A decoherence resilient quantum memory for topological quantum skyrmions: supplemental document

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1. ROBUSTNESS OF NONLOCAL SKYRMIONS AGAINST WHITE NOISE AND ATOMIC DIFFUSION

We theoretically investigate the resilience of nonlocal quantum skyrmions against three common sources of decoherence in quantum memories: isotropic white noise, atomic diffusion, and magnetic field-induced dephasing. While all mechanisms degrade conventional measures of quantum state fidelity, we demonstrate that the skyrmion number N-a discrete topological invariant-remains unchanged under these perturbations. This highlights the intrinsic topological protection of skyrmionic entanglement, offering advantages for quantum information processing in noisy environments.

(1) White Noise

White noise, such as background daylight or stray laboratory light, is modeled as isotropic noise mixed with the signal state. The stored density matrix becomes:

$$\rho = \eta|\Psi\rangle\langle\Psi| + (1-\eta)\frac{\mathbb{I}_4}{4}, \tag{S1}$$

where $\eta \in [0,1]$ is proportional to the storage efficiency (incorporating noise level), $|\Psi\rangle$ is the ideal skyrmionic state, and $\mathbb{I}_4$ is the identity operator on the bipartite Hilbert space (2D polarization for photon A × 2D OAM subspace for photon B).

The quantum Stokes parameters for the mixed state are:

$$S_i(\mathbf{r}) = \mathrm{Tr}\left[\rho\left(|\mathbf{r}\rangle\langle\mathbf{r}|_B \otimes \sigma_{A,i}\right)\right], \tag{S2}$$

Substituting ρ:

$$S_i(\mathbf{r}) = \eta \cdot S_i^{\text{pure}}(\mathbf{r}) + \frac{1-\eta}{4} \cdot \mathrm{Tr}\left[\mathbb{I}_4\left(|\mathbf{r}\rangle\langle\mathbf{r}| \otimes \sigma_i\right)\right], \tag{S3}$$

Where S_i^{pure} is the Stokes parameters of the pure state $|\Psi\rangle$. Since $\mathrm{Tr}_B\left[\sigma_i\right] = 0$, the noise term vanishes, yielding:

$$S_i(\mathbf{r}) = \eta \cdot S_i^{\text{pure}}(\mathbf{r}), \tag{S4}$$

The skyrmion number calculation requires normalized Stokes vectors: $\mathbf{S}_{\text{norm}}(\mathbf{r}) = \mathbf{S}(\mathbf{r})/|\mathbf{S}(\mathbf{r})|$. With noise:

$$|\mathbf{S}(\mathbf{r})| = \eta \cdot |\mathbf{S}^{\text{pure}}(\mathbf{r})|, \tag{S5}$$

$$\mathbf{S}_{\text{norm}}(\mathbf{r}) = \frac{\mathbf{S}(\mathbf{r})}{|\mathbf{S}(\mathbf{r})|} = \frac{\mathbf{S}^{\text{pure}}(\mathbf{r})}{|\mathbf{S}^{\text{pure}}(\mathbf{r})|}, \tag{S6}$$

The normalization eliminates the η-dependence. Consequently, the skyrmion density is:

$$\Sigma_z(\mathbf{r}) = \mathbf{S}_{\text{norm}} \cdot \left(\frac{\partial \mathbf{S}_{\text{norm}}}{\partial x} \times \frac{\partial \mathbf{S}_{\text{norm}}}{\partial y} \right), \tag{S7}$$

remains identical to the noiseless case. Integration yields:

$$N = \frac{1}{4\pi} \int_{\mathcal{R}^2} \Sigma_z(\mathbf{r}) d^2 r = \Delta\ell \cdot \text{sign}(|\ell_2| - |\ell_1|). \tag{S8}$$

which is independent of η.

Hence, isotropic noise contributes zero expectation value to Stokes parameters, only reducing their magnitude. Normalization discards magnitude information, preserving only the directional field. Thus, while fidelity decay with noise, the topological invariant N remains intact for any $\eta > 0$. Only at the singular point $\eta > 0$ (complete noise) does N become 0.

