Supplementary Materials

S1 Presentation of the GEMMES model

S1.1 Model Equations

S1.1.1 Nomenclature

Table 3: Variable name and description

Variable	Description	Type	Currency
Y^e	Expected sales	Real	
g_k	NFCs' capital stock growth rate	Real	
Y^P	Production of NFCs	Real	
$I^{V,d}$	Desired investment in inventories	Real	
V^d	Desired inventories	Real	
V	Inventories	Real	
I^V	Actual investment in inventories	Real	
$Y^{P,D}$	Domestic production of NFCs	Real	
Y^D	Aggregate demand	Nominal	Domestic
$Y^{D,r}$	Aggregate demand	Real	
C	Final consumption of goods	Nominal	Domestic
IC	Intermediate consumption	Nominal	Domestic
I^K	Total Investment	Nominal	Domestic
X	Total exports	Nominal	Domestic
X_C	Oil and coal exports	Nominal	Domestic
x_C	Oil and coal exports	Real	
X_{NC}	Non-oil and coal exports	Nominal	Domestic
$\sigma^T_{X_N}$	Target propensity to export non-commodity exports		
σ_{X_N}	Propensity to export non-commodity exports		
GDP_W	Foreign Gross Domestic Product	Real	
IM	Imports	Nominal	Domestic
M	Real imports	Real	
$\sigma^T_{M,C}$	Target propensity to import consumption goods		

$\sigma_{M,C}$	Propensity to import consumption goods		
$\sigma^T_{M,IC}$	Target propensity to import int. consumption goods		
$\sigma_{M,IC}$	Propensity to import int. consumption goods		
$\sigma^T_{M,I}$	Target propensity to import investment goods		
$\sigma_{M,I}$	Propensity to import investment goods		
a_D	Domestic productivity		
a_W	Foreign productivity	GO	
Y^B	Production of FCs	Nominal	Domestic
Y^G	Non-market production of the Government	Nominal	Domestic
G_C	Non-market consumption of the Government	Nominal	Domestic
GDP	Gross Domestic Product	Nominal	Domestic
UC	Unit cost of production	Nominal	Domestic
HUC	Historical unit cost of production	Nominal	Domestic
p^d	Desired production price level	Nominal	Domestic
μ	Mark-up on historical unit cost		
p	Production price level	Nominal	Domestic
p^C	Price of consumption goods	Nominal	Domestic
p^{IC}	Price of intermediate consumption goods	Nominal	Domestic
p^K	Price of investment goods	Nominal	Domestic
p^X	Price of export goods	Nominal	Domestic
p^W	Foreign price level	Nominal	Foreign
IC_F	Intermediate consumption of NFCs	Real	
INS_F	Insurance services paid by NFCs	Nominal	Domestic
COM_F	Financial commissions paid by NFCs	Nominal	Domestic
I^T_F	Target investment of NFCs	Real	
I_F	Investment of NFCs	Real	
K_F	Capital stock of NFCs	Real	
r_F	Profit rate	Nominal	
u	Utilisation rate of capital		
u^e	Expected utilisation rate of capital		
L_F	Employment in NFCs		
w_F	Wage paid by NFCs	Nominal	Domestic
GOS_F	Gross operating surplus of NFCs	Nominal	Domestic
MI_H	Mixed income	Nominal	Domestic
$GOS_{F,H}$	Gross operating surplus redistributed to Households	Nominal	Domestic
$GOS_{F,G}$	Gross operating surplus redistributed to the Government	Nominal	Domestic
GF_F	Gross profits of NFCs	Nominal	Domestic
F_F	Net profits of NFCs	Nominal	Domestic
F^{ND}_F	Net profits of NFCs net of deposits accumulation	Nominal	Domestic
D^{FX}_F	FX deposits of NFCs	Nominal	Foreign
D_F	Domestic deposits of NFCs	Nominal	Domestic
Div_F	Dividends paid by NFCs	Nominal	Domestic
$Div_{F,G}$	Dividends paid by NFCs to the Government	Nominal	Domestic
$Div_{F,W}$	Dividends paid by NFCs to the RoW	Nominal	Domestic
$Div_{F,H}$	Dividends paid by NFCs to Households	Nominal	Domestic

RE_F	Retained earnings of NFCs	Nominal	Domestic
TFN_F	Total financing needs of NFCs	Nominal	Domestic
$L_F^{FX,B,d}$	Desired FX loans of NFCs with FCs	Nominal	Foreign
$L_F^{FX,B}$	FX loans of NFCs with FCs	Nominal	Foreign
$L_F^{FX,W,d}$	Desired FX loans of NFCs with the RoW	Nominal	Foreign
$L_F^{FX,W}$	FX loans of NFCs with the RoW	Nominal	Foreign
L_F^d	Domestic currency loans of NFCs	Nominal	Domestic
$rat_F^{FX.B}$	Rationing of FCs to NFCs' FX loans demand		
$rat_F^{FX.W}$	Rationing of the RoW to NFCs' FX loans demand		
GOS_B	Gross operating surplus of FCs	Nominal	Domestic
L_B	Employment in FCs		
IC_B	Intermediate consumption of FCs	Real	
w_B	Wage paid by FCs	Nominal	Domestic
ST_B	Social transfers paid by FCs	Nominal	Domestic
I_B	Investment of FCs	Nominal	Domestic
K_B	Capital stock of FCs	Real	
GF_B	Gross profits of FCs	Nominal	Domestic
F_B	Net profits of FCs	Nominal	Domestic
OF_B^{CAR}	Capital required to comply with leverage regulations	Nominal	Domestic
RE_B	Retained earnings of FCs	Nominal	Domestic
OF_B	Own funds of FCs	Nominal	Domestic
Div_B	Dividends paid by FCs	Nominal	Domestic
$Div_{B,H}$	Dividends paid by FCs to Households	Nominal	Domestic
$Div_{B,W}$	Dividends paid by FCs to the RoW	Nominal	Domestic
L^d	Domestic currency loans	Nominal	Domestic
D^d	Domestic currency deposits	Nominal	Domestic
Rd	Domestic currency reserves	Nominal	Domestic
Bg_B	Government bonds held by FCs	Nominal	Domestic
$L_B^{FX,W,d}$	Desired FX loans of FCs with the RoW	Nominal	Foreign
$L_B^{FX,W}$	FX loans of FCs with the RoW	Nominal	Foreign
$rat_B^{FX.W}$	Rationing of the RoW to FCs' FX loans demand		
D_B^{FX}	FX deposits of FCs	Nominal	Foreign
$R_B^{FX,d}$	Desired FX reserves of FCs	Nominal	Foreign
R_B^{FX}	FX reserves held by FCs	Nominal	Foreign
TFN_B	Total financing needs of FCs	Nominal	Domestic
A	Liquidity Advances	Nominal	Domestic
i^D	Interest rate on domestic deposits		
md	Mark-down on monetary policy rate		
i_F^L	Interest rate on loans to NFCs		
AFC	Average funding cost of FCs		
$prem_F^T$	Target interest rate premium on loans to NFCs		
$prem_F$	Interest rate premium on loans to NFCs		
i_H^L	Interest rate on loans to Households		
$prem_H^T$	Target interest rate premium on loans to Households		
$prem_H$	Interest rate premium on loans to Households		

$i_B^{FX,W}$	Interest rate on FX loans of FCs with the RoW		
$prem_{FX}$	Premium on FX loans		
$i_F^{FX,B}$	Interest rate on FX loans of NFCs with FCs		
$i_F^{FX,W}$	Interest rate on FX loans of NFCs with the RoW		
$i_B^{FX,R}$	Interest rate on FX reserves of FCs		
$i^{P,T}$	Target monetary policy rate		
i^P	Monetary policy rate		
F_{CB}	Central Bank Profits	Nominal	Domestic
R_{CB}^{FX}	FX reserves held by the Central Bank	Nominal	Foreign
$i_{CB}^{FX,R}$	Interest rate on FX reserves of the Central Bank		
YD_H	Disposable income of Households	Nominal	Domestic
L	Total employment		
$unem$	Unemployment rate		
pop	Labour force		
wL	Wage income	Nominal	Domestic
ESC	Employers' social contributions	Nominal	Domestic
WSC	Workers' social contributions	Nominal	Domestic
INS_H	Insurance services paid by Households	Nominal	Domestic
COM_H	Financial commissions paid by Households	Nominal	Domestic
C_H^T	Target final consumption of Households	Nominal	Domestic
C_H	Final consumption of Households	Nominal	Domestic
m_1	Propensity to consume out disposable income		
m_2	Propensity to consume out wealth		
I_H^T	Target investment of Households	Nominal	Domestic
I_H	Investment of Households	Nominal	Domestic
γ_{IH}	Households' investment to disposable income ratio		
K_H	Capital stock of Households	Real	
S_H	Households' savings	Nominal	Domestic
TFN_H	Total financing needs of Households	Nominal	Domestic
L_H^d	Domestic currency loans of Households	Nominal	Domestic
$L_H^{d,C}$	Consumption loans of Households	Nominal	Domestic
υ_{LC}^T	Target consumption credit to disposable income ratio		
υ_{LC}	Consumption credit to disposable income ratio		
$L_H^{d,I}$	Mortgage loans of Households	Nominal	Domestic
TIR_H	Technical insurance reserves	Nominal	Domestic
D_H	Domestic currency deposits of Households	Nominal	Domestic
FR	Fiscal revenue	Nominal	Domestic
T_T	Tax revenue	Nominal	Domestic
T^I	Taxes on income	Nominal	Domestic
T^M	Taxes on imports	Nominal	Domestic
T^V	Value-added taxes	Nominal	Domestic
T^P	Other net taxes on products	Nominal	Domestic
T^Y	Taxes on production	Nominal	Domestic
Roy	Royalties	Nominal	Domestic
G_T	Total expenditures of the Government	Nominal	Domestic

G_P	Primary expenditures of the Government	Nominal	Domestic
C_G	Final consumption of the Government	Nominal	Domestic
w_G	Wage paid by the Government	Nominal	Domestic
L_G	Employment in the Government		
IC_G	Intermediate consumption of the Government	Real	
I_G^T	Target investment of the Government	Nominal	Domestic
I_G	Investment of the Government	Nominal	Domestic
K_G	Capital stock of the Government	Real	
ST	Social transfers received by Households	Nominal	Domestic
ST_G	Social transfers paid by the Government	Nominal	Domestic
G_{IP}	Interest payments on public debt	Nominal	Domestic
FD	Fiscal deficit	Nominal	Domestic
D_G^T	Target domestic currency deposits of the Government	Nominal	Domestic
D_G	Domestic currency deposits of the Government	Nominal	Domestic
$D_G^{CB,T}$	Target deposits of the Government at the Central Bank	Nominal	Domestic
D_G^{CB}	Deposits of the Government at the Central Bank	Nominal	Domestic
D_G^{FX}	FX deposits of the Government	Nominal	Foreign
TFN_G	Total Financing Needs of the Government	Nominal	Domestic
Bg^{FX}	FX bonds issued by the Government	Nominal	Foreign
Lg^{FX}	FX loans of the Government	Nominal	Foreign
$\Omega_G^{FX,T}$	Target share of FX debt in new public debt issued		
Ω_G^{FX}	Share of FX debt in new public debt issued		
Bg	Domestic currency bonds issued by the Government	Nominal	Domestic
i_G^B	Interest rate on domestic Government bonds	Nominal	Domestic
$prem_G^T$	Target Int. rate premium on domestic Government bonds	Nominal	Domestic
$prem_G$	Interest rate premium on domestic Government bonds	Nominal	Domestic
$i_G^{B,FX}$	Interest rate on FX Government bonds		
$i_G^{L,FX}$	Interest rate on FX Government loans		
CAD	Current account deficit	Nominal	Domestic
TB	Trade balance	Nominal	Domestic
IA	Income account of the balance of payments	Nominal	Domestic
Rem	Remittances	Nominal	Foreign
FDI	Foreign direct investment	Nominal	Domestic
FDI_F	Foreign direct investment in NFCs	Nominal	Domestic
FDI_B	Foreign direct investment in FCs	Nominal	Domestic
FDI_F^G	Greenfield foreign direct investment in NFCs	Nominal	Domestic
FDI_F^{NG}	Non-Greenfield foreign direct investment in NFCs	Nominal	Domestic
Bg_W	Domestic Government bonds held by the RoW	Nominal	Domestic
$L^{FX,W}$	FX loans with the RoW	Nominal	Foreign
D^{FX}	FX deposits	Nominal	Foreign
R^{FX}	FX reserves	Nominal	Foreign
S^{FX}	Foreign exchange supply	Nominal	Foreign
D^{FX}	Foreign exchange demand	Nominal	Foreign
e^N	Nominal exchange rate	Nominal	Domestic
$e^{N,e}$	Expected nominal exchange rate	Nominal	Domestic

e^R	Real exchange rate
rsk	Country risk
i_W	International reference interest rate

S1.1.2 Production, Aggregate Demand and GDP

The market for goods and services produced by Non-Financial Corporations (NFCs) exhibits disequilibrium dynamics because what is produced is not necessarily demanded. Hence, firms adjust their sales expectations (A1) based on the gap between real aggregate demand ($Y^{D,r}$) and real expected sales (Y^e) and a forward looking element proxied by their gross fixed capital accumulation rate (g_K).

$$\dot{Y}^e = \beta_y \cdot \left(Y^{D,r} - Y^e\right) + g_k \cdot Y^e \tag{A1}$$

g_k (A2) depends on the real investment of the firms sector (I_F^K) and the greenfield FDI (FDI_F^G) less the depreciation of the capital stock

$$g_K = \frac{I_F^K + \frac{FDI_F^G}{p^K}}{K_F} - \delta_F \tag{A2}$$

We assume that firms have a desired level of inventories (V_d, A3) given by a constant inventory to expected sales ratio α_v. Therefore, the desired investment in inventory replacement (A4) is modelled as a function of the gap between desired (V^d) and actual (V) inventories.

$$V^d = \alpha_v \cdot Y^e \tag{A3}$$
$$I^{V,d} = \beta_{IV} \cdot \left(V^d - V\right) \tag{A4}$$

The production of the firms (Y^P,A5) depends on real expected sales (Y^e) and the desired investment in inventories ($I^{V,d}$).

$$Y^P = Y^e + I^{V,d} \tag{A5}$$

The change in the actual level of inventories ($\dot{V}$, A6) is given by the gap between real production (Y^P) and real aggregate demand ($Y^{D,r}$). This is equal, following the SNA accounting rules, to the actual investment in inventories (I^V, A7).

$$\dot{V} = Y^P - Y^{D,r} \tag{A6}$$
$$I^V = \dot{V} \tag{A7}$$

Since we assume that all the imports are imported via the firms, domestic real production ($Y^{P,D}$, A8) is equal to total real production (Y^P) minus real imports (M).

$$Y^{P,D} = Y^P - M \tag{A8}$$

Aggregate demand, net of inventory accumulation, in nominal terms (Y^D, A9) is given by the sum of final consumption, intermediate consumption, investment and export expenditures. Accordingly, real aggregate demand ($Y^{D,r}$, A10) deflates each term by its respective price index.

$$Y^D = C + IC + I^K + X \tag{A9}$$

$$Y^{D,r} = \frac{C}{p^C} + \frac{IC}{p^{IC}} + \frac{I^K}{p^K} + \frac{X}{p^X} \tag{A10}$$

Households and the government are consuming final consumption goods (C, A11) produced by firms.

$$C = C_H + C_G \tag{A11}$$

The demand for intermediate consumer goods (IC, A12) produced by NFCs is driven by firms themselves, Financial Corporations (FCs) and the government.

$$IC = p^{IC} \cdot (IC_F + IC_B + IC_G) \tag{A12}$$

Investment (I^K, A13) is carried out by the firms, the government, the households sector, FCs and the rest of the world as greenfield FDI.

$$I^K = p^K \cdot I^K_F + I^K_G + I^K_H + I^K_B + FDI^G_F \tag{A13}$$

Exports (X) are split into two groups as shown in (A14)

$$X = X_C + X_{NC} \tag{A14}$$

On the one hand, oil and coal exports (X_C, A15) depend on the real quantity exported (x_c) and the international price index of these commodities (p^W_C) expressed in domestic currency using the nominal exchange rate (e^N). Then, we assume that real oil and coal exports grow at the exogenous rate α_x as presented in A16

$$X_C = x_C \cdot p^W_C \cdot \ e^N \tag{A15}$$

$$\dot{x_C} = \alpha_x \cdot x_C \tag{A16}$$

On the other hand, non-oil and non-coal exports (X_{NC}, A17) are determined by a time-varying fraction of world GDP. This propensity to export (σ_{X_N}) adjust towards its target value ($\sigma^T_{X_N}$, A18) at the speed β_{X_N} (A19). The target propensity to export depends on the real exchange rate (e^R) and the foreign tariff rate (τ_W) as a measurement of price competitiveness and the gap between foreign (a_W) and domestic (a_D) labour productivity as a proxy of non-price competitiveness.

$$X_{NC} = \sigma_{X_N} \cdot GDP_W \cdot p^W_X \cdot e^N \tag{A17}$$

$$\sigma^T_{X_N} = \sigma^X_p \cdot \left(\frac{e^R_X}{1+\tau_W}\right)^{\epsilon^X_p} + \sigma^X_a \cdot \left(\frac{a_D}{a_W}\right)^{\epsilon^X_a} \tag{A18}$$

$$\dot{\sigma}_{X_N} = \beta_{X_N} \cdot \left(\sigma^T_{X_N} - \sigma_{X_N}\right) \tag{A19}$$

Nominal imports (IM, A20) are equal to real imports multiplied by their respective foreign price index and the nominal exchange rate. Real imports (M, A21) are determined by time-varying propensities to import out consumption ($\sigma_{M,C}$), intermediate ($\sigma_{M,IC}$) and capital ($\sigma_{M,I}$) goods. As in the case of exports, these propensities adjust towards their target values ($\sigma^T_{M,i}$, A22 to A24) at the speed $\beta_{M,i}$ (A25 to A27). The target propensities to import are a function of the real exchange rate, the domestic tax rate on imports and the gap between foreign and domestic labour productivity.

$$IM = M \cdot p^W \cdot e^N \tag{A20}$$

$$M = \sigma_{M,C} \cdot \left(\frac{C}{p^C}\right) + \sigma_{M,IC} \cdot \left(\frac{IC}{p^{IC}}\right) + \sigma_{M,I} \cdot \left(\frac{I^K}{p^K}\right) \tag{A21}$$

$$\sigma^T_{M,C} = \sigma^M_{p,C} \cdot \left[e^R \cdot \left(1+\tau^M\right)\right]^{-\epsilon^M_{p,C}} + \sigma^M_{a,C} \cdot \left(\frac{a_W}{a_D}\right)^{\epsilon^M_{a,C}} \tag{A22}$$

$$\sigma^T_{M,IC} = \sigma^M_{p,IC} \cdot \left[e^R \cdot \left(1+\tau^M\right)\right]^{-\epsilon^M_{p,IC}} + \sigma^M_{a,IC} \cdot \left(\frac{a_W}{a_D}\right)^{\epsilon^M_{a,IC}} \tag{A23}$$

$$\sigma^T_{M,I} = \sigma^M_{p,I} \cdot \left[e^R \cdot \left(1+\tau^M\right)\right]^{-\epsilon^M_{p,I}} + \sigma^M_{a,I} \cdot \left(\frac{a_W}{a_D}\right)^{\epsilon^M_{a,I}} \tag{A24}$$

$$\dot{\sigma}_{M,C} = \beta_{M,C} \cdot \left(\sigma^T_{M,C} - \sigma_{M,C}\right) \tag{A25}$$

$$\dot{\sigma}_{M,IC} = \beta_{M,IC} \cdot \left(\sigma^T_{M,IC} - \sigma_{M,IC}\right) \tag{A26}$$

$$\dot{\sigma}_{M,I} = \beta_{M,I} \cdot \left(\sigma^T_{M,I} - \sigma_{M,I}\right) \tag{A27}$$

It is worth mentioning that domestic labour productivity grows at a constant rate α_D (A28).

$$\dot{a_D} = \alpha_D \cdot a_D \tag{A28}$$

FCs produce financial services other than Financial Intermediation Services Indirectly Measured (FISIM) on demand (A29), which can be split into insurance services and commissions. Both are demanded by firms and households, however, as intermediate consumption in the former and final consumption in the latter.

$$Y^B = INS_H + INS_F + Com_H + Com_F \tag{A29}$$

The government conducts non-market production (Y^G, A30) which refers to the goods and services provided by the public sector to households free of charge or at a price well below their market value, such as education and health. As in the SNA, the model calculates the value of non-market production at the cost of production which is equal to the sum of wages and employers' social contributions, intermediate consumption and capital consumption.

$$Y^G = (1 + \tau_{W,G}) \cdot w_G \cdot L_G + p^{IC} \cdot IC_G + \delta_G \cdot p^K \cdot K_G \tag{A30}$$

Following the SNA, the government demands its own non-market production as final consumption (G_C, A31).

$$G_C = Y^G \tag{A31}$$

GDP (A32) is equal to the sum of final consumption of NFCs and FCs products, the government market and non-market consumption, investment and the trade balance.

$$GDP = C + (INS_H + Com_H) + G_C + C_G + I^K + X - IM \tag{A32}$$

Pricing

Firms have a desired price level (p^d, A33), modelled as a time-varying mark-up over the historical unitary cost. The actual production price (p) follows its target level at the speed β_p as shown in A34.

$$p^d = (1 + \mu) \cdot HUC \tag{A33}$$

$$\dot{p} = \beta_p \cdot \left(p^d - p\right) \tag{A34}$$

The mark-up (μ, A35) reacts to the gap between the actual and target inventory-to-output ratio to capture demand pressures in the goods markets.

$$\mu = \mu_0 - \mu_1 \cdot \left(\frac{V}{Y^e} - \alpha_v\right) \tag{A35}$$

The historical unitary cost (HUC) adjusts towards the unitary cost of production (UC) for an open economy at the speed A36. The unit cost is made up of wages and employers' social contributions, intermediate consumption and taxes on production paid by the firms (A37).

$$\dot{HUC} = \beta_{HUC} \cdot (UC - HUC) \tag{A36}$$

$$UC = \frac{(1 + \tau_{W,F}) \cdot w_F \cdot L_F + p^{IC} \cdot IC_F + \tau_F^Y \cdot Y^{P,D}}{Y^{P,D}} \tag{A37}$$

The price indices for final consumption (p^C, A38), intermediate consumption (p^{IC}, A39) and investment (p^K, A40) are a linear combination of the domestic and foreign price levels weighted by their respective propensity to import. The domestic component levies the VAT rate (τ^v) on consumption and the other indirect taxes rate (τ^P) on intermediate and investment goods, while the foreign component always levies the import tax rate (τ^M).

$$p^C = \left(1 + \tau^V\right) \cdot \left[(1 - \sigma_{M,C}) \cdot p \ + \left(1 + \tau^M\right) \cdot \sigma_{M,C} \cdot p^W \cdot e^N\right] \tag{A38}$$

$$p^{IC} = \left(1 + \tau^P\right) \cdot \left[(1 - \sigma_{M,IC}) \cdot p \ + \left(1 + \tau^M\right) \cdot \sigma_{M,IC} \cdot p^W \cdot e^N\right] \tag{A39}$$

$$p^K = \left(1 + \tau^P\right) \cdot \left[(1 - \sigma_{M,I}) \cdot p \ + \left(1 + \tau^M\right) \cdot \sigma_{M,I} \cdot p^W \cdot e^N\right] \tag{A40}$$

The exports price index (p^X, A41) is calculated as the ratio of nominal to real exports.

$$p^X = \frac{X}{x_C + \sigma_{X_N} \cdot GDP_W} \tag{A41}$$

Non-Financial Corporations

Firms demand goods produced by themselves as intermediate consumption (IC_F, A42), which is modelled as a constant fraction of real production.[6]

$$IC_F = \theta_{IC,F} \cdot Y^P \tag{A42}$$

Financial services other than FISIM demanded by the firms are also intermediate consumption. Insurance services (INS_F, A43) depend on the firms' capital stock, while commissions (Com_F, A44) depend on the firms' stock of domestic currency loans.

$$INS_F = \theta_{Ins,F} \cdot p^K \cdot K_F \tag{A43}$$

$$Com_F = \theta_{Com,F} \cdot L_F^d \tag{A44}$$

[6]Note that in practice this should be expressed as a fraction of production net of imports but this would create a circularity which would prevent the simulation, we hence used total production instead.

