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## Supplementary Information (SI)

### **Beyond the Surface: Inferring Internal Battery Pressure and Temperature Using Optical Fibre Sensing and Machine Learning**

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15 **Supplementary Tables**

16 **Table S1** Optimised hyperparameters of the temporal models. Summary of the Temporal Convolutional  
 17 Network (TCN) configurations obtained via Bayesian optimisation for each chemistry, where a single model  
 18 jointly predicts internal temperature ( $T_{in}$ ) and internal pressure variation ( $\Delta P_{in}$ ). The table reports the input  
 19 window length, number of convolutional filters, kernel size, dense-layer units, dropout rate, batch size, and  
 20 learning rate used throughout the main text. Each optimisation comprised 500 trials per chemistry (seed =  
 21 42).

Chemistry	Window length	TCN filters	Dense units	Kernel size	Dropout	Batch size	Learning Rate
Li-ion	20	128	32	9	0.100	64	0.0014
Na-ion	30	64	48	3	0.285	16	0.0023

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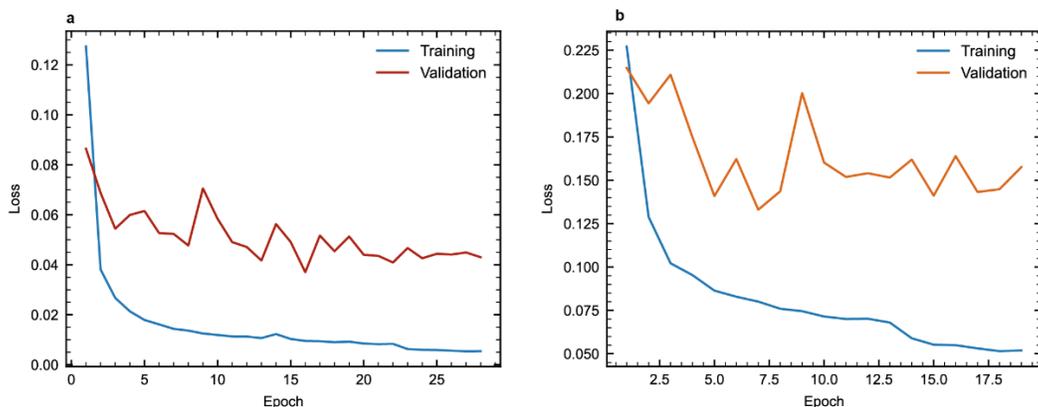
23 **Table S2** Global performance metrics for all models. Comprehensive error statistics for the LIB and SIB  
 24 virtual-sensing models. For each chemistry and target variable, the table reports mean absolute error (MAE),  
 25 root mean squared error (RMSE), median absolute error (MedAE), mean/standardised normalised RMSE  
 26 (including  $\text{NRMSE}_{\text{std}}$  and  $\text{NRMSE}_{\text{range}}$ ) and coefficient of determination ( $R^2$ ) computed on the held-out test  
 27 cycles. These values underpin the parity plots in Fig. 3.

Chemistry	Target	MAE (°C / bar)	RMSE (°C / bar)	MedAE (°C / bar)	$\text{NRMSE}_{\text{std}}$	$\text{NRMSE}_{\text{range}}$ (%)	$R^2$
Li-ion	$T_{in}$	0.058	0.075	0.047	0.059	1.17	0.996
Li-ion	$\Delta P_{in}$	0.028	0.048	0.012	0.170	4.25	0.971
Na-ion	$T_{in}$	0.202	0.288	0.143	0.102	1.87	0.990
Na-ion	$\Delta P_{in}$	0.011	0.013	0.009	0.299	5.34	0.911

