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Collision Avoidance Maneuver Planning and Optimization Considering Control Uncertainty and Mission Requirements

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Abstract

The rise in space traffic congestion has made collision avoidance maneuvers a crucial mode to ensure real-time safety of active satellites. However, performing a collision avoidance maneuver involves the usage of limited on-orbit fuel, losing mission time, incurring operational costs, etc. Therefore, the workload associated with conjunction analysis, the decision on the need to perform a collision avoidance maneuver, and, where appropriate, its design, has increased significantly. In this line, NorthStar has developed a two-step framework for both the decision-making and the design process of a collision avoidance maneuver. In the first step, a multi-objective problem is solved to identify the quantitative trade-offs among the various objectives. Once the suitable maneuver strategy has been identified, an optimal control problem is posed, the solution of which is compliant with high-fidelity dynamics and mission-specific constraints with the possibility of addressing additional safety considerations on top of respecting the stochastic and deterministic miss distance. These can include incorporating the effects of control uncertainty in the trajectory design alongside ensuring consideration for any unforeseen disruption of the propulsion system. The solution is also designed to incorporate orbit maintenance maneuvers and adhere to allotted GEO slots when necessary. The framework has been tested with case studies using CDMs. This paper presents the validation results of the system with emphasis on incorporating uncertainty on the propulsion system performance.

Keywords: Control uncertainty, Mission compliance, Non-linear programming, Collision Avoidance

1 Introduction

The current trends in space traffic have significantly increased the number of objects in near-Earth orbits and, accordingly, the number of conjunctions. Satellite operators must deal with this evolving situation with new tools that avoid high operating costs and maintain high safety standards. In this context, there are many challenges for improved conjunction assessment systems ranging from better support decision-making when performing collision avoidance maneuvers and streamlined design methods that provide efficient and safe maneuvers. In this article, we deal with the latter. Specifically, we deal with the design of conjunction avoidance maneuvers of spacecraft equipped with low-thrust propulsion systems. In this case, the limitation of the thrust makes the maneuver duration longer and the optimization more complex. In addition, there are certain characteristics of the maneuvers performed with these propulsion systems that should be considered in the optimization problem. Distinctively, this work includes the uncertainty associated with the realization of the maneuver in the probabilistic conjunction assessment, and ensures the prescribed maneuvers progressively increase the safety to reduce dependency on future requirements of control thrusts to better handle situations in case of a sudden propulsion system failure in the middle of an avoidance operation.

The field of optimization of collision avoidance maneuvers is broad and has been extensively explored in recent years. A comprehensive and convenient review of the methodologies for the design of collision avoidance maneuvers (CAM) has recently been published [1]. Methods range from providing analytical or semi-analytical, to numerical solutions. Analytical and semi-analytical methods ([2–5]) provide a significant advantage regarding the computation time; however, these come at the cost of quite a few assumptions and simplifications, which can be handled by the numerical methods ([6–20]) at a higher computation cost. Evolutionary algorithms, indirect optimization and direct optimization can be used to numerically solve the fully posed constrained optimization problem defined with accurate dynamics and intricate constraints. Evolutionary algorithms can be computationally expensive and are heuristic in nature, requiring careful adjustment of multiple parameters governing the selection, cross-over and mutation schemes; however, these can be used to search the entire domain to find the global optimal solution. Genetic algorithms have been implemented to solve for a single impulsive avoidance maneuver by Lee et al. [7] as well as to solve for the avoidance of multiple threats (c.f. Kim et al. [8]). To leverage the advantages of genetic algorithms without increasing the computation time, fast semi-analytical models can be used to calculate the optimum avoidance and return maneuvers for every candidate chosen by the genetic algorithm (NSGA-II), while exploring the entire search space as in an internal tool described in [19]. This tool uses Bombardelli’s analytical method [3] to generate a single impulsive avoidance maneuver and Lambert’s solution for the return to the final orbit. Also, the problem was posed as a multi-objective optimization in which the NSGA-II algorithm was used to generate a Pareto-front of optimal solutions, thus providing a quantitative trade-off analysis among the different objectives of CAM (Δv , time off nominal and collision probability). In this way, initial guesses of different CAMs strategies are available for a further refinement.

Gradient-based optimizations using direct or indirect methods remain the more robust optimization strategy capable of handling a much larger number of optimization variables compared to the performance of evolutionary algorithms. However, these are highly dependent on the initial guess and converge to a local optimum. There exists a long inheritance of approaches using gradient-based optimization solving for CAMs. In 2009, Martinson [12] solved for impulsive burns with the assumption of Clohessy-Wiltshire equations to solve for the avoidance of multiple encounters using a feedback mechanism. Therein, avoidance was ensured by maintaining pre-determined ellipsoidal regions around space objects, and the solution was based on a weighted objective function. Later, Sales [9] studied the effect of continuous as well as impulsive in-plane and out-of-plane thrusting for collision avoidance maneuvers using Radau’s pseudo-spectral method, implemented in GPOPS-II software. To circumvent the problems derived from the need for an initial guess, convex optimization has been used in different domains like collision avoidance, rendezvous or formation flying problems. In [11], a successive second-order cone programming was used to solve the convex rendezvous problem without the consideration of uncertainty, while the work in [21] proposed to solve the formation flying problem using linear programming. Robustness was accounted for by requiring the same avoidance constraint to be satisfied for several uncertain initial relative states. Li et al. [14] used a combination of a multi-resolution technique using convex optimization and mesh-refinement on Clohessy-Wiltshire equations. The avoidance region was handled by means of a tangent method similar to the one used in [21] for planning close-proximity operations. Armellin [15] used differential algebra and successive linearizations to solve a multi-impulse convex optimization problem concerning only the avoidance maneuvers without return to the original trajectory. Dutta and Misra [16–18] explored the effect of convex and non-convex optimization for avoidance trajectories along with return when considering a linear and nonlinear evolution of uncertainties. De Vittori et al. developed an analytical method concerning avoidance from short-term encounters with low thrust in [5] using indirect optimization, and in [20] solved the avoidance trajectory while incorporating Gaussian uncertainty and returning to nominal orbit for space objects in LEO. Pavanello et al. [22] in 2025, provided a thorough analysis for both short- and long-term encounters, with the possibility of addressing multiple encounters and using chance constraints to meet the maintenance of GEO slots to deal with the uncertainty of the primary. The method used an iterative approach involving both Sequential Quadratic Programming (SCP) and PP to help reduce time and variables while maintaining a desired level of accuracy and control, and although mostly successful, convergence could not be achieved on certain occasions.