The target real investment ($I_F^{K,T}$, A45) of the firms is a function of the profit rate net of inflation. The actual investment of the firms adjusts towards its target level at the speed β_{IF} (A47) and drives the evolution of the capital stock as presented in (A48).

$$I_F^{K,T} = \left[\kappa_0 + \kappa_1 \cdot \left(r_F - \frac{\dot{p}}{p}\right)\right] \cdot K_F \tag{A45}$$

$$\kappa_0 = \frac{1}{(1 + \exp(\kappa_{01} t - \kappa_{02}))(\kappa_{03} - \kappa_{03}\kappa_{04}) + \kappa_{03}\kappa_{04}} \tag{A46}$$

$$\dot{I_F^K} = \beta_{IF} \cdot \left(I_F^{K,T} - I_F^K\right) \tag{A47}$$

$$\dot{K_F} = I_F^K + \frac{FDI_F^G}{p^K} - \delta_F \cdot K_F \tag{A48}$$

The profit rate (r_F, A49) is defined as the ratio between net profits and the capital stock.

$$r_F = \frac{F_F}{p^K \cdot K_F} \tag{A49}$$

The employment level in NFCs (L_F, A50) depends on real domestic production and labour productivity, following a Leontieff production function.

$$L_F = \frac{Y^{P,D}}{a_D} \tag{A50}$$

The average nominal wage paid by the firms w_F grows as a function of the labour productivity growth rate, the employment rate relative to its average value $\omega_{F,2}$ and the inflation rate (A51). The employment gap term captures the tightness or slack in the labour market.

$$\dot{w_F} = \left(\omega_{F,0} \cdot \frac{\dot{a_D}}{a_D} + \omega_{F,1} \cdot \left(\frac{L}{\text{pop}} - \omega_{F,2}\right) + \omega_{F,3} \cdot \frac{\dot{p}}{p}\right) \cdot w_F \tag{A51}$$

The gross operating surplus of NFCs (GOS_F, A52) is equal to the aggregate demand after subtracting imports, indirect taxes (VAT, imports, and other taxes on products), taxes on production, intermediate consumption, wages and employers' social contributions.

$$GOS_F = Y^D - IM - T^P - T^M - T^V - \tau^{Y,F} \cdot Y^{P,D} - p^{IC} \cdot IC_F - INS_F \ldots$$
$$\ldots - Com_F - (1 + \tau_{W,F}) \cdot w_F \cdot L_F \tag{A52}$$

A share of the gross operating surplus of the firms is redistributed to households (A53 and A54) and the government (A55). We make the distinction in the former case between mixed-income (MI_H) and pure gross operating surplus ($GOS_{F,H}$). We follow this approach because in the model, unlike the SNA, NFCs carry out all production in the economy except for financial services and government non-market production.

$$MI_H = \theta_{MI} \cdot GOS_F \tag{A53}$$

$$GOS_{F,H} = \theta_{GH} \cdot GOS_F \tag{A54}$$

$$GOS_{F,G} = \theta_{GG} \cdot GOS_F \tag{A55}$$

Gross profits of NFCs (GF_F, A56) are given by the gross operating surplus minus net interest payments, royalties, and mixed-income and gross operating surplus redistributed.

$$\begin{aligned} GF_F = GOS_F + i_F^D \cdot D_F - i_F^L \cdot L_F^d - i_F^{FX,B} \cdot L_F^{FX,B} \cdot e^N \dots \\ \dots - i_F^{FX,W} \cdot L_F^{FX,W} \cdot e^N - Roy - MI_H - GOS_{F,H} - GOS_{F,G} \end{aligned} \tag{A56}$$

Net profits of NFCs (F_F, A57) are equal to gross profits minus the corporate income tax. Then, equation A58 shows net profits net of the accumulation of domestic and foreign currency deposits held as working capital (F_F^{ND}, A58).

$$F_F = \left(1 - \tau_F^I\right) \cdot GF_F \tag{A57}$$

$$F_F^{ND} = F_F - \dot{D}_F^{FX} \cdot e^N - \dot{D}_F \tag{A58}$$

NFCs accumulate domestic currency deposits ($\dot{D}_F$, A59) in order to have the short-term liquidity to cover a fraction $\eta_{D,F}$ of the wage bill.

$$\dot{D}_F = \beta_{DF} \cdot (\eta_{D,F} \cdot (1 + \tau_{W,F}) \cdot w_F \cdot L_F - D_F) \tag{A59}$$

On a similar basis, NFCs accumulate FX deposits ($\dot{D}_F^{FX}$, A60) to maintain in liquid form a fraction $\eta_{D,F}^{FX}$ of their stock of FX loans.

$$\dot{D}_F^{FX} = \beta_{DF}^{FX} \cdot \left(\eta_{D,F}^{FX} \cdot \left(L_F^{FX,B} + L_F^{FX,W}\right) - D_F^{FX}\right) \tag{A60}$$

NFCs save a constant fraction of their net profits net of deposits accumulation and distribute the remaining as dividends (Div_F, A61).

$$Div_F = (1 - s_F) \cdot F_F^{ND} \tag{A61}$$

Firms distribute dividends to the government ($Div_{F,G}$, A62), the rest of the world ($Div_{F,W}$, A63) and households ($Div_{F,W}$, A64). Dividend payments are sensitive to the ratio of oil and coal exports to GDP to account for the effect of Ecopetrol (i.e., the major state-owned oil company) and the fact that FDI are concentrated in oil and mining sectors, hence leading to return on investment for the rest of the world.

$$Div_{F,G} = \left[\zeta_{0G} + \zeta_{1G} \cdot \left(\frac{X_C}{GDP}\right)\right] \cdot Div_F \tag{A62}$$

$$Div_{F,W} = \left[\zeta_{0W} + \zeta_{1W} \cdot \left(\frac{X_C}{GDP}\right)\right] \cdot Div_F \tag{A63}$$

$$Div_{F,H} = Div_F - Div_{F,G} - Div_{F,W} \tag{A64}$$

Retained earnings of NFCs (RE_F, A65) are given by the profits saved minus other transfers.

$$RE_F = s_F \cdot F_F^{ND} - O_F \tag{A65}$$

The other transfers of NFCs (O_F, A66) are modelled as a constant fraction of domestic production. This item captures some minor SNA transactions not explicitly included in the model that affect the net lending/borrowing position of all the institutional sectors.

$$O_F = \nu_F \cdot p \cdot Y^{P,D} \tag{A66}$$

Since retained earnings allow to self-finance a portion of firms investment at the macroeconomic level, the total financing needs of NFCs (TFN_F, A67) are given by the difference between investment and retained earnings.

$$TFN_F = p^K \cdot I_F^K - RE_F \tag{A67}$$

Firms cover a share of their financing needs by borrowing in foreign currency. They demand a fraction $\eta_F^{FX,B}$ of their total financing needs as FX loans with FCs (A68). However, the desired FX loans demand ($L_F^{F\dot{X},B,d}$) is above the FX loans actually supplied ($L_F^{F\dot{X},B}$, A69) since FCs apply credit rationing standards (rat_B^{FX}, more details in A92) to the firms.

$$L_F^{F\dot{X},B,d} = \eta_F^{FX,B} \cdot \frac{TFN_F}{en} \tag{A68}$$

$$L_F^{F\dot{X},B} = \left(1 - rat_B^{FX}\right) \cdot L_F^{F\dot{X},B,d} \tag{A69}$$

Firms also demand a fraction $\eta_F^{FX,W}$ of their total financing needs as FX loans with foreign banks. Similar to the previous case, the desired FX loans demand ($L_F^{\dot{FX},W,d}$, A70) is above the actual supply ($L_F^{\dot{FX},W}$, A71). Here, the credit rationing parameter (rat_F^{FX}, A72) is modelled as a sigmoid sensitive to the country risk premium (rsk).

$$L_F^{\dot{FX},W,d} = \eta_F^{FX,W} \cdot \frac{TFN_F}{e^N} \tag{A70}$$

$$L_F^{\dot{FX},W} = \left(1 - rat_F^{FX}\right) \cdot L_F^{\dot{FX},W,d} \tag{A71}$$

$$rat_F^{FX} = LB_F^{FX} + \frac{1}{1 + exp\left(-\epsilon_F^{FX} \cdot \left(rsk - \chi_F^{FX}\right)\right)} \cdot \left(UB_F^{FX} - LB_F^{FX}\right) \tag{A72}$$

Firms' demand for domestic currency loans ($\dot{L_F^d}$, A73) acts as a buffer in the model. It is equal to the financing needs in excess of FX loans provided by domestic and foreign banks.

$$\dot{L_F^d} = TFN_F - L_F^{\dot{FX},B} \cdot e^N - L_F^{\dot{FX},W} \cdot e^N \tag{A73}$$

Financial Corporations

The gross operating surplus of FCs (GOS_B, A74) is given by the production of financial services already shown in A29 net of intermediate consumption, taxes on production, wages and employers' social contributions expenditures.

$$GOS_B = Y^B - p^{IC} \cdot IC_B - \tau_B^Y \cdot Y^B - (1 + \tau_{W,B}) \cdot w_B \cdot L_B \tag{A74}$$

The employment level in FCs (L_B, A75) grows at the constant rate λ_B.

$$\dot{L_B} = \lambda_B \cdot L_B \tag{A75}$$

We assume a constant technical relation between labour and inputs in FCs. Hence, real intermediate consumption (IC_B, A76) equals a constant fraction of the employment level.

$$IC_B = \theta_{IC,B} \cdot L_B \tag{A76}$$

The average nominal wage w_B paid by FCs grows (A77) in response to the labour productivity growth rate and the domestic inflation rate.

$$\dot{w_B} = \left(\omega_{B,0} \cdot \frac{\dot{a_D}}{a_D} + \omega_{B,1} \cdot \frac{\dot{p}}{p}\right) \cdot w_B \tag{A77}$$

Nominal investment (I_B, A78) is modelled as a constant fraction of FCs production and drives the evolution of the real capital stock as shown in A79.

$$I_B = \kappa_B \cdot Y^B \tag{A78}$$

$$\dot{K_B} = \frac{I_B}{p^K} - \delta_B \cdot K_B \tag{A79}$$

Social transfers paid by FCs (ST_B, A80) represent a constant fraction of the total transfers received by households (see A161 for more details).

$$ST_B = (1 - \varsigma_{ST,G}) \cdot ST \tag{A80}$$

Gross profits of FCs (GF_B, A81) equals the gross operating surplus minus net interest payments and social transfers paid to households. On the one hand, FCs receive interest on loans to households and NFCs, their holdings of government bonds and their FX reserve assets. On the other hand, FCs pay interest on deposits, their FX loans with the rest of the world, and the liquidity advances received from the central bank.

$$\begin{aligned} GF_B = {} & GOS_B + i_H^L \cdot L_H^d + i_F^L \cdot L_F^d + i_F^{FX,B} \cdot L_F^{FX,B} \cdot e^N + i_G^B \cdot Bg_B \ldots \\ & \ldots + i_B^{FX,R} \cdot R_B^{FX} \cdot e^N - i^D \cdot D_G - i_H^D \cdot D_H - i_F^D \cdot D_F \ldots \\ & \ldots - i_B^{FX,W} \cdot L_B^{FX,W} \cdot e^N - i^P \cdot A - ST_B \end{aligned} \tag{A81}$$

Then, net profits (F_B, A82) equals gross profits after paying the corporate income tax.

$$F_B = \left(1 - \tau_B^I\right) \cdot GF_B \tag{A82}$$

FCs must have a certain amount of own funds to meet solvency requirements (A83), which are given by a constant capital adequacy ratio *car* to total loans granted. Consequently, they retain profits (RE_B, A84) in response to the difference between target and current own funds. These retained earnings allow them to raise their own funds (A85).

$$OF_B^{car} = car \cdot \left(L_F^d + L_H^d + L_F^{FX,B} \cdot e^N\right) \tag{A83}$$

$$RE_B = \beta_{OF} \cdot (OF_B^{car} - OF_B) \tag{A84}$$

$$\dot{OF_B} = RE_B \tag{A85}$$

Dividends distributed to households ($Div_{B,H}$, A86) equals net profits exceeding retained earnings and other transfers.

$$Div_{B,H} = F_B - RE_B - O_B \tag{A86}$$

As in the case of NFCs, the other transfers (O_B, A87) are modelled as a constant fraction of domestic production

$$O_B = \nu_B \cdot p \cdot Y^{P,D} \tag{A87}$$

FCs respect a liquidity ratio and thus accumulate liquid reserves at the central bank (A88) as a constant fraction of the domestic deposits accumulated by NFCs, the household sector and the government (A89).

$$\dot{Rd} = lr \cdot \dot{D}^d \tag{A88}$$

$$\dot{D}^d = \dot{D}_F + \dot{D}_H + \dot{D}_G \tag{A89}$$

FCs have a desired demand for FX loans with foreign banks ($L_B^{\dot{FX},W,d}$, A90), which depends on their own funds acting as collateral and the stock of FX loans granted to NFCs. As in the case of NFCs, the FX loans actually supplied by foreign banks ($L_B^{\dot{FX},W}$, A91) are lower since they ration credit based on the country risk premium (A92).

$$L_B^{\dot{FX},W,d} = \eta_B^{FX,L} \cdot \frac{OF_B}{e^N} + L_F^{\dot{FX},B} \tag{A90}$$

$$L_B^{\dot{FX},W} = \left(1 - rat_B^{FX}\right) \cdot L_B^{\dot{FX},W,d} \tag{A91}$$

$$rat_B^{FX} = LB_B^{FX} + \frac{1}{1 + exp\left(-\epsilon_B^{FX} \cdot \left(rsk - \chi_B^{FX}\right)\right)} \cdot \left(UB_B^{FX} - LB_B^{FX}\right) \tag{A92}$$

FCs supply domestic currency loans ($\dot{L}^d$, A93) to NFCs and households on demand

$$\dot{L}^d = \dot{L}_F^d + \dot{L}_H^d \tag{A93}$$

FCs finance the government (A94) by purchasing all domestic bonds issued that are available in the market after foreign investors have made their respective purchases.

$$\dot{Bg}_B = \dot{Bg} - \dot{Bg}_W \tag{A94}$$

FCs accumulate FX deposits ($\dot{D_B^{FX}}$, A95) to keep in liquid form a constant share η_B^{FX} of the stock of FX loans borrowed from the rest of the world.

$$\dot{D_B^{FX}} = \beta_{DB}^{FX} \cdot \left[\eta_B^{FX} \cdot L_B^{FX,W} - D_B^{FX}\right] \tag{A95}$$

As a consequence of a strict no open position rule, the desired accumulation of FX reserves by FCs (A96) is equal to the desired demand for FX loans minus the FX deposits accumulated and the FX loans granted to NFCs.

$$R_B^{\dot{FX},d} = L_B^{\dot{FX},W,d} - D_B^{\dot{FX}} - L_F^{\dot{FX},B} \tag{A96}$$

To the extent that we account for credit rationing in the model, the actual FX reserves accumulated ($R_B^{\dot{FX}}$, A97) by FCs differ from those desired. Hence, in line with the balance of payments identity, the FX reserve assets accumulated by FCs equal the FX reserve assets accumulated by the whole economy minus those accumulated by the central bank.

$$R_B^{\dot{FX}} = \dot{R}^{FX} - \dot{R}_{CB}^{FX} \tag{A97}$$

The total financing needs of FCs (TFN_B, A98) arise from the difference between accrued assets (use of funds) and liabilities (sources of funds) in domestic and foreign currency.

$$\begin{aligned} TFN_B = \left[\dot{L^d} + \dot{Bg_B} + \dot{Rd} + I_B\right] - \left[(1+lr)\cdot \dot{D^d} + \dot{OF_B} + FDI_B + \dot{TIR_H}\right]\dots \\ \dots + \left[D_B^{\dot{FX}} \cdot e^N + L_F^{\dot{FX},B} \cdot e^N + R_B^{\dot{FX}} \cdot e^N - L_B^{\dot{FX},W} \cdot e^N\right] \end{aligned} \tag{A98}$$

The central bank acts as a lender of last resort in a way that it provides liquidity advances (A99) whenever FCs are running positive financing needs.

$$\dot{A} = TFN_B \tag{A99}$$

Regarding interest rates, the interest rate on domestic currency deposits (i^D, A100) is modelled as a mark-down over the monetary policy rate. This time-varying mark-down (md, *A*101) is sensitive to the ratio between the stock of advances and deposits collected as an indicator of liquidity in the banking sector.

$$i^D = (1 - md) \cdot i^P \tag{A100}$$

$$md = \rho_0 - \frac{\rho_1}{1 + \exp(-\rho_2 \left(\frac{A}{D^d} - \rho_3\right))} \tag{A101}$$

To better match the interest payments observed in the data for each sector, an additional constant mark-down over this rate is applied for households (A102) and NFCs (A103).

$$i_H^D = (1 - md_H) \cdot i^D \tag{A102}$$

$$i_F^D = (1 - md_F) \cdot i^D \tag{A103}$$

The interest rate charged on domestic currency loans to NFCs (i_F^L, A104) is given by a premium ($prem_F$) over the domestic average funding costs of FCs (AFC, A105). This time-varying premium adjusts towards its target value ($prem_F^T$, A106) at the speed β_{Pr}^F (A107). We model the target premium as sensitive to the ratio of firms' indebtedness to domestic production.

$$i_F^L = AFC \cdot (1 + prem_F) \tag{A104}$$

$$AFC = \frac{i^D \cdot D_G + i_H^D \cdot D_H + i_F^D \cdot D_F + i^P \cdot A}{D^d + A} \tag{A105}$$

$$prem_F^T = \phi_{F0}^{\text{Pr}} + \frac{\phi_{F1}^{\text{Pr}}}{1 + exp\left(-\phi_{F2}^{\text{Pr}} \cdot \left(\frac{L_F^d + L_F^{FX,B} \cdot e^N + L_F^{FX,W} \cdot e^N}{p \cdot Y^{P,D}}\right)\right)} \tag{A106}$$

$$\dot{prem}_F = \beta_{Pr}^F \cdot \left(prem_F^T - prem_F\right) \tag{A107}$$

The interest rate charged on FX loans to NFCs ($i_F^{FX,B}$, A108) is modelled as a constant premium over the interest rate on FX loans that foreign banks charge to FCs ($i_B^{FX,W}$).

$$i_F^{FX,B} = i_B^{FX,W} \cdot \left(1 + \varrho_F^{FX,B} \cdot prem_F\right) \tag{A108}$$

The interest rate charged on domestic currency loans to households (i_H^L, A109) is modelled as a premium ($prem_H$) over the interest rate on domestic loans to NFCs. This time-varying premium also follows its target value ($prem_H^T$, A110) at the speed β_{Pr}^H (A111). The target premium is sensitive to the households' debt to disposable income ratio.

$$i_H^L = i_F^L \cdot (1 + prem_H) \tag{A109}$$

$$prem_H^T = \phi_{H0}^{\text{Pr}} + \frac{\phi_{H1}^{\text{Pr}}}{1 + \exp\left(-\phi_{H2}^{\text{Pr}} \cdot \left(\frac{L_H^d}{YD_H}\right)\right)} \tag{A110}$$

$$\dot{prem}_H = \beta_{\text{Pr}}^H \cdot \left(prem_H^T - prem_H\right) \tag{A111}$$

