Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

The determination of the natural rate of interest: does the total factor productivity matter?

Ricardo Ramalhete Moreira

ricardo.moreira@ufes.br

Universidade Federal do Espírito Santo, Brazil

Article

Keywords: natural rate of interest, TFP, VEC, ARDL, Quantile regression, Brazil

Posted Date: March 3rd, 2026

DOI: https://doi.org/10.21203/rs.3.rs-8790550/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

The determination of the natural rate of interest: does the total factor productivity matter?

ABSTRACT

The joint dynamics of the natural rate of interest and total factor productivity (TFP) constitute a key structural determinant with significant economic relevance for households and firms, as well as for policy decision-making – particularly by central bank authorities. The hypothesis that increases in TFP are associated with declines in the natural rate of interest is tested and supported using time-series data for the Brazilian economy over the period from February 2003 to December 2022. The natural rate of interest is estimated through a parsimonious model employing the Kalman filter, while TFP is measured using growth accounting techniques. Incorporating additional control variables and cointegration methods (VEC and augmented ARDL, Sam et al., 2019), the empirical evidence suggests that Brazil's natural rate of interest is negatively affected by long-run changes in TFP, and positively influenced by the trend GDP growth rate, the sovereign risk premium, and - less robustly - by U.S. real interest rates. Besides, a quantile approach identified changes in the elasticities over the natural rate of interest's distribution.

Key-words: natural rate of interest; TFP; VEC; ARDL; Quantile regression; Brazil.

JEL.: E43; F43; O47.

1. Introduction

The natural rate of interest is defined as the real interest rate consistent with a stable economy – namely, one in which GDP grows at its potential rate and inflation remains steady (or aligned with the inflation target, in countries operating under inflation targeting regimes). Accordingly, inferring the time-varying level of the natural rate of interest is of particular relevance to central banks' monetary policy decisions. A poor estimation of this level may lead to a misalignment of real interest rates with economic activity and inflation dynamics (Moreira, 2024), as well as generate biases in the risk balance projected by monetary authorities.

The conventional definition of the natural interest rate is supported, for instance, by Laubach & Williams (2003) and Holston, Laubach & Williams (2017). Nonetheless, it is useful to revisit a classical definition that remains consistent with the modern concept: "*There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them*" (Wicksell, 1936, p. 102). According to Laubach & Williams (2003), one possible formalization for observed real interest rate levels should be grounded in economic growth models that incorporate intertemporal optimization of household consumption and savings decisions, such that:

$$(1)\ r = \frac{1}{\sigma} q + n + \rho$$

where r is the real interest rate, σ represents the intertemporal elasticity of consumption, q is the labor-augmenting technological change, n is the population growth rate, and ρ is the rate of time

preference[1]. This framework implies a positive long-run relationship between the real interest rate and the trend growth rate, population growth, and time preference, while the relationship with σ is negative.

Based on this first-order approximation of optimal real interest rates, Laubach & Williams (2003) argue that it would be inappropriate to treat the natural rate of interest, r^*, as a constant – particularly in light of strong evidence that its structural determinants, such as trend growth in selected countries, have varied over recent decades (Roberts, 2001), along with supply and/or demand-side shocks, as highlighted by Zhu et al. (2023) for the United States.

A notable aspect of equation (1) is that the trend growth component q is not decomposed into its structural elements, such as the growth in factor inputs (labor and capital), total factor productivity (TFP), and normal utilization rates. Therefore, the positive relationship between trend growth and the natural interest rate found by both Laubach & Williams (2003) and Holston, Laubach & Williams (2017) may reflect heterogeneous and possibly offsetting contributions from those components. For instance, a given trend growth rate in real GDP may largely reflect overutilization of productive resources, which would generate inflationary pressures and thereby push the natural rate of interest upward during that period.

Conversely, when trend growth is primarily driven by gains in total factor productivity (TFP), marginal costs (and inflation) may decelerate, allowing central banks to operate with lower levels of real interest rates. This interpretation aligns with Galí & Monacelli (2005), who, using a New Keynesian framework with micro-foundations, model the natural rate of interest as inversely related to productivity. Other variables are also considered in that context, such as the household discount rate, expected global growth, and structural or behavioral parameters, including the degree of international openness.

The rationale for the inverse relationship lies in the micro-founded determination of real marginal costs – assumed to decline with rising productivity – and of potential output, which increases with productivity. Positive productivity shocks thus raise potential output, narrow the output gap, and reduce real marginal costs. This combination of responses is consistent with macroeconomic stability at lower real interest rates, resulting in a decline in the natural rate of interest. Galí & Monacelli (2005) illustrate this mechanism through simulations calibrated to Canadian data from the 1960s to the early 2000s, and their results confirm the theoretical predictions.

Simultaneously, a positive productivity shock justifies a drop in inflation, driven by efficiency gains in allocation and production, as well as declining marginal costs. A growing body of literature provides empirical support for these transmission channels through inflation dynamics (Ball & Moffitt, 2001; Karanassou & Sala, 2012; Tesfaselassie & Wolters, 2018). For instance, Dos Santos & Moreira (2020) estimated a set of regressions for the period from January 2003 to December 2015 in Brazil, using OLS and GMM methods, and found statistical evidence supporting an inverse relationship between TFP – measured through different proxies – and monthly consumer inflation.

It is worth highlighting that the international literature on the determinants of the natural interest rate has been largely motivated by the stylized fact of its secular decline across most advanced economies. Despite a range of potential drivers, the most frequently cited explanatory factors include: (a) declining potential (or trend) growth, which reduces investment opportunities and

[1] The rate of time preference is assumed to be inversely related to the savings rate, that is, ρ represents a measure of the so-called subjective discount rate of households.

lowers returns; (b) excess savings in developed economies (savings glut); (c) increased demand for safe or risk-free assets; (d) slower productivity growth, often attributed to diminishing marginal returns from already high productivity levels; and (e) declining population growth (IMF, 2014; CEA, 2015).

The study of these relationships – particularly that between total factor productivity (TFP) and the natural rate of interest – is of special relevance for developing economies. This is because such economies typically exhibit higher structural levels of real interest rates and lower levels of total factor productivity when compared with advanced and developed economies (Hall & Jones, 1999; Hsieh & Klenow, 2009; United Nations, 2022). Within this context, the present study employs the Brazilian economy as a representative case for empirical investigation. Given the literature reviewed above, the main objective of this article is to estimate the long-run relationship between TFP in Brazil and a latent measure of the natural interest rate (r^*), over the sample period from February 2003 to December 2022. The theoretical framework can be expressed as:

$$(2) \quad r^* = \beta_0 + \beta_1 tfp + \sum_{j=1}^{v} \beta_{j+1} V_j$$

where tfp denotes total factor productivity, and $\sum_{j=1}^{v} \beta_{j+1} V_j$ represents a vector of control variables – including the trend growth rate of the economy, a proxy for global liquidity conditions, and a variable capturing sovereign risk perception. The central hypothesis derived from the theoretical literature is that $\beta_1 < 0$, indicating a negative long-run relationship between the natural interest rate and TFP. To our knowledge, this article provides the first empirical test of that hypothesis specifically for the Brazilian case, one of the most important emerging markets.

The remainder of the article is organized as follows. Section 2 presents a brief overview of the international literature, as well as the evidence regarding the Brazilian case; Section 3 outlines the methodological foundations, including the construction of the natural interest rate series, the measurement of TFP, and the selection of control variables. In turn, Section 4 presents the empirical findings based on time series models – initially using VAR/VEC specifications, followed by robustness checks employing the augmented ARDL approach proposed by Sam et al. (2019). The latter methodology is particularly well-suited to our case study, as it allows for the estimation of long-run relationships regardless of the integration order of the variables and even when the dependent variable is I(0), while also addressing potential endogeneity concerns. Finally, Section 5 implements a quantile regression approach to assess tail behavior and asymmetries across the distribution of the natural interest rate. The final discussion, appendix, and references follow.

2. Brief Literature Review

2.1. Assessment of the international literature

The notion of a natural rate of interest emerged from the classical framework proposed by Wicksell (1936), who defined it as the real interest rate consistent with stable prices and equilibrium between savings and investment. In Wicksell's vision, deviations of the market rate from its "natural" counterpart generate cyclical pressures on output and inflation: an interest rate below the natural rate would induce credit expansion and inflationary tendencies, whereas a rate above it would contract credit and output. This conceptual foundation became a cornerstone for monetary theory, shaping later developments in business cycle analysis and policy design.

Over the decades, the concept evolved from a purely theoretical construct to a central empirical object in modern macroeconomics. Early neoclassical and Keynesian interpretations treated the

natural rate as exogenous and relatively stable. However, the postwar expansion of macroeconometric modeling, combined with advances in time-series econometrics, revealed that the natural rate is not static but varies with structural and institutional factors. The resurgence of the Wicksellian idea in the late 20th century was driven by the recognition that unobservable equilibrium rates could explain persistent deviations of inflation and output from target levels, even in regimes of active monetary policy.

The empirical revival of the natural rate concept owes much to the seminal work of Laubach and Williams (2003), who estimated *it* for the United States using a semi-structural model with unobserved components. In their approach, r^* is determined jointly with potential output and the output gap, enabling real-time inference about the stance of monetary policy. This framework, later expanded by Holston, Laubach, and Williams (2017), revealed a long-term decline in the natural rate across advanced economies, which they attributed to lower potential growth, demographic aging, and shifts in global saving and investment behavior. These findings reshaped the debate on "secular stagnation," echoing earlier hypotheses by Summers (2015) that persistent demand deficiencies could anchor real interest rates at historically low levels.

