High-Fidelity Spatial Photonic Ising Machines via Precise Wavefront Shaping

Supplementary Material

Verification Test for Phase Correction

To verify the result before and after the application of the retrieved phase correction, two different target images are observed. The first is the point spread function affected by the rectangular aperture of our SLM, and the second is the LG_{10} , also known as the vortex phase. In the literature, the vortex phase is documented as being more sensitive to phase distortions [1]. As illustrated in Figure S1, the experimental outcomes of the phase correction are presented, employing both the linear and logarithmic scales.

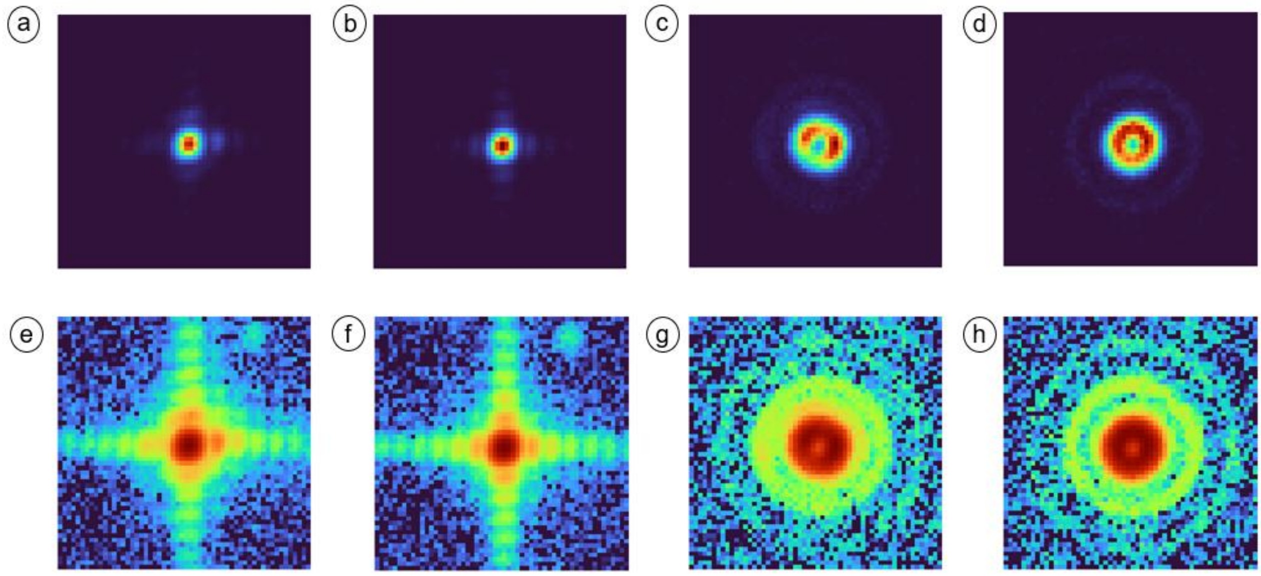

Figure S1. *Comparative results of the phase correction for point spread function and LG01 beam*. Results are presented in both linear (a, b, c, d) and logarithmic scale (e, f, g, h). *PSF:* Respective results for the PSF without (a, e) and with (b, f) phase correction. *LG01 mode*: The LG mode on the camera plane without use of correction mask (c, g) and with the correction applied (d, h).

Noise Effect on Detection

In the domain of optical systems, noise emerges as a pivotal element that warrants meticulous consideration. In this particular case, the presence of noise has the effect of limiting our ability to detect single-spin flips. As presented in Figure S2, the impact of averaging on the capacity to detect spin flips is illustrated across varying macropixel sizes.

