

1 **Supplementary Figures and Supplementary Table for the**
2 **manuscript entitled**
3 **“Quantifying climate-mode–driven ocean variability reveals**
4 **intensified sea-level rise”**

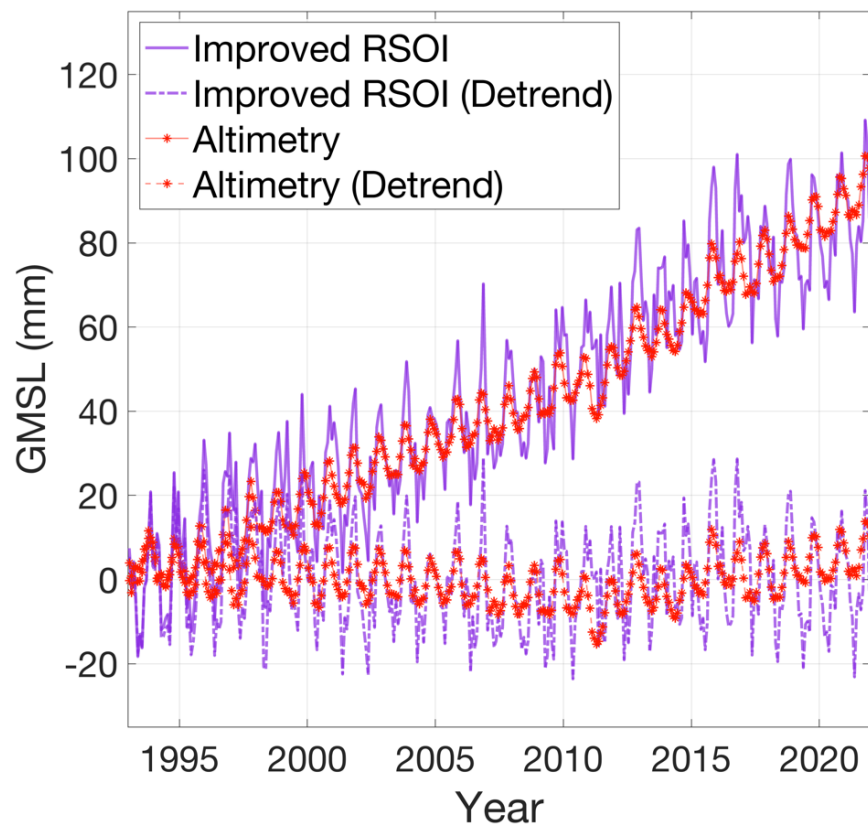
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