Uncertainty in collision avoidance maneuvers does not only come from the lack of knowledge on the state of the secondary object, it is also due to the off-nominal performance of the propulsion system. Accounting for this source of uncertainty usually deserves a new approach and specific methodologies. J.T. Betts in 2006 [23] presented a seminal work on the incorporation of statistical constraints to a Nonlinear Programming Problem (NLP) for space trajectory optimization. In this line, model predictive control (MPC) approaches have been extensively used to deal with the input optimization of systems with non-linear dynamics. Moreover, evolved versions of MPC,

such as Stochastic MPC [24] or Tube-based MPC [25], have been developed to deal with uncertainty in a rigorous manner. One of the other advantages of these methods is their potential to be implemented on board if required. Although space applications incorporating these approaches are scarce, it is worth mentioning the work to autonomously deal with navigation uncertainty in deep space missions [26, 27], or rendezvous maneuvers [28]. To the authors' knowledge, the design of CAM considering input uncertainty has not been addressed in the literature.

Thus, in this work, we present a framework for the design of optimal CAM maneuvers that accounts for the uncertainty on the propulsion system and allows for the incorporation of additional constraints, e.g., ensure that the maneuver does not increase the collision risk in case of failure. The initial guess is provided by the global search performed by the genetic algorithm (with analytical performance evaluators) presented in [19]. The gradient-based optimizer incorporates high-fidelity dynamics with additional constraints and considerations. This paper advances the methodology proposed in [19] by considering additional constraints and mission requirements so that the formulation better adapts to real-life mission requirements. The uncertainty associated with both the states and control is considered in the second step for the evaluation of the associated risk. Propulsion constraints, scheduling requirements, and any orbit maintenance needs are also addressed in this step, while maintaining the chosen minimum miss distance and maximum risk tolerance from the previous step. The final optimization result is enforced to respect the time window chosen by the operator in the first step. The proposed method is tested in realistic scenarios involving different orbit regimes. The following sections provide an overview of the general framework (Section 2) the novel methodology to incorporate the uncertainty (Section 3), which is followed by the results (Section 4) and conclusions (Section 5).

2 Problem Statement. Non linear Programming Problem

This section presents the general framework for the optimal design of CAMs. The problem design is determined by its objective and various terminal and path constraints within a the nonlinear programming problem. The incorporation of control uncertainties in risk assessment and collision avoidance is discussed in Section 3. Within this framework, additional operational constraints can be included. Alongside the avoidance, orbit maintenance can also be addressed when planning the maneuvers, if needed. When considering the return maneuvers, it can be decided to locate the satellite in its nominal (predicted) position or another desired orbit, thus incorporating orbit maintenance.

In that way, the collision avoidance maneuver is designed as the solution to an optimal control problem with a single objective function, as shown in Eqn. 1. This equation shows the generalized cost function which is designed to be able to incorporate multiple objectives, such as, to minimize control acceleration, maximize statistical and euclidean distances between the primary and secondary, and return to the required

states within the time period $[t_0, t_f]$ chosen by the user/operator.

$$\begin{aligned} \min_{\mathbf{u}(t)} J(\mathbf{u}(t), t_f) = & w_1 \int_{t_0}^{t_f} \|\mathbf{u}(t)\|_2 dt - w_2 \min_t [(\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))^T \mathbf{P}(t)^{-1} (\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))] - \\ & - w_3 \min_t [(\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))^T (\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))] + \dots \end{aligned} \quad (1)$$

subject to

$$\begin{aligned} \delta \mathbf{x}_1(t_0) &= \mathbf{0} && \text{Initial conditions} \\ \delta \mathbf{x}_1(t_f) &\in \chi_{\text{goal}} && \text{Terminal conditions} \\ \delta \dot{\mathbf{x}}_1(t) &= \mathbf{f}(\delta \mathbf{x}_1(t), \mathbf{u}(t), t) && \text{System dynamics (2.1)} \\ [(\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))^T \mathbf{P}(t)^{-1} (\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))] &\geq M_{d,th}, \forall t \in [t_0, t_f] && \text{Minimum Mahalanobis distance (2.2)} \\ [(\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))^T (\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))] &\geq E_{d,th}, \forall t \in [t_0, t_f] && \text{Minimum Euclidean distance (2.2)} \\ \mathbf{u}(t) &\in \mathcal{U}(\mathbf{x}(t), t) && \text{Control feasibility (2.3)} \\ [\delta \phi, \delta \lambda] &\leq [\delta \phi_{th}, \delta \lambda_{th}] && \text{Maintenance of GEO slot (2.5)} \\ \{t \mid \mathbf{u}(t) \neq \mathbf{0}\} &\in \mathcal{T}(t) && \text{Operator schedule compliance} \end{aligned}$$

where $\mathbf{x}_1 = \mathbf{x}_{\text{ref},1} + \delta \mathbf{x}_1$ is the state of the primary, $\mathbf{x}_{\text{ref},1}$ is the nominal trajectory of the primary without performing the collision avoidance maneuver, $\delta \mathbf{x}_1$ is the relative state of the primary with respect to the nominal, $\mathbf{x}_{\text{ref},2}$ is the nominal state vector of the secondary, χ_{goal} represents the set of desired terminal conditions, $\mathbf{P}(t)$ is the combined covariance of primary and secondary at time t , $M_{d,th}$ is the user-defined threshold for the allowed Mahalanobis distance, $E_{d,th}$ is the user-defined threshold for the allowed Euclidean distance, $\mathcal{U}(\mathbf{x}(t), t)$ represents the set of allowed controls for a given state and time, ϕ is the geocentric latitude, and λ is the corresponding longitude. Finally, $\mathcal{T}(t)$ represents the set of allowed time intervals to execute the maneuver according to the operator schedule constraints.