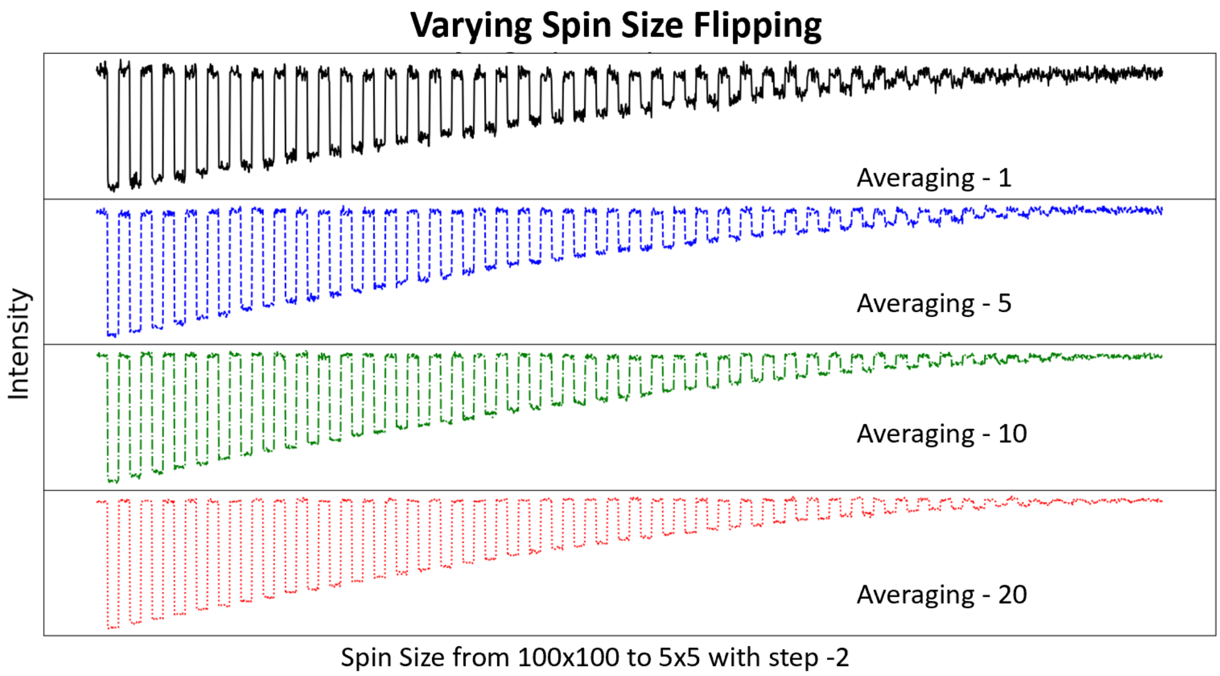

Figure S2. *Noise study on the detection plane.* In the figure we conduct a study regarding the sensitivity of our system to detect small spin flips, varying the spin size from 100x100 to 5x5 with a step of -2, observing at the same time the enhancement of the spin flip detection over averaging, hence noise suppresion.

Integration Region Study

In the study by Sakellariou et al. [2], the authors demonstrate that by measuring only the k=0 of the image plane, the intensity exhibits a correspondence with the Ising Hamiltonian. In practice, the k=0 is not a point but rather a region, its limits defined by the diffraction limit of the experimental setup. In this study, the argument is evaluated by applying the interaction normalization to the intensity retrieval method. The objective of this application is to cancel out the Gaussian beam curvature. This process is repeated for various values of the pixel integration radius, ranging from 0 to 4. Subsequent to the retrieval process, the root mean square error (RMSE) of the retrieved profile is calculated as an additional factor to determine the ideal region of integration. In Figure S3a, the outcomes of the process are illustrated, showing that the central 3X3 pixel region around the maximum intensity pixel would be the optimal choice. As demonstrated in Figure S3b, the cross-section of the Gaussian beam is presented, along with the various pixel integration regions. It is evident that the region ±1 pixel constitutes the maximum area falling within the FWHM. Additionally, as the number of pixels increases, additional frequency components are obtained.