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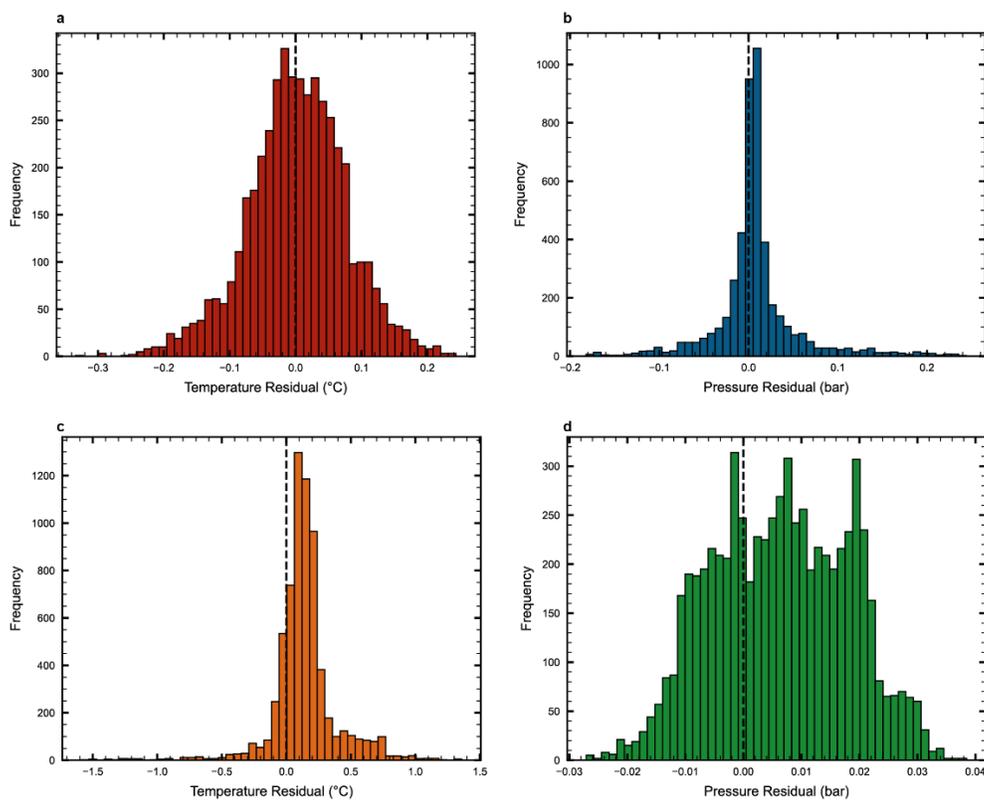
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## 30 Supplementary Figures