2.1 System Dynamics

The propagation of the nominal trajectories of both primary and secondary is performed using a high-fidelity dynamical model

$$\dot{\mathbf{x}}_{1,2}(t) = \mathbf{f}_{HF}(\mathbf{x}_{1,2}, t), \quad \mathbf{x}_{1,2}(t_0) = \mathbf{x}_{10,20}, \quad (2)$$

where the subscripts 1 and 2 have been used to denote primary and secondary, respectively. The dynamics considered in the optimal control problem are the analytical relative motion approximation with respect to the nominal trajectory of the primary,

$\mathbf{x}_{\text{ref},1} = [\mathbf{r}_{\text{ref},1}^T, \mathbf{v}_{\text{ref},1}^T]^T$, as described in [29], which read

$$\mathbf{f} = \begin{bmatrix} \delta\dot{x} \\ \delta\dot{y} \\ \delta\dot{z} \\ \left(\frac{2\mu}{r_{\text{ref},1}^3} + \frac{h_{\text{ref},1}^2}{r_{\text{ref},1}^4}\right)\delta x - \frac{2(\mathbf{v}_{\text{ref},1} \cdot \mathbf{r}_{\text{ref},1})h_{\text{ref},1}}{r_{\text{ref},1}^4}\delta y + 2\frac{h_{\text{ref},1}}{r_{\text{ref},1}^2}\delta\dot{y} \\ \left(\frac{h_{\text{ref},1}^2}{r_{\text{ref},1}^4} - \frac{\mu}{r_{\text{ref},1}^3}\right)\delta y + \frac{2(\mathbf{v}_{\text{ref},1} \cdot \mathbf{r}_{\text{ref},1})h_{\text{ref},1}}{r_{\text{ref},1}^4}\delta x - 2\frac{h_{\text{ref},1}}{r_{\text{ref},1}^2}\delta\dot{x} \\ -\frac{\mu}{r_{\text{ref},1}^3}\delta z \end{bmatrix},$$

with $\delta\mathbf{x}_1 = [\delta x, \delta y, \delta z, \delta\dot{x}, \delta\dot{y}, \delta\dot{z}]^T$, where μ is the Earth's gravitational constant, and $h_{\text{ref},1}$ represents the modulus of the specific angular momentum of the primary reference trajectory. The state transition matrix can be defined as

$$\dot{\Phi}(t, t_0) = \mathbf{A}(t)\Phi(t, t_0), \quad \Phi(t_0, t_0) = \mathbf{I}, \quad (3)$$

where the Jacobian matrix $\mathbf{A} = \partial\mathbf{f}/\partial(\delta\mathbf{x}_1)$, and is used for the propagation of the covariances, where required.

A non-dimensional version of the dynamics is used in the implementation of the prototype to avoid scalability issues in the non-linear programming problem. Thus, the non-linear version of the dynamics reads

$$\delta\dot{\hat{\mathbf{x}}}_1 = \hat{\mathbf{f}}(\delta\mathbf{x}_1) + [\mathbf{0}_{3 \times 1}, \mathbf{u}_1^T]^T, \quad (4)$$

where the states involved are scaled according to a characteristic length and time.

2.2 Collision avoidance constraint

Collision avoidance constraints are dealt with as a Keep Out Zone (KOZ) constraint defined using Mahalanobis distance, which provides an ellipsoidal avoidance region. The KOZ is defined as the complement of the free space, in terms of the covariances of the primary and secondary, as

$$\chi_{\text{KOZ}} = \chi_{\text{free}}^C \quad (5)$$

$$\chi_{\text{free}} = \{\mathbf{x} | (\mathbf{x} - \mathbf{x}_2)^T \mathbf{P}^{-1} (\mathbf{x} - \mathbf{x}_2) \geq M_{d_0}^2\} \quad (6)$$

where $\mathbf{P} = \mathbf{C}_1 + \mathbf{C}_2$, with $\mathbf{C}_i$, $i = 1, 2$, the covariance matrices of primary and secondary, respectively and $M_{d_0}^2$ is the desired Mahalanobis threshold to be maintained.

Additionally, when considering more than one CDM (when two CDMs A and B are available) in the avoidance plan, the feasible space is defined as

$$\chi_{\text{free}} = \{\mathbf{x} | (\mathbf{x} - \mathbf{x}_2^A)^T (\mathbf{P}^A)^{-1} (\mathbf{x} - \mathbf{x}_2^A) \geq M_{d_0}^2\} \cap \{\mathbf{x} | (\mathbf{x} - \mathbf{x}_2^B)^T (\mathbf{P}^B)^{-1} (\mathbf{x} - \mathbf{x}_2^B) \geq M_{d_0}^2\}$$

Avoidance can also be enforced using Euclidean distance, and in that case $\mathbf{P}_i$ is replaced by an identity matrix $\mathbf{I}_{3 \times 3}$.

2.3 Control feasibility

These constraints deal with the limitations and capabilities of the thruster model, such as the maximum thrust capability and maximum steer rate capabilities for the rotation of the control thruster. The corresponding controls are shown below, where u is the magnitude of the acceleration and α and β are the two control angles

$$u(t) \leq u_{max} \quad (7)$$

$$\dot{\alpha}(t) \leq \dot{\alpha}_{max} \quad (8)$$

$$\dot{\beta}(t) \leq \dot{\beta}_{max}. \quad (9)$$

The angles relate in the following way to the accelerations in the RTN frame of the primary satellite

$$u_r = u \cos(\alpha) \cos(\beta) \quad (10)$$

$$u_t = u \sin(\alpha) \cos(\beta) \quad (11)$$

$$u_n = u \sin(\beta) \quad (12)$$

$$(13)$$

2.4 Fail-safe maneuver constraint

An additional constraint is included to ensure that the CAM is a fail-safe maneuver, i.e. the collision risk is not increased at any point of the maneuver with respect to the no maneuver case. Thus, it ensures that the probability of collision is continuously lowered (and the miss distance increased) even in the event of a mid-operation propulsion failure. The condition is checked at or in a time window around the nominal TCA (t_c) and imposed both for the Mahalanobis and Euclidean distance (in which case $\mathbf{P}$ is replaced by a suitable identity matrix). The constraint is defined as follows for a time step t_{c_i} in the time window around t_c

$$\begin{aligned} & (\Phi(t_{c_i}, t_k) \mathbf{x}(t_k) - \mathbf{x}_2(t_{c_i}))^T \mathbf{P}(t_{c_i}) (\Phi(t_{c_i}, t_k) \mathbf{x}(t_k) - \mathbf{x}_2(t_{c_i}))^T \leq \\ & (\Phi(t_{c_i}, t_{k+1}) \mathbf{x}(t_{k+1}) - \mathbf{x}_2(t_{c_i}))^T \mathbf{P}(t_{c_i}) (\Phi(t_{c_i}, t_{k+1}) \mathbf{x}(t_{k+1}) - \mathbf{x}_2(t_{c_i}))^T \\ & \forall k \in [0, c_i], \end{aligned}$$

and ensures that control at every time step improves both the deterministic and statistical miss distance when compared to the cumulative control up to the previous time step.