Parallel to these developments, dynamic stochastic general equilibrium (DSGE) models provided a new theoretical foundation for understanding r^*. In contrast to semi-structural approaches, DSGE frameworks endogenize the natural rate as the equilibrium outcome of intertemporal consumption and investment decisions under rational expectations. Key contributions by Smets and Wouters (2007), Lubik and Schorfheide (2007), and Justiniano and Primiceri (2008) incorporated productivity, preference, and policy shocks into forward-looking New Keynesian structures. These models demonstrated that fluctuations in r^* are linked to the same disturbances driving the business cycle, suggesting that real and nominal rigidities, as well as the policy rule itself, can amplify or dampen equilibrium rate movements.

The global financial crisis of 2008 marked another turning point. Empirical evidence from Del Negro et al. (2019) and Rachel and Summers (2019) showed that the post-crisis persistence of low real rates was not merely cyclical but structural. Explanations converged around several long-term forces: slower total factor productivity (TFP) growth, demographic headwinds, rising demand for safe assets, and higher inequality in wealth and income distributions – all contributing to excess savings and a reduced natural rate. Central banks, confronted with this "low r^* environment", faced new challenges for monetary policy transmission, as traditional interest-rate instruments approached their effective lower bounds.

2.2 Assessing the evidence for Brazil

The empirical investigation of the natural rate of interest in Brazil mirrors this international evolution, though with distinct institutional and structural characteristics. Brazil's transition to an inflation-targeting regime in 1999 created a fertile environment for estimating r^*, as the newly autonomous monetary policy sought to align short-term rates with long-run equilibrium conditions. Early efforts – such as those by Kanczuk (2002), Muinhos and Nakane (2006), Barcellos Neto and Portugal (2009) and Duarte (2010) – employed reduced-form or Kalman filter techniques, deriving time-varying estimates of r^* consistent with inflation and output dynamics. These studies consistently identified a gradual decline in the natural rate during the 2000s, coinciding with improved macroeconomic stability, falling risk premia, and rising policy credibility.

However, these early approaches faced challenges regarding the separation of cyclical and structural components and the identification of exogenous shocks. Subsequent work incorporated richer

structural dynamics. Palma (2016), for instance, estimated the natural interest rate for Brazil using a parsimonious AR-trend-bound model – proposed by Chan, Koop and Potter (2013) – which considers a time varying autoregressive process for the interest rate gap and a stochastic volatility. The findings indicated a fall in r^* from July 2001 to December 2015, totaling 174 observations. Besides, the results demonstrated a positive interest rate gap until mid 2009, which indicated a restrictive policy for this first period. Since then, the gap was predominantly negative, which indicated an expansionist policy.

In turn, the methodological frontier in this literature is well represented by Costa Júnior et al. (2022), which employed a calibrated DSGE model for Brazil, spanning 2000Q1–2018Q4, explicitly estimating r^* and decomposing its movements through impulse-response and variance analyses. They observe that the natural rate fell consistently until around 2012, rose temporarily amid fiscal deterioration and credibility shocks between 2013 and 2016, and then stabilized at moderate levels thereafter. The authors identify three key mechanisms shaping Brazil's r^* dynamics: (i) the Central Bank's reputation-building process, affecting expectations and the interest-rate gap; and (ii) productivity and savings behavior, which determine the long-run equilibrium between investment and consumption.

3. Methodological Aspects and Data

3.1. Extracting the Natural Rate of Interest

A key empirical challenge in working with the natural rate of interest lies in its latent nature – it is not directly observable. However, the premise that real interest rates must converge to their natural level in the medium term provides a foundation for extracting this intrinsic component. From an empirical standpoint, and using a time series approach, the underlying model for the dynamic behavior of the real interest rate can be specified as:

$$(3)\ rr_t = \beta rr_{t-1} + (1-\beta){r^*}_t + \varepsilon_t$$

where rr_t denotes the short-term real interest rate, ${r^*}_t$ the natural rate of interest, and ε_t a white noise term with zero mean and constant variance. This first-order autoregressive (AR(1)) specification for the real interest rate is consistent with Favero (2001). The parameter β captures the degree of persistence in the real interest rate.

While the real interest rate can be observed and calculated via a standard Fisher equation, the natural rate is an unobserved component that must be estimated or extracted. One common approach to retrieving this latent variable (state variable) is the application of the Kalman Filter (Harvey, 1989). A state-space model can be defined as:

$$(4)\ \mathrm{J}_t = \mathrm{z'}_t\theta_t + \mu_t$$

$$(5)\ \theta_t = \mathrm{T}_t\theta_{t-1} + \eta_t$$

In this representation, J_t denotes the observable variables, related to a vector θ_t of dimension m × 1, referred to as the state vector (t = 1, . . . , T); $\mathrm{z'}_t$ is an N × m matrix, and μ_t is an N × 1 vector of disturbances with zero mean and covariance G_t. Although the components of θ_t are latent, they evolve according to a first-order Markov process, while T_t is an m × m matrix, and η_t is a b × 1 vector of uncorrelated errors with zero mean and covariance Q_t. The estimation of the natural rate

of interest can therefore be structured within this state-space framework and specified through the following expressions:

$$(6)\ \mathrm{J}_t = rr_t = \beta rr_{t-1} + (1-\beta){r^*}_t + \varepsilon_t$$

$$(7)\ \mathrm{z'}_t = [rr_{t-1}]$$

$$(8)\ \theta_t = \begin{bmatrix} \beta_t \\ {r^*}_t \end{bmatrix}$$

$$(9)\ \mu_t = \varepsilon_t$$

$$(10)\ \mathrm{T}_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(11)\ \mathrm{Q}_t = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}$$

$$(12)\ \begin{bmatrix} \beta_t \\ {r^*}_t \end{bmatrix} = \mathrm{T}_t \begin{bmatrix} \beta_{t-1} \\ {r^*}_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}$$

The latent variable of interest in this study is the time-varying behavior of ${r^*}_t$. In the Brazilian case, the real interest rate (rr_t) was computed using a standard Fisher equation, defined as the difference between the effective annual Selic rate (Selic over) and expected inflation over the next 12 months, based on IPCA forecasts (ex-ante real interest rate). Figure 1 presents the time series of both the real interest rate and the natural rate extracted via the Kalman Filter, for the period from February 2003 to December 2022.

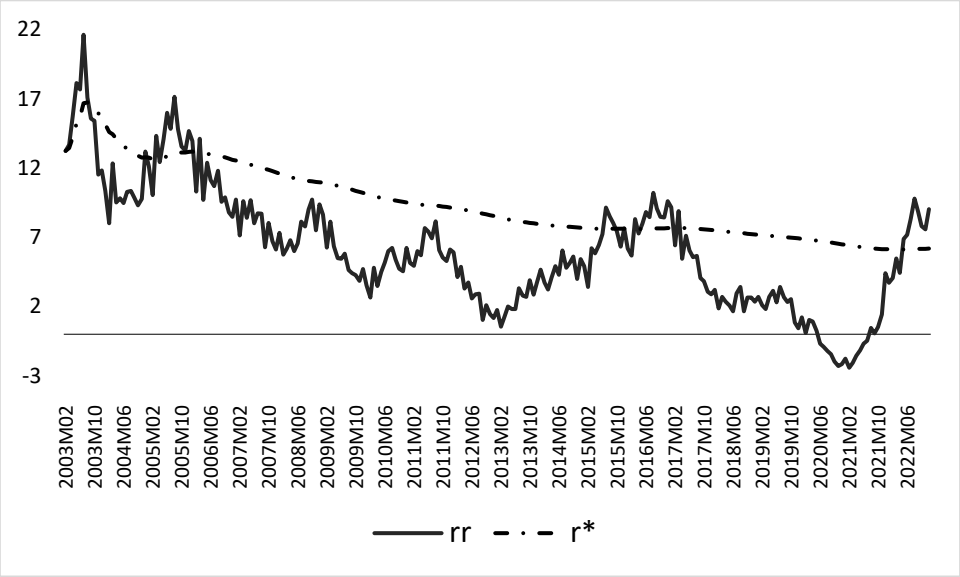
The results show a clear downward trend in the natural rate of interest (${r^*}_t$) over the sample period, with a steeper decline up to the end of 2013, and a slower pace thereafter. However, the deviation of the real interest rate from its natural level – i.e., the gap between rr_t and ${r^*}_t$ – varies significantly over time as a result of macroeconomic fluctuations and monetary policy decisions by the Central Bank of Brazil[2].

Certain periods were characterized by restrictive real interest rates (i.e., real rates above the natural rate), such as in the early years of the sample (2003 and 2005), and again in 2015, 2017, and from mid-2022 onward. Conversely, accommodative real interest rates (i.e., real rates below the natural level) were observed for most of the sample period. This does not imply that interest rates were low in absolute terms (which would run counter to the well-established stylized fact of structurally high real rates in Brazil—see Favero & Giavazzi, 2002), but rather that they were, nonetheless, below the estimated natural rate during those specific intervals.

The primary goal of this study is not to explain the short-term drivers of real interest rate cycles, but rather to investigate the long-term structural components that may account for the behavior of the natural rate of interest in Brazil. The central hypothesis is that Brazil's natural interest rate has, to some extent, been shaped by movements in total factor productivity (TFP). Accordingly, the second methodological step involves the measurement of TFP in Brazil.

[2] This refers to the well-established literature on monetary policy rules, or the Taylor Rule (Taylor, 1993). A discussion of such monetary policy reactions for the Brazilian economy can be found in Moreira (2015).

Figure 1 – Real interest rate and natural rate of interest in Brazil: Feb/2003 to Dec/2022.



Source: authors' elaboration.