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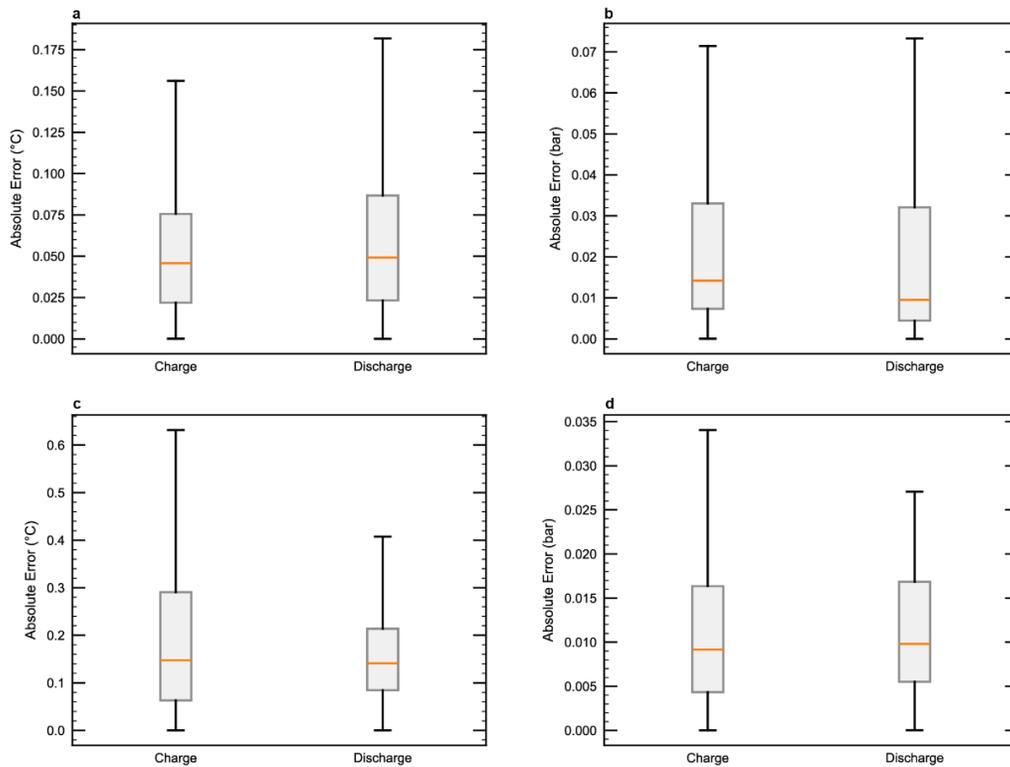
32 **Fig. S1** Training and validation loss curves for LIB and SIB models. **a.** Evolution of training and validation  
 33 losses for the LIB models predicting internal temperature and pressure over the training epochs. **b.**  
 34 Corresponding loss curves for the SIB models. In both chemistries, training loss decreases steadily, whereas  
 35 validation loss improves primarily during the early epochs and then fluctuates around a plateau, revealing a  
 36 train-validation gap but no late-epoch instability.



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38 **Fig. S2** Histograms of residuals for internal temperature and pressure. **a.** Distribution of residuals (prediction  
 39 minus ground truth) for LIB internal temperature on the test set. **b.** Residual distribution for LIB internal  
 40 pressure. **c.** and **d.** Corresponding histograms for SIB internal temperature and pressure, respectively.  
 41 Histograms are centred approximately around zero for all tasks, with narrower spreads for temperature than

42 for pressure, and slightly heavier tails for pressure residuals. These distributions complement the parity plots  
43 in Fig. 3 and the residual maps in Fig. 4, illustrating the overall symmetry and magnitude of the errors.