2.5 Maintenance of designated slot for GEO satellites

This section outlines the procedure implemented to maintain the satellite within the required longitude band for GEO operations. Let ρ_0 be the geocentric distance, α_0

be the local sidereal time based on the mean longitude of the satellite and β_0 be the nominal latitude of the satellite. The actual radial position vector ($\mathbf{r}$) can then be represented using perturbations for the orbital distance ($\delta\rho$), local sidereal time ($\delta\alpha$) and latitude($\delta\beta$) around the corresponding nominal values.

$$(\mathbf{r}_0 + \delta\mathbf{r})_{ECI} = (\rho_0 + \delta\rho)[\cos(\alpha_0 + \delta\alpha)\cos(\beta_0 + \delta\beta)\mathbf{I} + \sin(\alpha_0 + \delta\alpha)\cos(\beta_0 + \delta\beta)\mathbf{J} + \sin(\beta_0 + \delta\beta)\mathbf{K}] \quad (14)$$

Neglecting second-order perturbation terms in the expansion and equating and solving for $\delta\alpha$ and $\delta\beta$, we obtain

$$\delta\alpha = \frac{\cos(\alpha_0)\delta r_{ECI_J} - \sin(\alpha_0)\delta r_{ECI_I}}{\rho_0 \cos(\beta_0)}, \quad (15)$$

$$\delta\beta = \frac{\cos(\alpha_0)\cos(\beta_0)\delta r_{ECI_K} - \sin(\beta_0)\delta r_{ECI_I} - \rho_0 \sin(\beta_0)\cos(\beta_0)\sin(\alpha_0)\delta\alpha}{\rho_0 \cos(\alpha_0)}. \quad (16)$$

Therefore, $\delta\alpha$ and $\delta\beta$ are kept within the desired thresholds based on the tolerable deviations from the mean latitude and longitude of the satellite.

3 Uncertainty on Control Inputs

In this section, the uncertainty in the input maneuver is taken into account. Starting from the non-dimensional relative dynamics, ($\delta\dot{\mathbf{x}}_1 = \hat{\mathbf{f}}(\delta\mathbf{x}_1) + [\mathbf{0}_{3 \times 1}, \mathbf{u}_1^T]^T$), the control input is the non-dimensional acceleration produced by the propulsion system, $\mathbf{u}_1 = \hat{\mathbf{a}}_T$. The physical source of uncertainty to be modeled in a collision avoidance maneuver is related to the thrust produced by the propulsion system (a_T) and the pointing of the spacecraft (τ), with $\mathbf{a}_T = a_T\tau$. The projection of thrust direction in the RTN and ECI reference frame reads

$$\tau = \cos\alpha^{ECI}\cos\beta^{ECI}\mathbf{i} + \sin\alpha^{ECI}\cos\beta^{ECI}\mathbf{j} + \sin\beta^{ECI}\mathbf{k}, \quad (17)$$

$$\tau = \cos\alpha^{RTN}\cos\beta^{RTN}\mathbf{u}_r + \sin\alpha^{RTN}\cos\beta^{RTN}\mathbf{u}_\theta + \sin\beta^{RTN}\mathbf{u}_h, \quad (18)$$

$$(19)$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ is the orthonormal basis of the ECI reference frame. Depending on the frame in which the covariance is represented, the choice of projection of τ can be made accordingly, but in general, the RTN reference frame is preferred.

In addition, it is assumed that there is a reference temporal evolution for each of the three variables $a_{T,\text{ref}}(t), \alpha_{\text{ref}}^q(t), \beta_{\text{ref}}^q(t)$ ($q = RTN, ECI$). Note from an optimal control problem standpoint, the reference corresponds to the problem's (optimal) solution. As a first approximation, the difference between the actual value and the reference value, Δa_T , can be characterized as a Gaussian error

$$\Delta a_T \sim \mathcal{N}(\Delta \bar{a}_T, \sigma_{\Delta a_T})$$

$$\Delta \mathbf{y}^q = \{\Delta \alpha^q, \Delta \beta^q\}^T \sim \mathcal{N}(\Delta \bar{\mathbf{y}}^q, \Sigma_\Delta^q)$$

The dynamical model can then be expanded in terms of the Δ values as follows

$$\delta \dot{\mathbf{x}}_1 = \hat{\mathbf{f}}(\delta \mathbf{x}_1) + [\mathbf{0}_{3 \times 1}, (\mathbf{a}_{T,\text{ref}} + \Delta \mathbf{a}_T) \tau]^T ,$$

with

$$\begin{aligned} \tau = & (\cos \alpha_{\text{ref}}^q \cos \Delta \alpha^q - \sin \alpha_{\text{ref}}^q \sin \Delta \alpha^q) (\cos \beta_{\text{ref}}^q \cos \Delta \beta^q - \sin \beta_{\text{ref}}^q \sin \Delta \beta^q) \mathbf{u}_r + \\ & + (\sin \alpha_{\text{ref}}^q \cos \Delta \alpha^q + \cos \alpha_{\text{ref}}^q \sin \Delta \alpha^q) (\cos \beta_{\text{ref}}^q \cos \Delta \beta^q - \sin \beta_{\text{ref}}^q \sin \Delta \beta^q) \mathbf{u}_\theta + \\ & + (\sin \beta_{\text{ref}}^q \cos \Delta \beta^q + \cos \beta_{\text{ref}}^q \sin \Delta \beta^q) \mathbf{u}_h . \end{aligned}$$

Assuming: $\Delta \alpha \ll 1$ and $\Delta \beta \ll 1$, then $\cos \Delta \alpha \sim 1 \sim \cos \Delta \beta$, $\sin \Delta \alpha \sim \Delta \alpha$, $\sin \Delta \beta \sim \Delta \beta$ and $\Delta \alpha \Delta \beta \ll 1$. Thus, $\tau = \tau_r + \Delta \tau$ with

$$\begin{aligned} \tau_r = & \cos \alpha_r \cos \beta_r \mathbf{u}_r + \sin \alpha_r \cos \beta_r \mathbf{u}_\theta + \sin \beta_r \mathbf{u}_h \\ \Delta \tau = & -(\cos \alpha_r \sin \beta_r \Delta \beta + \cos \beta_r \sin \alpha_r \Delta \alpha) \mathbf{u}_r + (\cos \alpha_r \cos \beta_r \Delta \alpha - \sin \alpha_r \sin \beta_r \Delta \beta) \mathbf{u}_\theta + \cos \beta_r \Delta \beta \mathbf{u}_h , \end{aligned}$$

so that $\Delta \tau = S \Delta \mathbf{y}$ in the RTN reference frame, where

$$S = \begin{bmatrix} -\cos \beta_r \sin \alpha_r & \cos \alpha_r \sin \beta_r \\ \cos \alpha_r \cos \beta_r & -\sin \alpha_r \sin \beta_r \\ 0 & \cos \beta_r \end{bmatrix} .$$