3.2. Measuring TFP in Brazil: a brief contextualization

Total factor productivity (TFP) is also a latent variable and must be inferred through estimation or calculation methods, as there is no official statistical measure for it. Following the procedure adopted by Barbosa Filho and Pessoa (2014), and using a Hicks-neutral Cobb-Douglas production function as the theoretical benchmark, technological progress in the economy at period *t* can be measured as:

$$(13)\quad ln\left(\frac{A_{t+1}}{A_t}\right) = ln\left(\frac{Y_{t+1}}{Y_t}\right) - \alpha \ln\left(\frac{K_{t+1}}{K_t}\right) - (1-\alpha)\ln\left(\frac{L_{t+1}}{L_t}\right)$$

The growth rate of technology $\left(\frac{A_{t+1}}{A_t}\right)$ is generally interpreted as the growth rate of total factor productivity (TFP), which equals the output growth $\left(\frac{Y_{t+1}}{Y_t}\right)$ net of the weighted growth in capital $\left(\frac{K_{t+1}}{K_t}\right)$ and labor $\left(\frac{L_{t+1}}{L_t}\right)$, with weights α and (1 − α) representing their respective long-run shares in total output.

The capital stock time series was constructed based on the perpetual inventory method[3]. Following Barbosa Filho and Pessoa (2014) and Gomes, Pessoa, and Veloso (2003), a capital-to-output ratio of 2.7 was assumed for December 1996 (the base month), consistent with estimates for the 1990s[4], along with a capital depreciation rate of 3.5% and a capital income share of 40% (i.e., α = 0.4). Additionally, monthly investment was estimated via exponential interpolation of quarterly data, using the quarterly series on gross fixed capital formation growth available in the national accounts system. This approach is similar to that employed by Dos Santos & Moreira (2020).

Hence, the percentage change in TFP (g_{tfp}) was derived as a residual from the growth rates of monthly GDP (Y), estimated capital stock (K), and the economically active population (L) (sourced from PNAD/IBGE). Moreover, the capital stock was adjusted by industrial capacity utilization (u_1),

[3] The perpetual inventory method is defined as: $K_{t+1} = (1-\delta)K_t + I_t$, where δ is the capital depreciation rate. See Gomes, Pessoa, and Veloso (2003).
[4] This ratio is required to determine the initial capital stock, based on GDP statistics for the base period.

and the labor series was corrected using the unemployment rate (u_2) reported by the IBGE. It is worth noting that omitting such adjustments could lead to mismeasurement of TFP variation. Thus:

$$(14)\ g_{tfpt} = ln\left(\frac{Y_t}{Y_{t-1}}\right) - 0{,}4\ln\left(\frac{u_{1_t}K_t}{u_{1_{t-1}}K_{t-1}}\right) - 0{,}6\ln\left[\frac{(1-u_{2_t})L_t}{(1-u_{2_{t-1}})L_{t-1}}\right]$$

It is also important to highlight that g_{tfpt} can be computed while controlling for the additional role of human capital (H), typically proxied by average years of schooling (Barro & Lee, 1993), in the process of economic growth. Within the canonical framework of endogenous growth models, by considering labor augmented by skills (LH), equation (13) may be extended to:

$$(15)\ \ ln\left(\frac{A_{t+1}}{A_t}\right) = ln\left(\frac{Y_{t+1}}{Y_t}\right) - \alpha\ln\left(\frac{K_{t+1}}{K_t}\right) - (1-\alpha)\left[\ln\left(\frac{L_{t+1}}{L_t}\right) + \ln\left(\frac{H_{t+1}}{H_t}\right)\right]$$

In this specification, the relative contribution of skill-augmented labor is (1 − α). However, the implementation of equation (15) for the Brazilian economy faces practical limitations: (i) data on years of schooling are only available at annual or decennial frequency (IBGE/Ipeadata); (ii) available data extend only from 2016 onward. Given these constraints for monthly Brazilian time series (though this limitation does not apply to panel data studies), the estimation in this paper relies on equations (13) and (14). Therefore, in this context, TFP reflects productivity improvements arising both from human capital accumulation and technological progress.

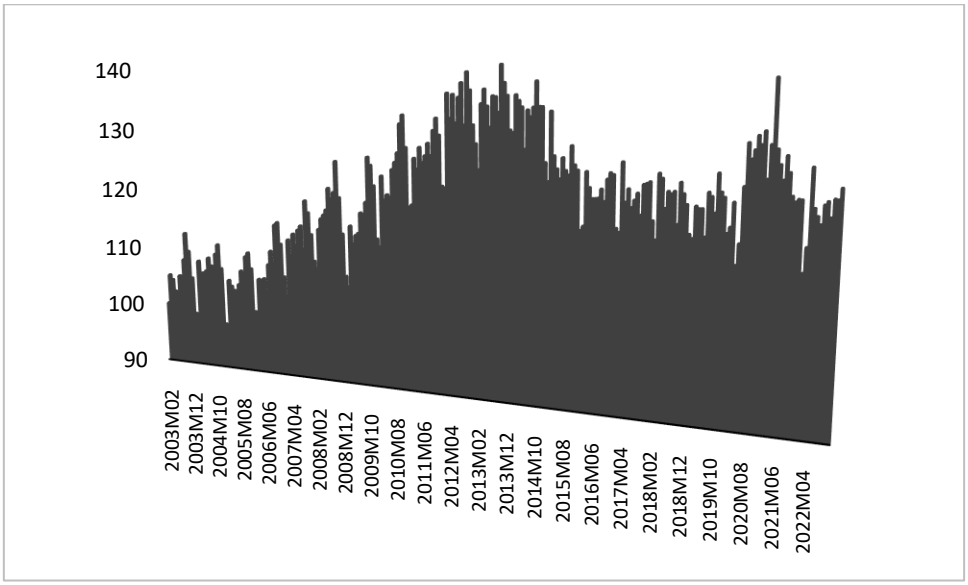
Based on the extracted TFP growth rates over the sample period, it is possible to compute a TFP index, taking the first month of the sample (February 2003) as the base period. Figure 2 displays the time series behavior of TFP in Brazil, estimated using the approach described above and the corresponding time series data.

A brief contextualization is warranted. The TFP series exhibits a cyclical pattern over time, albeit with an underlying upward trend. From the beginning of the sample through mid-2013, Brazil experienced a period of rapid TFP growth, reaching a peak index value of 143 in October 2013. This corresponds to a cumulative increase of 43% over approximately ten years, or a compound annual growth rate of 3.6%. However, following the economic slowdown starting in 2014 – particularly the GDP contraction in 2015–2016 – TFP entered a period of sharp decline, reaching a trough of 119 in June 2019, reflecting a cumulative decline of 16% from its 2013 peak.

From July 2019 to March 2021, a new phase of accelerated TFP growth was observed, with the index reaching a maximum of 144. Interestingly, contrary to what occurred with GDP in 2020 (the first year of the COVID-19 recession), TFP ended that year at 136 – above the December 2019 level of 126. For the full sample period, with the index ending at 128 in December 2022, the implied compound annual growth rate of TFP was approximately 1.2%. This represents a relatively modest rate of productivity growth for an emerging market economy with significant development needs.

Thus, a key point to highlight is the downward trend in the long-term TFP growth rate (g_{tfpt}) over the sample period. While the early years (February 2003 to October 2013) were marked by strong productivity growth, the subsequent period (November 2013 to December 2022) saw a noticeable slowdown. This pattern indicates a declining trend in Brazil's productivity growth—a trend that has also been observed in advanced economies (IMF, 2014; CEA, 2015).

Figure 2 – Total Factor Productivity in Brazil: Feb/2003 to Dec/2022.



Source: authors' elaboration.

3.3. Control variables

The long-term (10-year) real interest rate on U.S. Treasury securities was included as one of the control variables ($us10$), given its relevance to global liquidity conditions and, via interest rate parity, *ceteris paribus*, to interest rates in both advanced and emerging market economies. This series was retrieved from the FRED Economic Data system. In order to allow for subsequent estimations in natural logarithms, a linear shift was applied to the original series based on its minimum observed value in August 2020 (-0.40).

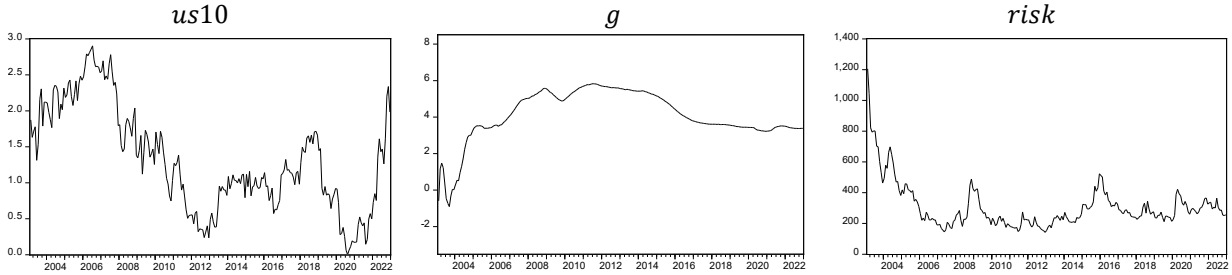
The second control variable was the estimated trend growth rate (g) of the Brazilian economy. This determinant is suggested by Laubach & Williams (2003) and Holston, Laubach & Williams (2017). Since the trend growth rate is also a latent variable, it was estimated through a Kalman Filter, following the same methodological procedure adopted for the estimation of the natural rate of interest. The time series used in the state-space model was the monthly growth rate of real GDP (source: Central Bank of Brazil – deflated by the IPCA), measured as the year-over-year percentage change. As in the previous case, the series was linearly shifted based on its minimum observed value (-0.90 in September 2003) to allow for the use of natural logarithms.

