications for the digital restorative workflow; it suggests that distinct "male" versus "female" ratio libraries may not be necessarily “uniform” or “constant.” However, in some select cases, owing to unforeseen constraints, a unified proportional framework can likely be applied regardless of gender, thereby simplifying the design process for restorative dentists.

Conclusion

The Golden Proportion is not a universal standard for the Pakistani anterior dentition. The RED proportion concept is valid on the central-to-lateral incisor transition but needs to be adapted to the lateral-to-canine transition that is generally broader in this population irrespective of gender. Clinicians should apply esthetic ratios as flexible guidelines rather than rigid mathematical laws.

This "Variable Proportion" method prevents the formation of unnatural and constrictive buccal corridors when rehabilitating in the esthetic rehabilitation.

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Data availability

The datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

Declarations Ethics approval and consent to participate

The Institutional Ethical *Review Committee* ***(ERC),*** *Islamic International Dental College & Hospital, Riphah International University, Islamabad, Pakistan*, a local independent institutional body, approved the study on 30th November, 2024, prior to its start (Ref. No: IIDC/IRC/2024/011/006). All subjects gave written consent before their participation. All procedures involving human participants were carried out in accordance with the ethical standards of the relevant institutional and/or national research committees and with the 1964 Declaration of Helsinki and its later amendments.

Consent for publication

Written informed consent for publication of the Photographs was obtained from the participants.

Competing interests

The authors declare no competing interests.

Author Contributions

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575 *R.A. performed the main research work, conceptualized the study, conducted data collection,*
576 *assisted in the primary data analysis, interpreted the results, wrote the original manuscript draft*
577 *and reviewed and finalised the final draft, performed the revisions and finalised the revised draft.*
578 *M.A.G.C. supervised the study, approved the research topic, and critically reviewed the*
579 *manuscript and approved the final version.*
580 *S.K.F. assisted with statistical analysis and dataset preparation.*
581 *A.M. contributed to manuscript formatting and reference management.*
582 *H.A. assisted in manuscript writing.*
583 *J.A. supervised the research, performed data analysis and provided guidance in manuscript*
584 *writing, critically reviewed the manuscript, and approved the final version.*
585 *All authors read and approved the final manuscript.*

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