Finally, assuming $\Delta a_T \ll a_T$, the dynamical model can be expressed as

$$\delta \dot{\mathbf{x}}_1 = \hat{\mathbf{f}}(\delta \mathbf{x}_1) + [\mathbf{0}_{3 \times 1}, (\mathbf{a}_{T,\text{ref}} \tau + \mathbf{a}_{T,1} + \mathbf{a}_{T,2})]^T ,$$

The terms $\mathbf{a}_{T,1}, \mathbf{a}_{T,2}$ are affine transformations of univariate or multivariate normal distributions, with:

$$\begin{aligned} \mathbf{a}_{T,1} = & \Delta a_T \tau_r \sim \mathcal{N}(\Delta \bar{a}_T \tau_r, \sigma_{\Delta a_T}^2 \tau_r \tau_r^T) \\ \mathbf{a}_{T,2} = & a_{T,r} \Delta \tau = a_{T,r} S \Delta \mathbf{y} \sim \mathcal{N}(a_{T,r} S \Delta \bar{\mathbf{y}}, a_{T,r}^2 S \Sigma_\Delta S^T) . \end{aligned}$$

In general, $\Delta \bar{a}_T = 0$ and $\Delta \bar{\mathbf{y}} = \mathbf{0}$, i.e. it is assumed there is no bias with respect to the optimal control solution. The resulting covariance for the control input reads

$$\mathbf{R}(t) = \sigma_{\Delta a_T}^2 \tau_r \tau_r^T + a_{T,r}^2 S \Sigma_\Delta S^T \quad (20)$$

This uncertainty should be mapped from the instant at which the control acceleration is applied to TCA, so that

$$\mathbf{R}(t_c) = \sum_i \Phi(t_i, t_c) \mathbf{B} \mathbf{R}(t_i) \mathbf{B}^T \Phi^T(t_i, t_c) \Delta t_i , \quad (21)$$

where the instants t_i are those with a non-null control input and $\mathbf{B}$ is the control matrix relating the effects of the control inputs on the states in the equation describing the linearized relative state dynamics.

4 Results

The proposed method is validated for both LEO and GEO orbital regimes. Using the LEO case, it is shown how consideration of the control uncertainty and fail-safe requirements (in addition to the necessary avoidance and return requirements) affects the cost and the maneuver trajectory.

For this, the reference case is presented first, which involves the following:

- Avoiding the collision
- Returning to its nominal location within a time period
- In a fuel-optimal manner
- While maintaining the threshold deterministic and statistical distance from the secondary all along
- And adhering to the control constraints with respect to maximum magnitude and steer rates.

While for this case, uncertainty stemming from the control actuation is not considered, the following case is set up to explore this particular effect and ensure trajectories with bounded erroneous controls are still maintained to be safe. Thereafter, the effect of the fail-safe constraint is also explored.

Finally, in the GEO scenario, the methodology's ability to provide a solution that includes a station-keeping maneuver close to the CAM execution is demonstrated, and how the optimization solution respects the longitude and latitude limits established by the operator, while meeting the other requirements of safety, timely return and control capacities.

4.1 Scenario in LEO

This scenario considers a close approach between two LEO satellites with TCA at midnight UTC on January 1, 2007.

The nominal orbit of the primary has the following Keplerian parameters: $a = 7004.7$ km; $e = 0.00137$; $i = 97.4869$ deg; $RAAN = -49.2587$ deg; $\omega = 58.071$ deg. The orbit of the secondary has the following Keplerian parameters: $a = 7004.656$ km; $e = 0.001346$; $i = 89.51$ deg; $RAAN = -74.589$ deg; $\omega = 60.07$ deg. The mass of the primary is 156 kg, and the relative distance at TCA is 357.4 m, which stems from the following relative position vector in ECI coordinates: $[-163.71649735, 290.44110637, 128.83620463]$ m.

The primary and secondary position covariances at TCA in the RTN frame read

$$\mathbf{P}_p = \begin{bmatrix} 9.0 & 0 & 0 \\ 0 & 106276.0 & 0 \\ 0 & 0 & 64.0 \end{bmatrix} \text{ m}^2 \quad \mathbf{P}_s = \begin{bmatrix} 144.0 & 0 & 0 \\ 0 & 131044.0 & 0 \\ 0 & 0 & 100.0 \end{bmatrix} \text{ m}^2.$$

4.1.1 Case 1a: Reference case

In this case (Case 1a), the goal is to design a collision avoidance maneuver for the primary, followed by a return to the nominal location. The avoidance constraint is enforced by requiring a Mahalanobis distance of 6 to be maintained between the two RSOs over the entire time window. In turn, the terminal constraint is defined in a way such that the primary satellite is constrained to return to its nominal orbit at its nominal true anomaly by matching the nominal states by the end of the avoidance trajectory. Finally, the control is also constrained by a maximum thrust and a maximum steer rate for the thrust control angles.