In addition, the country risk measure (EMBI+ Brazil) was also included, as the perception of sovereign credit risk by international investors can affect capital inflows, the exchange rate, and thereby the sustainable level of real interest rates required to achieve inflation targets in Brazil. The time series ($risk$) was obtained from the Ipeadata system and is compiled by JP Morgan.

Figure 3 presents the time plots of the three control variables: $us10$, g, and $risk$. For the purpose of preliminary inference on the joint dynamics between the natural rate of interest and the variables tfp, $us10$, g, and $risk$, Figure 4 displays the corresponding scatter plots over the sample period from February 2003 to December 2022. A negative relationship is observed between the natural rate of interest and total factor productivity (TFP), and a positive relationship with long-term U.S. interest rates – both of which approximate a linear pattern.

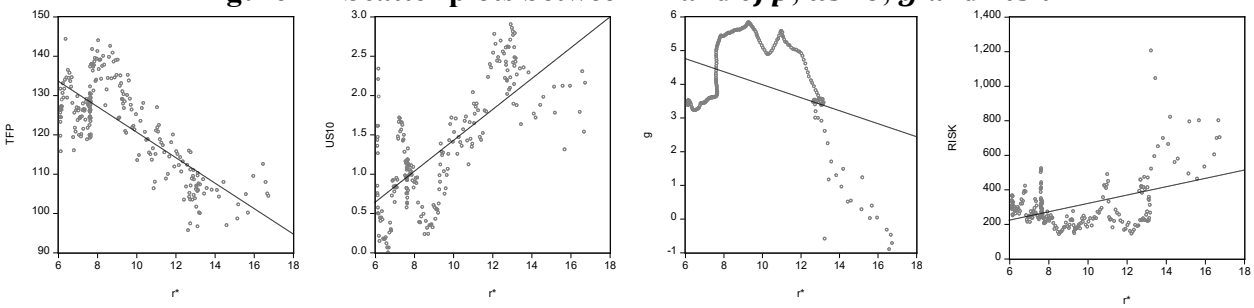
In contrast, the empirical relationship between the natural rate of interest and the trend growth rate does not appear to follow a well-defined or linear pattern throughout the sample. It appears to be positive when the natural rate is equal to or below 10%, and negative when r^* exceeds 10%. Lastly, the relationship between r^* and the country risk indicator appears to be positive at first glance, although it exhibits substantial dispersion around the regression line. These scatter plot relationships should be regarded as inconclusive and are considered merely as initial evidence for the subsequent estimation.

Figure 3 – Time behavior of $us10$, g, and $risk$: Feb/2003 to Dec/2022



Source: authors' elaboration.

Figure 4 – Scatter plots between r^* and tfp, $us10$, g and $risk$



Source: authors' elaboration.

4. Long-Run Relationships and Error Correction

4.1. Identifying the Order of Integration

The first step in any empirical study involving time series data is to assess and determine the order of integration of the variables. The literature offers a wide range of unit root and stationarity tests, including those based on the null hypothesis of a unit root, those based on the null of stationarity, as well as tests that allow for structural breaks or consider discrete or fractional orders of integration. Perhaps the most commonly employed tests in applied economics are the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, both of which assume a null hypothesis of a unit root. In this study, we adopt both tests to identify the integration order of the variables under analysis.

The results suggest that the natural rate of interest and the country risk premium are stationary in levels – i.e., both reject the unit root null hypothesis in the ADF and PP tests – thus being classified as I(0). The U.S. long-term interest rate ($us10$), on the other hand, displays evidence of a unit root in both tests. The results for total factor productivity (TFP) and Brazil's trend growth rate are inconclusive. Consequently, we apply the unit root tests to the first differences of these variables ($us10$, tfp and g) to assess whether they are at most I(1). This step is essential, as the ARDL modeling strategy employed later in the paper requires that none of the variables be integrated of

order two [I(2)], a condition that would invalidate the estimation procedure (Sam et al., 2019; Pesaran et al., 2001).

Table 1 – Identifying the Order of Integration: **ADF/PP tests (level)**

	ADF		PP	
Variable	**t stat**	**prob.**	**adj t stat**	**prob.**
r^*	-5.627$^{(I)}$	0.000	-1.972$^{(I)}$	0.046
tfp	-2.091$^{(III)}$	0.248	-3.944$^{(II)}$	0.011
$us10$	-2.058$^{(III)}$	0.262	-1.846$^{(III)}$	0.357
g	-2.330$^{(II)}$	0.415	-3.401$^{(II)}$	0.053
$risk$	-7.179$^{(III)}$	0.000	-6.880$^{(III)}$	0.000

Notes: lag length defined based on the Schwarz Information Criterion. (III) only constant is statistically significant; (II) both constant and trend are statistically significant; (I) baseline equation without exogenous components. Source: authors' elaboration based on the underlying data and applied methods.

Table 2 – Identifying the Order of Integration: **ADF/PP tests (First difference)**

	ADF		PP	
Variable	**t stat**	**prob.**	**adj t stat**	**prob.**
$tfp^{(I)}$	-4.211	0.000	-27.176$^{(III)}$	0.000
$us10^{(I)}$	-16.522	0.000	-16.657$^{(III)}$	0.000
$g^{(II)}$	-5.572	0.000	-13.332$^{(III)}$	0.000

Notes: lag length defined based on the Schwarz Information Criterion. (III) only constant is statistically significant; (II) both constant and trend are statistically significant; (I) baseline equation without exogenous components. Source: authors' elaboration based on the underlying data and applied methods.

The test results confirm that *us10*, tfp, and g are stationary in first differences according to both the ADF and PP tests, which lends confidence that no variable in the model is I(2). Therefore, the use of the ARDL methodology is appropriate in the present case. Importantly, the presence of variables with mixed orders of integration – I(0) and I(1) – does not preclude the possibility of cointegration relationships, as will be further discussed.

4.2. VAR/VEC estimates: preliminaries

According to the original interpretation by Engle and Granger (1987), time series variables must be integrated of the same order (typically I(1)) for a cointegration relationship to be tested. However, Campbell and Perron (1991) proposed a more flexible view, suggesting that a set of variables may be cointegrated as long as at least two of them are integrated of the same order and that order corresponds to the highest integration order among the variables. For instance, if in a given system three time series are I(0) and two are I(1), it is still possible for the full set of five variables to exhibit cointegration, provided the appropriate statistical tests are satisfied.

This broader approach is adopted in the present study by applying the Vector Error Correction (VEC) methodology as an initial empirical strategy (Johansen, 1988; Johansen & Juselius, 1990). Within the set of time series variables – r^*, tfp, $us10$, g and $risk$ – two are stationary in levels (r^* and $risk$), while the other three are unambiguously stationary in first differences (tfp , $us10$ and g). This configuration fulfills the Campbell-Perron criterion.

The standard procedure begins with selecting the optimal lag length for an unrestricted Vector Autoregression (VAR) model. Next, the existence of at least one cointegrating equation is evaluated

using the trace and maximum eigenvalue tests, under a parsimonious specification with intercept but no deterministic trend—consistent with the theoretical model specified in Equation (2).

It is important to note that the VEC model estimates the coefficient matrix using the first differences of the time series. Accordingly, the standard practice is to specify the VEC model with *n–1* lags relative to the unrestricted VAR, in order to preserve degrees of freedom. The optimal lag length for the VAR model was selected based on the Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SIC), and Hannan-Quinn Criterion (HQ). Table 3 presents the results, which point to a VAR model with 5 lags. Consequently, the VEC specification includes 4 lags (*n–1*).

Table 3 – Optimal lag identification

Lag	AIC	SC	HQ
1	-8.599688	-8.230529	-8.450843
2	-9.414989	-8.676672	-9.117300
3	-10.59787	-9.490391*	-10.15133
4	-10.89445	-9.417814	-10.29907
5	-11.32468*	-9.478891	-10.58046*

Note: (*) for optimal choice; Source: authors' elaboration.

The cointegration test was then carried out using the trace and maximum eigenvalue statistics, under the null hypothesis of no cointegrating relationship among the variables (see Table 4). Both statistics reject the null hypothesis, indicating that cointegration is present in the system. Given that the present study focuses on a single dependent variable (r^*) and one primary relationship of interest – between r^* and tfp – the model is estimated with a single cointegrating equation, which is consistent with the test results[5].

Table 4 – Cointegration tests

Trace				
	Eigenvalue	Stat	Critical, 5%	Prob.
H_0: no coint	0.352	189.82	76.972	0.000
Maximum Eigenvalue				
	Eigenvalue	Stat	Critical, 5%	Prob.
H_0: no coint	0.352	101.79	34.805	0.000

Source: authors' elaboration.

The long-run log-log empirical specification is given by:

$$(16)\quad \ln(r^*_t) = \beta_0 + \beta_1 \ln(tfp_t) + \beta_2 g_t + \beta_3 \ln(risk_t) + \beta_4 \ln(us10_t) + \varepsilon_t$$

Our theoretical expectations are: $\beta_1 < 0, \beta_2 > 0, \beta_3 > 0$ and $\beta_4 > 0$. The estimated coefficients are consistent with the study's central hypothesis ($\beta_1 < 0$), statistically significant at the 1% level. That is, based on Johansen's preliminary cointegration method, the evidence suggests that in Brazil, over the sample period, increases in total factor productivity are associated with reductions in the natural

[5] The Johansen method is versatile insofar as it allows for the joint estimation of two or more cointegrated equations. However, when a set of time series admits more than one cointegrating relationship, this does not preclude the estimation of a parsimonious model with only one. In our view, such a choice should be grounded either in theory or in the underlying evidence.

rate of interest. This implies a potential for inflation stabilization at lower structural real interest rates and reduced capital costs for firms. The estimated long-run equation for the expected natural interest rate is as follows (with *t*-statistics in brackets):

$$(17)\quad \ln \widehat{r^*} = 8.98 - 1.93\ln(tfp_t) + 0.54\ln(g_t) + 0.24\ln(risk_t) + 0.09\ln(us10_t)$$
$$[3.155] \quad [-3.642] \quad [4.895] \quad [1.843] \quad [2.154]$$

The results imply that a 1% increase in TFP leads to a 1.93% reduction in the natural interest rate in Brazil, with strong statistical significance (1% level). In turn, the U.S. long-term real interest rate has a positive but relatively inelastic effect: a 1% increase in $us10$ raises the Brazilian r^* by just 0.09%, significant at the 5% level. Although this aligns with theoretical expectations, it suggests a limited sensitivity of Brazil's structural interest rate to international real rates. Additionally, a 1% increase in Brazil's trend growth rate is associated with a 0.54% rise in r^*, while a 1% increase in country risk raises r^* by 0.24%. These elasticities are statistically significant at the 1% and 10% levels, respectively – both consistent with theoretical predictions[6].