Central Bank

The central bank has a target monetary policy rate ($i^{P,T}$, A112) that follows a simple inflation-targeting Taylor Rule. Then, the actual policy rate follows its target value at the speed β_{ip} as shown in A113.

$$i^{P,T} = \iota_0 + \iota_1 \cdot \left(\frac{\dot{p}}{p} - \iota_2\right) \tag{A112}$$

$$\dot{i^p} = \beta_{ip} \cdot \left(i^{P,T} - i^P\right) \tag{A113}$$

Central bank profits (F_{CB}, A114) are equal to the interest received on liquidity advances and FX reserve assets minus the interest paid to the government on their deposit holdings. The profits are fully transferred to the government

$$F_{CB} = i^P \cdot A + \ i_{CB}^{FX,R} \cdot R_{CB}^{FX} - i_{CB}^D \cdot D_G^{CB} \tag{A114}$$

The central bank has a target of FX reserves equal to a constant fraction $\theta_{FX,M}$ of imports in normal time, plus a reaction term in case the reserve ratio approaches a precautionary lower boundary. When the actual stock of FX reserves is below the normal-time level,or approaches the prudential reserve ratio the central bank intervenes in the FX market to accumulate FX-denominated assets (A115). This reaction term was added to be able to run the model on extreme regions of the sensitivity space. Together with Equation A146, it mimics the behaviour witnessed in Turkey over recent balance-of-payment crises, and supposes the existence of swap lines akin to those that prevailed during the Covid crises (REF). The reaction curve shows in Figure 8.

$$\dot{R_{CB}^{FX}} = \max \left[\theta_{FX,M} \cdot M \cdot p^W - R_{CB}^{FX} \cdot (1 - S_{FX,cb} \tanh \gamma_{FX} (\overline{res}_{FX} - res)) \, , 0\right] \tag{A115}$$

Households

Equation (A116) describes the different components of households' disposable income, wage income, financial income, social contributions and social transfers and remittances as income and social contribution, interest payments and financial consumption as expenses.

$$\begin{aligned} YD_H = \left(1 - \tau_W^I\right) \cdot wL + MI_H + GOS_H + ESC + ST + i_H^D \cdot D_H + Div_{F,H} \dots \\ \dots + Div_{B,H} + Rem \cdot e^N - WSC - i_H^L \cdot L_H^d - INS_H - Com_H + O_H \end{aligned} \tag{A116}$$

The unemployment rate ($unem$, A117) depends on the level of employment relative to the labour force. Total employment equals the sum of employees in NFCs, FCs and the government (A118), while the labour force grows at the constant rate α_P (A119).

$$unem = 1 - \frac{L}{pop} \tag{A117}$$

$$L = L_F + L_B + L_G \tag{A118}$$

$$\dot{pop} = \alpha_P \cdot pop \tag{A119}$$

Accordingly, the total wage bill (wL, A120) and the total employers' social contributions (ESC, A121) received by the household sectors are paid by NFCs, FCs and the Government.

$$wL = w_F \cdot L_F + w_B \cdot L_B + w_G \cdot L_G \tag{A120}$$

$$ESC = \tau_{W,F} \cdot w_F \cdot L_F + \tau_{W,B} \cdot w_B \cdot L_B + \tau_{W,G} \cdot w_G \cdot L_G \tag{A121}$$

Workers' social contributions (WSC, A122) are paid to the government. They equal employers' social contributions plus an additional levy charged to households as a fraction τ_{SC} of the total wage bill.

$$WSC = ESC + \tau_{SC} \cdot wL \tag{A122}$$

As in previous cases, the other transfers of households (O_H, A123) are modelled as a constant fraction of total domestic production.

$$O_H = \nu_H \cdot p \cdot Y^{P,D} \tag{A123}$$

Similar to what has been done for NFCs, households' demand for financial services is divided into insurance (INS_H, A124) and commissions (Com_H, A125). The first term depends on the capital stock and the second on the stock of loans.

$$INS_H = \theta_{Ins,H} \cdot p^K \cdot K_H \tag{A124}$$

$$Com_H = \theta_{Com,H} \cdot L_H^d \tag{A125}$$

Households' target consumption (C_H^T, A126) depends on a time-varying propensity to consume out of disposable income, a time-varying propensity to consume out of wealth, and the flow of consumer credit. Then, the actual consumption level slowly moves towards its target level at the speed β_C as presented in (A127).

$$C_H^T = m_1 \cdot YD_H + m_2 \cdot \left(D_H + TIR_H + p^K \cdot K_H\right) + L_H^{\dot{d},C} \tag{A126}$$

$$\dot{C_H} = \beta_C \cdot \left(C_H^T - C_H\right) \tag{A127}$$

The time-varying marginal propensities to consume, m_1 (A128) and m_2 (A129), are modelled as sigmoids sensitive to the real deposit rate.

$$m_1 = LB_H^{YD} + \frac{1}{1 + exp\left(-\epsilon_H^{YD} \cdot \left(\left(i_H^D - \frac{\dot{p}}{p}\right) - \chi_H^{YD}\right)\right)} \cdot \left(UB_H^{YD} - LB_H^{YD}\right) \tag{A128}$$

$$m_2 = LB_H^W + \frac{1}{1 + exp\left(-\beta_H^W \cdot \left(\left(i_H^D - \frac{\dot{p}}{p}\right) - \chi_H^W\right)\right)} \cdot \left(UB_H^W - LB_H^W\right) \tag{A129}$$

Households' target investment in housing (I_H^T, A130) is modelled as a time-varying fraction (κ_H, A131) of the disposable income. This fraction negatively depends on the interest rate and the unemployment rate since they are indicators of borrowing costs and the business cycle, respectively. As in previous cases, the actual investment adjusts toward its target value at the speed β_{IH} (A132) and drives the dynamic of the capital stock (A133) .

$$I_H^T = \kappa_H \cdot YD_H \tag{A130}$$

$$\kappa_H = \kappa_{H,0} - \kappa_{H,1} \cdot i_H^L - \kappa_{H,2} \cdot unem \tag{A131}$$

$$\dot{I_H} = \beta_{IH} \cdot \left(I_H^T - I_H\right) \tag{A132}$$

$$\dot{K_H} = \frac{I_H}{p^K} - \delta_H \cdot K_H \tag{A133}$$

We now define savings (S_H, A134) as the difference between disposable income and consumption.

$$S_H = YD_H - C_H \tag{A134}$$

It can be seen in (A135) that households will exhibit positive total financing needs whenever investment exceeds savings, while they will show a net lending position in the opposite case.

$$TFN_H = I_H - S_H \tag{A135}$$

The demand of households for domestic currency loans (A136) is split into consumer and mortgage loans. In the first case (A137), the demand for credit is adjusted to keep the ratio of consumer loans to disposable income stable. In the second case (A138), mortgage credit demand is a constant fraction of housing investment.

$$\dot{L_H^d} = \dot{L_H^{d,C}} + \dot{L_H^{d,I}} \tag{A136}$$

$$\dot{L_H}^{d,C} = \beta_{L,CH} \cdot \left(\eta_{LC} \cdot YD_H - L_H^{d,C}\right) \tag{A137}$$

$$\dot{L_H^{d,I}} = \eta_{LI} \cdot I_H \tag{A138}$$

Households allocate their savings in two assets. First, in the form of insurance, pensions and standardised guarantee schemes (SGS), which is modelled as a constant fraction of the wage bill (A139). Second, in the form of domestic currency deposits (A140), the value of which is determined as residual from the difference between the sources of funds (savings (or minus total financing needs) and loans) and the uses of funds (investment and insurance, pensions and SGS).

$$\dot{TIR_H} = \eta_{TIR} \cdot wL \tag{A139}$$

$$\dot{D_H} = -TFN_H + \dot{L_H^d} - TI\dot{R}_H \tag{A140}$$

Government

Fiscal revenue (FR, A141) is made up of tax revenue, royalties, gross operating surplus redistributed by NFCs, workers' social contributions, interest payments on deposit holdings, and dividends paid by NFCs.

$$FR = T_T + Roy + GOS_G + WSC + i^D \cdot D_G + i^D_{CB} \cdot D^{CB}_G + Div_{F,G} \tag{A141}$$

Tax collection (T_T, A142) is divided into five types of taxes.

$$T_T = T^I + T^M + T^V + T^P + T^Y \tag{A142}$$

Taxes on income (T^I, A143) are levied on the wage bill and gross profits of NFCs and FCs with differential taxes rates.

$$T^I = \tau^I_W \cdot wL + \tau^I_F \cdot GF_F + \tau^I_B \cdot GF_B \tag{A143}$$

Import taxes (T^M, A146) are levied at a uniform tax rate on real imports plus a premium if the reserve ratio of the economy approaches a lower bound. This equation also aims at stabilising the model in border cases, together with Equation A115. Reaction curve shows in Figure 8

$$\tau^T_M = \tau_{M_0}(1 + \overline{\tau_M}\tanh(\gamma_{\tau_M}(\overline{res}_{\tau_M} - res))) \tag{A144}$$

$$\dot{\tau_M} = \beta_{\tau_M}(\tau^T_M - \tau_M) \tag{A145}$$

$$T^M = \tau^M \cdot IM \tag{A146}$$

VAT (T^V, A147) is levied only on consumer goods at the τ^V rate

$$T^V = \tau^V \cdot \frac{C \cdot \left[(1 - \sigma_{M,C}) \cdot p \ + \left(1 + \tau^M\right) \cdot \sigma_{M,C} \cdot p^W \cdot e^N\right]}{p^C} \tag{A147}$$

The revenue of other taxes on products net of subsidies (T^P, A148) depends on a uniform tax rate on final consumption, intermediate and investment goods

$$\begin{aligned} T^P = \tau^P \cdot \frac{C \cdot \ \left[(1 - \sigma_{M,C}) \cdot p \ + \ \left(1 + \tau^M\right) \cdot \sigma_{M,C} \cdot p^W \cdot e^N\right]}{p^C} \dots \\ \dots + \tau^P \cdot \frac{IC \cdot \ \left[(1 - \sigma_{M,IC}) \cdot p \ + \ \left(1 + \tau^M\right) \cdot \sigma_{M,IC} \cdot p^W \cdot e^N\right]}{p^{IC}} \dots \\ \dots + \tau^P \cdot \frac{I^K \cdot \ \left[(1 - \sigma_{M,I}) \cdot p \ + \ \left(1 + \tau^M\right) \cdot \sigma_{M,I} \cdot p^W \cdot e^N\right]}{p^K} \end{aligned} \tag{A148}$$

Taxes on production net of subsidies (T^Y, A149) are paid by NFCs and FCs.

$$T^Y = \tau_F^Y \cdot p \cdot Y^{P,D} + \tau_B^Y \cdot Y^B \tag{A149}$$

Royalty collection (Roy, A150) is modelled as a constant fraction θ^R of oil and coal exports.

$$Roy = \theta^R \cdot X_C \tag{A150}$$

Total government expenditure (G_T, A151) is divided into primary expenditure, interest payments on public debt and other transfers.

$$G_T = G_P + G_{\text{IP}} + O_G \tag{A151}$$

Primary expenditures (G_P, A152) consist of final market consumption, the wage bill, employers' social contributions, intermediate consumption, public investment and social transfers paid to households.

$$G_P = C_G + (1 + \tau_{W,G}) \cdot w_G \cdot L_G + p^{IC} \cdot IC_G + p^K \cdot I_G + ST_G \tag{A152}$$

The government has a target demand for final consumption goods (C_G^T, A153) that is modelled as a constant fraction $\theta_{G,C}$ of GDP. As in previous cases, the actual value of consumption moves towards the target at the speed β_{CG} as presented in (A154).

$$C_G^T = \theta_{G,C} \cdot GDP \tag{A153}$$

$$\dot{C_G} = \beta_{CG} \cdot \left(C_G^T - C_G\right) \tag{A154}$$

Regarding the determinants of the wage bill, the nominal wage paid by the government (w_G) grows in relation to the labour productivity growth and the inflation rate (A155). Whereas the level of employment (L_G, A156) is determined as a constant fraction of the labour force.

$$\dot{w_G} = \left(\omega_{G,0} \cdot \frac{\dot{a_D}}{a_D} + \omega_{G,1} \cdot \frac{\dot{p}}{p}\right) \cdot w_G \tag{A155}$$

$$L_G = \theta_{G,L} \cdot pop \tag{A156}$$

We assume a constant technical relation between the real intermediate consumption (IC_G, A157) and employment in the government sector.

$$IC_G = \theta_{IC,G} \cdot L_G \tag{A157}$$

The government has as a target to invest a constant fraction κ_G of the capital stock (A158). This target serves as an attractor for the actual level of investment which converges at the speed β_{IG} (A159) and, simultaneously, determines the dynamics of the capital stock (A160).

$$I_G^T = \kappa_G \cdot K_G \tag{A158}$$

$$\dot{I_G} = \beta_{IG} \cdot \left(I_G^T - I_G\right) \tag{A159}$$

$$\dot{K_G} = I_G - \delta_G \cdot K_G \tag{A160}$$

Total social transfers received by households (ST, A161) depend on population and unemployment dynamics. Of this amount, the government takes over the constant fraction $\varsigma_{ST,G}$ (A162), while the remaining part is paid by FCs as shown in (A80).

$$ST = \theta_{G0,ST} \cdot w_F \cdot (pop - L) + \theta_{G1,ST} \cdot w_F \cdot pop \tag{A161}$$

$$ST_G = \varsigma_{ST,G} \cdot ST \tag{A162}$$

The government pays interest (G_{IP}, A163) on domestic currency bonds, FX bonds and FX loans.

$$G_{IP} = i_G^B \cdot Bg + i_G^{B,\ FX} \cdot Bg^{FX} \cdot e^N + i_G^{L,FX} \cdot Lg^{FX} \cdot e^N \tag{A163}$$

As in previous cases, the other transfers (O_G, A164) of the government is modelled as a constant fraction of total domestic production.

$$O_G = \nu_G \cdot p \cdot Y^{P,D} \tag{A164}$$

At this point, we define the fiscal deficit (FD, A165) as the difference between total expenditures and revenues after taking into account the central bank profits.

$$FD = G_T - FR - F_{CB} \tag{A165}$$

Consequently, the total financing needs of the government (TFN_G, A166) are equal to the fiscal deficit plus the deposits accumulated as cash funds.

$$TFN_G = FD + \dot{D_G} + \dot{D_G^{CB}} + \dot{D_G^{FX}} \cdot e^N \tag{A166}$$

The government accumulates domestic currency deposits at FCs (A167) and the central bank (A168) to hold in liquid form a fraction η_{DG} and η_{DG}^{FX} of its total expenditures, respectively.

$$\dot{D_G} = \beta_{DG} \cdot (\eta_{DG} \cdot G_T - D_G) \tag{A167}$$

$$\dot{D_G^{CB}} = \beta_{DG}^{CB} \cdot \left(\eta_{DG}^{CB} \cdot G_T - D_G^{CB}\right) \tag{A168}$$

Similarly, the government accumulates FX deposits (A169) to retain in liquid form a constant fraction η_{DG}^{FX} of the stock of FX debt.

$$\dot{D_G^{FX}} = \beta_{DG}^{FX} \cdot \left(\eta_{DG}^{FX} \cdot \left(Bg^{FX} + Lg^{FX}\right) - D_G^{FX}\right) \tag{A169}$$

Domestic currency bonds issuance ($\dot{Bg}$, A170) is determined as a residual from the difference between total financing needs and the currency financing raised from abroad via bonds and loans. The latter flows will be explained in the next subsection.

$$\dot{Bg} = TFN_G - \dot{Bg^{FX}} \cdot e^N - \dot{Lg^{FX}} \cdot e^N \tag{A170}$$

Domestic currency bonds pay the interest rate (i_G^B, A171), which depends on the monetary policy rate and a time-varying risk premium ($prem_G$). The risk premium moves gradually towards its target value ($prem_G^T$, A172) at the speed β_{Pr}^G (A173). Lastly, the target premium is modelled as a sigmoid sensitive to the government debt-to-GDP ratio.

$$i_G^B = i^P + prem_G \tag{A171}$$

$$prem_G^T = \phi_{G0}^{\text{Pr}} + \frac{\phi_{G1}^{\text{Pr}}}{1 + exp\left(-\phi_{G2}^{\text{Pr}} \cdot \left(\frac{Bg + Bg^{FX} \cdot e^N + Lg^{FX} \cdot e^N}{GDP}\right)\right)} \tag{A172}$$

$$\dot{prem_G} = \beta_{Pr}^G \cdot \left(prem_G^T - prem_G\right) \tag{A173}$$

Rest of the World

World real GDP grows (A174) at the constant rate α_G^W.

$$\dot{GDP_W} = \alpha_{GW} \cdot GDP_W \tag{A174}$$

World labour productivity (A175) grows at the constant rate α_W.

$$\dot{a_W} = \alpha_W \cdot a_W \tag{A175}$$

In terms of prices, the foreign GDP deflator (p^W) and the price index for non-oil and non-coal exports in FX (p_X^W) have an inflation rate of α_p^W (A176) and $\alpha_{p_X^W}$ (A177), respectively. Similarly, the international oil and coal price index (p_C^W) has an inflation rate of $\alpha_{p_C^W}$ (*A*178).

$$\dot{p^W} = \alpha_{p^W} \cdot p^W \tag{A176}$$

$$\dot{p_X^W} = \alpha_{p_X^W} \cdot p_X^W \tag{A177}$$

$$\dot{p}_C^W = \alpha_{p_C^W} \cdot p_C^W \tag{A178}$$

The current account (CAD, A179) deficit is given by the trade balance and the income account.

$$CAD = -(TB + IA) \tag{A179}$$

The trade balance (TB, A180) equals exports minus imports.

$$TB = X - IM \tag{A180}$$

The income account (IA, A181) is equal to receipts from remittances, interest on reserve assets and other transfers minus outflows from interest payments on the bond holdings of foreign investors, interest payments on FX loans and dividend payments.

$$\begin{aligned} IA = {} & Rem \cdot e^N + i_{CB}^{FX,R} \cdot R_{CB}^{FX} \cdot e^N + i_B^{FX,R} \cdot R_B^{FX} \cdot e^N - i_G^{B,\ FX} \cdot Bg^{FX} \cdot e^N \dots \\ & \dots - i_G^{L,FX} \cdot Lg^{FX} \cdot e^N - i_G^B \cdot Bg_W - i_F^{FX,W} \cdot L_F^{FX,W} \cdot e^N \dots \\ & \dots - i_B^{FX,W} \cdot L_B^{FX,W} \cdot e^N - Div_{F,W} - Div_{B,W} + O_W \end{aligned} \tag{A181}$$

Remittances (Rem, A182) are modelled as a constant fraction θ_{REM} of nominal world GDP.

$$Rem = \theta_{REM} \cdot GDP_W \cdot p^W \tag{A182}$$

Other transfers of the rest of the world (O_W, A183), as done for the other institutional sectors, are modelled as a constant fraction of total domestic production.

$$O_W = \nu_W \cdot p \cdot Y^{P,D} \tag{A183}$$

To begin with the description of flows of funds with the rest of the world, FDI (A184) is assumed to be a constant fraction ψ^{FDI} of the investment carried out by the firms.

$$FDI = \psi^{FDI} \cdot p^K \cdot I_F \tag{A184}$$

ψ_F^{FDI} represents the share of FDI allocated into the NFCs sector (A185). This flow can take the form of greenfield FDI (A186) which adds to the capital stock and non-greenfield FDI (A187) which is only a source of funding

$$FDI_F = \psi_F^{FDI} \cdot FDI \tag{A185}$$

$$FDI_F^G = \psi_G^{FDI} \cdot FDI_F \tag{A186}$$

$$FDI_F^{NG} = \left(1 - \psi_G^{FDI}\right) \cdot FDI_F \quad \text{(A187)}$$

The remaining share $(1 - \psi_F^{FDI})$ of total FDI inflows is allocated into FCs (A188) as non-greenfield FDI

$$FDI_B = \left(1 - \psi_F^{FDI}\right) \cdot FDI \quad \text{(A188)}$$

The stock-flow consistency can be seen in the fact that FDI allows the rest of the word to accumulate equities against NFCs (A189) and FCs (A190).

$$\dot{EQ}_F^W = FDI_F \quad \text{(A189)}$$
$$\dot{EQ}_B^W = FDI_B \quad \text{(A190)}$$

We assume that the rest of the world purchases domestic currency bonds ($\dot{Bg}_W$, A191) issued by the government on the basis of a constant relation with the trade balance.

$$\dot{Bg}_W = -\eta_{GW}^D \cdot TB \quad \text{(A191)}$$

We also assume that the government tends to seek more currency financing when the economy faces a stronger balance of payment imbalances as a way to cope with and ease the external constraint. Precisely, Ω_G^{FX} determines the percentage of the trade deficit that the government is willing to finance through FX public debt operations. This fraction moves towards its target value ($\Omega_G^{FX,T}$, A192) at the speed $\beta_{FX,G}$ (A193), which depends positively on the current account deficit as a percentage of GDP.

$$\Omega_G^{FX,T} = \Omega_{G0} + \Omega_{G1} \cdot \frac{CAD}{GDP} \quad \text{(A192)}$$
$$\dot{\Omega_G^{FX}} = \beta_{FX,G} \cdot \left(\Omega_G^{FX,T} - \Omega_G^{\mathrm{FX}}\right) \quad \text{(A193)}$$