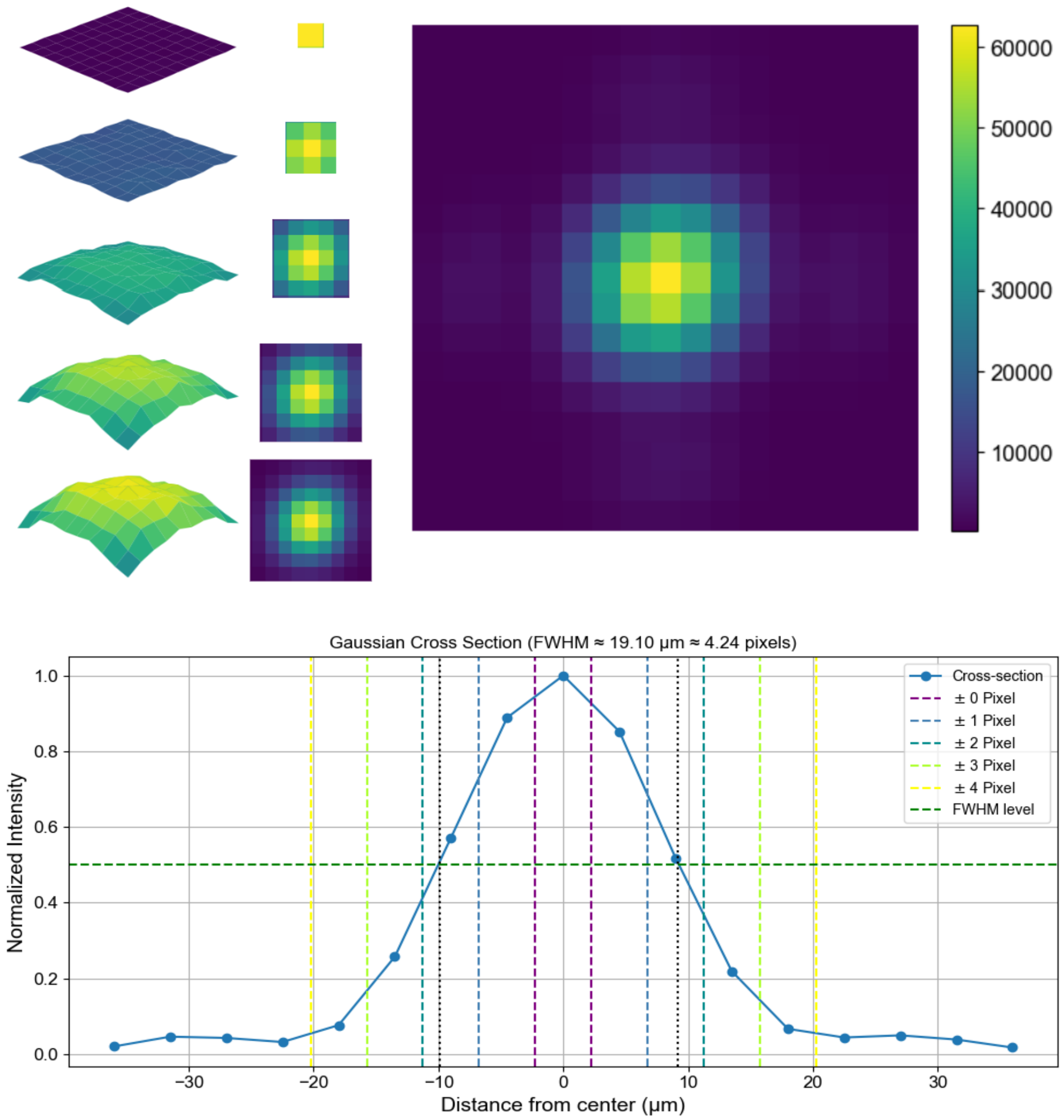

Figure S3. *Integration Region Study*. a) Different intensity retrieval results based on the used integration radius of the PSF in the Image plane b) Cross section of the gaussian profile showing that the region of ± 1 pixel is the largest region we can use to be inside the FWHM of the beam

Wavefront Retrieval and Correction

Phase accuracy is crucial for faithful representations of spin configurations in implementations of SPIMs. As the laser beam propagates through the optical system, it accumulates phase distortions from lenses, mirrors, and other optical components. These aberrations disrupt the precise values of desired encoded phases, leading to inaccuracies in the spin representation. Correcting these phase aberrations is essential to maintain a flat laser phase front, ensuring that the encoded spins remain precise across the entire SLM surface. In addition, the optical field incident on the SLM plane exhibits a Gaussian amplitude profile. When using the full SLM area, this non-uniformity introduces spatial variations in the effective spin-spin couplings, leading to systematic errors in encoding the Ising Hamiltonian. Previous implementations have primarily operated within the central SLM region to maintain a uniform amplitude profile [24, 29, 30, 31, 32]. To enable full-area operation, two critical steps are required: precise beam profiling to characterize the intensity distribution, followed by a coupling normalization scheme to compensate for spatial variations. Failure to correct this profile leads to position-dependent errors which become increasingly important for spins near the SLM edges, fundamentally compromising photonic simulations.