The VEC model also enables an analysis of short-run dynamics in response to deviations from long-run equilibrium. This is examined through the short-term responses of the first-differenced variables to a positive deviation ($\varepsilon_t > 0$) in Equation (16). Table 5 summarizes the results. The key short-run adjustment variables – those with statistically significant coefficients – are the natural rate of interest and the trend growth rate, with the latter displaying the highest speed of adjustment. Specifically, the trend growth rate adjusts downward by 0.29% per month in response to a 1% deviation from the long-run relationship. Meanwhile, r^* exhibits a high degree of inertia (or exogeneity), with only a 0.006% adjustment following a 1% shock to the long-run equilibrium.

Table 5 – Short-run and error correction

Error correction:	$\Delta(\ln(r^*))$*	$\Delta(\ln(us10))$	$\Delta(\ln(tfp))$	$\Delta(ln(g))$*	$\Delta(\ln(risk))$
Coint eq	-0.006347	0.208461	-0.011913	-0.294349	-0.003727
	(0.00061)	(0.17204)	(0.00725)	(0.06993)	(0.02528)
	[-10.4425]	[1.21174]	[-1.64407]	[-4.20946]	[-0.14742]

Notes: standard errors in parentheses; *t*-statistics in brackets. (*) denotes statistical significance at the 1% level. Source: authors' elaboration.

4.3. Long-run relationships and augmented ARDL estimates: robustness test

The main advantage of the Autoregressive Distributed Lag (ARDL) model proposed by Pesaran et al. (2001) is that long-run estimation does not require compliance with the Engle and Granger (1987) condition—that is, that all variables must be integrated of first order. In a sense, Pesaran et al. (op. cit.) confirm the earlier approach of Campbell and Perron (1991). In addition, the common issue of endogeneity is mitigated in the ARDL model, which jointly estimates short- and long-run coefficients and employs both dependent and explanatory variables in lagged form (Murthy & Okunade, 2016). Nevertheless, under a conventional interpretation, Pesaran et al. (op. cit.) require that the dependent variable be I(1). This would rule out estimation in the present study, since r^* is I(0). However, Sam et al. (2019) propose an augmented ARDL bounds test. Their framework incorporates an additional F-test on the lagged levels of the independent variables in the ARDL

[6] It is possible to speak of elasticities because, in log-log models, the coefficients are interpreted as the elasticity of the dependent variable with respect to the regressors (Greene, 2012; Wooldridge, 2019).

equation. The main advantage of the augmented ARDL bounds test is that the dependent variable need not be I(1); it is thus possible to estimate a long-run relationship even when the ARDL model's dependent variable is stationary in levels, I(0).

For the estimated relationship to display cointegration, Sam et al. (op. cit.) propose a procedure that requires not only that the F-statistic on all variables exceed the upper critical bound I(1) based on the values in Pesaran et al. (2001), but also that: (a) the t-statistic on the lagged level of the dependent variable exceeds that same upper bound; and (b) the F-statistic on the lagged independent variables exceeds the critical upper bound reported in Sam et al. (2019). The most general empirical ARDL model estimated follows the specification proposed by Pesaran et al. (2001), where Δ is the difference operator, α_0 a constant, and μ_t the error term (the logarithm operator is omitted for brevity):

$$(18)\ \Delta {r^*}_t = \alpha_0 + \sum_{i=1}^{n} \alpha_{1i} \Delta {r^*}_{t-i} + \sum_{i=0}^{n} \alpha_{2i} \Delta tfp_{t-i} + \sum_{i=0}^{n} \alpha_{3i} \Delta g_{t-i} + \sum_{i=0}^{n} \alpha_{4i} \Delta risk_{t-i} + \sum_{i=0}^{n} \alpha_{5i} \Delta us10_{t-i} + \beta_0 {r^*}_{t-1} + \beta_1 tfp_{t-1} + \beta_2 g_{t-1} + \beta_3 risk_{t-1} + \beta_4 us10_{t-1} + \mu_t$$

The null hypothesis of no cointegration corresponds to $\beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. If cointegration exists, an error correction model (ECM) can be estimated, where θ is the adjustment coefficient (or error-correction speed), and ECEC are the residuals from equation (18):

$$(19)\ \Delta {r^*}_t = \alpha_0 + \sum_{i=1}^{n} \alpha_{1i} \Delta {r^*}_{t-i} + \sum_{i=0}^{n} \alpha_{2i} \Delta tfp_{t-i} + \sum_{i=0}^{n} \alpha_{3i} \Delta g_{t-i} + \sum_{i=0}^{n} \alpha_{4i} \Delta risk_{t-i} + \sum_{i=0}^{n} \alpha_{5i} \Delta us10_{t-i} + \theta EC_{t-1} + \varepsilon_t$$

Table 6 reports the long-run evidence, which is automatically deduced from (18)–(19). For robustness, three ARDL specifications (I, II, and III) were estimated. As shown, the hypothesis that increases in TFP are associated with reductions in the Brazilian natural rate of interest ($\beta_1 < 0$) is confirmed at the 1% significance level. The long-run sensitivity of the natural rate to changes in TFP lies between –3.22% and –3.48%. In the general model (I), which includes all controls, the estimate was $\beta_1 = -3.22$, implying an elasticity of –3.226% in the natural rate for every 1% increase in TFP.

With the exception of long-term real interest rates in the U.S., the other control variables also displayed the expected sign and statistical significance. Both trend growth and country risk were positively related to the Brazilian natural rate, lending support to the view that they are conditioning factors. In particular, in model I, the natural rate exhibited an elasticity of 1.43% with respect to g and 0.66% with respect to country risk, for each 1% increase in these variables.

Regarding the statistics from Sam et al. (2019), the results support cointegration among the variables across all three specifications. Furthermore, to ensure the reliability of the estimates, parameter stability must be verified. Two stability tests were employed: recursive residuals and the cumulative sum (CUSUM) test (Appendix). Both assess whether, as new information is incorporated over the sample, the residuals retain white-noise properties and the estimated coefficients remain stable. For

simplicity, both tests were applied only to the general model (I). For the recursive residuals test (left panel), the residuals remain within the upper and lower bounds (±2\pm 2 S.E.); and for the CUSUM test, the cumulative residuals also remain within the 5% significance band.

Table 6 – Long-run relationships (augmented ARDL) / Dependent: ln(r*)

	I(4, 2, 4, 0, 0)	***II(4, 2, 4, 0)***	***III(4, 2, 4, 0)***
Intercept	11.338***	13.492***	17.340***
	(2.890)	(1.711)	(3.645)
ln(tfp)	-3.226***	-3.402***	-3.481***
	(0.819)	(0.751)	(0.786)
ln(g)	1.433***	1.464***	0.922***
	(0.421)	(0.420)	(0.196)
ln($risk$)	0.669**	0.704**	-
	(0.345)	(0.350)	-
ln($us10$)	0.022	-	0.004
	(0.024)	-	(0.022)
bound F-test	43.107***	51.845***	43.829***
t lagged DP	-4.063**	-4.036**	-5.475***
F lagged IDPs	45.72***	45.98***	107.36***

Notes: *** denotes statistical significance at the 1% level; ** at 5%; * at 10%. Standard errors in parentheses; DP for dependent variable; IDP for independent variables. Source: authors' own elaboration based on data series and applied methods.

The short-run relationships among the model's variables operate with the purpose of eliminating potential disturbances in the long-run equilibrium. The error-correction model (19) makes it possible to capture such dynamics. The general ARDL specification (I) was employed to extract the short-run estimates. Table 7 presents the evidence obtained. The first point to highlight is the identification of a short-run persistence process in the natural rate of interest. Variations occurring in months *t–2* and *t–3* are positively correlated with the current variation in the natural rate, Δln(r*). With statistical significance, these two lagged components indicate an accumulated loading of 0.855 (= 0.709 + 0.146) onto the contemporaneous variation. This result emphasizes that, once a shift in r* takes place, the process tends to persist in the short run and in the same direction.

Table 7 – ARDL: error correction model / short-run (Model I)

Regressor	Coef	St-dev	t-Stat	p-value
Δln(r* (-1))	-0.059	0.0500	-1.185	0.237
Δln(r* (-2))	0.709	0.0543	13.047	0.000
Δln (r* (-3))	0.146	0.0477	3.055	0.002
Δln(tfp)	0.002	0.0054	0.504	0.614
Δln (tfp (-1))	0.022	0.0055	3.971	0.000
Δln (g)	0.005	0.0005	10.599	0.000
Δln (g(-1))	-0.001	0.0005	-2.448	0.015
Δln (g(-2))	0.0005	0.0004	1.165	0.245
Δln (g(-3))	0.001	0.0004	2.597	0.010
EC(-1)	-0.005	0.0003	-16.264	0.000

Fonte: elaboração própria.