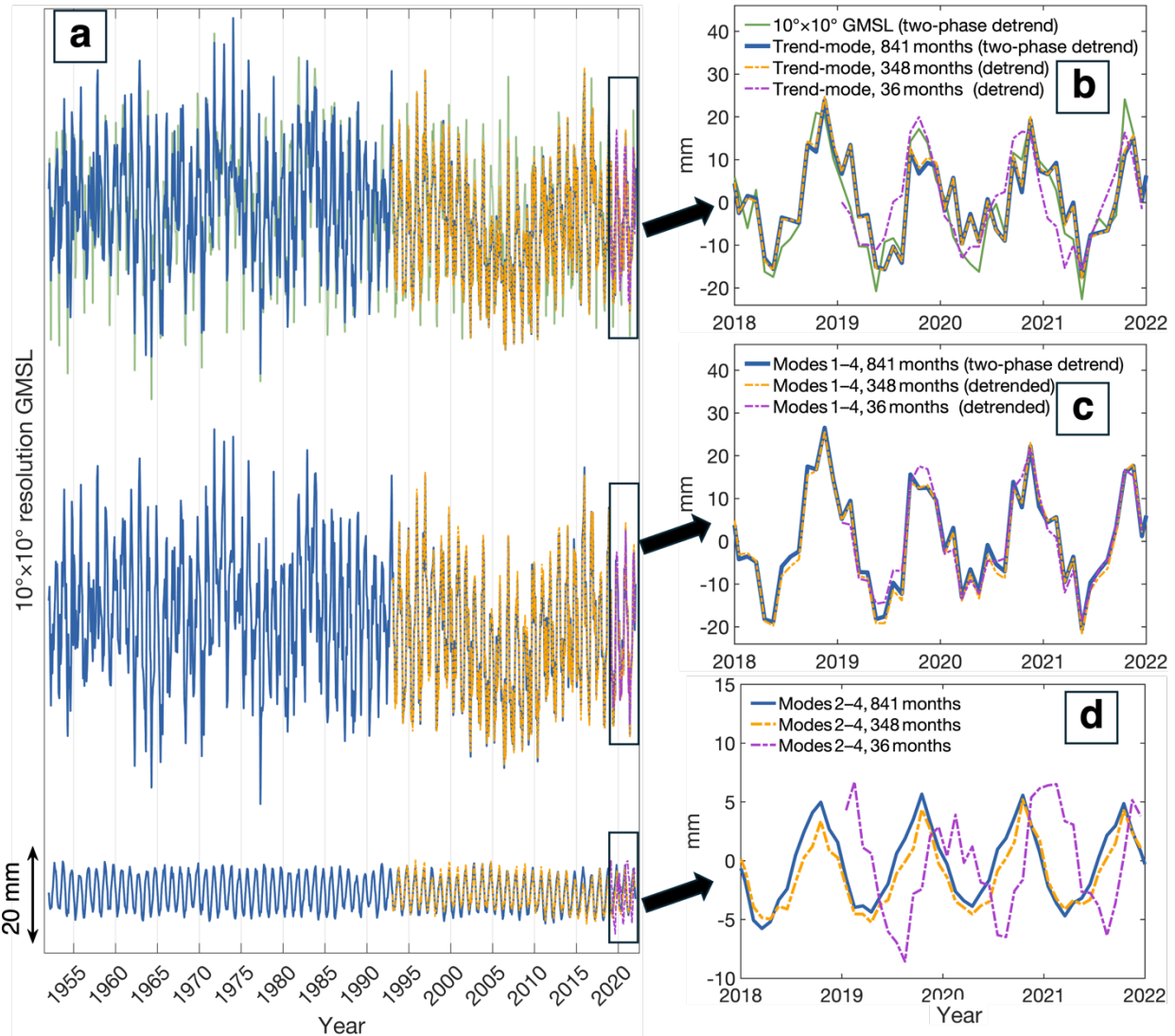
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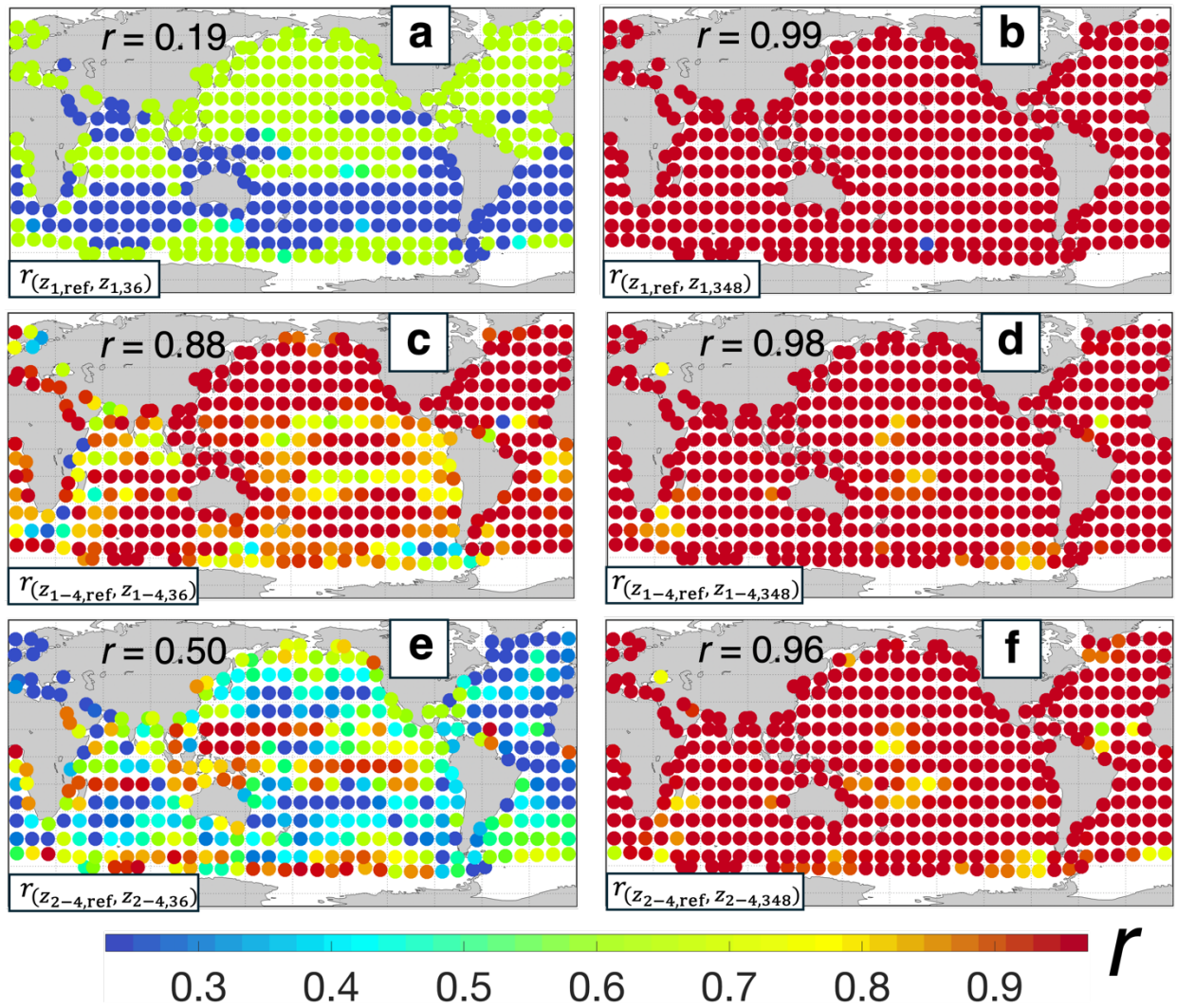