Hence, equation (A194) shows the FX bonds issued by the government ($\dot{B_G^{FX}}$), which are fully purchased by foreign investors. While equation (A195) indicates the FX loans demanded by the government ($\dot{L_G^{FX}}$), all of which are granted (i.e., no credit rationing) by foreign banks.

$$\dot{Bg^{FX}} = -\phi_{BG}^{FX} \cdot \left(\Omega_G^{FX} \cdot \frac{TB}{e^N}\right) \quad \text{(A194)}$$
$$\dot{Lg^{FX}} = -\ \left(1 - \phi_{BG}^{FX}\right) \cdot \left(\Omega_G^{FX} \cdot \frac{TB}{e^N}\right) \quad \text{(A195)}$$

To simplify further definitions, equation (A196) shows the total flow of FX loans supplied by foreign banks ($L^{\dot{FX},W}$) to NFCs, FCs and the government.

$$L^{\dot{FX},W} = L_F^{\dot{FX},W} + L_B^{\dot{FX},W} + L_G^{\dot{FX},W} \tag{A196}$$

Conversely, equation (A197) presents the total accumulation of FX deposits ($D^{\dot{FX}}$) by these three sectors against the rest of the world.

$$D^{\dot{FX}} = D_F^{\dot{FX}} + D_B^{\dot{FX}} + D_G^{\dot{FX}} \tag{A197}$$

The balance of payments identity is guaranteed by (A198). Basically, it specifies that the economy will accumulate FX reserve assets when in the balance of payments inflows exceed outflows, while FX reserve assets will fall in the opposite case.

$$\dot{R}^{FX} = \frac{TB}{e^N} + \frac{IA}{e^N} + \frac{FDI}{e^N} + B\dot{g}^{FX} + L\dot{g}^{FX} + L^{\dot{FX},W} + \frac{B\dot{g}_W}{e^N} - D^{\dot{FX}} \tag{A198}$$

In line with the above, the demand for foreign exchange (D^{FX}, A199) is driven by the outflows in the current and financial accounts of the balance of payments.

$$\begin{aligned} D^{FX} = & \frac{IM}{e^N} + i_G^{B,\ FX} \cdot Bg^{FX} + i_G^{L,FX} \cdot Lg^{FX} + \frac{i_G^B \cdot Bg_W}{e^N} \dots \\ & \dots + i_F^{FX,W} \cdot L_F^{FX,W} + i_B^{FX,W} \cdot L_B^{FX,W} + \frac{Div_{F,W}}{e^N} + \frac{Div_{B,W}}{e^N} + \dot{R}_B^{FX,d} \end{aligned} \tag{A199}$$

The supply of foreign exchange (S^{FX}, A200) , on the other hand, is given by the inflows in the current and financial accounts of the balance of payments.

$$\begin{aligned} S^{FX} = & \frac{X}{e^N} + Rem + O_W + i_{CB}^{FX,R} \cdot R_{CB}^{FX} + i_B^{FX,R} \cdot R_B^{FX} \dots \\ & \dots + \frac{FDI}{e^N} + B\dot{g}^{FX} + L\dot{g}^{FX} + L^{\dot{FX},W} + \frac{B\dot{g}_W}{e^N} - \dot{R}_{CB}^{FX} \end{aligned} \tag{A200}$$

The dynamic of the nominal exchange rate ($\dot{e^N}$, A201) is driven by the relative excess demand for foreign exchange. Thus, the exchange rate will increase with an excess FX demand and it will decrease with an excess FX supply.

$$\dot{e^N} = \beta_{e^N} \cdot \left(\frac{D^{FX} - S^{FX}}{S^{FX}} \right) \cdot e^N \tag{A201}$$

Accordingly, the real exchange rate (e^R, A202), as the ratio of the foreign price index expressed in domestic currency to the domestic price level.

$$e^R = \frac{p^W \cdot e^N}{p} \tag{A202}$$

Now, before starting the description of interest rates, it is important to mention that the country risk premium (rsk, A203) depends on the ratio of imports to FX reserve assets.

$$rsk = \xi_0 \cdot \left(\frac{M \cdot p^W}{R^{FX}} \right)^{\xi_1} \tag{A203}$$

In turn, the country risk premium affects the overall premium on FX borrowing ($prem_{FX}$, A204) that affects the borrowing costs of all the sectors, albeit to different magnitudes

$$prem_{FX} = \phi_0^{FX} + \phi_1^{FX} \cdot (rsk)^{\phi_2^{FX}} \tag{A204}$$

The interest rate charged by foreign banks to FCs ($i_B^{FX,W}$, A205) depends on the international risk-free rate (i_W) and the overall premium ($prem_{FX}$) on FX borrowing.

$$i_B^{FX,W} = i_W + prem_{FX} \tag{A205}$$

The interest rate charged by foreign banks to NFCs ($i_F^{FX,W}$, A206) also depends on the international risk-free rate and a multiple $\varrho_F^{FX,W}$ greater than one of the overall premium on FX borrowing. Our assumption departs from the fact that firms are normally charged with higher borrowing costs in comparison to financial institutions

$$i_F^{FX,W} = i_W + \varrho_F^{FX,W} \cdot prem_{FX} \tag{A206}$$

The interest rates on FX reserve assets held by the central bank (A207) and FCs (A208) are given by some constant premiums, $\varrho_B^{FX,R}$ and $\varrho_{CB}^{FX,R}$, over the international risk-free interest rate, respectively.

$$i_B^{FX,R} = i_W + \varrho_B^{FX,R} \tag{A207}$$

$$i_{CB}^{FX,R} = i_W + \varrho_{CB}^{FX,R} \tag{A208}$$

The interest rate on FX government bonds ($i_G^{B,FX}$, A209) also depends on the international risk-free rate and a multiple $\zeta_G^{B,FX}$ lower than one of the overall premium on FX borrowing. This is consistent with the fact that governments set the floor on borrowing costs and, accordingly, are charged with lower interest rates than the private sector.

$$i_G^{B,FX} = i_W + \zeta_G^{B,FX} \cdot prem_{FX} \tag{A209}$$

Last but not least, the interest rate on FX government loans ($i_G^{L,FX}$, A210) applies a mark-down $\zeta_G^{L,FX}$ over the FX bonds rate. We assume this since multilateral and development banks that typically provide FX loans tend to offer more favourable borrowing costs vis-a-vis raising currency financing in capital markets.

$$i_G^{L,FX} = \left(1 - \zeta_G^{L,FX}\right) \cdot i_G^{B,FX} \tag{A210}$$

Indicators

$$perCapita = \frac{GDP}{pop \cdot e^N \cdot p^w} \tag{A211}$$

Per capita income (Nominal)

$$inflation = \frac{\dot{p}}{p} \tag{A212}$$

Inflation rate (Nominal)

$$reserves = \frac{R^{fx} \cdot e^N}{GDP} \tag{A213}$$

Foreign reserves (Nominal, %GDP)

$$foreignDebt = \frac{B_g^{FX} \cdot e^N + L_g^{FX} \cdot e^N + L_F^{FX,B} \cdot e^N + L_F^{FX,W} \cdot e^N + L_g^{FX,tr} \cdot e^N}{GDP} \tag{A214}$$

Foreign debt (Nominal, %GDP)

$$privateDebt = \frac{L_F^d + L_F^{FX} \cdot e^N + L_F^{FX,W} \cdot e^N + L_H^d}{GDP} \tag{A215}$$

Private debt (Nominal, %GDP)

$$pubDebt = \frac{Bg + B_g^{FX} \cdot e^N + L_g^{FX} \cdot e^N + L_g^{FX} tr \cdot e^N + B_g^{TR}}{GDP} \tag{A216}$$

Public debt (Nominal, %GDP)

$$fiscalDef = \frac{FD}{GDP} \tag{A217}$$

Fiscal deficit (Nominal, %GDP)

$$hhFrag = \frac{i_H^L \cdot L_H^d}{S_H} \tag{A218}$$

Household fragility (Nominal)

$$firmsFrag = \frac{i_F^L \cdot L_F^d + i_F^{L,FX,B} \cdot L_F^{FX,B} \cdot e^N + i_F^{FX,w} \cdot L_F^{FX,W} \cdot e^N}{F_F} \tag{A219}$$

Firms fragility (Nominal)

S1.2 Reaction Curves

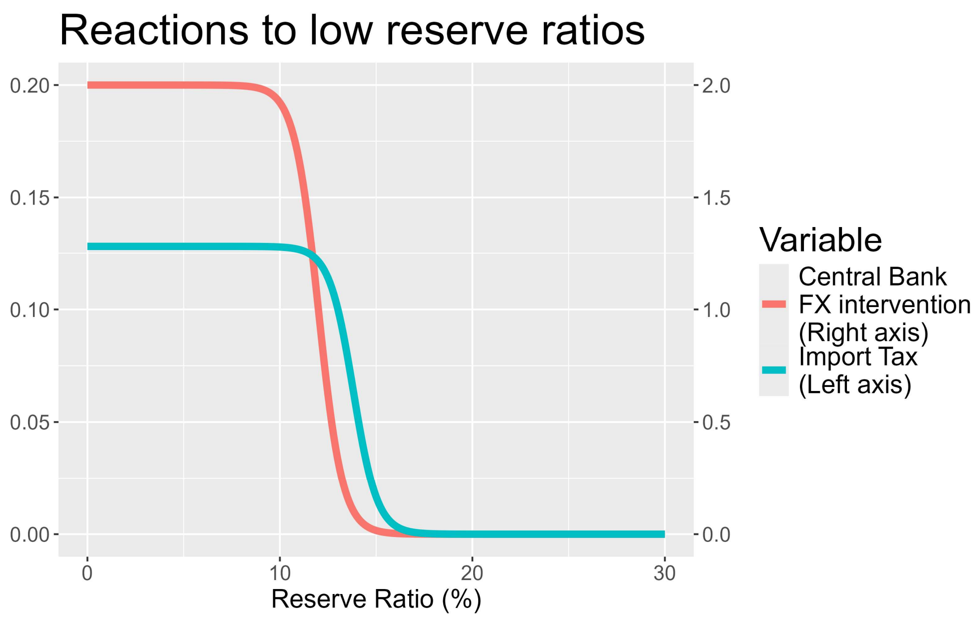


Fig. 8: Reaction curves for central bank FX intervention and import taxes

S1.3 Calibration values

S1.3.1 Initial values

Variable	Value	Meaning
y^e	1674.564	Real expected sales (*Real*, *Flow*)
v	127.1373	Inventories (*Real*, *Stock*)
k_f	2213.68	NFCs' capital stock (*Real*, *Stock*)
k_{tr}	0	Green capital stock (*Real*, *Stock*)
ik_f	104.17	NFCs' investment demand (*Real*, *Flow*)
σ_{mc}	0.12	Propensity to import final consumption goods (*Real*, *%*)
σ_{mic}	0.095	Propensity to import intermediate consumption goods (*Real*, *%*)
σ_{mk}	0.29	Propensity to import investment goods (*Real*, *%*)
$\sigma_{m,tr}$	0.574	Propensity to import transition goods (*Real*, *%*)
$\sigma_{x,n}$	0.0012	Propensity to export non-oil and coal (*Real*, *%*)
huc	0.715	Historical unitary cost (*Nominal*, **)
p	1.087	Producer price level (*Nominal*, **)
W_f	11.443	Average wage paid by NFCs (*Nominal*, **)
D_f	82.931	NFCs' domestic deposits (*Nominal*, *Stock*)
D_f^{FX}	16.738	NFCs' FX deposits (*Nominal*, *Stock*)
$L_{f,b}^{FX}$	21.544	NFCs' FX loans with FCs (*Nominal*, *Stock*)
$L_{f,w}^{FX}$	97.223	NFCs' FX loans with the Rest of the World (*Nominal*, *Stock*)
L_f^D	234.187	NFCs' domestic currency loans (*Nominal*, *Stock*)
L_b	0.348	FCs' employment level (*Real*, **)
k_{rb}	9.9171	FCs' capital stock (*Real*, *Stock*)
W_b	47.09	Average wage paid by FCs (*Nominal*, **)
R_b^{FX}	40.273	FCs' FX reserves (*Nominal*, *Stock*)
D_b^{FX}	11.718	FCs' FX deposits (*Nominal*, *Stock*)
$L_{b,w}^{FX}$	69.915	FCs' FX loans with the Rest of the World (*Nominal*, *Stock*)
A_d	8.55	Liquidity advances (*Nominal*, *Stock*)
B^{gb}	254.865	Government domestic currency bonds held by FCs (*Nominal*, *Stock*)
R_d	102.482	Cash plus bank reserves at the Central Bank (*Nominal*, *Stock*)
OF_b	105.95	FCs' own funds (*Nominal*, *Stock*)
π_f	1.66	NFCs' premium on domestic currency loans rate (*Nominal*, *%*)
π_h	0.44049	Households' premium on domestic currency loans rate (*Nominal*, *%*)
i_p	0.058	Monetary policy rate (*Nominal*, *%*)
C_h	686.604	Desired households' consumption (*Nominal*, *Flow*)
k_{rh}	368.889	Households' capital stock (*Real*, *Stock*)

Variable	Value	Meaning
Ik_h	49.722	Households' investment demand (*Nominal*, *Flow*)
D_h	173.869	Households' domestic deposits (*Nominal*, *Stock*)
L_h^D	198.497	Households' domestic currency loans (*Nominal*, *Stock*)
IPS_h	506.068	Households' insurance, pensions and SGS (*Nominal*, *Stock*)
$L_h^{D,i}$	62.111	Households' mortgage loans (*Nominal*, *Stock*)
θ_{lh}	0.322	Consumption loans to households' disposable income ratio (*Nominal*, *%*)
W_g	29.596	Average wage paid by the Government (*Nominal*, **)
Ik_g	36.683	Government's investment demand (*Nominal*, *Flow*)
C_g	21.601	Government's market-consumption (*Nominal*, *Flow*)
k_{rg}	498.487	Government's capital stock (*Real*, *Stock*)
D_g	50.705	Government's domestic currency deposits at FCs (*Nominal*, *Stock*)
$D_{cb,g}$	7.943	Government's domestic currency deposits at the Central Bank (*Nominal*, *Stock*)
D_g^{FX}	8.6	Government's FX deposits (*Nominal*, *Stock*)
B_g	338.851	Government domestic currency bonds (*Nominal*, *Stock*)
B_g^{FX}	147.843	Government FX bonds (*Nominal*, *Stock*)
B_{gw}	83.986	Government domestic currency bonds held by the Rest of the World (*Nominal*, *Stock*)
L_g^{FX}	59.731	Government FX loans (*Nominal*, *Stock*)
$L_g^{FX,tr}$	0	FX green loans (*Nominal*, *Stock*)
B_g^{tr}	0	Domestic green bonds (*Nominal*, *Stock*)
B_{gw}^{tr}	0	Domestic green bonds held by the RoW (*Nominal*, *Stock*)
B_{gb}^{tr}	0	Domestic green bonds held by FCs (*Nominal*, *Stock*)
π_{gd}	0.012	Premium on public domestic bonds rate (*Nominal*, *%*)
R^{FX}	171.885	Total FX reserves (*Nominal*, *Stock*)
D_w^{FX}	32.2015	Total FX deposits (*Nominal*, *Stock*)
R_{cb}^{FX}	131.612	Central Bank's FX reserves (*Nominal*, *Stock*)
e_n	1.195	Nominal exchange rate (*Nominal*, **)
a	74.45	Domestic labour productivity (*Real*, **)
a_w	80	Foreign labour productivity (*Real*, **)
a_{gr}	74.45	Domestic labour productivity in green industries (*Real*, **)
$a_{w,gr}$	80	Foreign labour productivity in green industries (*Real*, **)
p_w	0.951	Foreign imports price level (*Nominal*, **)
p_{wx}	0.555	Foreign GDP deflator (*Nominal*, **)
p_O	4.038	Implied oil and coal price level (*Nominal*, **)
GDP_w	88952.636	Foreign real GDP (*Real*, *Flow*)
pop	49.319	Scaled due to population definition
LF_o	24.405	Labour force (*Real*, **)
$L_h^{D,c}$	236.386	Households' consumption loans (*Nominal*, *Stock*)
$i^{w,st}$	0.0232	Short-term external interest rate (*Nominal*, *%*)

Variable	Value	Meaning
σ_{FX}	0.04	Share of public debt issuance in FX (*Nominal*, *%*)
xr_O	19.093	Fossil fuel exports (*Real*, *Flow*)
adj_ik_{tr}	0	Adjustment parameter for green capital investment
τ_m	0.0641	Tariff or import tax rate

NB: All values are rounded to 1e-3 or 1e-4 depending on the value

S1.3.2 Parameter values

Non-financial corporations

Parameter	Value	Explanation
β_{y}	3	Speed of convergence of expected demand
α_{v}	0.0783	Ratio of desired inventories to expected sales
β_{ivd}	0.1630	Speed of convergence for desired inventory accumulation
λ_{icf}	0.4063	Intermediate consumption technical coefficient from NFCs
κ_0	0.2	Autonomous parameter in NFCs investment function
κ_1	0.5	Sensitivity of NFCs investment to real profit rate
β_{ikf}	1	Speed of convergence for NFCs investment
δ_{f}	0.04	Depreciation rate of NFCs' capital stock
σ_{pc}	0.1297	Linear term in price effect on propensity to import final consumption goods
σ_{pcNew}	1	Size of the shock on the propensity to import final consumption goods
ϵ_{1c}	0.7543	Price elasticity for propensity to import final consumption goods
σ_{ac}	0.00025	Linear term in productivity effect on propensity to import final consumption goods
ϵ_{2c}	1.5758	Productivity elasticity for propensity to import final consumption goods
$\beta_{\sigma_{\mathrm{mc}}}$	0.5858	Speed of adjustment for propensity to import final consumption goods
σ_{pic}	0.1009	Linear term in price effect on propensity to import intermediate consumption goods
σ_{picNew}	1	Size of the shock on the propensity to import intermediate consumption goods
ϵ_{1ic}	0.6912	Price elasticity for propensity to import intermediate consumption goods
σ_{aic}	0.00085	Linear term in productivity effect on propensity to import intermediate consumption goods
ϵ_{2ic}	2.0637	Productivity elasticity for propensity to import intermediate consumption goods
$\beta_{\sigma_{\mathrm{mic}}}$	0.6411	Speed of adjustment for propensity to import intermediate consumption goods

σ_{pk}	0.3049	Linear term in price effect on propensity to import investment goods
σ_{pkNew}	1	Size of the shock on the propensity to import investment goods
ϵ_{1k}	0.4454	Price elasticity for propensity to import investment goods
σ_{ak}	0.00022	Linear term in productivity effect on propensity to import investment goods
ϵ_{2k}	0.241	Productivity elasticity for propensity to import investment goods
$\beta_{\sigma_{\text{mk}}}$	1.7765	Speed of adjustment for propensity to import investment goods
σ_{mSpeed}	0.3	Speed of the shock on propensities to import
σ_{mInit}	4	Initial period of the shock on propensities to import
σ_{xnp}	0.0014	Linear term in price effect on the propensity to export
ϵ_{xn1}	0.6	Price elasticity for propensity to export
σ_{xna}	0.00025	Linear term in productivity effect on the propensity to export
ϵ_{xn2}	1.3745	Productivity elasticity for propensity to export
$\beta_{\sigma_{\text{xn}}}$	1	Speed of adjustment for propensity to export
σ_{xnpNew}	1	Size of the shock on the propensity to export
σ_{xnSpeed}	0.4	Speed of the shock on the propensity to export
σ_{xnInit}	5	Initial period of the shock on propensities to export
μ_0	0.5914	Autonomous parameter in mark-up function
μ_1	0.0132	Sensitivity of the mark-up to inventory accumulation
β_{huc}	15	Speed of convergence of historical unitary cost
β_{p}	0.75	Speed of convergence of prices
θ_{Gh}	0.0972	Share of NFCs' gross operating surplus distributed to Households
θ_{Gg}	0.0057	Share of NFCs' gross operating surplus distributed to the Government
β_{Hmi}	0.3800	Share of NFCs' gross operating surplus distributed as mixed income to Households
ω_{f0}	1	Sensitivity of NFCs' wage curve to productivity
ω_{f1}	0	Sensitivity of NFCs' wage curve to unemployment
ω_{f2}	0.8797	Employment rate as reference for wage curve
ω_{f3}	1	Sensitivity of NFCs' wage curve to prices
θ_{wf}	0.1642	Share of NFCs' wage bill paid as social security contributions to Households

NB: All values are rounded to 1e-3 or 1e-4 depending on the value

Financial Corporations

Parameter	Value	Explanation
β_{y}	3	Speed of convergence of expected demand
α_{v}	0.0783	Ratio of desired inventories to expected sales
β_{ivd}	0.1630	Speed of convergence for desired inventory accumulation
λ_{icf}	0.4063	Intermediate consumption technical coefficient from NFCs
κ_0	0.2	Autonomous parameter in NFCs investment function
κ_1	0.5	Sensitivity of NFCs investment to real profit rate
β_{ikf}	1	Speed of convergence for NFCs investment
δ_{f}	0.04	Depreciation rate of NFCs' capital stock
σ_{pc}	0.1297	Linear term in price effect on propensity to import final consumption goods
σ_{pcNew}	1	Size of the shock on the propensity to import final consumption goods
ϵ_{1c}	0.7543	Price elasticity for propensity to import final consumption goods
σ_{ac}	0.00025	Linear term in productivity effect on propensity to import final consumption goods
ϵ_{2c}	1.5758	Productivity elasticity for propensity to import final consumption goods
$\beta_{\sigma_{\text{mc}}}$	0.5858	Speed of adjustment for propensity to import final consumption goods
σ_{pic}	0.1009	Linear term in price effect on propensity to import intermediate consumption goods
σ_{picNew}	1	Size of the shock on the propensity to import intermediate consumption goods
ϵ_{1ic}	0.6912	Price elasticity for propensity to import intermediate consumption goods
σ_{aic}	0.00085	Linear term in productivity effect on propensity to import intermediate consumption goods
ϵ_{2ic}	2.0637	Productivity elasticity for propensity to import intermediate consumption goods
$\beta_{\sigma_{\text{mic}}}$	0.6411	Speed of adjustment for propensity to import intermediate consumption goods
σ_{pk}	0.3049	Linear term in price effect on propensity to import investment goods
σ_{pkNew}	1	Size of the shock on the propensity to import investment goods
ϵ_{1k}	0.4454	Price elasticity for propensity to import investment goods
σ_{ak}	0.00022	Linear term in productivity effect on propensity to import investment goods
ϵ_{2k}	0.241	Productivity elasticity for propensity to import investment goods
$\beta_{\sigma_{\text{mk}}}$	1.7765	Speed of adjustment for propensity to import investment goods