Furthermore, although TFP is inversely related to r* in the long run, such a relationship is not observed in the short run. In practice, an increase in Δln(ptf(-1)) is associated with a rise in Δln(r*),

albeit with low elasticity (0.022). Likewise, short-run variations in the trend component, based on the statistically significant coefficients, are followed by an increase in $\Delta\ln(r^*)$, with an accumulated elasticity of 0.005. These impacts of small magnitude are likely explained by the strong short-run inertia in changes of the natural rate. In this sense, the ARDL evidence is consistent with the findings obtained through the VEC model previously presented. Finally, the error-correction term (EC) is statistically significant and carries the expected negative sign. However, its low coefficient (–0.005) suggests that the natural rate adjusts only very slowly in the short run when faced with disruptions in the estimated long-run relationship.

5. Quantile regression estimates: nonlinearities in the relationship between TFP and the natural rate

Since the linear cointegration methods employed rely on estimates for the conditional mean of the natural rate of interest, there may be underlying nuances regarding possible differences in the estimated correlations across the distribution of the dependent variable. One may suppose that, as realizations of r^* occur in regions of the distribution further from the median, structural changes may arise in the impacts of variations in the regressors—particularly total factor productivity (TFP). Put differently, do the correlations estimated in the previous sections change (in terms of magnitude and/or sign) as r^* moves away from its median?

Adopting quantile regression estimates can help address this issue. Among the recognized advantages of quantile regression are: (i) reduced sensitivity to extreme values or tail events (outliers) in the dependent variable, making it more robust than linear regression; and (ii) by examining different quantiles, quantile regression reveals how predictors affect different parts of the response distribution, offering a richer view of the relationships among variables (Koenker and Bassett, 1978; Buchinsky, 1998; Yu et al., 2003). The formal and parsimonious structure of a quantile regression model can be expressed as:

$$(20)\ \ Q_\tau(r^*: X) = \mathrm{B}_\tau X, \tau \in (0,1)$$

where $Q_\tau(r^*: X)$ denotes the quantile response of the natural rate of interest to the regressor matrix X, conditional on quantile τ. The quantile distribution of the natural rate lies in the interval $(0,1)$, with $\tau = 0.5$ representing its median. Finally, the coefficient vector B_τ is defined for each quantile, capturing potential structural changes in the relationship between the natural rate and the set of regressors. The covariance matrix and standard errors are estimated via minimization of a Tukey/Huber loss function for each quantile of the distribution. The sparsity estimation follows Siddiqui (1960), allowing the present study to obtain the different $\beta's$ for the contemporaneous relationships between r^* and ptf, g, $risk$ and $us10$ (all in natural logarithms), across 10 deciles of the natural rate distribution. The estimated functional form is:

$$(21)\ \ \ln({r^*}_\tau) = \beta_{0\tau} + \beta_{1\tau}\ln(tfp_\tau) + \beta_{2\tau}g_\tau + \beta_{3\tau}\ln(risk_\tau) + \beta_{4\tau}\ln(us10_\tau) + \varepsilon_\tau$$

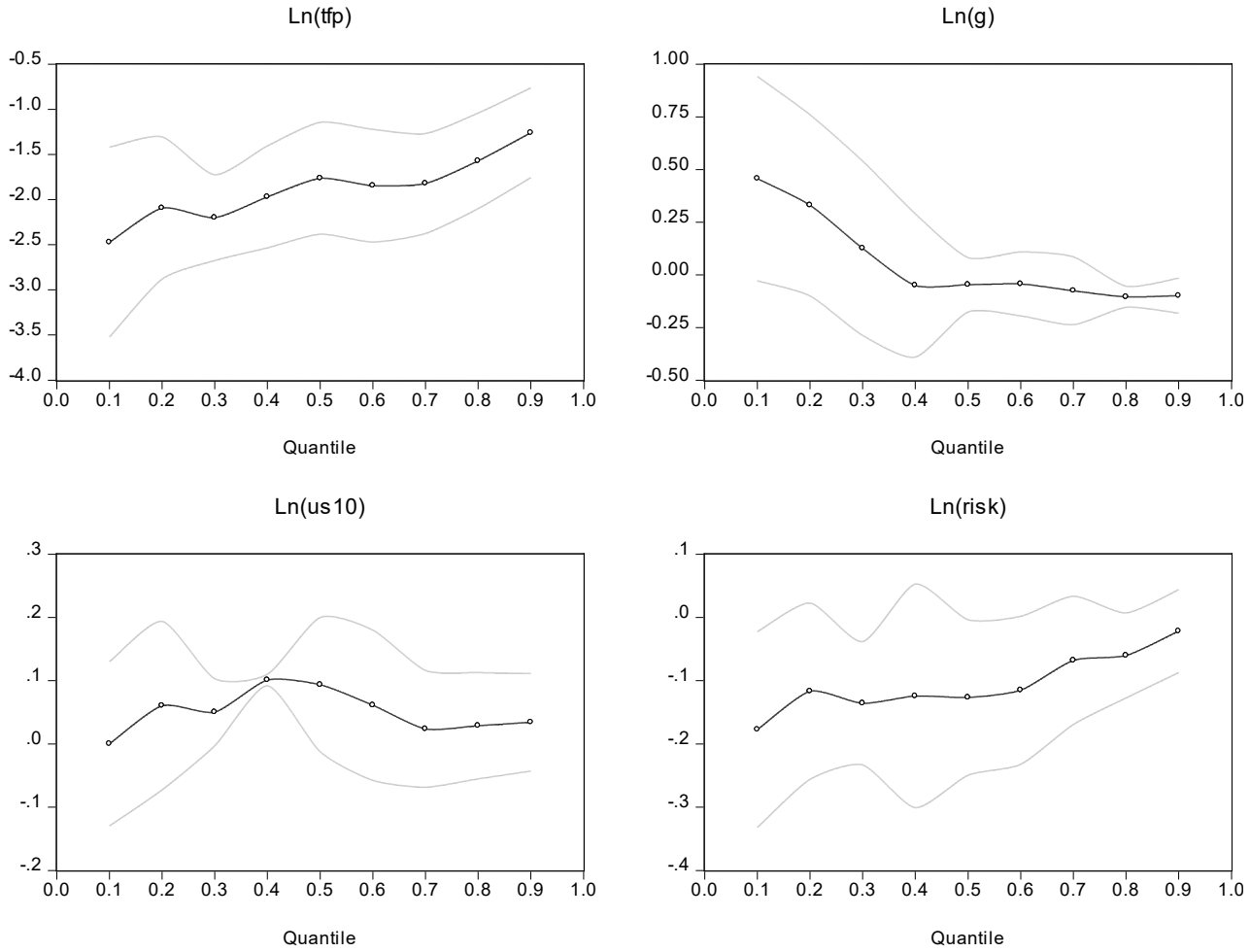
with $\tau = 0{,}1, 0{,}2, \ldots, 0{,}9$. Figure 5 reports the evidence obtained. Importantly, the inverse relationship between TFP and the natural rate holds across the entire quantile distribution (upper-left panel). In this case, the hypothesis that $\beta_{1\tau} < 1$ is confirmed for all τ. Nevertheless, the potential impact of TFP is stronger at the lower deciles of the natural rate distribution. In other words, the elasticity is -2.47\% for the bottom 10% of r^* (first decile) and decreases in absolute value as the deciles rise, reaching -1.26% at the tenth decile (top 10% of r^*). This implies a relevant nonlinearity: differences in the responses of the natural rate at the tails of the distribution compared with the conditional median response (-1.76\% when $\tau = 0.5$).

The practical implication for policy is that the elasticity of the structural real interest rate to TFP tends to increase when the natural rate itself is low. In other words, the gains from productivity improvements appear greater (in terms of reducing real interest rates) in more recent years, compared with the earlier part of the sample (where the highest deciles of the natural rate lie). Regarding trend growth, estimates show a sign reversal across the distribution: positive (as found in the previous sections) at the lower deciles, but turning negative from the fourth decile onwards.

By contrast, the long-term U.S. real interest rate retained a positive sign throughout the distribution, while country risk displayed a negative sign across all deciles – opposite to the earlier linear estimates. Thus, regarding the controls employed, the results suggest sensitivity to the estimation method and caution in interpreting the evidence. In contrast, the relationship of interest (TFP and the natural rate) consistently remained negative and statistically significant across all quantiles, despite differences in magnitude.

Finally, the standard test for equality of quantile coefficients was conducted. The resulting Wald-type χ^2 statistic (=302.95) is statistically significant at the 1% level (Appendix). This leads to the rejection of the null hypothesis that coefficients are equal across the distribution, supporting the adequacy of the quantile approach adopted here.

Figure 5 – Quantile process estimates for tfp, g, $us10$ and $risk$



Note: changes in the response coefficients of r* to the independent variables, across each decile of the dependent variable distribution. Confidence intervals at the 1% level. Source: authors' own elaboration.

6. Final Discussion and Policy Implications

Monitoring the natural rate of interest is a fundamental activity in calibrating monetary policy decisions (Moreira, 2024), especially for horizons that extend beyond the short term (above 24 months). Misestimation of this rate may result in current and expected real interest rates being misaligned with the objectives of stabilizing both economic activity and inflation.

As technological innovation and human capital improvement unfold over the medium and long run, there is a natural tendency for national TFP to rise (in this case, assuming the measure adopted in this study, where TFP also captures developments in human capital). This implies structurally lower levels of real interest rates, i.e., a lower natural rate of interest. This is the main empirical result of the present study for the period February 2003 to December 2022 in Brazil. It can be suggested that most of the observed increase in TFP was associated with gains in human capital and the resulting improvements in labor productivity, with the dimension of technological progress per se falling considerably short (Veloso et al., 2024).