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17 **Supplementary Fig. 1 | Monthly global-mean sea level (GMSL) from the improved RSOI**
18 **reconstruction and AVISO satellite altimetry during the common data span (January 1993–December**
19 **2021). Solid lines show the original time series; dashed lines show the series after linear detrending.**



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 21 **Supplementary Fig. 2 | Record-length sensitivity (global mean) of EOF modes using $10^\circ \times 10^\circ$ -**
 22 **resolution reconstructed sea level. a**, GMSL component of Mode 1 (trend mode; two-phase detrended
 23 pre- and post-January 1993), the sum of Modes 1–4, and the sum of Modes 2–4, each computed from
 24 EOF analysis on the full record (January 1952–January 2022, 841 months), the altimetry-era subset
 25 (January 1993–December 2021; 348 months), and a short subset (January 2019–December 2021; 36
 26 months). **b–d**, Zoom in for 2019–2021 showing (b) Mode 1, (c) sum of Modes 1–4, and (d) sum of Modes
 27 2–4. The 348-month solutions closely track the 841-month reference in the common data span.

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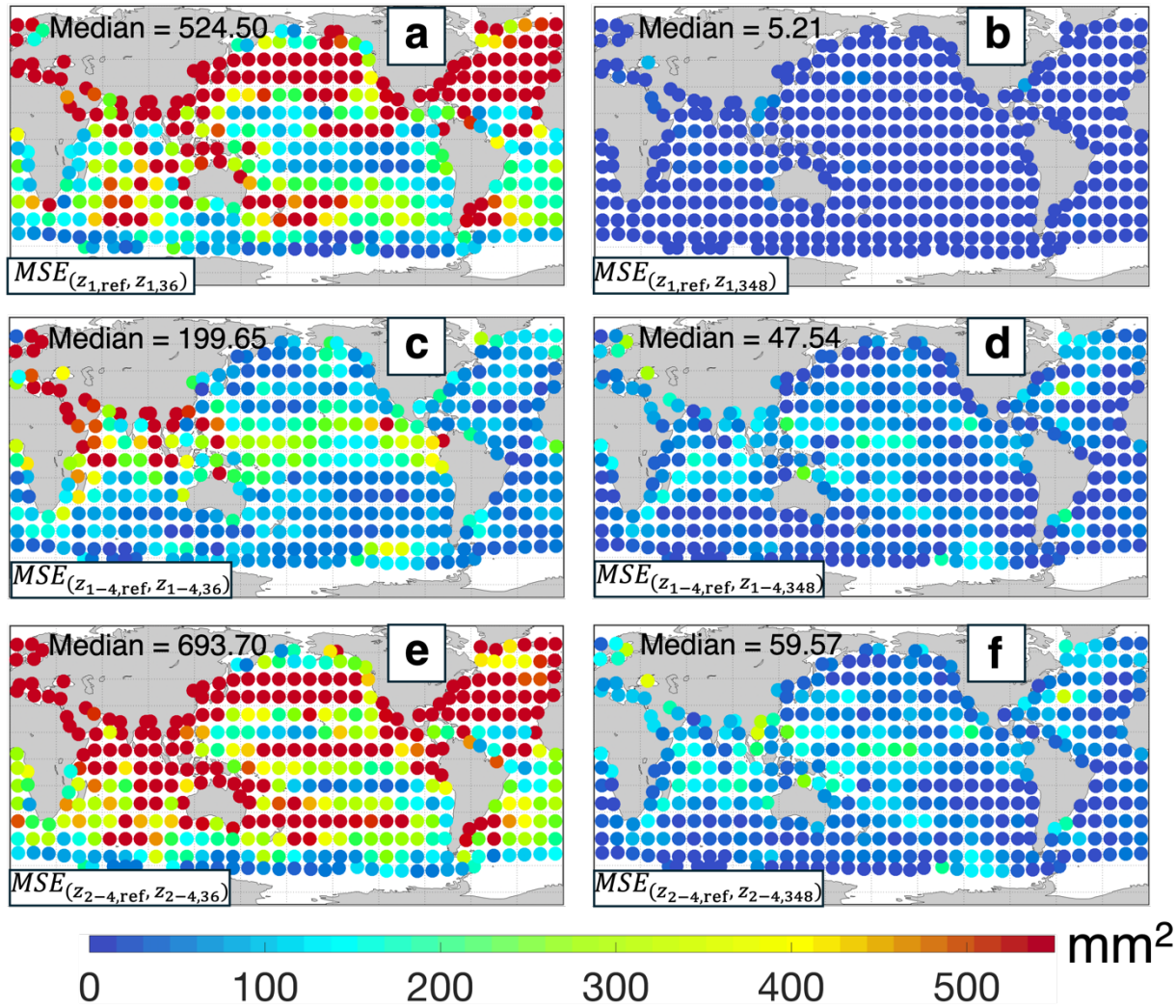
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33 **Supplementary Fig. 3 | Correlation coefficients (r) between truncated records and the 841-month**
 34 **reference on each $10^\circ \times 10^\circ$ grid. a,b, Mode 1: correlations between truncated windows and the reference**
 35 **(January 1952-January 2022) over the common data span - a, 36 months (January 2019- December**
 36 **2021); b, 348 months (January 1993- December 2021). c,d, Sum of Modes 1-4 for the same two windows.**
 37 **e,f, Sum of Modes 2-4 for the same two windows. Averaged correlation r is shown at the top left of each**
 38 **panel. Mode 1 is two-phase-detrended (pre- and post January 1993) before comparison; 36- and 348-**
 39 **month records use trends matched to the detrended Mode 1 within the common span.**

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44 *Supplementary Fig. 4 | Mean squared error (MSE) with the 841-month reference on the $10^\circ \times 10^\circ$ grid.*