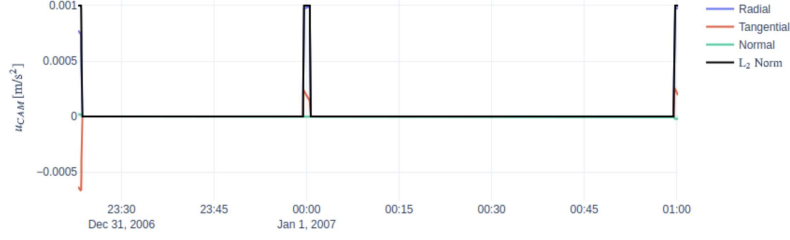


Fig. 1: The control acceleration profile for Case 1a

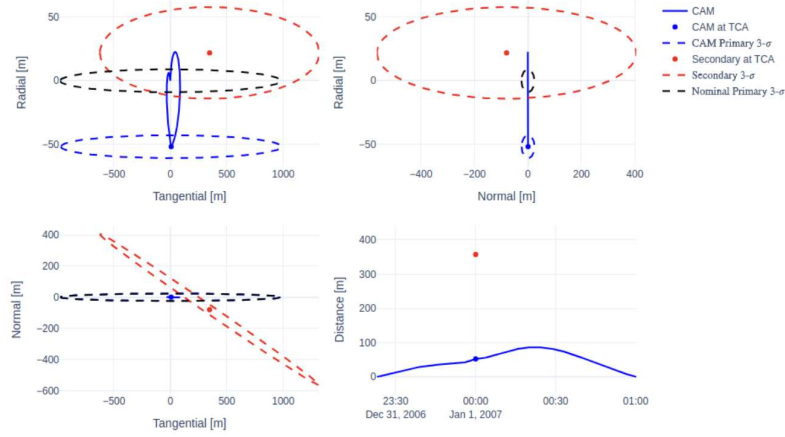


Fig. 2: Relative coordinates of the CAM trajectory with respect to the reference trajectory (without maneuver) in blue. The red dot and ellipsoid show the location and error ellipsoid of the secondary, respectively, at TCA in red.

The convergence to a locally optimal solution is depicted by the resulting ‘bang-bang’ solution for the optimal control acceleration magnitude, whereby the magnitude of the control acceleration switches between the maximum and the minimum values

(see Fig. 1). A ‘bang-bang’ solution is desirable because it produces the maximum available thrust acceleration at the locations of the orbit where it is effective and does not thrust at all when it is not effective. Fig.2 shows the optimized trajectory of the primary in terms of deviation from its nominal $[0,0,0]$ presented in projections in the local frame and its return at the end. The bottom right graph in Fig.2 shows the norm of the distance for completeness. The blue and red dot shows the optimized location of the primary and the location of the secondary at TCA, respectively. Hence, the blue dot should be outside the $3\text{-}\sigma$ error ellipsoid of the secondary (red), which can be observed in Fig.2. The total Δv requirement for this equals 0.137 m/s.

In the reference case, the time window allotted for the entire avoidance mission, along with the return, was one orbital period. The optimized trajectory is seen to use the entire time window to complete the ‘avoidance-and-return’ mission. However, when the operator’s choice allows for a larger time window for the avoidance and return, it was observed that the satellite does not need to use the full time window and can complete the mission in a time lower than the total allowed time window, thus flexibly reducing the avoidance time when possible for a fuel-optimal trajectory. To test this, the window for the operation is increased to 3 orbital periods. This larger allowed time window finds the Δv requirement to be much lower than that required in the reference case, as it is reduced from 0.1377 m/s to 0.03997 m/s.

4.1.2 Incorporation of the control uncertainty

This case incorporates the possibility of an error in the magnitude and pointing of the thruster. The standard deviations of which are assigned to 10% of maximum thrust capability and 0.1 deg/s for the magnitude and steer rate, respectively, for this case. With this incorporation, it can be seen that the trajectory can be slightly adjusted with a very slight increase in cost, as Δv increases from 0.1377 m/s to 0.1391 m/s, to ensure that the planned trajectory is safe even with uncertainty in the propulsion module. To verify the control uncertainty incorporation and covariance formulation, 100 random control sequence errors are generated based on the user-defined/pre-defined standard deviations. Thereafter, the deviations of trajectories with these random erroneous controls from the reference CAM trajectory (with no control error) are presented in red in Fig. 3. These deviations are seen to respect the $3\text{-}\sigma$ bounds obtained from the calculated control covariance, thus validating the control covariance calculation presented in this work. Moreover, the $3\text{-}\sigma$ bounds are also not overly optimistic, since the highest random deviations are close to these bounds. Perhaps due to the small control action, the uncertainty induced by the control itself is rather small for the radial and normal components, with a slightly higher standard deviation along the tangential direction of around 12 m at the end of the analysis window.

4.1.3 Incorporation of the fail-safe constraint

The fail-safe constraint ensures that the control at each time step makes the primary progressively safer with respect to the evolving TCA. Thus, there would be no control scheme allowed which would increase the risk at TCA in case of sudden thruster failure mid-operation. To ensure this, the satellite state (position and velocity) from every time step is propagated with and without the control at that time step up to

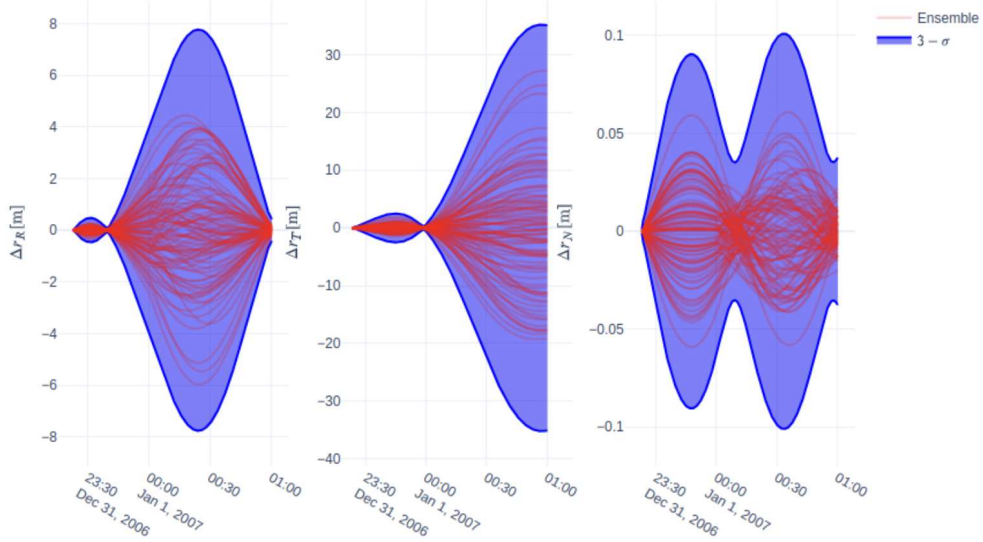


Fig. 3: Deviations in CAM trajectory due to control uncertainty

a certain time window around the original TCA (where the orbits of primary and secondary are in proximity) and it is ensured that the Euclidean and Mahalanobis distance between the primary and the secondary with the control is not less than that without the control input. The result of this can be verified in Fig. 4, where the x-axis denotes the epoch of a sudden thruster failure and the y-axis shows the resulting value of the risk parameters obtained upon propagating the states at the failure epoch to the resulting TCA. Therefore, the trajectory with the fail-safe constraint (purple curve) in Fig. 4 prevents the Euclidean or Mahalanobis distance from the secondary decrease with control at any stage, unlike for the reference case without the fail-safe requirement (blue). So, while both cases ensure that the optimized trajectories are better than the nominal values, with the fail-safe, the values of the risk parameters at the resulting TCA are constrained to constantly improve with each control timestep. The Δv requirement in this case is observed to increase from 0.1377 m/s to 0.1572 m/s, and the scenario is seen to require more contribution from the normal component of thrust than when the fail-safe constraint is off, that could be attributed to the along-track and radial increase in uncertainty that is not experience in the along-track direction.