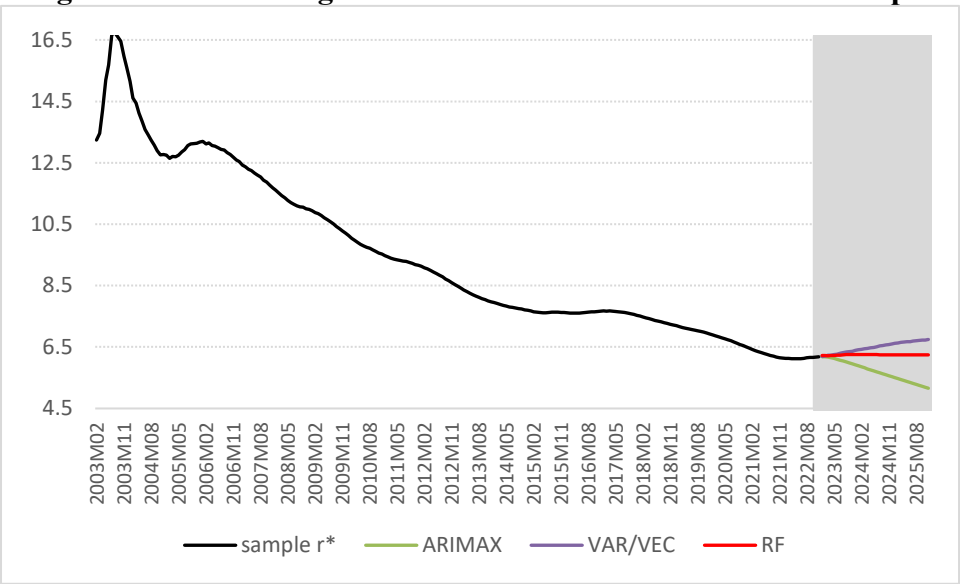
Key events over the past decade — such as labor market and pension reforms, the introduction of *Pix* (certainly one of the most important technological innovations in Brazil's financial history), and even the gradual adoption of artificial intelligence (AI) by firms and institutions — may help in disinflating the economy at lower social costs. Assuming that such institutional and technological changes are accompanied by persistent gains in TFP, then, yes, long-term real interest rates tend to fall, although this trajectory is neither linear nor devoid of volatility. In 2025, with the nominal Selic reaching 15% per year, the real Selic rate (discounted by expected 12-month ahead inflation/BCB) would be around 9% annually — undoubtedly highly restrictive and above the natural rate (at least, as estimated in this study for December 2022 at 6.18%). Such deviations are themselves a countercyclical instrument of monetary policy to stabilize inflation.

Indeed, we developed a parsimonious forecasting exercise for the natural rate of interest, extending the sample period by 36 months (covering the horizon from January 2023 to December 2025). Three forecasting methods were employed – ARIMAX, VAR/VEC, and Random Forest with temporal lags. A concise methodological summary and the theoretical underpinnings of each approach are presented in the Appendix.

The forecasts are displayed in Figure 6. It can be observed that the projected trajectories vary depending on the method applied. The most optimistic forecast was obtained from the ARIMAX model, which predicts a natural interest rate of 5.15% in December 2025, whereas the most pessimistic projection arises from the VAR/VEC model, yielding 6.74% at the end of the forecasting horizon. The Random Forest (RF) model produced an intermediate estimate of 6.24%, slightly above the level estimated for December 2022 (6.18%), the last month within the sample. Nonetheless, regardless of the forecasting method considered, the results suggest a benign outlook for Brazil's natural rate of interest, at least when compared with the levels observed over the past two decades.

At the same time, fiscal, credit, political, and legal risks are also crucial factors to allow for this improved future scenario. Improvements in Brazil's fiscal framework, together with advances in corporate governance – by improving the overall business environment – could allow the natural rate to decline via lower country risk, with potential direct or indirect effects on TFP itself. In this regard, Donnat et al. (2025) found an inverse relationship between the public debt-to-GDP ratio and TFP, based on estimates for a panel of 25 OECD countries between 1980 and 2019.

Figure 6 – Forecasting models for r^* over 36-months out-of-sample



Note: filled area for forecasted r^* Source: authors' own elaboration.

Equally important, our estimates indicate that the natural rate responds positively to the trend growth rate of GDP, in line with theoretical predictions (e.g., Laubach & Williams, 2003; Holston, Laubach & Williams, 2017). *Ceteris paribus*, the higher the trend GDP growth rate, the higher r^*. Since g (trend growth) can be influenced by a range of demand- and supply-side factors, a concrete example may be useful in the current Brazilian context. An expansionary fiscal policy may boost g by persistently raising social spending and transfers that stimulate household consumption. However, if such an effect is not accompanied by a simultaneous increase in TFP, the natural rate of interest is expected to rise. This constrains BCB's monetary policy, as a higher real interest rate is then required to meet the inflation target and close any output gap. Ignoring this trajectory (i.e., the path of the neutral interest rate) may lead to poor calibration of the monetary policy risk balance and undesirable business cycle outcomes (Moreira, 2024).

By contrast, when aggregate demand stimulus comes primarily from investment in fixed capital formation, even if g rises, there is greater likelihood that such an environment will also foster higher levels of TFP. This may, in turn, allow a disinflationary process (via lower marginal costs of production), thereby reducing the neutral rate of interest. As suggested by our estimates (Model I), the elasticity of the natural rate with respect to TFP is greater than its elasticity with respect to g. In approximate terms, a 1% impulse in the latter, coupled with a 1% increase in TFP, should lead to a decline in the natural rate of interest of -1.79% (= -3.226 + 1.433), holding other regressors constant.

In this sense, the research agenda on the medium- and long-term determinants of economic policy — particularly monetary policy — still faces challenges, given the scarcity of related evidence (especially applied to Brazil), as well as the technical difficulties in estimating and measuring both the natural rate and TFP itself. From our perspective, the contribution of this paper is to partially fill this gap.

Finally, this study employed a quantile regression approach to test for differences in estimated elasticities along the distribution of the natural rate of interest. The evidence obtained for a 10-decile distribution suggests that the elasticity of the natural rate with respect to TFP was stronger in the lower deciles, indicating that lower values of r^* are associated with greater sensitivity to further productivity changes (elasticity of –2.47% for the bottom 10% of r^*). Since the lowest realizations

of the natural rate occurred in the final years of the sample, it may be inferred that productivity gains have been yielding increasing returns in terms of reducing Brazil's real interest rates.

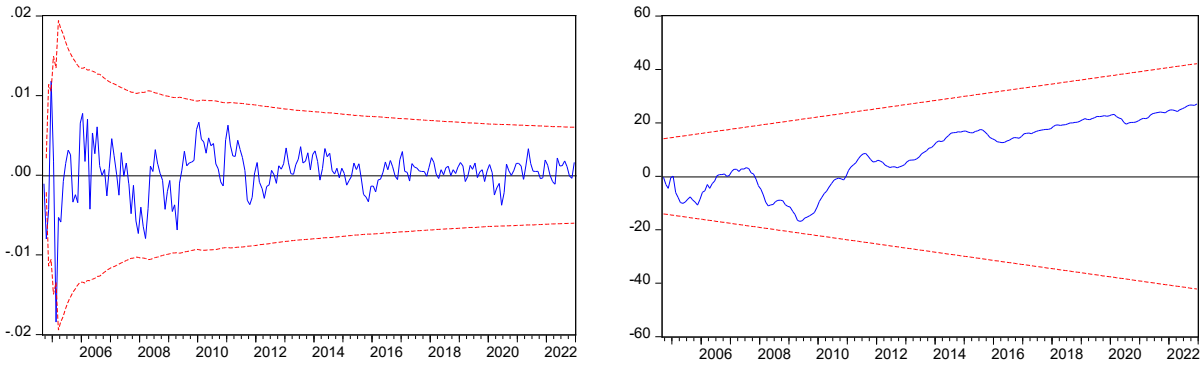
The message is straightforward: for Brazil to gradually and consistently achieve declining levels of real interest rates, broad-based economic planning must be anchored in gains from technological progress (and human capital). Future research may expand both the scope and the quality of evidence in this line of inquiry, and such efforts would be most welcome.

Data Availability

The full EViews file and data were shared.

Appendix

A1. Recursive Residuals and CUSUM



Source: authors' elaboration.

A2. Quantile Coefficients Equality Test

Test Summary	Chi-Sq. Statistic	Chi-Sq. D.f.	Prob.
Wald Test	302.9530	32	0.0000

Source: authors' elaboration.

A3. Methodology of the Forecasting Models

The methodological procedures adopted for forecasting the variable r^* (the natural rate of interest), based on three complementary econometric approaches – ARIMAX with four exogenous variables (tfp, g, $risk$, and $us10$), VAR/VEC involving the five series simultaneously, and Random Forest with up to twelve lags – are widely recognized in the international forecasting literature and offer distinct advantages in terms of structure, flexibility, and statistical robustness.

First, the ARIMAX model (*AutoRegressive Integrated Moving Average with Exogenous Variables*) combines the autoregressive and moving-average structure with the explicit inclusion of exogenous regressors, allowing the model to capture the effects of external shocks on the dependent variable (r^*). Estimation was carried out using maximum likelihood, with identification of the parameters p, d, and q based on information criteria (AIC and BIC). The exogenous variables tfp, g, $risk$, and $us10$ were included as contemporaneous regressors. For out-of-sample forecasting, each exogenous variable was projected using univariate ARIMA models, ensuring dynamic consistency across simulated trajectories. This approach is well documented in Hyndman and Athanasopoulos (2021), who highlight its effectiveness for conditional forecasting in partially structured systems.