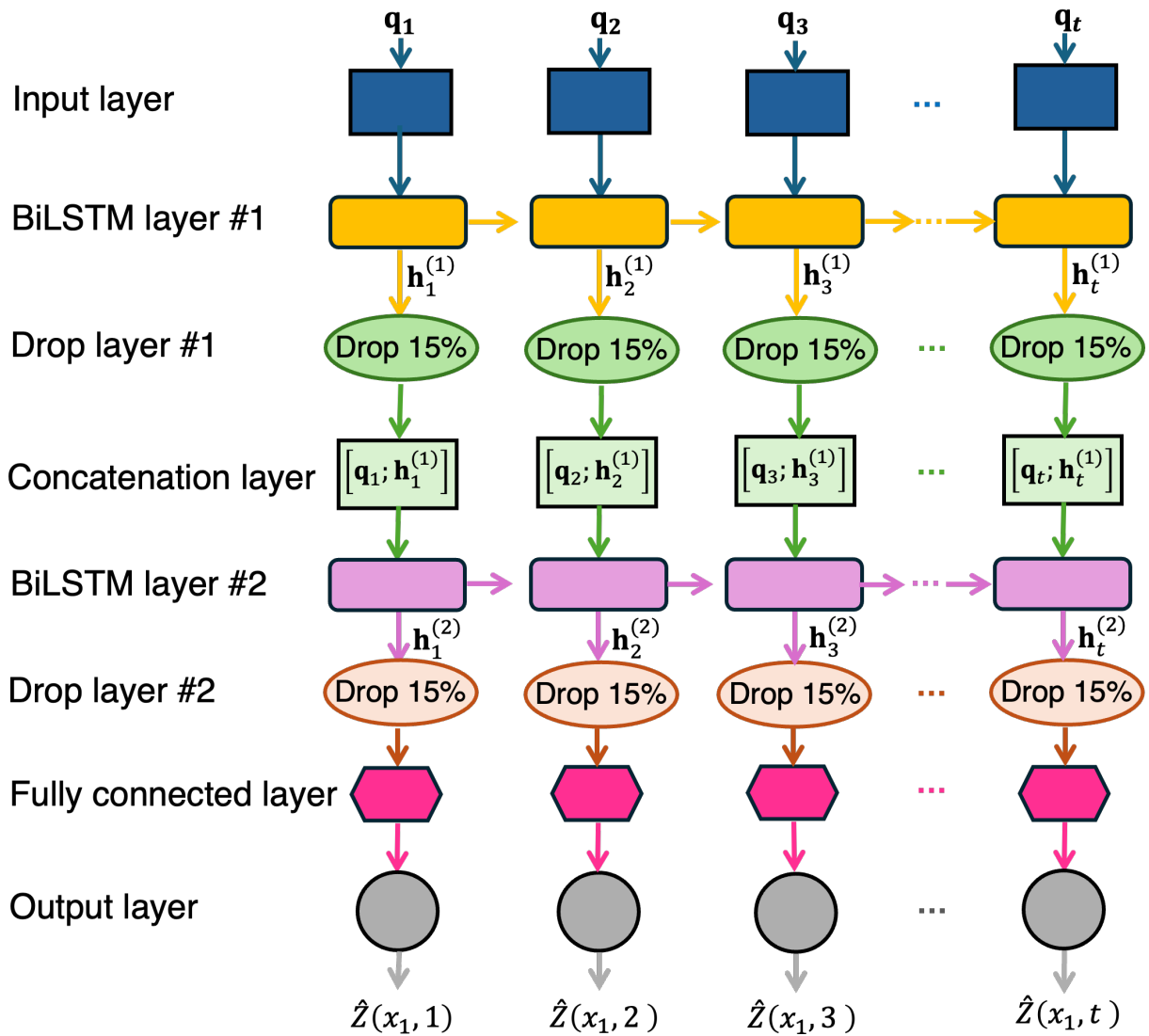
45 *a,b, Mode 1 MSE between truncated records and the reference: a, 36 months (January 2019–December*

46 *2021); b, 348 months (January 1993–December 2021). c,d, Sum of Modes 1–4 for the same windows. e,f,*

47 *Sum of Modes 2–4 for the same windows. Median MSE is shown at the upper left of each panel.*