σ_{mSpeed}	0.3	Speed of the shock on propensities to import
σ_{mInit}	4	Initial period of the shock on propensities to import
σ_{xnp}	0.0014	Linear term in price effect on the propensity to export
ϵ_{xn1}	0.6	Price elasticity for propensity to export
σ_{xna}	0.00025	Linear term in productivity effect on the propensity to export
ϵ_{xn2}	1.3745	Productivity elasticity for propensity to export
$\beta_{\sigma_{\text{xn}}}$	1	Speed of adjustment for propensity to export
σ_{xnpNew}	1	Size of the shock on the propensity to export
σ_{xnSpeed}	0.4	Speed of the shock on the propensity to export
σ_{xnInit}	5	Initial period of the shock on propensities to export
μ_0	0.5914	Autonomous parameter in mark-up function
μ_1	0.0132	Sensitivity of the mark-up to inventory accumulation
β_{huc}	15	Speed of convergence of historical unitary cost
β_{p}	0.75	Speed of convergence of prices
θ_{Gh}	0.0972	Share of NFCs' gross operating surplus distributed to Households
θ_{Gg}	0.0057	Share of NFCs' gross operating surplus distributed to the Government
β_{Hmi}	0.3800	Share of NFCs' gross operating surplus distributed as mixed income to Households
ω_{f0}	1	Sensitivity of NFCs' wage curve to productivity
ω_{f1}	0	Sensitivity of NFCs' wage curve to unemployment
ω_{f2}	0.8797	Employment rate as reference for wage curve
ω_{f3}	1	Sensitivity of NFCs' wage curve to prices
θ_{wf}	0.1642	Share of NFCs' wage bill paid as social security contributions to Households
α_{a}	0.02	Productivity growth rate
α_{pop}	0.005	Labour force growth rate
ι_{0w}	0.02	Autonomous term on NFCs' dividends distribution towards the Rest of the World
ι_{1w}	1.0956	Sensitivity of dividends distribution to fossil fuel exports towards the Rest of the World
ι_{0g}	0.0190	Autonomous term on NFCs' dividends distribution towards the Government
ι_{1g}	0.8483	Sensitivity of dividends distribution to fossil fuel exports towards the Government
s_f	0.4610	Saving rate of NFCs
η_{df}	0.35	Share of NFCs' wage bill used to determine NFCs' target domestic deposits
β_{Df}	1	Speed of convergence of NFCs' domestic deposits
η_{dfxf}	0.14	Share of NFCs' FX loans used to determine NFCs' target FX deposits
β_{Dfx}	1	Speed of convergence of NFCs' FX deposits
η_{lfxfb}	0.04	Share of NFCs' financing needs asked in the form of FX loans from FCs

η_{lfxfw}	0.13	Share of NFCs' financing needs asked in the form of FX loans from the Rest of the World
β_{riskFFX}	30	Elasticity of NFCs' rationing to country risk
UBFFX	1.1188	Upper bound for rationing of NFCs' FX loans
LBFFX	0.0299	Lower bound for rationing of NFCs' FX loans
MPFFX	0.08	Country risk reference for NFCs rationing
l_r	0.08	Country risk reference for NFCs rationing

NB: All values are rounded to 1e-3 or 1e-4 depending on the value

Central Bank

Parameter Name	Value	Explanation
ι_0	0.04852755	Autonomous term for monetary policy rate
ι_1	2	Linear term for monetary policy rate on inflation
ι_2	0.03	Target inflation for policy rate
β_{ip}	0.5	Speed of convergence for monetary policy rate
π_{rfx}	-0.003146882	Mark-up on official reserves return rate
π_{rfxb}	0.01935873	Mark-up on FCs reserve return rate
σ_{Rfxb}	0.74	Share of imports to determine FX reserves target of the Central Bank

NB: All values are rounded to 1e-3 or 1e-4 depending on the value

Households

Parameter Name	**Value**	**Explanation**
ϕ_{sc}	0.0159	Social contribution rate on households wage bill
β_{con}	1	Speed of convergence for Households consumption
λ_{l0}	-6	Elasticity of marginal propensity to consume to the real interest rate
λ_{l1}	0.013	Target real deposit rate in propensity to consume
mpc_{UB}	0.97	Upper bound of the marginal propensity to consume
mpc_{LB}	0.87	Lower bound of the marginal propensity to consume
β_{Ldch}	12	Speed of convergence for consumption credit demand
θ_{l3}	0.127	Share of Households' investment borrowed
κ_{h0}	0.073	Autonomous term in Households' propensity to invest
κ_{h1}	0	Sensitivity to the interest rate of Households' propensity to invest
κ_{h2}	0	Sensitivity to the unemployment rate of Households' propensity to invest
β_{Ih}	1	Speed of convergence for Households' investment
δ_h	3.2e-06	Depreciation rate of Households' capital stock
ζ_{itr}	0.035	Share of Households' wage bill saved as insurance, pensions and SGS

NB: All values are rounded to 1e-3 or 1e-4 depending on the value

Rest of the world

Parameter	**Value**	**Explanation**
σ_{Rem}	0.0002356	Share of World GDP distributed as remittances
ς_{fdi1}	0.2	Speed of the shock on the FDI entering the economy
ς_{fdi2}	1.3	Initial period of the shock on the FDI entering the economy
ς_{fdi3}	0.3666	Initial FDI entering the economy as a share of NFCs investment
ς_{fdi4}	0.68	New FDI entering the economy as a share of NFCs investment
ζ_{aff}	0.7763	Share of FDI towards NFCs
$\text{shr}_{\text{GreenField}}$	0.4841	Share of NFCs FDI that is greenfield
υ_1	0.0085	Linear term in country risk premium
υ_2	2	Elasticity of country risk to the imports to official FX reserves ratio
ζ_{bg}	0.016	Share of the trade balance determining the purchase of domestic public bonds by the Rest of the World
β_{en}	2.5	Speed of convergence of nominal exchange rate
β_{iwst}	1	Speed of convergence of external short-term interest rate
α_{w}	0.02	Foreign productivity growth rate
α_{gw}	0.03	World real GDP growth rate
α_{pw}	0.03	Foreign inflation rate
α_{pO}	0.03	Inflation rate of oil and coal prices

NB: All values are rounded to 1e-3 or 1e-4 depending on the value

Calibration Procedure

The calibration of GEMMES-Colombia involves setting initial conditions and parameter values so that the model replicates the average behaviour of empirical time series in levels and moments. Following [2], initial conditions match observed state variables. Parameters with available data (e.g., tax rates, productivity growth) are calibrated by averaging over 2014–2019, while others are estimated using CMA-ES [39] to minimise deviations from observed dynamics. Additional parameters, such as the fossil fuel export decay rate, are calibrated to fit the baseline scenario.

Validation

We propose a two-step validation procedure. First, we assess how GEMMES-Colombia approximates the moments of historical series, and provide some sense of the performance of the CMA-ES algorithm. We then compare the short-run forecasting performances of GEMMES against a well-established macroeconometric model used by the Colombian Economic Ministry [40, 41].

Historical Validation. We validate GEMMES against the moments of historical data (2005–2019) at annual and quarterly frequencies, focusing on autocorrelation and cross-correlation structures of key macroeconomic indicators. All GDP components are logged, while unemployment and inflation and kept as ratios. We trim the start of the model simulation (2019-2021) to control for model haggling. In accordance with common practice, moment matching is performed on the cyclical component of the filtered data. We use a Hodrick-Prescott filtering [42]. Note that we are aware of the discussions on the general merits of the Hodrick-Prescott filter compared to other alternatives, like the procedure proposed by Hamilton [43], which has been favoured in many studies. Halmilton famously highlighted that the HP procedure was subject to spurious relationships across variables, while performing badly at endpoints. However, the literature has highlighted that the Hamilton filter also has important drawbacks, notably its need for very well-behaved time series. Hodrick [42] notes that it might notably be unsuited to small datasets or to time series that may fall short of classical properties, like being generated by classical ARIMA or random walks. Given the small time-series length on historical data (14 years), the HP procedure might be preferred. [44] also highlights the superiority of the HP decomposition on aggregate economic outputs, most of all at a quarterly frequency. Hence, we stick to the HP procedure, with a smoothing parameter λ set to 1600 for quarterly data and to 6.25 for annual data, as in [45].

Figures 9 and 10 present GEMMES's performance on annual data. Autocorrelation is generally preserved, with GEMMES showing greater persistence, especially for investment, likely reflecting the model's slow adjustment dynamics.

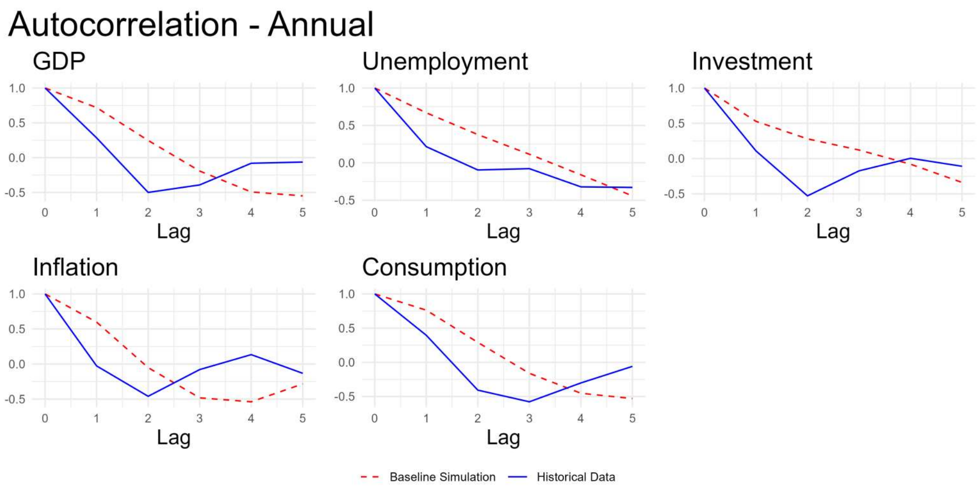


Fig. 9: Autocorrelation structure – Annual

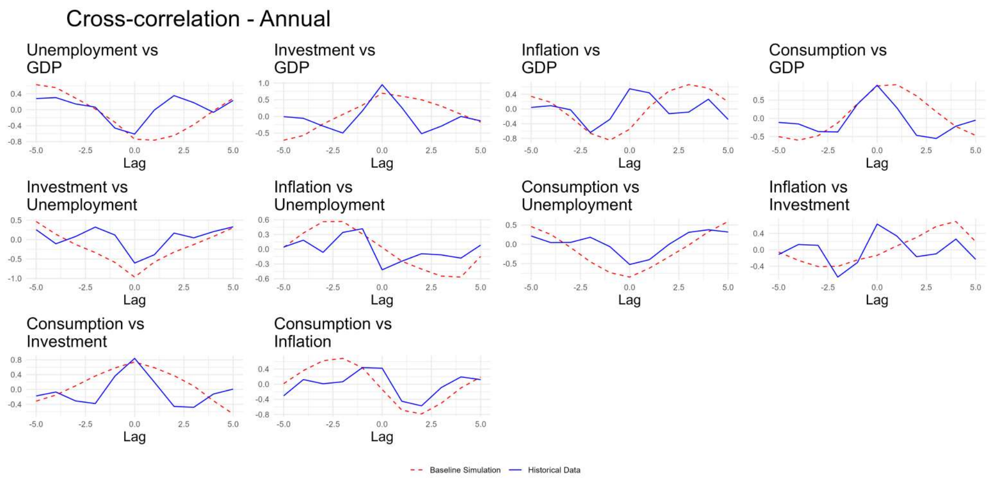


Fig. 10: Cross-correlation structure – Annual

The cross-correlation time structure is also broadly preserved. Performance is somewhat weaker for relationships involving inflation, GDP components, and unemployment. While the overall structure remains similar, the absence of a Phillips-curve-type mechanism in the model—which is difficult to identify in Colombian data [46] and is therefore not included—may limit the model's ability to reproduce these correlations accurately. It is also possible that, because the model is in continuous time and calibrated at a short time step, its precision for annual variations is mechanically lower.

This conjecture is verified on quarterly data. Figures 11 and 12 show the moment structures for quarterly data. The picture is the same as for annual data, with much more precision on both auto-correlation and cross-correlation. The baseline displays some temporal shifts with respect to historical behaviour, but these are within ranges reported by other well-established SFC models [47]. All in all, GEMMES Colombia matches moments remarkably well, giving confidence in the projections underlying our MORDM exercise.

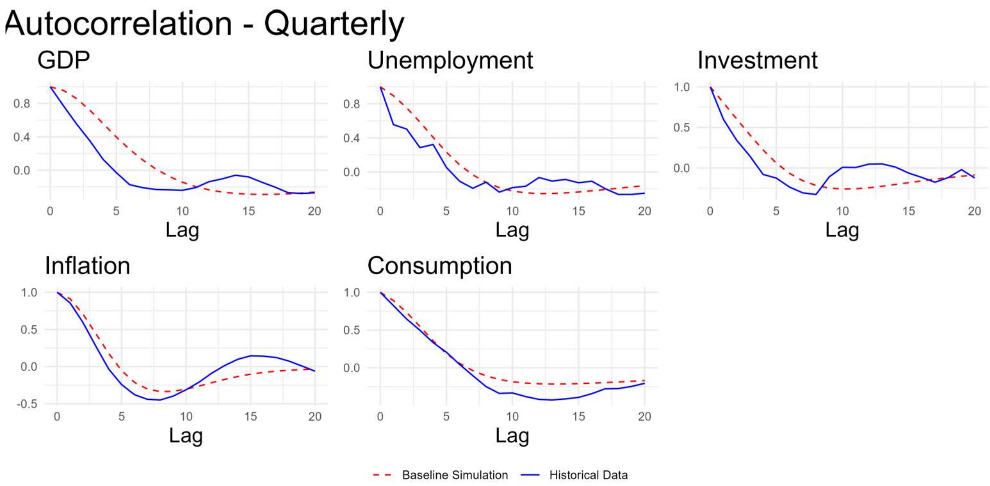


Fig. 11: Autocorrelation structure – Quarterly

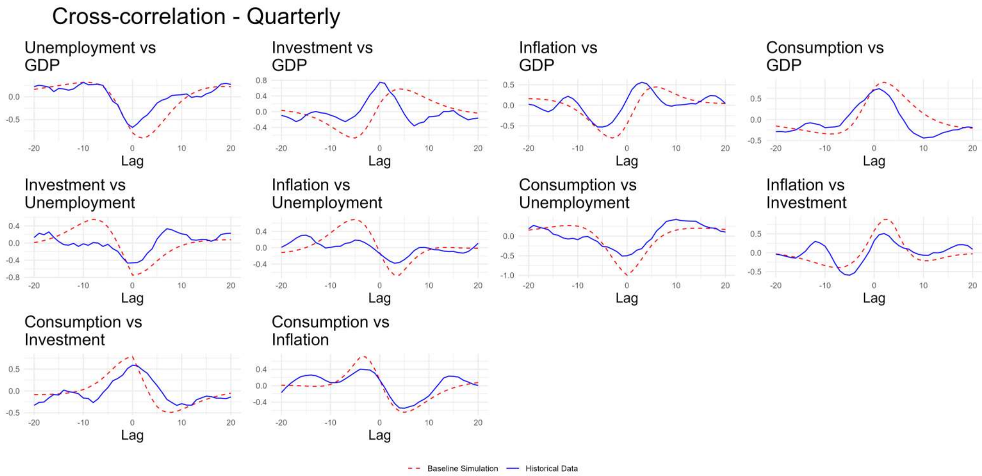
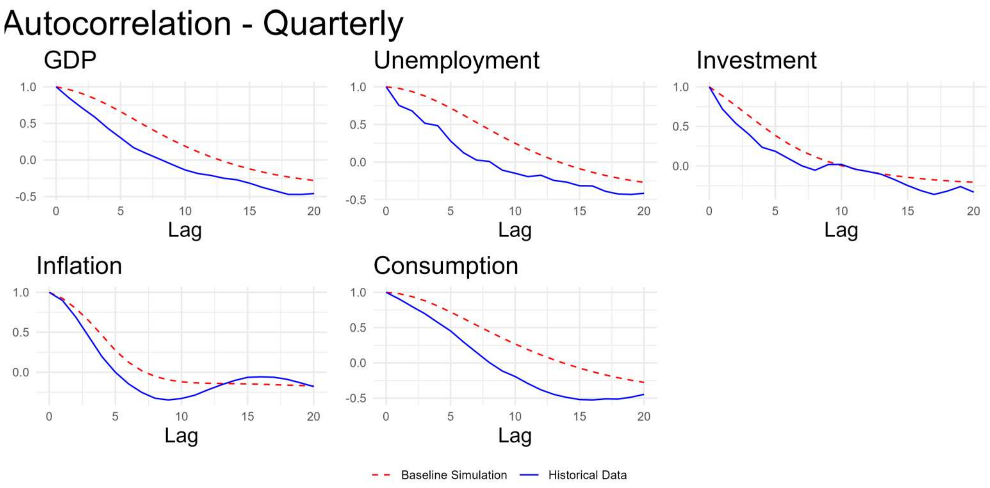
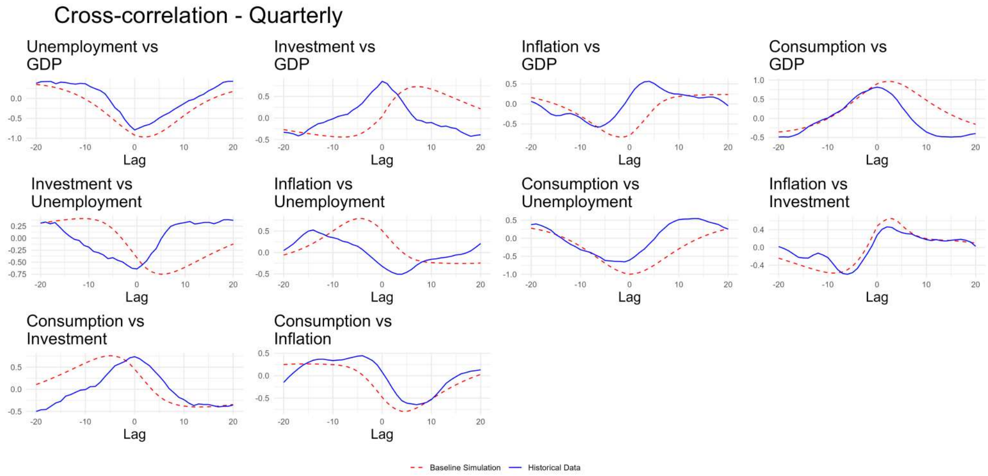


Fig. 12: Cross-correlation structure – Quarterly

Note that the smoothing parameter could significantly affect our results. For quarterly data, we repeated the exercise with two additional λ values drawn from [44], 12016 and 18736, which are much more restrictive. With these, correlation structures are less well-preserved, but the results are still very satisfactory (Figures 13 and 14).



(a) Autocorrelation structure $\lambda = 12016$

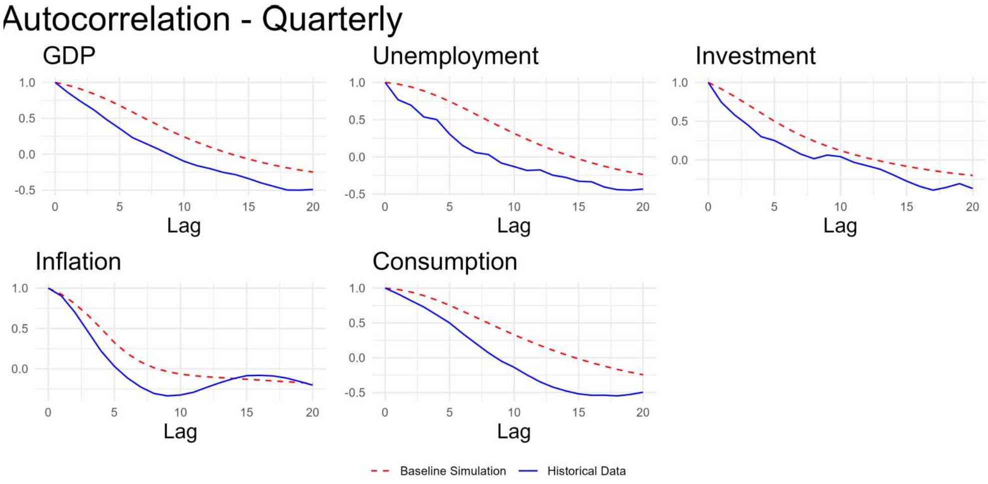


(b) Cross-correlation structure $\lambda = 12016$

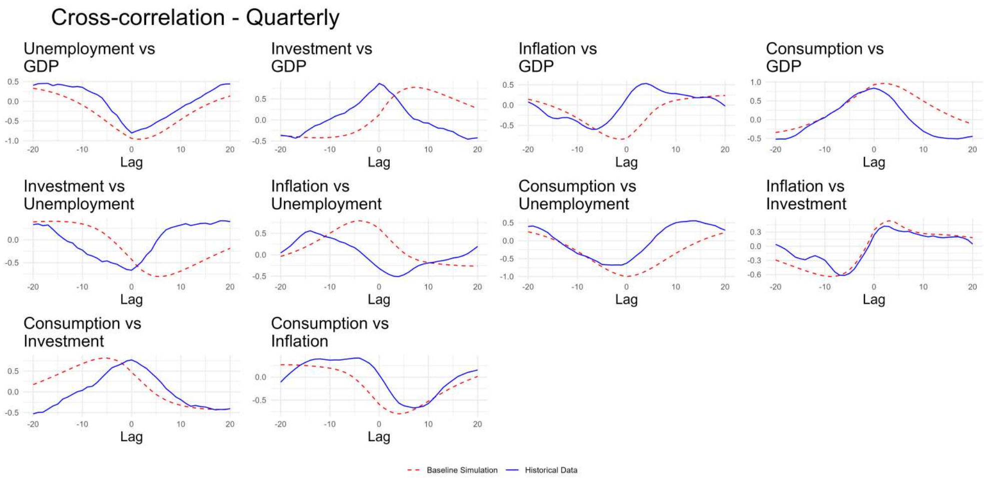
Fig. 13: Moment structure – $\lambda = 12016$

Projection validation

We complement this validation with a projection exercise. More precisely, we compare GEMMES's short-to-medium run projection performances against those provided by a well-established macroeconometric model of the Colombian economy [40] under the Ministry's MFMP (Marco Fiscal Mediano Plazo) process [41]. The latter aims to provide medium-run projections to guide Colombian policymakers. Results are shown in Figure 15.