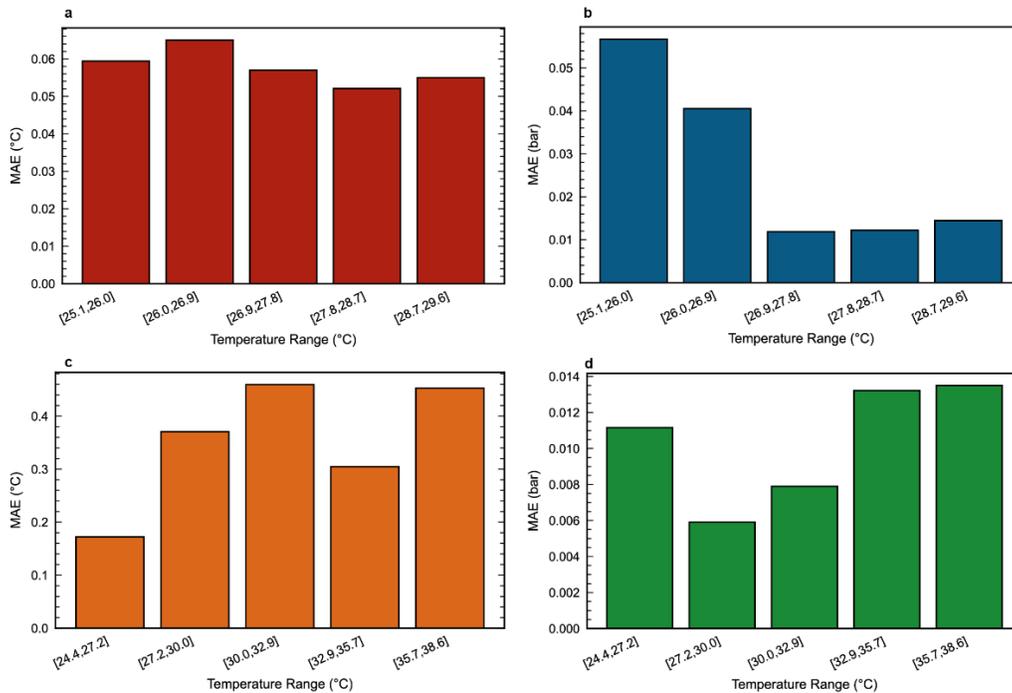


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45 **Fig. S3** Charge/discharge-resolved absolute errors. **a.** Boxplots of the absolute error in LIB internal  
46 temperature, separated into charge and discharge segments for the test cycles. **b.** Equivalent boxplots for LIB  
47 internal pressure. **c.** and **d.** Corresponding charge/discharge-resolved boxplots for SIB internal temperature  
48 and pressure. The distributions show comparable median errors and variability between charge and discharge,  
49 indicating that model performance is robust across operating directions and is not dominated by a specific  
50 regime. These results support the discussion of regime-independent accuracy in Section 2.4.

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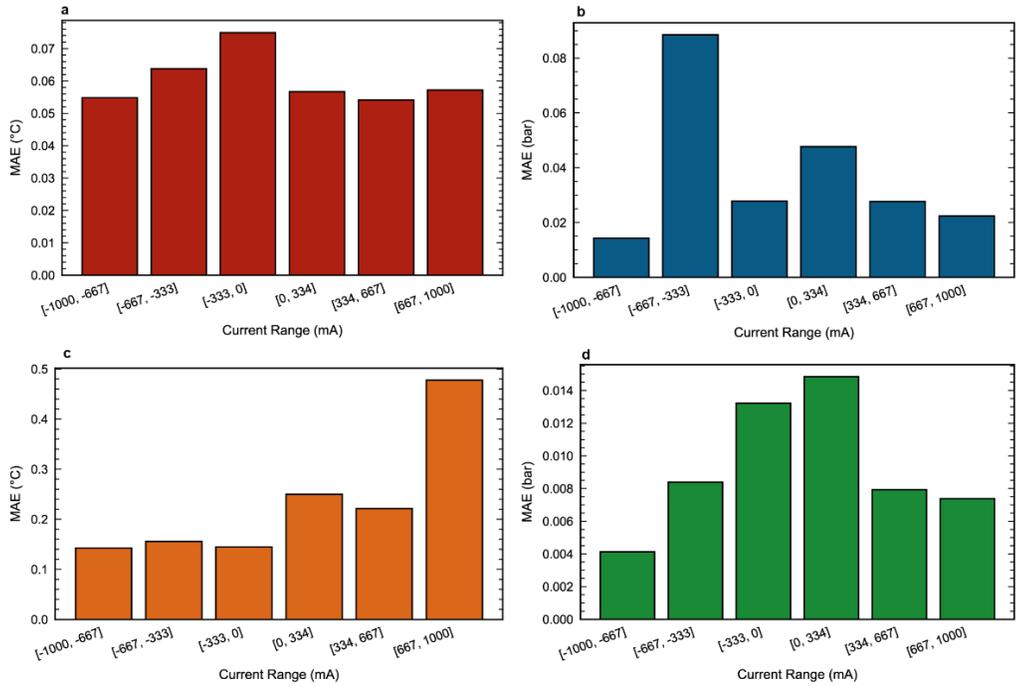
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54 **Fig. S4** Range-wise error as a function of internal temperature. **a.** MAE of LIB internal temperature  
 55 predictions as a function of true internal-temperature range. **b.** Range-wise error profiles for LIB internal  
 56 pressure. **c.** and **d.** Analogous temperature-binned error profiles for SIB internal temperature and pressure.  
 57 The curves reveal how prediction errors evolve across the explored temperature range, showing that LIB  
 58 errors remain nearly uniform, whereas SIB temperature errors exhibit a moderate increase at higher  
 59 temperatures. These observations are referenced in Section 2.4 when analysing error dependence on thermal  
 60 conditions.