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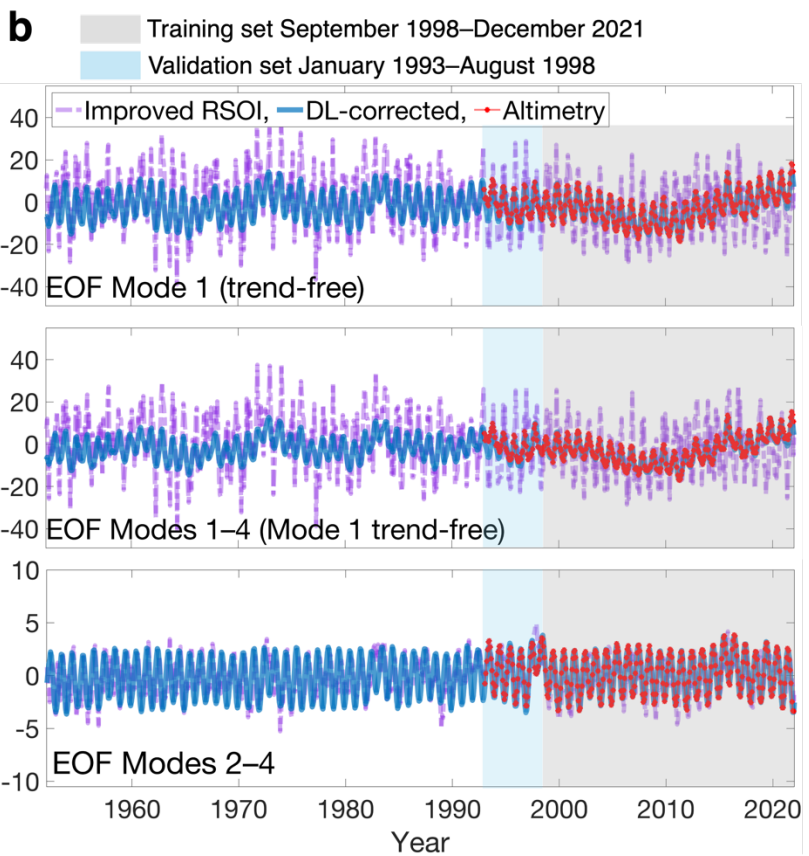
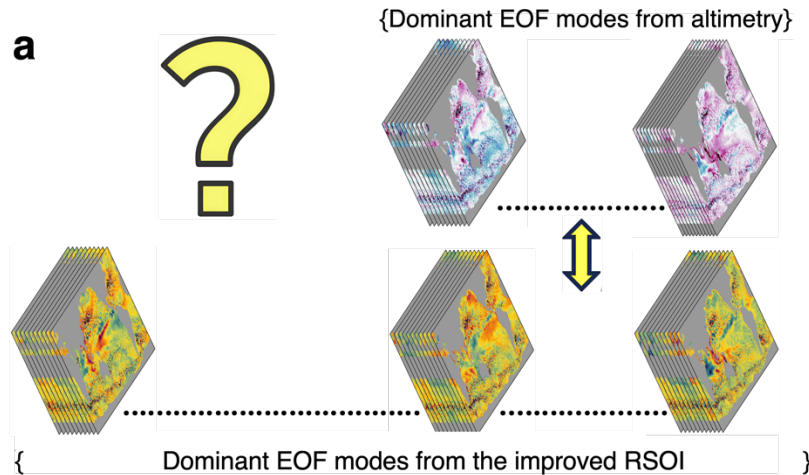
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51 *Supplementary Fig. 5 | Architecture of the two-layer skip-fusion BiLSTM (2SF-BiLSTM) used to*
 52 *calibrate sea-level EOF-mode estimates.*

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57 **Supplementary Fig. 6 | Deep-learning training framework and global-mean EOF comparisons before**
 58 **and after calibration. a, Schematic: majority EOF modes (upper) from altimetry sea-level anomaly and**
 59 **the corresponding EOF modes from the improved RSOI reconstruction (lower). b, Global-mean time**
 60 **series for Mode 1, sum of Modes 1–4, and sum of Modes 2–4 from improved RSOI before and after**
 61 **applying 2SF-BiLSTM deep-learning calibration. Mode 1 from improved RSOI is two-phase detrended,**
 62 **with a breakpoint at January 1993; the altimetry Mode 1 trend matches the two-phase-detrended**
 63 **improved RSOI Mode 1 during their overlap. Training and validation period highlighted in shading.**

64 **Supplementary Table 1 | Performance of deep-learning models at each $1^\circ \times 1^\circ$ grid for the validation**
 65 **set (January 1993–August 1998).** The deep-learning models (TCN, self-attention, and 2SF-BiLSTM) are
 66 trained to calibrate the reconstructed sum of EOF Modes 1–4 (Mode 1 detrended) against the
 67 corresponding altimetry-derived series. Metrics (r^2 , MSE, and MAE) are computed at each grid cell and
 68 then averaged globally. The uncalibrated improved RSOI results are shown for comparison.

<i>Algorithms</i>	<i>TCN</i>	<i>Self-attention</i>	<i>2SF-BiLSTM</i>	<i>Improved RSOI</i>
<i>Comparison</i>				
<i>Averaged r^2 with target altimetry series</i>	0.87	0.91	0.95	0.70
<i>Averaged MSE with target altimetry series</i>	139.18 mm²	243.17 mm²	77.21 mm²	432.97 mm²
<i>Averaged MAE with target altimetry series</i>	8.21 mm	9.68 mm	5.94 mm	15.32 mm

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