(a) Autocorrelation structure $\lambda = 18736$



(b) Cross-correlation structure $\lambda = 18736$

Fig. 14: Moment structure – Alternative Smoothing parameters

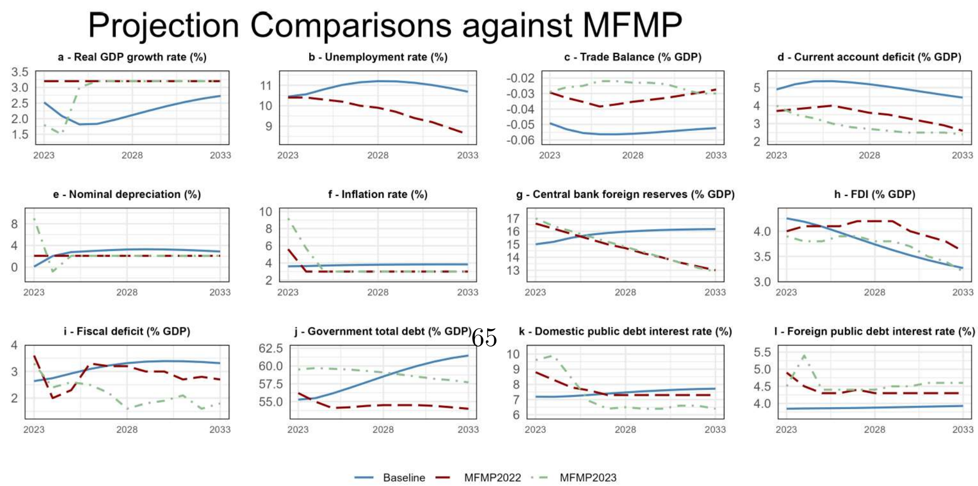


Fig. 15: MFMP Comparison

Overall, GEMMES exhibits projections in line with it macro econometric counterpart, with relatively more convergence with the MFMP2022. For most indicators, GEMMES stands between the MFMP2022 and 2023, notably on public finance metrics. For monetary indicators like inflation or nominal depreciation, GEMMES yields similar results to its counterpart. However, GEMMES' estimates of read growth, unemployment and external constraints are not in line with the MFMP. For real GDP and unemployment, this can be explained by GEMMES's long-run nature, which makes for slower adjustments – and indeed GEMMES converges in the long run to values close to the MFMP's.

Conclusion

In summary, despite inherent limitations in calibration, GEMMES-Colombia achieves satisfactory validation performance. Given the model's purpose of long-term prospective analysis, some deviation from the exact replication of historical moments and levels is acceptable. In contrast to short-term forecasting exercises, where precise projections are critical, our focus lies in capturing broad dynamic patterns. While further improvements to the calibration procedure are under consideration, pursuing an excessively precise fit may constrain the model's ability to explore dynamics not present in historical data, thereby narrowing the range of potential trajectories. Moreover, as emphasised by [48] and [49], the replication of historical moments is not necessarily a robust validation criterion, particularly given the substantial degrees of freedom available in many economic models, including those achieving high precision.

S2 Additional Methods

S2.1 Clustering method

Clustering was performed on the set of performances with Z-score scaling. Given the poor performance of linear K-means and the non-normal distribution of performances, we used kernel clustering. The Laplacian kernel with automated sigma estimation showed the best results. To choose the best number of clusters, we applied the algorithm for three to six kernels ten times and chose the clustering that exhibited the highest Calinsky-Harabasz Index [50]. This index is a synthesis of the usual within-sum of square and Dunn index that respectively measure the density of a clusters and the ratio of the minimum distance between clusters and the maximum distance across clusters. The number of minimum and maximum clusters was chosen to avoid too many clusters and to impose a minimum degree of contrast across policy options by avoiding a two-way classification. We then post-processed this best cluster by reapplying the Laplacian kernel method with the chosen clusters as centres ten times, and selecting again the clustering that yielded the highest Calinski-Harabasz Index. This post-processing step allowed us to handle outliers that may have been incorrectly assigned to a given cluster. Figure 16 shows our best clustering across policy levers in a 3D scatterplot for our main exercise.

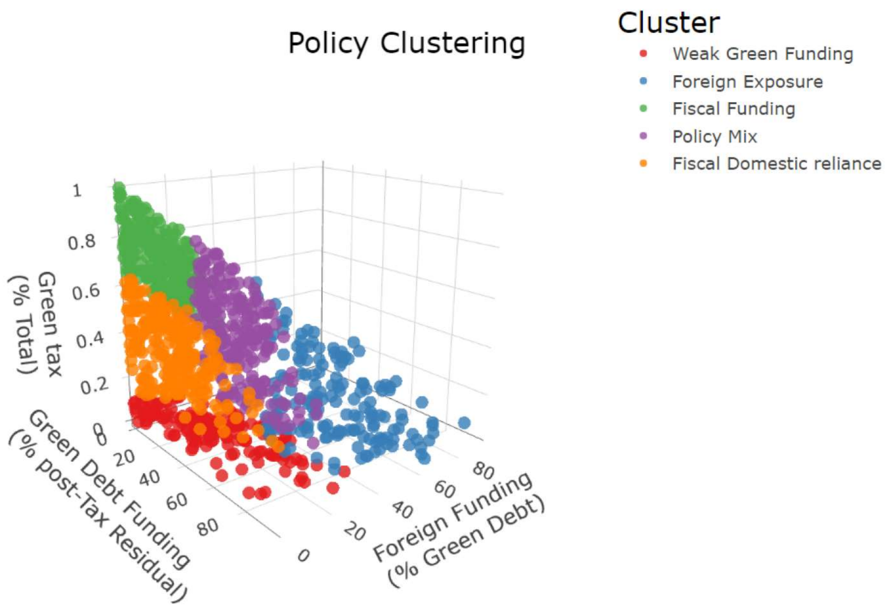


Fig. 16: Policy space delineation after policy clustering

S2.2 Definition of the cost function

We reproduce Table 1 in the main body of the text that describes our Safe Operating Space (SOS). This SOS consists of bandwidths. These can first be either static of dynamic.

Static bandwidths describe policy goals for stationary outcomes, like ratios or relevant growth rates in macroeconomic, like inflation. They can be can be uni-directional or bi-directional.

Uni-directional bandwidths gather objectives that should only remain below or above a given threshold. For this proof of concept, these thresholds are largely arbitrary, but they would be refined to match relevant decision-makers' preferences or actual legal constraints. For instance, we assume that the Colombian Foreign Debt (in % of GDP) is safe below 40%, and that the country becomes more fragile above this threshold.

For an objective X, let $x_{t,i,p}$ be the value of the corresponding quantity at time t for state of world i and instrument p. Supposing that X is problematic only from an upper threshold $\overline{x}$ onward, let us define the cost $C_{X,t,i,p}$:

$$C_{X,t,i,p} = \begin{cases} -\sqrt{100(\frac{\overline{x}-x_{t,i,p}}{\overline{x}})} & \text{if } x_{t,i,p} \leq \overline{x} \\ 100\left(\frac{x_{t,i,p}-\overline{x}}{\overline{x}}\right) & \text{otherwise.} \end{cases} \tag{A220}$$

In other words, the cost associated with going beyond the threshold at time t is expressed as percentage deviation from the threshold. Costs are assumed to be linear above the threshold to keep values within workable ranges. Conversely, we reward remaining below the threshold, but only concavely, by applying a square root operator. This cost could be defined symmetrically for a downward threshold $\underline{x}$:

$$C_{X,t,i,p} = \begin{cases} -\sqrt{100(\frac{x_{t,i,p}-\underline{x}}{\underline{x}})} & \text{if } x_{t,i,p} \geq \underline{x} \\ 100\left(\frac{\underline{x}-x_{t,i,p}}{\underline{x}}\right) & \text{otherwise.} \end{cases} \tag{A221}$$

To compute our total cost metric, we consider:

$$C_{X,i,p} = \frac{1}{T}\int_0^T C_{X,t,i,p}dt. \tag{A222}$$

The average performance over the run as our main cost metric. We purposefully do not discount to avoid under-weighting late periods.[7]

For bidirectional bandwidths, we adopt the same approach, but by considering that, in this case, the economy is safest when it is the farthest away from the bandwidth's boundaries, i.e., closer to the barycenter of the interval. Defining $\underline{x}$ and $\overline{x}$ the lower and upper bounds, we can define

$$\tilde{x} = \frac{\underline{x}+\overline{x}}{2}.$$

the barycenter of the bandwidths. The cost metric is therefore defined as:

$$C_{X,t,i,p} = \begin{cases} 100\left(\frac{\underline{x}-x_{t,i,p}}{\overline{x}-\underline{x}}\right) & \text{if } x_{t,i,p} \leq \underline{x} \\ -\sqrt{100(\frac{x_{t,i,p}-\underline{x}}{\overline{x}-\underline{x}})} & \text{if } x_{t,i,p} \geq \underline{x} \text{ and } x_{t,i,p} < \tilde{x} \\ -\sqrt{100(\frac{\overline{x}-x_{t,i,p}}{\overline{x}-\underline{x}})} & \text{if } x_{t,i,p} > \tilde{x} \text{ and } x_{t,i,p} \leq \overline{x} \\ 100\left(\frac{x_{t,i,p}-\overline{x}}{\overline{x}-\underline{x}}\right) & \text{if } x_{t,i,p} \geq \overline{x}. \end{cases} \tag{A223}$$

And taking the average over time as in the above.

Finally, *dynamic* thresholds are meant to evaluate non-stationary variables, or variables whose stationarisation does not lend itself to an easy interpretation. In the problem at hand, per-capita GDP and real consumption fall require a moving SOS. To do so, we consider the value for both variables at the start of the transition x_0, and apply a constant growth rate g_X, defining a reference trajectory $\overline{\overline{x_t}} = (1+g_x)^{t-t_0}x_0$. Costs will be computed as relative deviations from this reference trajectory. The threshold are downward, and therefore measure the Colombian economy's ability to achieve a minimum growth requirement

[7] Note that the cost metric is capped at 100% upward to ensure that it remains within workable ranges.

$$C_{X,t,i,p} = \begin{cases} -\sqrt{100(\frac{\overline{\overline{x_t}} - x_{t,i,p}}{\overline{\overline{x}}})} & \text{if } x_{t,i,p} \leq \overline{x} \\ 100\left(\frac{x_{t,i,p} - \overline{\overline{x_t}}}{\overline{\overline{x_t}}}\right) & \text{otherwise.} \end{cases} \tag{A224}$$

This cost is then averaged over time.

Note that we normalise in this case by the length of the bandwidth – dynamic or static, and costs are expressed as percentages of the bandwidth's length.[8] In what follows, we consider any positive cost metric calculated as above as "overshooting". Averaging is on the period 2023-2050, with 2023 the effective thart of the transition in the model.

S2.3 Robustness metric

For this proof of concept, we adopt a rather conservative definition of regret proposed by [10]. For a given policy in a SOW, regret on an SOS dimension is defined as a deviation, taken as a Z-score for normalisation purposes, from the best-performing policy in the SOW. For each SOS dimension, we consider the 90th percentile of the regret distribution across SOW to enforce a conservative criterion, and we consider the maximum value of these SOS-dimension-specific 90th percentiles.

$$R_\ell = \max_{s \in S} q_i^{90} \frac{C_{\ell,s,i} - \min_\ell C_{\ell,k,i}}{\sigma(C_{i,s})}. \tag{A225}$$

The choice of taking a maximand instead of, for instance, an average, is meant to increase the conservativeness of the metric. Defined as it is, this regret approach selects policies that will perform acceptably across all SOS dimensions and all SOWs, without any prior knowledge of the probability of a given SOW occurring.

S3 Diagnostics

S3.1 Diagnostics

We develop here a diagnostic of our results by breaking down key macroeconomic channels. Notably, we explore how our two main causality channels, exchange rate depreciation and the changes in domestic interest rates, affect our SOS outcomes. We display these corresponding variables in Figure 19.

Currency depreciation (Panel a) is the largest in the conventional-funding baseline. The Fiscal Domestic Reliance and Weak Green Funding clusters limit it modestly, while other clusters exhibit a stronger relative appreciation. This relative appreciation is attributable to increased foreign borrowing in the "Policy mix" and "Foreign Exposure" clusters, hence leading to more foreign currency entering the country, easing the tension on the foreign currency market. This lower level of depreciation of the currency drives import demand higher, which worsens the trade deficit (Panel (b)). Furthermore, it reduces the value of dollar inflows, which is not compensated for by the higher

[8] This normalisation is equivalent to the one we used for unidirectional bandwidths if we consider that unidirectional bandwidths are defined implicitly on a $(0, \underline{x})$ or $(0, \overline{x})$ interval. Note that for downward bandwidths, this would require adopting a different, but equivalent cost definition to the one above, by penalising values within the bandwidth and rewarding values that are outside.

overall inflow of currency. As a result, the income account also worsens (Panel (c)). Since the Trade and Income accounts are components of the Current Account Deficit (CAD), the clusters favouring a stronger currency mechanically exacerbate the CAD. This deterioration leads to a decline in household employment (Panel (d)), which in turn reduces consumption and slows GDP growth (Panel (e)). The appreciation also directly affects per capita GDP in foreign currency growth (Panel (k)).

The appreciation also affects debt emission. As modelled (see Supplementary Material S1.1.2, Equation A192), the government responds to the current account deficit by issuing more foreign debt (Panels (f) and (g)) to secure foreign currency inflows and limit exchange rate depreciation. However, this also increases interest payments (Panel (h)) and Total Financing Needs (Panel (i)) in the medium to long run. Furthermore, as public indebtedness increases, the premium on government debt augments (Panel (o)), leading to higher total financing needs of the government (TFNG), interest payment and, eventually, debt. Note that this goes hand in hand with a reduction in country risk on private debt (Panel (p)), confirming that the government pays the cost of a higher peso. As a result, the fiscal deficit and public debt increase, pushing also upward total domestic foreign debt upward, as the government emits more debt in foreign currency. This contrasts with tax-based clusters, which mechanically decrease financing needs and, therefore, public debt and deficits.

Regarding the decrease in domestic interest rates, the precise causality depends on the cluster. For debt-based strategies, the effect goes more through currency appreciation. This reduces the total financing needs of banks by making foreign funding cheaper. This reduces the need for central bank advances and therefore reduces banks' average funding costs. This effect is increased by the change in the structure of government debt emissions, which is more directed towards foreign investors (Panels (f) and (g)). For tax-based strategies, the effect is higher simply because overall debt emission decreases thanks to higher tax proceeds. Average funding costs serving as the basis for interest rate setting, these RMSs have, in general, the effect of reducing domestic interest rates (Panel (l), which lowers fragility indicators. These channels are summarised in Figure 17.

Finally, the effect of tax policies, because they directly affect consumer prices, have an increasing effect on inflation and depresses real consumption. Mechanically, strong peso strategies decrease inflation and allow per-capita consumption to recover by making imports cheaper (Panels (n), (m)). These channels are summarised in Figure 18.

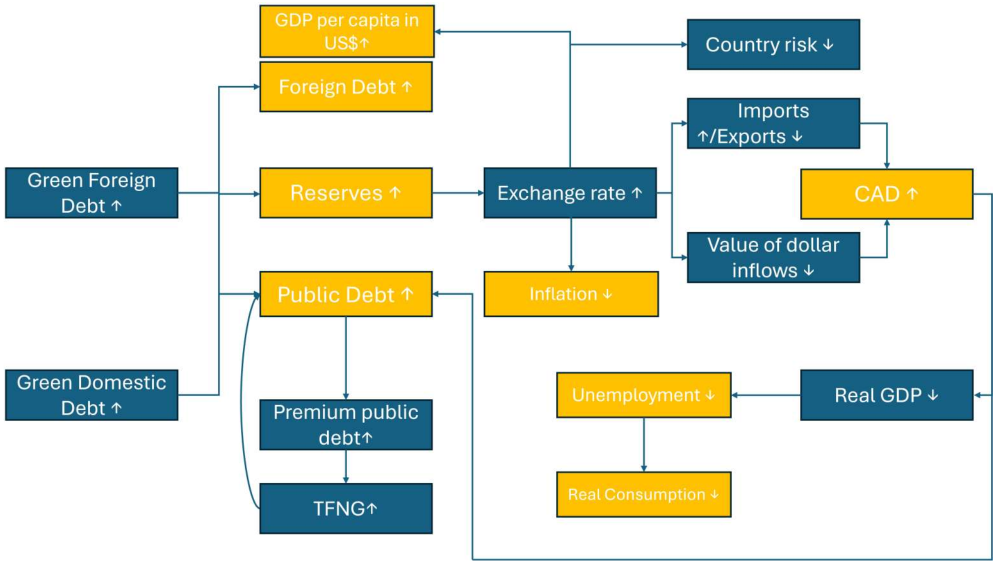


Fig. 17: Causality channels – Tax policies

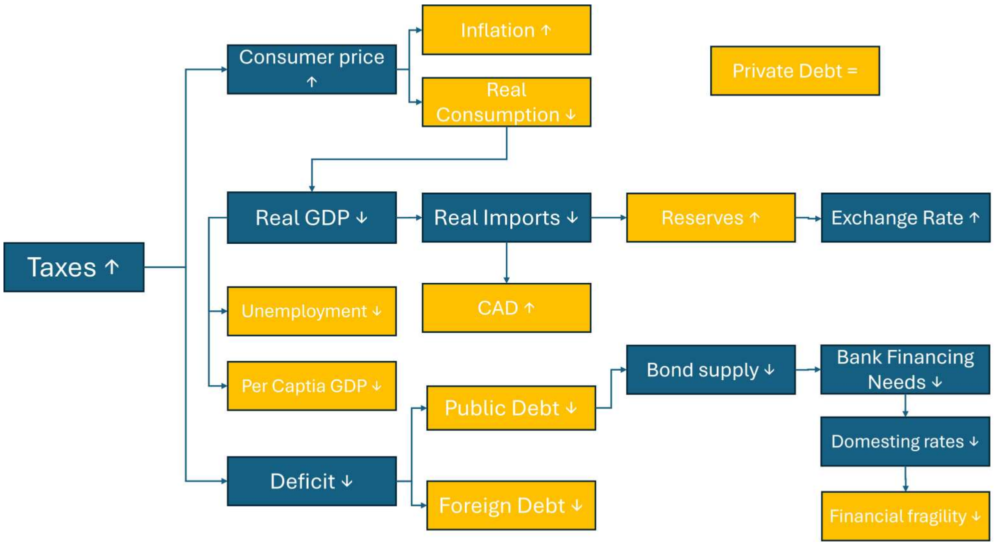


Fig. 18: Causality channels – Tax policies

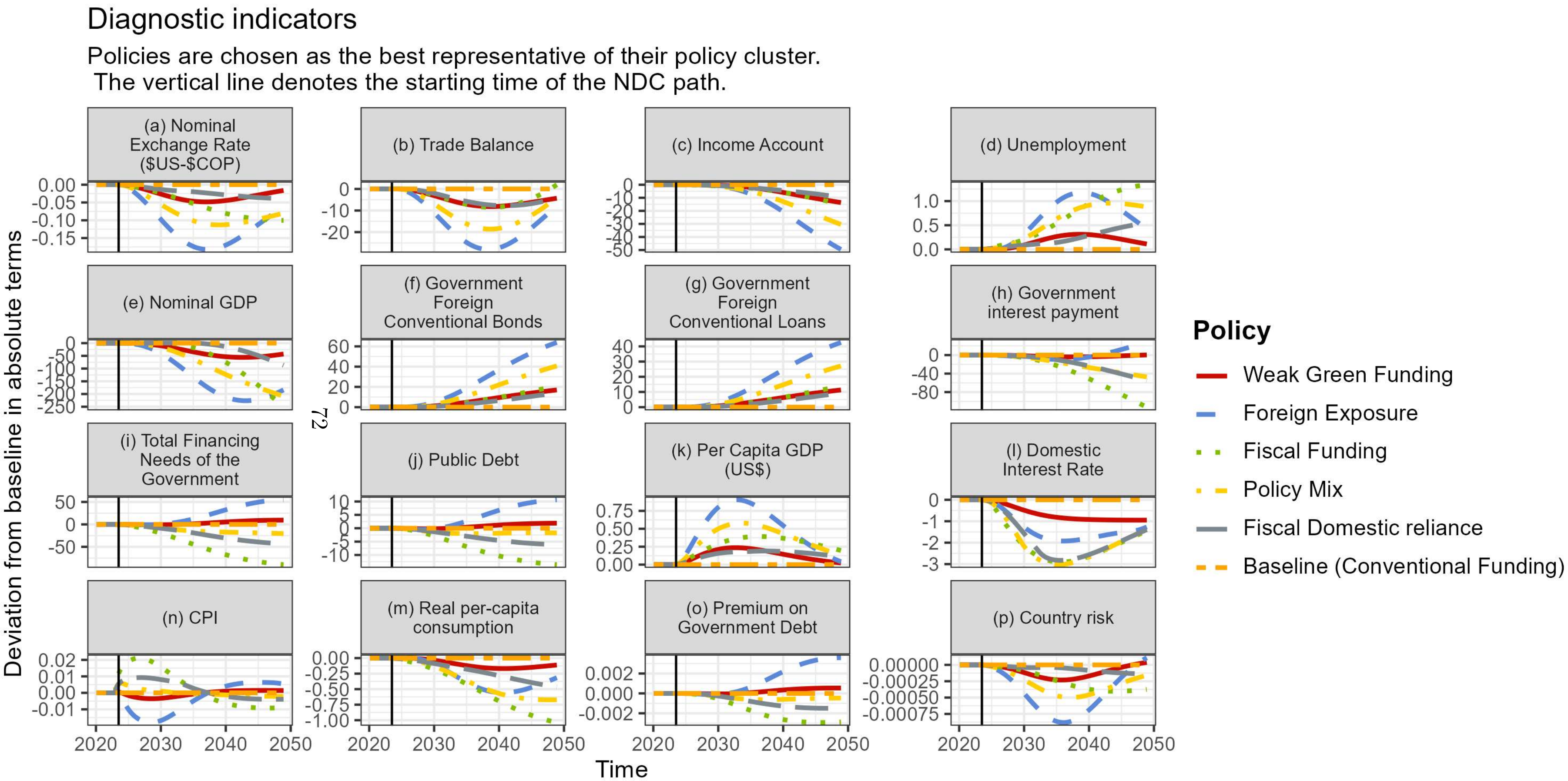


Fig. 19: Diagnostic Indicators

S4 Additional results

S4.1 Performances on SOS indicators

We display here the dynamic performances of cluster representatives, both on row SOS indicators and on SOS costs.