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63 **Fig. S5** Range-wise error as a function of current. **a.** MAE of LIB internal temperature as a function of current  
 64 range, computed over test-set segments grouped by applied current. **b.** Corresponding current-binned errors  
 65 for LIB internal pressure. **c.** and **d.** Analogous current-range error profiles for SIB internal temperature and  
 66 pressure. The plots demonstrate that temperature errors tend to increase at higher currents, reflecting stronger  
 67 dynamical excitation, while pressure errors show limited dependence on current across the investigated  
 68 ranges. These trends complement the temperature-binned analysis in Supplementary Fig. S4 and support the  
 69 statements in Section 2.4 regarding the operating-condition dependence of the residuals.

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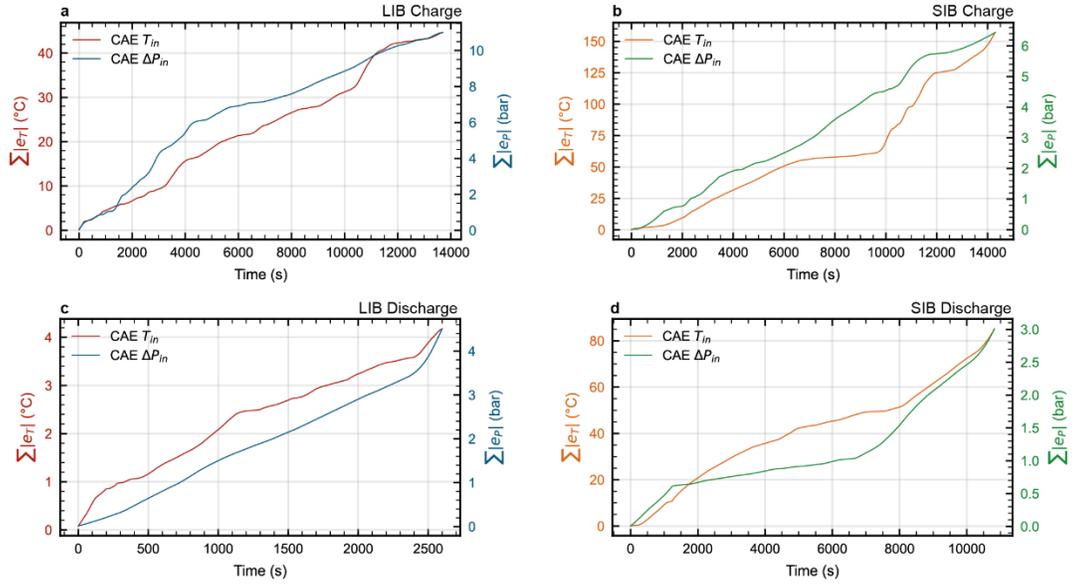
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80 **Fig. S6 a.** Cycle-resolved cumulative absolute error (CAE) for internal temperature and pressure. CAE over  
 81 time for  $T_{in}$  and  $\Delta P_{in}$  during the same representative charge and discharge test cycles shown in Fig. 5. Panels  
 82 show charge (a,b) and discharge (c,d) cycles for LIB (a,c) and SIB (b,d) cells. For each target, the CAE is  
 83 computed as the running sum of  $|e(t)|$  along the cycle. The approximately linear growth and absence of abrupt  
 84 jumps indicate that errors remain bounded and do not accumulate catastrophically over time, supporting  
 85 temporal stability over unseen test cycles.

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