4.1.4 Incorporation of flexible return requirements

Finally, the flexibility of the approach to reach different terminal conditions is tested. The satellite can return to both nominal orbit or a desired orbit at a specific true anomaly or at any true anomaly. When the satellite is free to conveniently choose any location on the nominal orbit for its return, the required Δv is seen to reduce to 0.056 m/s from 0.1377 m/s required in the reference case. The relative distance to the

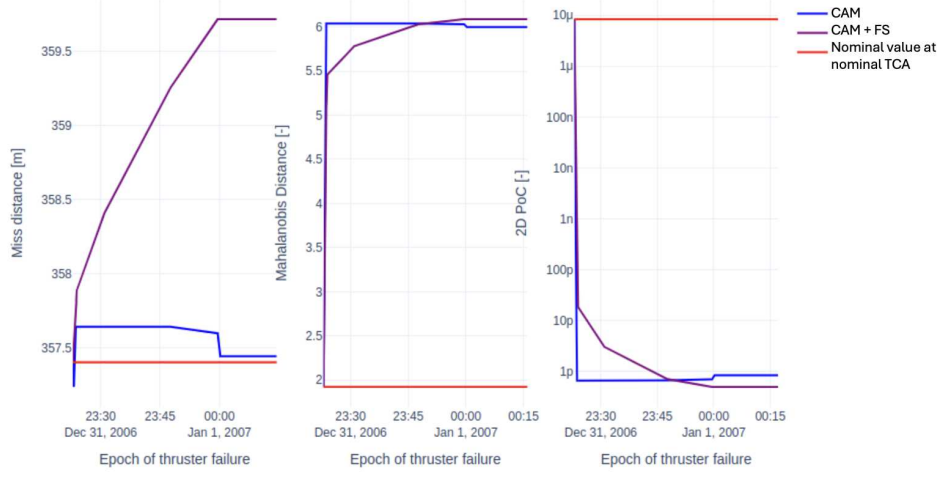


Fig. 4: Evolution of the risk metrics with different propulsion failure epochs before TCA

reference trajectory (trajectory without maneuvers) of the two solutions is shown in Fig. 5.

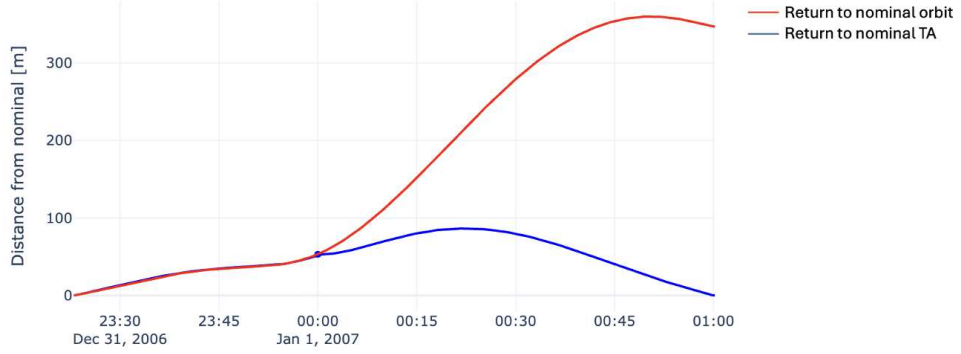


Fig. 5: Relative distance of the CAM trajectory with respect to the nominal trajectory with different terminal conditions: at a given true anomaly (labeled as “Return to nominal TA”) or at any true anomaly in the nominal orbit (labeled as “Return to nominal orbit”).

Table 1 provides a concise comparison among the different cases.

	Nominal	Reference Case (4.1.1)	With control uncertainty (4.1.2)	With fail-safe (4.1.3)	Return to non-nominal location (4.1.4)
TCA	2007-01-01T00:00:00.00 UTC	2006-12-31T23:59:59.99 UTC	2006-12-31T23:59:59.99 UTC	2007-01-01T00:00:00.00 UTC	2006-12-31T23:59:59.99 UTC
Mahalanobis Distance	1.9238	5.9986	6.0083	6.0857	6.0635
2D Collision Probability	8.3700e-06	8.3147e-13	7.8004e-13	4.9138e-13	5.6246e-13
Δv (m/s)	-	0.1377	0.1391	0.1572	0.056

Table 1: Comparison of the cost and risk parameters across the different LEO scenarios

4.2 Scenario in GEO

This example considers a low velocity conjunction in the geosynchronous regime ¹. Alongside the necessary constraints involving avoidance, return criteria and control limitations, the primary is also constrained to remain within the designated latitudes and longitudes at all times of the operation.

The nominal orbit of the primary has the following Keplerian parameters: $a = 42249.8026$ km; $e = 0.000395$; $i = 8.1395$ deg; RAAN = 56.91 deg; $\omega = -100.8778$ deg. The orbit of the secondary has the following Keplerian parameters: $a = 42168.1684$ km; $e = 0.00158$; $i = 14.74$ deg; RAAN = 3.623 deg; $\omega = -207.489$ deg. The nominal relative distance at TCA is 17.765 km, which stems from the following relative position vector in RTN coordinates: [807.50, -17643.40, -1910.07] m.