(2) Atomic motion-induced Diffusion

During storage, thermal motion causes atoms to diffuse with a coefficient D. This blurs the spatial spin-wave pattern encoding the OAM information. The diffusion is modeled by convolution with a Gaussian kernel[1]:

$$G_t(\mathbf{r}) = \frac{1}{4\pi D t} e^{-\mathbf{r}^2/(4Dt)}, \tag{S9}$$

where t is the storage time.

Each OAM mode ℓ_k with radial profile $f_k(r)$ evolves as

$$a_k^{\text{diff}}(\mathbf{r}, t) = [f_k(r) e^{i\ell_k \phi}] * G_t(\mathbf{r}), \tag{S10}$$

For isotropic diffusion, the Gaussian kernel is radially symmetric. Convolution preserves the angular phase factor:

$$a_k^{\text{diff}}(\mathbf{r}, t) = \tilde{f}_k(r, t) e^{i\ell_k \phi}, \tag{S11}$$

where $\tilde{f}_k(r,t) = [f_k * G_t](r)$ is the diffused radial envelope.

The two-photon state becomes:

$$|\Psi_{\text{diff}}\rangle = \int d^2 r\, |\mathbf{r}\rangle \left[\tilde{f}_1(r) e^{i\ell_1 \phi} |H\rangle + \tilde{f}_2(r) e^{i(\ell_2 \phi + \delta)} |V\rangle \right], \tag{S12}$$

Normalizing the conditional polarization state yields:

$$|\psi_A(\mathbf{r})\rangle = \cos\theta_{\text{diff}}(r) |H\rangle + \sin\theta_{\text{diff}}(r) e^{i(\Delta\ell\phi + \delta)} |V\rangle, \tag{S13}$$

where $\tan\theta_{\text{diff}}(r) = \tilde{f}_2(r)/\tilde{f}_1(r)$.

The skyrmion density is:

$$\Sigma_z^{\text{diff}}(\mathbf{r}) = \Delta\ell \cdot \frac{2\sin(2\theta_{\text{diff}})\, \theta'_{\text{diff}}(r)}{r}, \tag{S14}$$

Integrating this density over the whole plane and substituting the explicit form of the Laguerre–Gaussian modes yields the Skyrmion number:

$$N_{\text{diff}} = \frac{\Delta\ell}{2} \int_0^\infty \sin(2\theta_{\text{diff}})\, \theta'_{\text{diff}}(r) dr = \Delta\ell \cdot \text{sign}(|\ell_2| - |\ell_1|) = N. \tag{S15}$$

(3) Magnetic field-induced phase shift

The initial nonlocal skyrmion state is:

$$|\Psi_{\text{initial}}\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_A |\ell_1\rangle_B + e^{i\delta} |V\rangle_A |\ell_2\rangle_B \right), \tag{S16}$$

During storage, both OAM modes ℓ_1 and ℓ_2 are mapped onto the same ground-state coherence between the $|g\rangle$ and $|s\rangle$ states of ^{87}Rb atomic ensemble. For simplicity, we assume the ensemble is subjected to a linear spatial gradient of the magnetic field, given by $B(\mathbf{r}) = B_0 + Gx$, inducing a position-dependent Zeeman phase[2]

$$\Phi(\mathbf{r}) = \frac{\mu_B \Delta g}{\hbar} \int_0^T B(\mathbf{r}, t)\, dt = \gamma(B_0 + Gx)T, \tag{S17}$$

where $\gamma = \mu_B \Delta g / \hbar$ and $\Delta g = g_s M_s - g_g M_g$. Here, μ_B is the Bohr magneton, $\hbar$ is the reduced Planck constant, g_s and g_g are the Landé g-factors corresponding to the specific energy states, M_s and M_g are their respective magnetic quantum numbers.

Each OAM mode acquires the same local phase factor $e^{i\Phi(\mathbf{r})}$, resulting in the stored spin-wave modes:

$$S'_k(\mathbf{r}) = \psi_k(\mathbf{r})\, e^{i\Phi(\mathbf{r})} |g\rangle\langle s|, \quad k = 1, 2, \tag{S18}$$

with $\psi_k(\mathbf{r}) = f_k(r) e^{i\ell_k \phi}$ being the spatial wavefunction of the OAM mode. Although the two OAM modes have different transverse intensity profiles $f_k(r) \neq f_k(r)$, they experience identical local phase modulation $\Phi(\mathbf{r})$ because they share the same atomic transition.