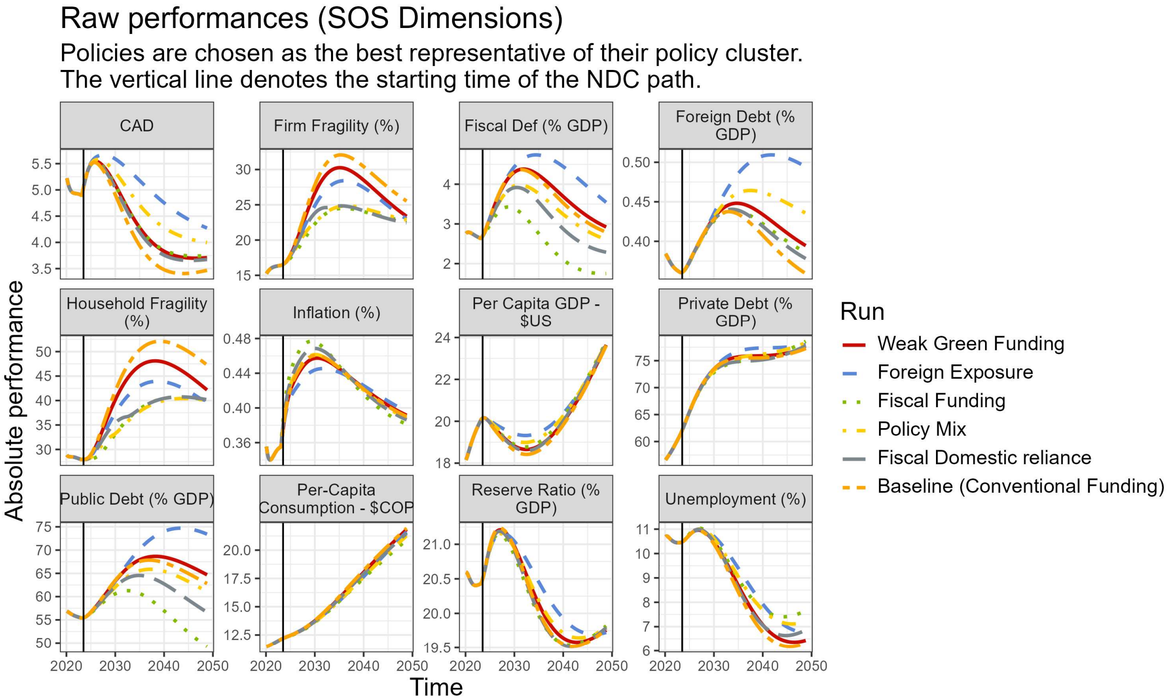


Fig. 20: Performance on SOS indicators

S4.2 Alternative Regret Measure

In the main body of the text, we focused on a particular definition of Regret, which considers robust a decision that minimises regret over all possible states of the world:

$$R_\ell = \max_{s \in S} q_i^{90} \frac{C_{\ell,s,i} - \min_\ell C_{\ell,k,i}}{\sigma(C_{i,s})}. \tag{A226}$$

This definition implies that the policymaker does not hold any prior on future states of the world. Hence, it is naturally a very conservative metric. Yet, policymakers do hold priors. In particular, the baseline calibration on which robust candidates were designed can be considered as having more weight than other possible states of the world. It can therefore serve as a basis of comparison:

$$R^b_\ell = \max_{k \in K} \frac{q^{90}_i C_{\ell,s,i} - C_{\ell,k,b}}{\sigma(C_k)}. \tag{A227}$$

Where b indicates baseline, and where, unlike for R_ℓ, we scale by the standard deviation cost performances across all SOWs. Although it is not as homogeneous as the normalisation of the main regret metric, since we compare performances from different distributions, this regret metric can still be interpreted as a multiple of the whole-SOW item-specific standard deviation. This method also has the advantage of controlling for items with a very low standard deviation in baseline, like reserves.[9] Again, this choice is made to retain some comparability across objectives, while allowing for negative cost performances. We report results in Figure 21.

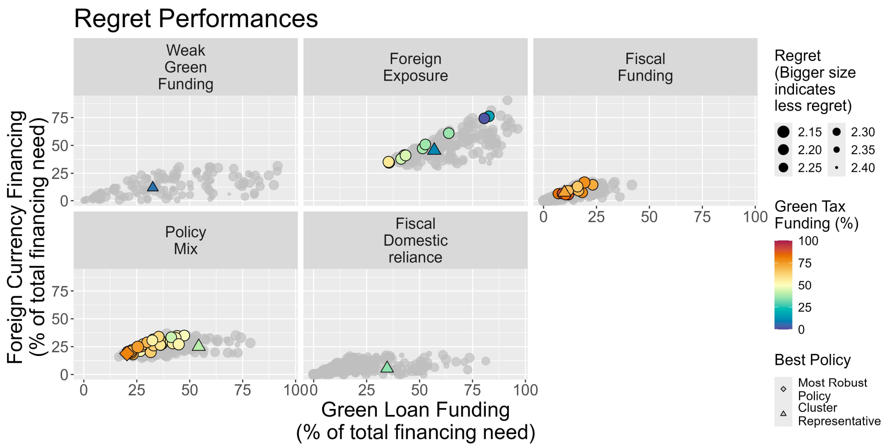


Fig. 21: Robustness - Regret I

In this case, relevant RMSs belong to the "Foreign Exposure", "Fiscal Funding" and "Policy mix" clusters, with the MRP being in the latter cluster – however, given its positioning, it is most likely on the border between the "Policy mix" and the "Fiscal Funding" clusters. Compared to the our main regret metric, this Policy Mix measure exhibits a much higher tax rate (around 80%) and with a close to full foreign exposure on the residual, which amounts to 20% of the funding. Hence, while it relies more on taxes, this strategy relinquishes conventional funding. Figure 22 shows that the main source of regret for this formulation is clearly reserves. While more foreign exposure, by allowing foreign inflows, can help keep the reserve ratio high, better robustness performances seem to be achieved by instead preventing strong dollar outflows, which would occur notably through higher import demand. This is precisely what high-tax strategies do. By limiting real consumption, they temper the multiplier effect of NDC funding and therefore limit import demand growth.

As a result, a policymaker with a higher prior on the baseline, and therefore desirous to remain as close as possible to the baseline's performances, should consider

[9] Scaling by the standard deviation across all SOWs did not change the results

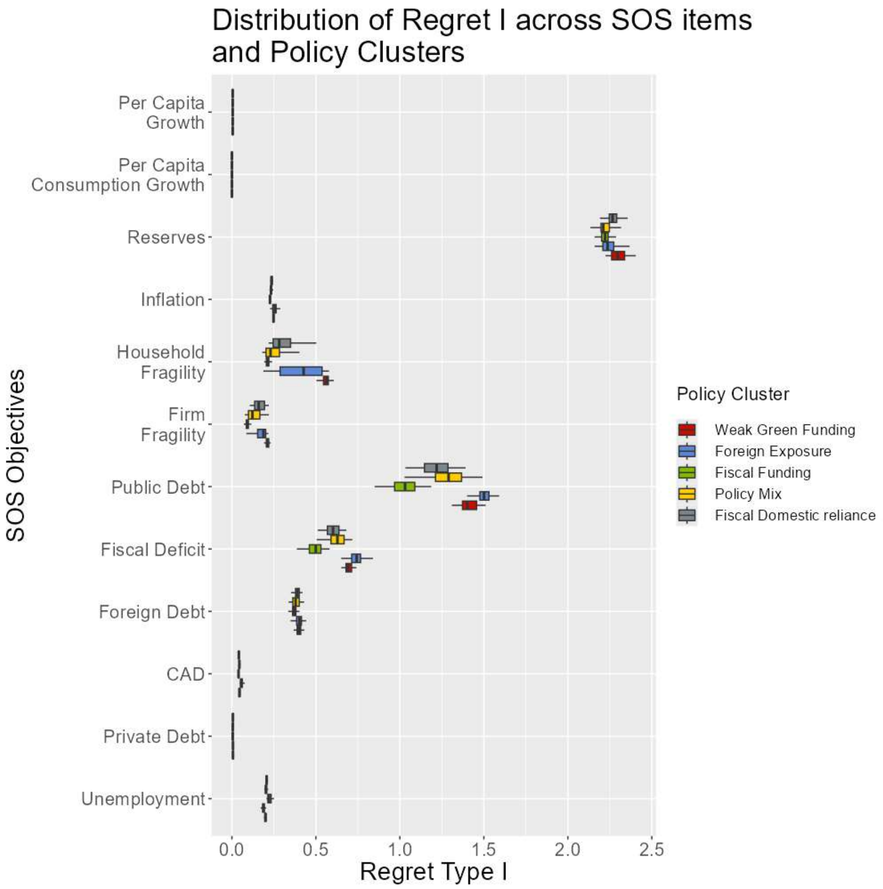


Fig. 22: Robustness - Regret I

higher tax rates. This suggests that the baseline assumption might be optimistic in terms of external constraint, and that it would be much higher in other SOWs. A sensible hedging strategy, in this case, is to limit the overheating coming from NDC spending.

S4.3 Satisficing

Another way to think about robustness is to measure a policy's ability to fulfil some minimum threshold, or at least to maximise the probability to meet said thresholds. For illustrative purposes, we define a quite restrictive satisficing function as the probability for an RMS to remain within the SOS across all states of the world. With the indicator function:

$$I_{k,\ell,i} = \mathbb{1}\{C_{k,\ell,i} < 0\}.$$

Taking the sum, we obtain the "sustainability value" (SV) of the RMS on SOW i:

$$SV_i = \sum_k I_{k,\ell,i}.$$

We define this other indicator function:

$$S_{\ell,i} = \mathbb{1}\{SV_i = 12\}.$$

We consider the expectation value of this indicator function over all SOWs:

$$S_\ell = \frac{1}{N} \sum_i S_{\ell,i},$$

where N is the size of our sensitivity space.

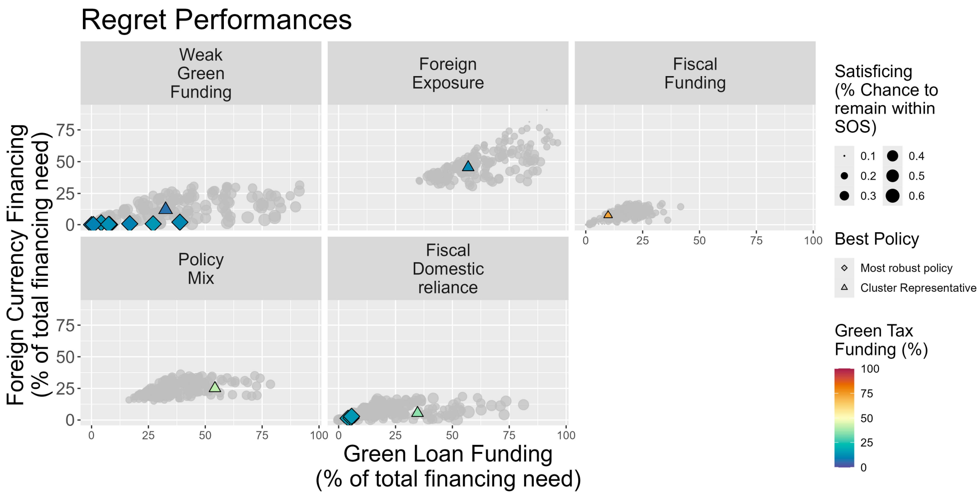


Fig. 23: Satisficing performances

Results are displayed in Figure 23. It is obvious that since our criterion is highly restrictive, its scores are very low. Indeed, we reach no more than a 0.2% chance to remain within the SOS across our 1,000 sensitivity calibrations. This indicates once again that the policies we study are at best of second order, and that maximising this satisficing metric would require much more ambitious and concrete policies. However, it is interesting to see that a population of RMSs has non-negligibly higher satisficing performances than others. Strikingly, they are mostly found in clusters that were left aside in previous studies: Weak green funding and Fiscal Domestic Reliance. These RMS groups are closer to the conventional funding baseline: low taxes and a modest to high green loan taking. This suggests that the plain NDC spending could well be "satisficing" in terms of SOS compliance across SOWs, or at least close to it, and that a certain level of greenium can favour this SOS compliance. However, the robustness analysis showed that these strategies were very inferior in terms of robustness. This finding highlights the trade-off that may exist between maximising SOS compliance and allowing for some overshoot to avoid too high SOS variation. In the present case, given the very low SOS-compliance probabilities, this trade-off is easy to solve, but it could be more intricate in the presence of other policies.

S4.4 Adding a discount rate

S4.4.1 Cost transformation

We finish our analysis by adding a discount rate ρ to our cost function. For all costs defined in SupMats S2.2, we apply the following transformation:

$$\tilde{C}_{X,t,i,p} = \exp(-\rho t) C_{X,t,i,p} \tag{A228}$$

We apply a recent estimate of the social discount rate for Latin American countries. [52] reports a value of 12% for Colombia. With this adjustment, we reproduce the same analysis as in the main body of the text. We conserve the time averaging: as a result, the discount rate simply underweights late periods.

S4.4.2 Results from Borg

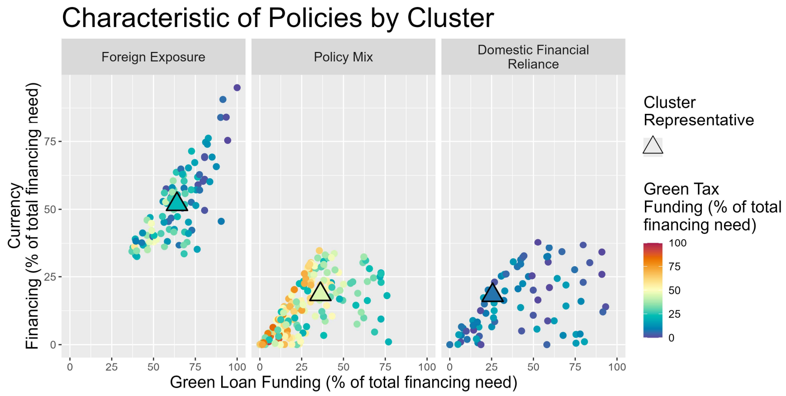
Figure 24 shows the outcome of the Borg optimisation and the characterisation of the policies.

The overall cost profile is similar to the exercise without a discount rate, while absolute levels are lower due to discounting. Only three clusters are necessary, which nonetheless span the arbitrages of the main exercise. These results highlight that most of the effects detected in the main specification belong to the short run, and that long-run dynamics do not matter much in the characterisation of the qualitative trade-offs. Note, nonetheless, that fewer policies are chosen, highlighting that the account of long-run dynamics gives more degrees of freedom in minimising SOS costs as the unwinding of the NDC investment compensates for short-run losses by bringing the Colombian economy closer to a steady-state path.

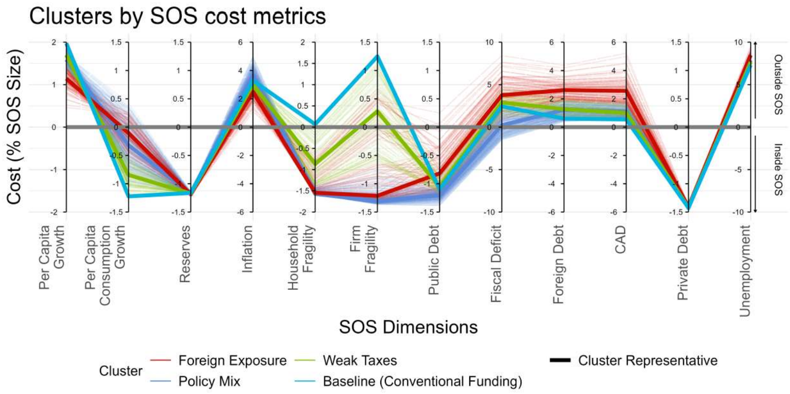
S4.4.3 Regret Analysis

We also provide a regret analysis, using our main regret metric. Results are displayed in Figure 25.

Results highlight that the MRP is very close to conventional funding. In other words, prioritising a resilience to short-run development leads to a policy with very little green funding, very little taxes and very little foreign exposure. This high sensitivity of results to the discount rate is hardly surprising, as it is indeed a common issue in climate economics and beyond [53]. That RMSs under a high discount rate be conservative is highly reminiscent of the sensitivity of climate action itself to the discount rate [54]. RMSs beyond conventional funding do generate short-run costs to the economy, and they indeed entail fragility over a transitory period. However, in comparison to the results obtained in our main exercise, policy mixes yield long-run resilience gains. These results highlight the importance of the temporal nature of our SOS performances.



(a) Characterisation of RMS candidates - With 12% Discounting. Colours indicate the extent of the green tax funding



(b) Performance of RMS candidates - With 12% Discounting.

Fig. 24: Resource Mobilisation Strategies - With Discounting. Cluster representatives were determined by considering the policy closest to the cluster's centroid,*i.e*, minimising distance to the set of average within-cluster performances.

This directly questions the viability of such high discount rates like the one we used: with such a number, outcomes beyond 2030 are discounted at more than 70%, which greatly diminishes their significance. Meanwhile, the low-carbon transition is a long-run effort, which may call for lower interest rates, closer to 5%, similar to global market rates [55], or even market rates with a markdown, as it is often done for public policy projects [56]. Social rates have been notoriously considered as lower than market rates [57]. Finally, justice dimensions may call for even lower rates [54], or even zero over finite time spans, like we do in the main exercise.

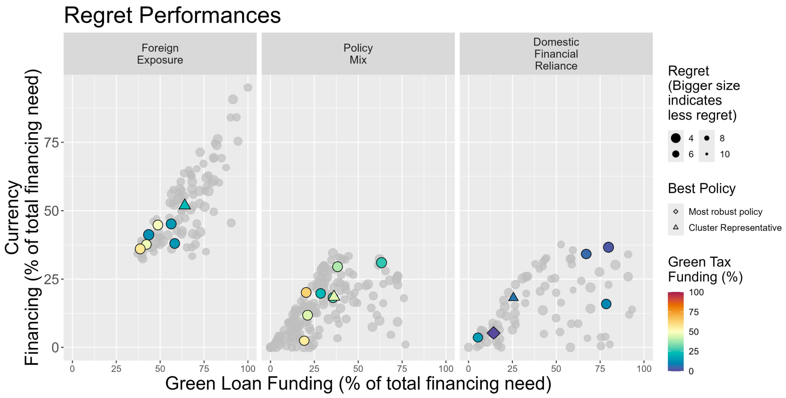


Fig. 25: Result of regret analysis - 12% discounting

S4.5 Alternative NDC Trajectories

In our exercise, the NDC investment path is exogenous. Although it was calibrated to match existing plans [21] reasonably, it is somewhat arbitrary, as other curves could have been used. We therefore provide robustness results on three other NDC investment paths. Note that we are aware that MORDM, in principle, is amenable to differentiated scenarios within the process itself [59]. For clarity, and in the sake of space, we preferred to stick to parametric uncertainty in our main robustness analysis, while using differentiated scenarios to assess more clearly how the RMS choice could be affected by different NDC investment shapes.

S4.5.1 Alternative Shapes

Figure 26 contrasts our NDC curves with our main specification. All these curves were generated by modulating the parameters ruling the investment curve's law of motion in GEMMES under the constraint that the integral of investment should correspond to 3% of GDP in real term. This process allowed us to iron out three alternative plans.