The second method employed was the VAR/VEC model, as estimated in Section 4.2, considering the five endogenous series (r^*, tfp, g, $risk$, and $us10$). For the out-of-sample forecasting exercise, all variables were jointly projected, preserving the coherence of the multivariate system so that the trajectories of each exogenous series were generated endogenously by the model itself. According to Lütkepohl (2005), this feature gives the VAR/VEC framework strong aggregate forecasting power and the ability to analyze the transmission of economic policy shocks.

The Random Forest (RF) model, in turn, was employed as a nonparametric and machine-learning–based approach. Up to twelve lags of each series were used to construct the feature matrix, ensuring that the temporal structure was embedded in the learning process. The model was fitted through an ensemble of decision trees, each built on random subsamples of the data and subsets of predictors, following the aggregation principle proposed by Breiman (2001). Model performance was evaluated based on out-of-sample error metrics.

The implementation of the Random Forest model was carried out in the Python programming language using the `scikit-learn` library (https://scikit-learn.org/stable/), which is widely recognized in the machine-learning literature applied to econometrics (Breiman, 2001). Compared with the ARIMAX and VAR/VEC methods, Random Forest has the advantage of modeling complex nonlinear relationships and variable interactions without the need for parametric specification, resulting in superior predictive performance in economic environments subject to structural changes or high volatility.

References

Ball, L., & Moffitt, R. (2001). Productivity growth and the Phillips Curve (NBER Working Paper No. 8421, pp. 1–50). National Bureau of Economic Research. https://doi.org/10.3386/w8421

Barbosa Filho, F. H., & Pessoa, S. A. (2014). Pessoal ocupado e jornada de trabalho: Uma releitura da evolução da produtividade no Brasil. *Revista Brasileira de Economia*, 68(2), 149–169.

Barcellos Neto, P. C. F. D., & Portugal, M. S. (2009). The natural rate of interest in Brazil between 1999 and 2005. *Revista Brasileira de Economia*, 63, 103–118.

Barro, R. J., & Lee, J. W. (1993). International comparisons of educational attainment. *Journal of Monetary Economics*, 32(3), 363–394.

Breiman, L. (2001). Random Forests. *Machine Learning*, 45(1), 5–32.

Buchinsky, M. (1998). Recent advances in quantile regression models: a practical guideline for empirical research. *Journal of Human Resources,* 33(1), 88-126.

Campbell, J. Y., & Perron, P. (1991). Pitfalls and opportunities: What macroeconomists should know about unit roots. In O. J. Blanchard & S. Fischer (Eds.), NBER Macroeconomics Annual (pp. 141–201). MIT Press.

CEA (Council of Economic Advisors) (2015). Long-Term Interest Rates: A Survey. https://obamawhitehouse.archives.gov/sites/default/files/docs/interest_rate_report_final.pdf

Chan, J. C. C., Koop, G., & Potter, S. M. (2016). A Bounded Model of Time Variation in Trend Inflation, Nairu and the Phillips Curve. *J. Appl. Econ.*, 31,551–565

Costa Junior, C. J., Garcia-Cintado, A. C., & Marques Junior, K. (2023). A taxa de juros natural em modelos DSGE: o caso do Brasil. *Economia Aplicada*, *27*(4), 433-453.

Del Negro, M., Giannone, D., Giannoni, M. P., & Tambalotti, A. (2019). Global trends in interest rates. *Journal of International Economics*, 118, 248-262,

Donnat, G., Menuet, M., Minea, A., & Villieu, P. (2025). Does public debt impair total factor productivity? Documents de travail GREDEG, Working Papers Series, n. 2025-26.

Dos Santos, L. B., & Moreira, R. R. (2020). Nominal effects of changes in total factor productivity: Evidence for an emerging economy. *International Journal of Economics and Finance*, 12(12), 89–99.

Duarte, J. (2010). Measuring the natural interest rate in Brazil. Institute of Brazilian Business and Public Management Issues.

Engle, R. F., & Granger, C. W. J. (1987). Cointegration and error correction: Representation, estimation, and testing. *Econometrica*, 55(2), 251–276.

Favero, C. A. (2001). *Applied macroeconometrics*. Oxford University Press.

Favero, C. A., & Giavazzi, F. (2002). Why are Brazilian interest rates so high? Mimeo, IGIER-Bocconi, Milan.

Galí, J., & Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, 72(3), 707–734.

Gomes, V., Pessoa, S. A., & Veloso, F. (2003). Evolução da produtividade total dos fatores na economia brasileira: Uma análise comparativa. *Pesquisa e Planejamento Econômico*, 33(3), 389–434.

Greene, W. H. (2012). *Econometric Analysis*. 7th ed, Prentice Hall, Cambridge, MA.

Harvey, A. C. (1989). *Forecasting, structural time series models and the Kalman filter*. Cambridge University Press.

Hall, R. E., & Jones, C. I. (1999). Why Do Some Countries Produce So Much More Output Per Worker Than Others? *The Quarterly Journal of Economics*, *114*(1), 83–116.

Holston, K., Laubach, T., & Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, 108(1), S39–S75.

Hyndman, R. J., & Athanasopoulos, G. (2021). Forecasting: Principles and Practice (3rd ed.). OTexts.

Hsieh, C., & Klenow, P. (2009). Misallocation and Manufacturing TFP in China and India. The Quarterly Journal of Economics, 124(4), 1403-1448.

IMF (International Monetary Fund) (2014). *Perspectives on Global Real Interest Rates*. World Economic Outlook, April.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2–3), 231–254.

Johansen, S., & Juselius, K. (1990). Maximum likelihood estimation and inference on cointegration: With application to the demand for money. *Oxford Bulletin of Economics and Statistics*, 52(2), 169–210.

Justiniano, A., & Primiceri, G. E. (2008). The Time-Varying Volatility of Macroeconomic Fluctuations. *American Economic Review,* 98(3), 604–41.

Kanczuk, F. (2002). Juros reais e ciclos reais brasileiros. *Revista Brasileira de Economia*, 56, 249–267.

Karanassou, M., & Sala, H. (2012). Productivity growth and the Phillips curve: A reassessment of the US experience. *Bulletin of Economic Research*, 64(3), 344–366.

Koenker, R.; Basset, G. (1978). Regression quantiles. *Econometrica*, 46, 33-50.

Laubach, T., & Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, 85(4), 1063–1070.

Lubik, T. A., & Schorfheide, F. (2007). Do central banks respond to exchange rate movements? A structural investigation, *Journal of Monetary Economics*, 54(4), 1069-1087.

Lütkepohl, H. (2005). New Introduction to Multiple Time Series Analysis. Springer-Verlag.

Moreira, R. R. (2015). Reviewing Taylor rules for Brazil: was there a turning-point?. *Journal of Economics and Political Economy*, *2*(2), 276–289.

Moreira, R. R. (2024). Neutral real interest rate, monetary policy and business cycle: Using the Kalman filter and a counterfactual structural approach for a large emerging market. *International Journal of Economics and Finance*, 16(12), 62–72.

Muinhos, M., & Nakane, M. (2006). Comparing equilibrium real interest rates: Different approaches to understand Brazilian rates. *Banco Central do Brasil* Working Paper 101.

Murthy, V. N. R., & Okunade, A. A. (2016). Determinants of U.S. health expenditure: Evidence from autoregressive distributed lag (ARDL) approach to cointegration. *Economic Modelling*, 59, 67–73.

Palma, A. A. (2016). Natural interest rate in Brazil: further evidence from an AR-trend-bound model. *Economics Bulletin*, 36(3), 1306-1314

Pesaran, M. H., Shin, Y., & Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics*, 16(3), 289–326.

Rachel, L., & Summers, L. H. (2019). On falling neutral real rates, fiscal policy, and the risk of secular stagnation. *Brookings Papers on Economic Activity*, 2019(1), 1–76.

Roberts, J. M. (2001). Estimates of the productivity trend using time-varying parameter techniques. *Contributions to Macroeconomics*, 1(1), 1–27.

Sam, C. Y., McNown, R., & Goh, S. K. (2019). An augmented autoregressive distributed lag bounds test for cointegration. *Economic Modelling*, 80, 130–141.

Siddiqui, M. (1960). Distribution of quantiles from a bivariate population. *Journal of Research of the National Bureau Standards*, 64, 145-150.

Smets, F., & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review,* 97(3), 586–606**.**

Summers, L. H. (2015). Demand Side Secular Stagnation. *American Economic Review,* 105(5), 60–65**.**

Taylor, J. (1993). Discretion versus Policy Rules in Practice. Carnegie-Rochester Conference Series Public Policy, 39, 195-214.

Tesfaselassie, M. F., & Wolters, M. H. (2018). The impact of growth on unemployment in a low vs. a high inflation environment. *Review of Economic Dynamics*, 28, 34–50.

United Nations (2022). Inter-agency Task Force on Financing for Development, *Financing for Sustainable Development Report 2022*. (New York: United Nations, 2022), available from: https://developmentfinance.un.org/fsdr2022.

Veloso, F., Feijó, J., Barbosa Filho, F. H., & Ruhe, A. P. N. (2024). Mensuração do capital humano e suas implicações sobre a produtividade da economia brasileira. Policy Paper, Abril.

Wicksell, K. (1936). *Interest and prices* (R. F. Kahn, Trans.). Macmillan. (Original work published 1898)

Wooldridge, J. M. (2019). *Introductory Econometrics: A Modern Approach*. 7th ed, Cengage Learning, Boston, MA.

Yu, K.; Lu, Z.; Stander, J. (2003). Quantile regression: applications and current research areas. *The Statistician*, 52(3), 331-350.

Zhu, S., Kavanagh, E., & Gao, J. (2023). The natural real rate of interest and monetary policy: New evidence for the US. *International Journal of Finance & Economics*, 1–17.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- naturalrateinbrazil2023.wf1