To correct for optical aberrations, we first measure the phase of the optical field at the SLM plane and subsequently apply its conjugate as a compensating phase mask. In our approach, we adopted the wavefront retrieval method introduced by Zupancic et al.[3] and subsequently adapted by Schroff [4,5], which provides a robust and straightforward technique for recovering the phase across the SLM display. This method retrieves the phase by fitting sinusoidal functions to 2D interferograms, which are acquired by the interference of a central reference aperture on the SLM and a sampled aperture across the whole surface. It offers a direct and accurate measurement of the phase profile at the SLM plane, enabling compensation for optical aberrations with a precision below λ/40 root-mean-square (RMS) wavefront error.

For the amplitude normalization we begin by considering a complex field with both spatially varying amplitude and phase[6,7]:

$$A(x,y)e^{i\,\varphi(x,y)} \qquad (1)$$

We then consider the sum of two additional fields, each sharing the same spatially varying amplitude but possessing different spatially varying phases, such that their interference results in an intensity equal to the minimum intensity of the initial field :

$$C(x,y)e^{i\,\lambda(x,y)} + C(x,y)e^{i\,\mu(x,y)} \quad (2)$$

where

$$\lambda(x,y) = \phi(x,y) + \cos^{-1}\big(A_{min}/A(x,y)\big) \qquad (3a)$$

$$\mu(x,y) = \phi(x,y) - \cos^{-1}\big(A_{min}/A(x,y)\big) \qquad (3b)$$

and

$$C(x,y) = \frac{A(x,y)}{2} \qquad (4)$$

The encoding is then obtained by spatial multiplexing using a single phase element $\beta(x,y)$ as

$$M_1(x,y)\, e^{i\,\lambda(x,y)} + M_2(x,y)\, e^{i\,\mu(x,y)} = e^{i\,\beta(x,y)} \qquad (5)$$

where

$$\beta(x,y) = M_1(x,y)\,\lambda(x,y) + M_2(x,y)\,\mu(x,y) \qquad (6)$$

Here $M_1(x,y)$, $M_2(x,y)$ are complementary two-dimensional binary gratings (Boolean checkerboard patterns) that cover the SLM area, satisfying the condition: $M_1(x,y) + M_2(x,y) = 1$.

Thus, we aim to redistribute the excess intensity from the central region of the SLM to areas far from the region of interest on the camera. Our proposed method aims to suppress this Gaussian curvature by ensuring that all regions contribute equally, matching the contribution of the region with the minimum initial amplitude. This normalization method directly corrects the system curvature to encode a constant ferromagnetic interaction. In the case of a non-constant interactions we use the term $A_{min}/A(x,y)$ as a corretion coefficient, multiplied to the intended encoded interactions. By this suppression we effectively eliminate the intrinsic interaction layer embedded within the beam.

Supplementary Material References

1. Jesacher, A. *et al.* Wavefront correction of spatial light modulators using an optical vortex image. *Opt. Express* **15**, 5801 (2007).
2. Sakellariou, J., Askitopoulos, A., Pastras, G. & Tsintzos, S. I. Supplemental material: Encoding arbitrary Ising Hamiltonians on Spatial Photonic Ising Machines.

3. Zupancic, P. *et al.* Ultra-precise holographic beam shaping for microscopic quantum control. *Opt. Express* **24**, 13881 (2016).
4. Schroff, P., La Rooij, A., Haller, E. & Kuhr, S. Accurate holographic light potentials using pixel crosstalk modelling. *Sci. Rep.* **13**, 3252 (2023).
5. Schroff, P., Haller, E., Kuhr, S. & La Rooij, A. Rapid stochastic spatial light modulator calibration and pixel crosstalk optimization. *Opt. Express* **32**, 48957 (2024).
6. Mendoza-Yero, O., Mínguez-Vega, G. & Lancis, J. Encoding complex fields by using a phase-only optical element. *Opt. Lett.* **39**, 1740 (2014).
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