The alternative plans offer a gradient of transition sharpness. While in our main exercise we chose a relatively smooth path, the alternative plans show stronger effort in the short run. Note that we do not change the starting point of the transition, as we set ourselves in the situation where the decision maker commits to their plan from today onwards. We display some insights on a delayed transition later on.

In what follows, we proceed as in the main body of the text. First, optimal candidates will be derived from Borg, and then we will assess their robustness properties. We do not disclose the vulnerability analysis, which showed comparable results to those of the main exercise.

S4.5.2 Borg Results

Figures 27-29 show that different NDC paths give rise to various clustering patterns, which, however, do not differ too much from the base case. The most different one, for the Moderate scenario, retains the clear separation between Foreign Exposure,

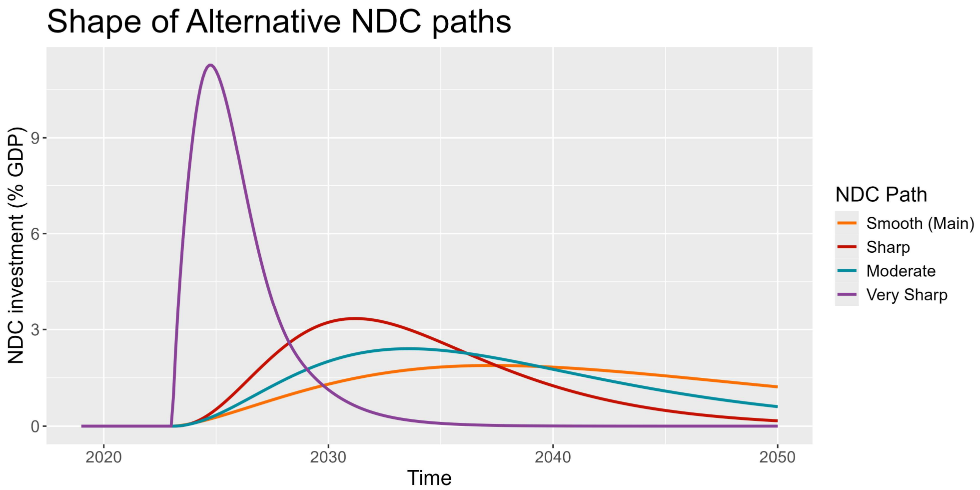


Fig. 26: Alternative NDC paths

Fiscal Funding and domestic debt-based funding, the latter two cases being divided into two more clear-cut categories in the main exercise. Otherwise, the underlying arbitrages are very similar across NDC path shapes, with the only changes being on the magnitude of the SOS-overshoot scores and, mechanically, on the policies' ability to remain within the SOS. It confirms that while funding policies can affect scores somehow, the bulk of the effect transits through the NDC path.

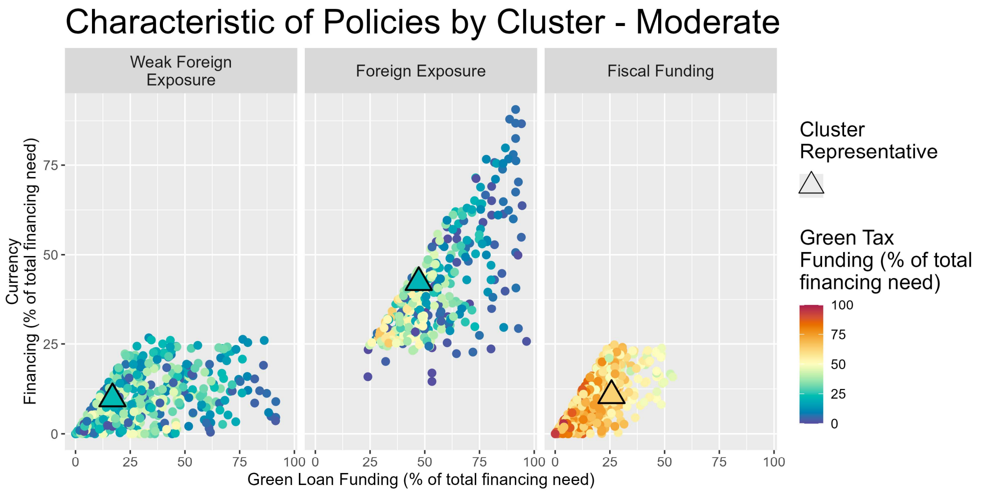
S4.5.3 Regret Analysis

Figures 30-32 show our regret analysis for various transition shapes. Strikingly, the RMS choice is quite different from our main result with a smoother transition path. In all cases, a sharper transition, even moderate, calls rather for a maximisation of foreign exposure and a low tax financing.

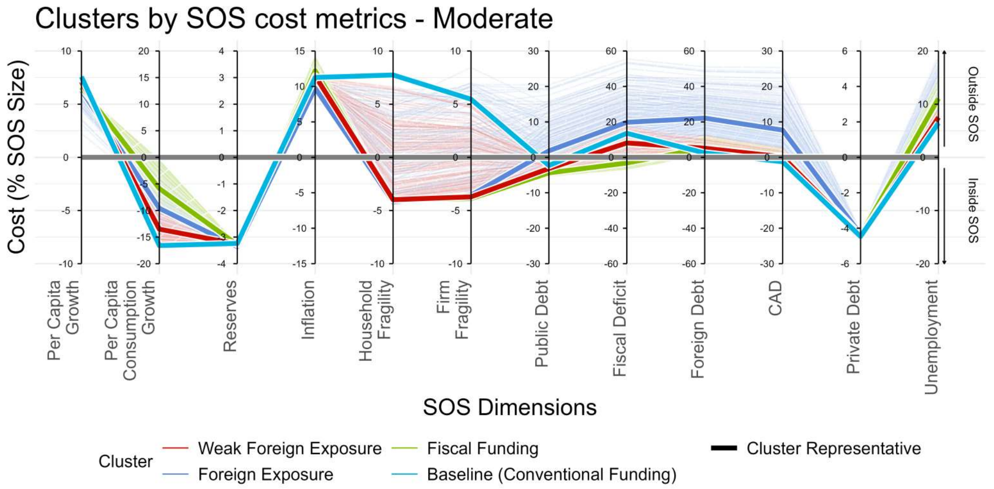
This comes first for the overall lower costs to GDP in these trajectories. Indeed, an ambitious investment pathway in the short run implies significant multiplier effects, which translate in turn into higher inflation and fragility indices. As a result, arbitrages across regret dimensions go more in the direction of these metrics, which are minimised altogether by maximising foreign inflows by the emission of foreign bonds on the market.

S4.6 Extension of the MORDM Analysis to the alternative trajectories

Finally, we adopt a different angle by considering the NDC pledge as a source of uncertainty. To do so, we consider the policies derived from the base NDC path and run them on the uncertainty space, but with the parameterisation of our alternative NDC paths. This creates 3,000 additional uncertainty points on which can compute robustness, either altogether or NDC path by NDC path.



(a) Characterisation of RMS candidates. Colours indicate the extent of the green tax funding



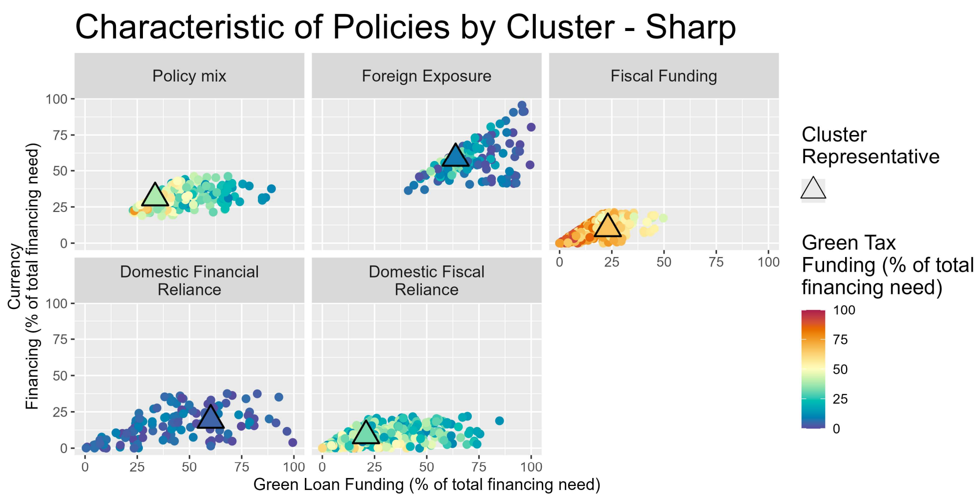
(b) Performance of RMS candidates.

Fig. 27: Resource Mobilisation Strategies – Moderate NDC Path

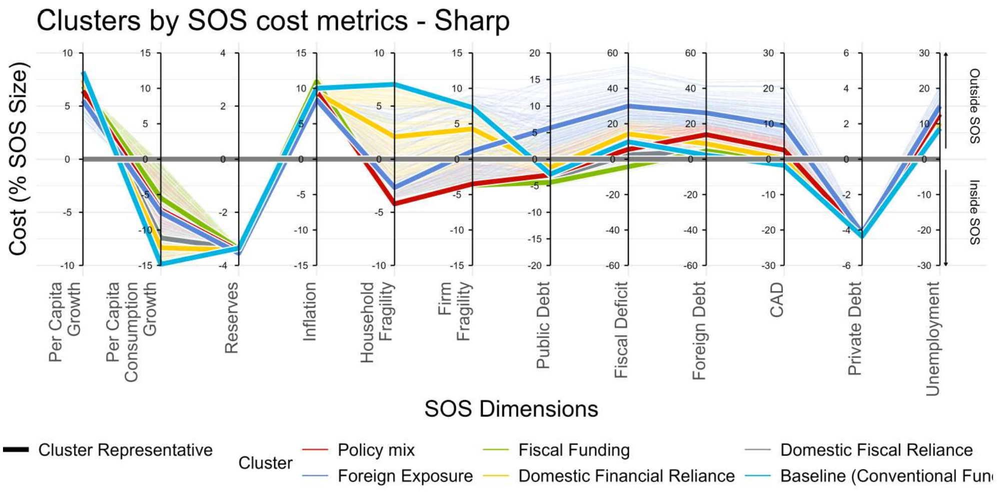
S4.6.1 NDC path-wise regret analysis

We first examine the regret properties of our robust candidates across scenarios, *i.e*, we do as if the NDC shape changed unexpectedly without the possibility to reconsider the RMS. This approach bears resemblance to the scenario-based MORDM proposed by [60]. Results are shown in Figure 33.

Results are close to those of subsection S4.5.1. As soon as a sharper NRD investment path is implemented, most-bust RMSs move to the "Foreign Exposure" cluster.



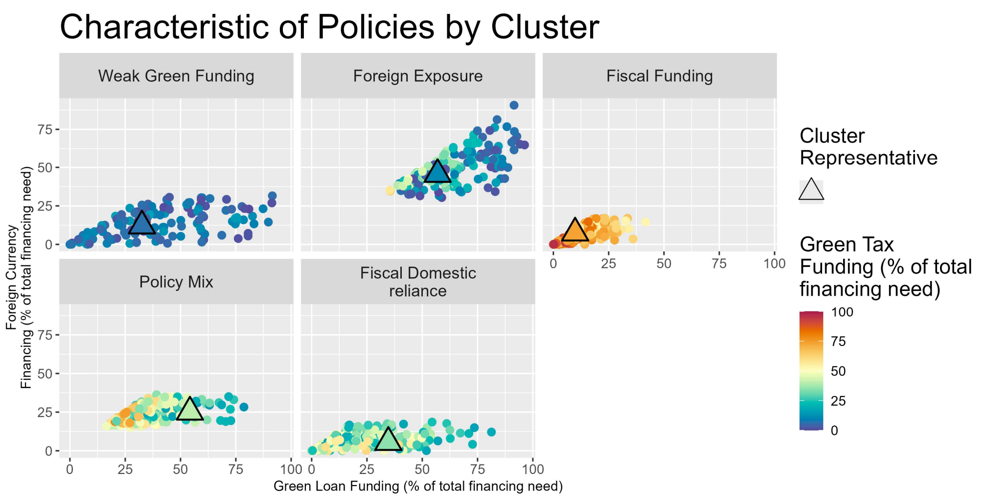
(a) Characterisation of RMS candidates. Colours indicate the extent of the green tax funding



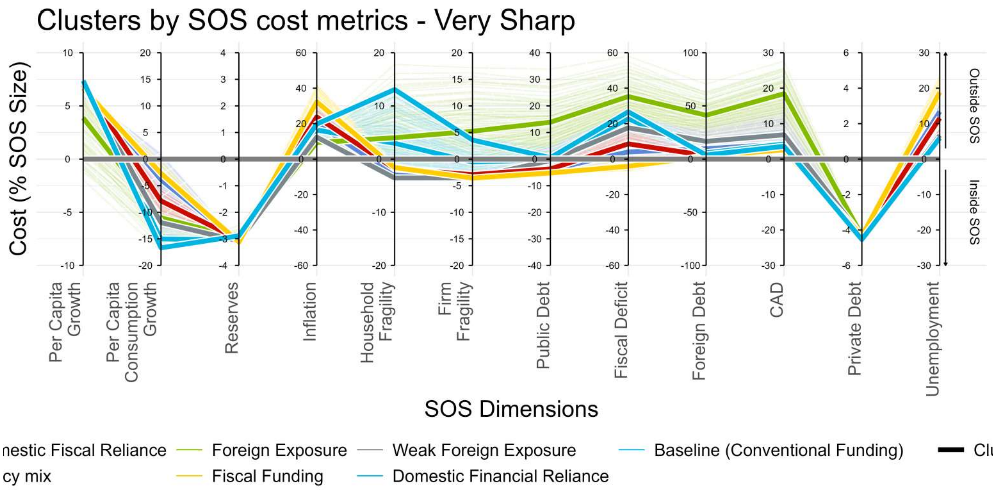
(b) Performance of RMS candidates.

Fig. 28: Resource Mobilisation Strategies – Sharp NDC Path

Mechanisms are similar: investment pathways introduce important multiplier effects in the short run, shifting the pressure on inflation and financial fragility. Hedging policies thus favour a stronger peso. These results suggest that if the policymaker is uncertain of the pace of its NDC path, notably if they push for more ambition in the short run, a safer strategy is to increase foreign exposure and use green debt as instruments.



(a) Characterisation of RMS candidates. Colours indicate the extent of the green tax funding



(b) Performance of RMS candidates.

Fig. 29: Resource Mobilisation Strategies – Very Sharp NDC Path

S4.6.2 All-paths robustness analysis

We close by looking for the least-regret RMS policy while taking NDC investment paths as uncertainty, *i.e*, we aggregate all results from sensitivity calibrations across NDC investment paths and consider the minimum-regret RMS based on our main metric. This corresponds to a total of 3,900,00 simulations (1000 sensitivity calibrations times 975 policies times 4 pathways – we include the smooth path used in the main exercise). Results are displayed in Figure 34.

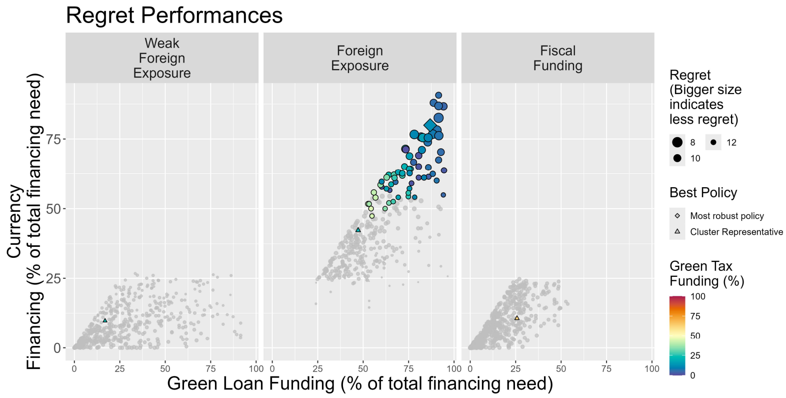


Fig. 30: Regret performances across RMSs - Moderate Sharpness. Only policies in the bottom 90% of regret are highlighted.

Results are very close to the above, with the same interpretation.

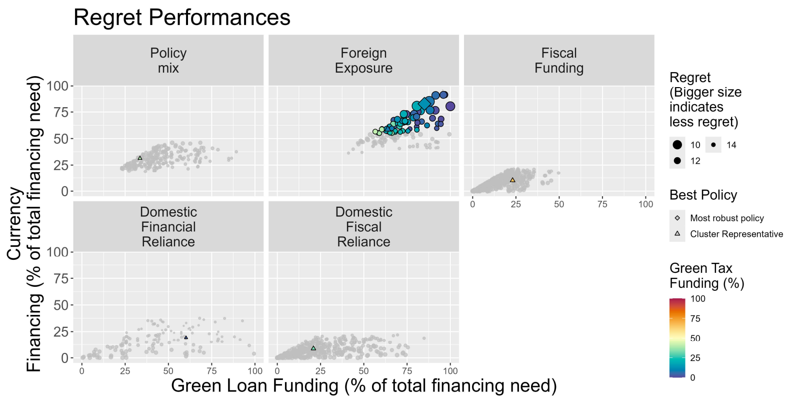


Fig. 31: Regret performances across RMSs - Strong Sharpness. Only policies in the bottom 90% of regret are highlighted.

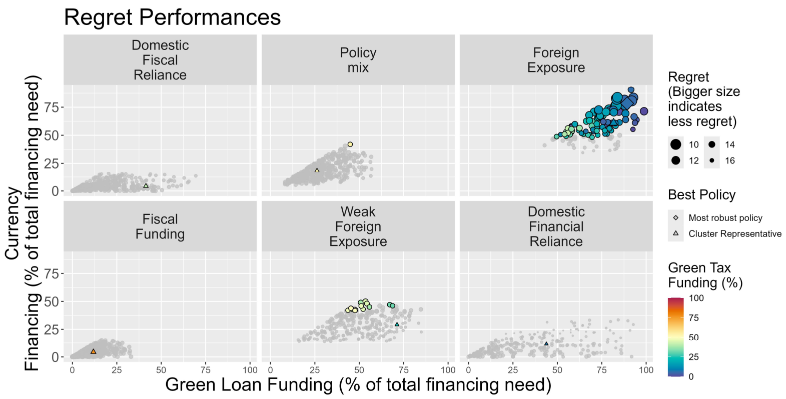


Fig. 32: Regret performances across RMSs - Very Strong Sharpness. Only policies in the bottom 90% of regret are highlighted.

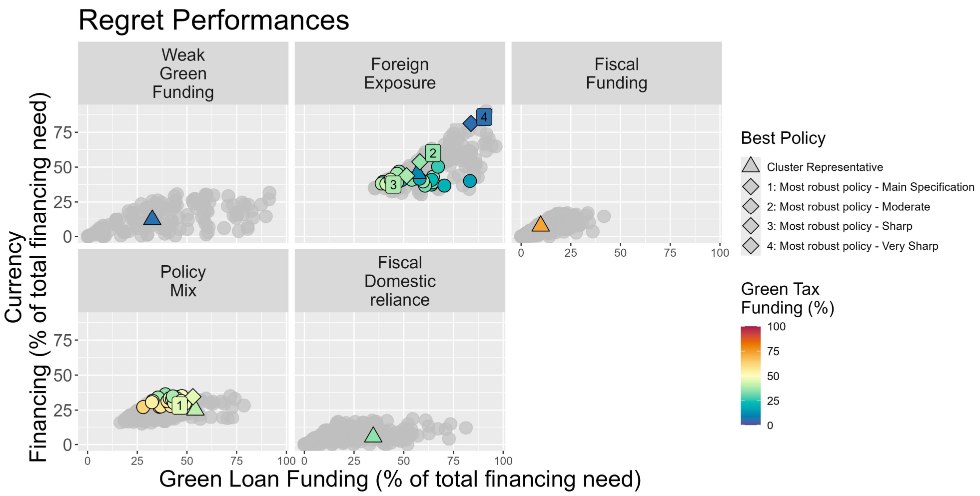


Fig. 33: Regret Analysis – Across NDC paths

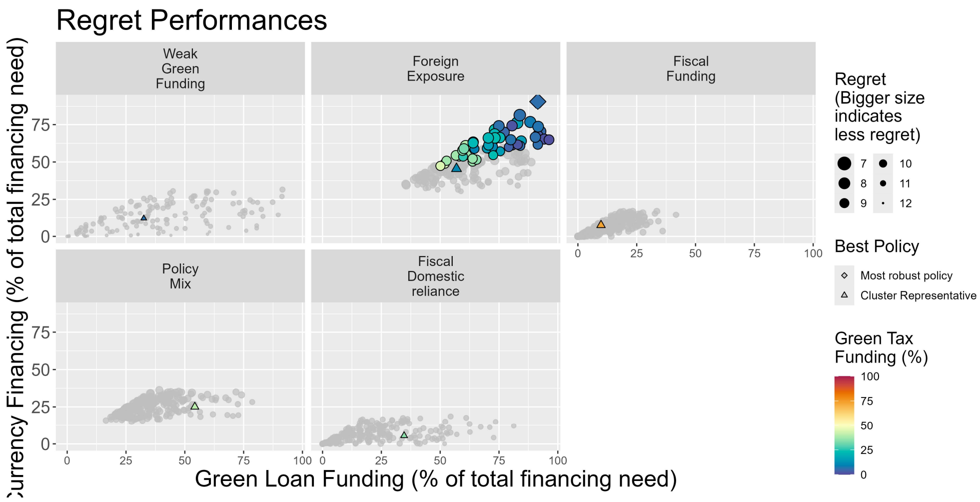


Fig. 34: Regret Analysis – All NDC paths as uncertainties

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