Primary covariance at TCA in the RTN frame:

$$\mathbf{P}_p = \begin{bmatrix} 6281.643918129001 & -55231.492974595 & 119.50993485100001 \\ -55231.492974595 & 3036698.785727172 & 1278.353123232 \\ 119.50993485100001 & 1278.353123232 & 1127.828296788 \end{bmatrix} \text{ m}^2$$

Secondary covariance at TCA in the RTN frame :

$$\mathbf{P}_s = \begin{bmatrix} 60190546.9937632 & -1220388780.110406 & -253984953.6512747 \\ -1220388780.110406 & 25141993089.16686 & 5234478222.84274 \\ -253984953.6512747 & 5234478222.84274 & 1090211417.611783 \end{bmatrix} \text{ m}^2.$$

As can be noted, the secondary is characterized by a high uncertainty, and to avoid the resulting error ellipsoid, the radial distance is changed significantly by over 5 km by the optimization result. The new miss distance components in RTN frame become [6691.87, -18019.7, -1974.47] m. The overall miss distance increases by over 1.5 km. The resulting optimized trajectory places the primary at about a Mahalanobis distance of 6 from the secondary, reducing the collision probability by about 7 orders

¹ According to the definition in the update of the ESA Space Debris Mitigation Handbook (section 1.3.3, page 12)

of magnitude. The required Δv for this operation is calculated to be 1.1632 m/s and is found to be larger than a typical value East-West station keeping maneuver (0.1 m/s), but smaller than North-South maneuvers (2-5 m/s). A different initial guess or a global optimal solution would help reduce the Δv further. The corresponding "bang-bang" (on-off) control profile is shown in Fig. 6.

The other aspect of this maneuver is to stay within the designated latitude-longitude slot. For the nominal trajectory of the primary, the mean latitude is 0 deg and the range of allowed nominal latitudes is ± 8.148 deg, while the mean longitude is -129.5156 deg, and the range of allowed nominal longitudes is ± 0.983 deg. The optimized trajectory is only allowed a maximum deviation of 0.25 deg in longitude and 0.005 deg in latitude from the nominal location at any given time. These bounds for the satellite in the geosynchronous region are seen to be respected in Figure 7, and the optimized trajectory starts from and returns to the nominal locations as constrained by the problem set-up.

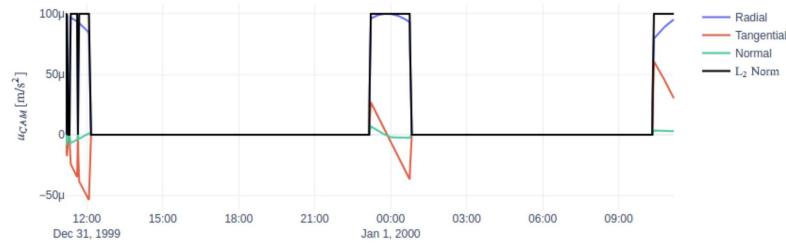


Fig. 6: Control acceleration profile

5 Conclusions

This work presents a framework that is geared to provide optimized CAM solutions starting from an initial guess coming from a global search with a genetic algorithm, followed by a refined optimization plan incorporating mission-specific flight dynamics requirements along with the avoidance. The primary global search presents the user/operator with a fast quantitative trade-off analysis across several objectives, such as cost, time and risk. The chosen maneuver plan with the desired combination of objectives is then tuned further using the described nonlinear programming problem formulation to design Collision Avoidance Maneuvers (CAMs) compatible with satellite operations, forming the second phase of a decision-making tool for CAM execution and design.

The framework allows tailored solutions to meet operational needs while aligning with dynamics, collision avoidance procedures, and performance constraints. Key constraints include executing CAMs alongside station-keeping (SK) operations and meeting desired return requirements while mitigating collision risks during propulsion

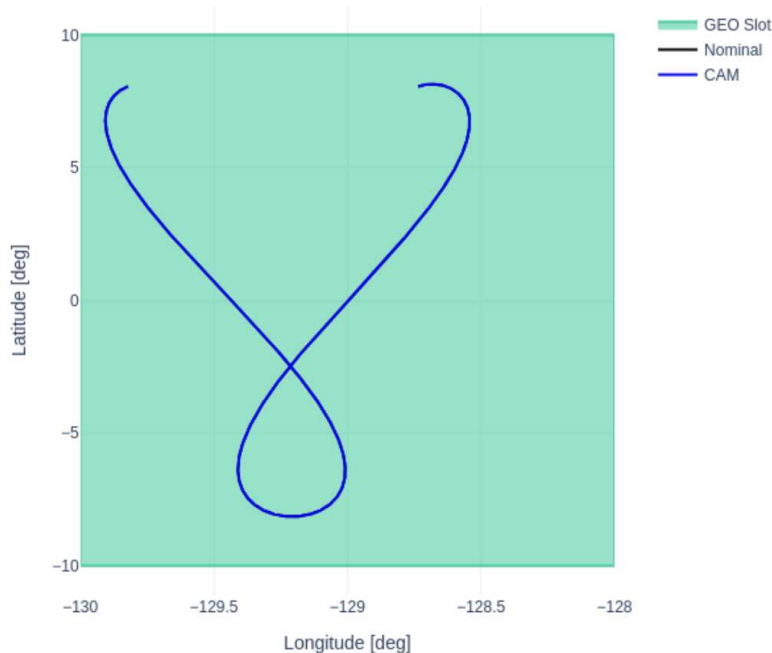


Fig. 7: Latitude-longitude evolution for the nominal and the maneuvered trajectory

system disruptions. It also addresses uncertainties in propulsion operations, acknowledging that maneuver execution can increase uncertainty and affect risk assessment during conjunctions.

The results show that, by incurring a very small increase in cost, the optimized avoidance trajectory can be secured against any uncertainty stemming from erroneous thrust or pointing. Trajectories with such uncertainty are also shown to be confined within the $3 - \sigma$ bounds predicted by the derived control covariance, thus validating the approach. Although the overall cost may be only slightly affected, the control strategy and avoidance trajectory can be influenced by this factor. A constraint to ensure the decrease in risk even in the case of a propulsion failure has also been tested and validated, getting the solutions that allow us to measure the additional cost in Δv in which the operator must incur. Additionally, the method's ability to execute CAMs with SK maneuvers in Geostationary Orbit (GEO) has been validated, imposing specific longitude and latitude limits. This work provides a robust tool for designing CAMs that integrate dynamics and satellite subsystem requirements while managing conjunction uncertainties and non-standard propulsion operations. However, the discretization required in the formulation allows the avoidance constraints to be imposed only at the fixed nodes, which can lead to missing the exact location of the updated TCA with the maneuvers. To avoid this, the number of nodes is required to be large, but that risks an increase in computation time. Also, the non-convex nature of the problem leads to convergence to a local minima, and thus the initial guess proves to be critical in reaching the lowest Δv or the desired objective.

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