After retrieval, the two-photon state becomes:

$$|\Psi'\rangle = \frac{1}{\sqrt{2}} \int d^2r\, |\mathbf{r}\rangle_A \left[f_1(r) e^{i\Phi(\mathbf{r})} |H\rangle_A + f_2(r) e^{i(\ell_2\phi + \delta + \Phi(\mathbf{r}))} |V\rangle_A \right], \tag{S19}$$

For a given detection position $\mathbf{r}$ of photon B, the unnormalized conditional state of photon A is:

$$|\tilde{\psi}_A(\mathbf{r})\rangle = f_1(r) e^{i\Phi(\mathbf{r})} |H\rangle + f_2(r) e^{i(\ell_2\phi + \delta + \Phi(\mathbf{r}))} |V\rangle, \tag{S20}$$

Factoring out $e^{i\Phi(\mathbf{r})}$:

$$|\tilde{\psi}_A(\mathbf{r})\rangle = e^{i\Phi(\mathbf{r})} \left[f_1(r)|H\rangle + f_2(r) e^{i(\ell_2\phi + \delta)} |V\rangle \right], \tag{S21}$$

Normalization removes the common phase factor $e^{i\Phi(\mathbf{r})}$, yielding:

$$|\psi_A(\mathbf{r})\rangle = \frac{f_1(r)|H\rangle + f_2(r) e^{i(\ell_2\phi + \delta)} |V\rangle}{\sqrt{f_1(r)^2 + f_2(r)^2}}. \tag{S22}$$

Thus, the magnetic phase $\Phi(\mathbf{r})$ cancels out in the normalized conditional state, leaving the skyrmion number N unchanged despite the spatially varying magnetic field.

2. THE ENERGY LEVEL STRUCTURE

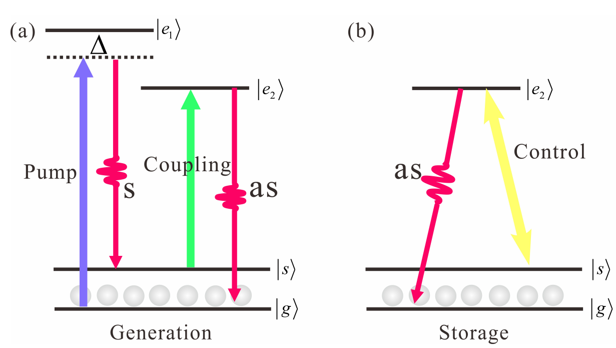

Fig. S1. The energy levels for generating and the storage of non-local skyrmions.

The energy levels for generating and the storage of non-local skyrmions are depicted in Fig. S1. Initially, all atoms are prepared in the ground state $|g\rangle = |5S_{1/2}, F = 1\rangle$. A weak pump laser ($\Omega_p$), red-detuned by 60 MHz from the $|5S_{1/2}, F = 1\rangle \rightarrow |5P_{3/2}, F' = 2\rangle$ transition, excites the atoms to state $|e_1\rangle = |5P_{1/2}, F' = 2\rangle$. Via spontaneous Raman scattering, a low-frequency Stokes (S) photon is emitted, and the atoms decay to $|s\rangle = |5S_{1/2}, F = 2\rangle$. A strong coupling laser (Ω_c), resonant with the $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{1/2}, F' = 2\rangle$ transition, then pumps the atoms to excited state $|e_2\rangle = |5P1/2, F' = 2\rangle$. Through a second spontaneous Raman process, a high-frequency anti-Stokes (AS) photon is emitted, returning the atoms to the initial ground state $|1\rangle$. This process produces photon pairs exhibiting non-classical correlations along all directions satisfying the phase-matching condition. In our experiment, we select two symmetric biphoton modes emitted at $\pm 0.2^\circ$ relative to the long axis of the atomic ensemble.

The electromagnetically induced transparency (EIT) memory [3] energy level is shown in Fig. S1(b). The AS photon can be stored and retrieved by switching on and off the Control beam, which have the same frequency as the Coupling beam.

The atomic ensemble is initially prepared and optically pumped into the ground state $|g\rangle$, thereby suppressing uncorrelated noise photons that would otherwise scatter from the intermediate state $|s\rangle$.

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