Supplementary Material: Technical Derivations and Extended Stability Analysis

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1 Effective action and variational structure

We consider a covariant effective field theory (EFT) extension of General Relativity in which the gravitational action is supplemented by curvature-dependent operators encoding leading-order geometric corrections,

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \alpha \mathcal{I}_q + \beta \mathcal{I}_2 + \gamma \mathcal{I}_{\nabla R} \right], \tag{1}$$

where the operators $\mathcal{I}_q$, $\mathcal{I}_2$, and $\mathcal{I}_{\nabla R}$ denote scalar curvature invariants. Their explicit covariant form is not required for the background analysis presented here; rather, they are characterized by the fact that, on a homogeneous and isotropic spacetime, they reduce to effective contributions proportional to a^{-6}, H^2, and $\dot{H}$, respectively.

The action is constructed such that all resulting field equations remain second order in time derivatives. This requirement guarantees the absence of Ostrogradsky instabilities and ensures that no additional propagating ghost degrees of freedom are introduced within the EFT regime [2,3]. The coefficients (α, β, γ) therefore parametrize controlled geometric corrections rather than new fundamental degrees of freedom.

2 Field equations and FLRW reduction

Variation of the action (1) with respect to the metric yields modified Einstein equations of the form

$$G_{\mu\nu} + \Delta^{(\alpha)}_{\mu\nu} + \Delta^{(\beta)}_{\mu\nu} + \Delta^{(\gamma)}_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \tag{2}$$

where each correction tensor $\Delta^{(i)}_{\mu\nu}$ arises from the corresponding EFT operator and contains at most second derivatives of the metric.

Specializing to a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric,

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2, \tag{3}$$

the background equations reduce to the modified Friedmann equation

$$(1-\beta)H^2 - \gamma \dot{H} = \frac{1}{3}\left(\rho + \alpha a^{-6}\right), \tag{4}$$

supplemented by the standard continuity equation for matter. In the high-curvature regime, the term proportional to αa^{-6} dominates the dynamics and provides an effective repulsive contribution that prevents the divergence of curvature invariants. As a result, the classical cosmological singularity is replaced by a smooth, nonsingular bounce.

3 Geometric memory and temporal correlations

The presence of the $\gamma \dot{H}$ term introduces a form of geometric memory into the cosmological dynamics. In contrast with standard General Relativity, where the Friedmann equation is algebraic in H^2 and the evolution is local in time, the modified equation (4) dynamically couples the expansion rate to its temporal variation. This structure correlates nearby stages of the cosmological evolution and gives rise to an effective form of dissipation at the level of the background geometry.

Such behavior is naturally interpreted within nonequilibrium extensions of gravitational dynamics, where irreversible evolution emerges from coarse-grained geometric degrees of freedom rather than from microscopic time-reversal breaking [4, 5]. The memory effect discussed here is therefore purely classical and geometric in origin: it does not rely on quantum decoherence, stochastic sources, or nonlocal dynamics. Instead, it reflects the macroscopic imprint of curvature memory encoded in the effective gravitational action.

4 Perturbative stability

4.1 Scalar perturbations

Scalar perturbations around the homogeneous and isotropic background are conveniently described in terms of the comoving curvature perturbation ζ. Expanding the effective action to quadratic order in ζ yields the standard form

$$S^{(2)} = \int dt\, d^3x\, a^3 \left[Q_s \dot{\zeta}^2 - \frac{c_s^2}{a^2} (\nabla\zeta)^2 \right], \tag{5}$$

where Q_s denotes the effective kinetic coefficient and c_s^2 the squared sound speed of scalar perturbations.

The absence of ghost and gradient instabilities requires

$$Q_s > 0, \qquad c_s^2 > 0, \tag{6}$$

ensuring that the scalar sector propagates healthy degrees of freedom with well-posed initial value dynamics.

Within the EFT-consistent parameter domain

$$\beta > 0, \qquad 3\alpha + \beta > 0, \tag{7}$$

these conditions are satisfied throughout the cosmological evolution, including across the nonsingular bounce. Importantly, the geometric slip parameter γ does not introduce additional propagating degrees of freedom and therefore does not modify the structure of the quadratic action for perturbations. Its role is confined to the background dynamics, where it induces effective curvature memory and damping without affecting the perturbative spectrum directly.

5 Bounded curvature operators

Bounded curvature operators provide a natural mechanism for regulating high-curvature regimes within an effective field theory description. A representative example is given by operators of the schematic form $\sin(R/R_c)$, where R_c denotes a characteristic curvature scale below the EFT cutoff [1]. Such operators admit a controlled low-curvature expansion,

$$\sin\left(\frac{R}{R_c}\right) = \frac{R}{R_c} - \frac{1}{6}\left(\frac{R}{R_c}\right)^3 + \mathcal{O}(R^5), \tag{8}$$

and therefore fit naturally within a local curvature expansion for $R \ll R_c$.

When truncated consistently, bounded curvature operators do not introduce higher-order equations of motion or additional dynamical degrees of freedom. In the present framework, they should be understood as effective regulators that motivate the inclusion of curvature-bounded contributions at the level of the background dynamics, such as the αa^{-6} term. Their role is to control curvature growth in the high-energy regime while preserving covariance, second-order field equations, and EFT consistency.

6 Remarks on black-hole geometries

Because the effective action considered in this work is fully covariant and constructed from curvature invariants, the same geometric corrections responsible for regularizing cosmological singularities are expected to contribute in other strong-curvature configurations, including black-hole spacetimes. In particular, higher-curvature and bounded corrections have long been known to soften or regulate curvature divergences in the deep interior of black holes, while leaving the exterior geometry essentially unchanged at low curvatures [1, 2].

Within the present framework, curvature-bounded terms and geometric memory effects may therefore modify the interior structure of black-hole solutions in a manner analogous to their role in the early-universe cosmological dynamics. Such modifications are expected to become relevant only in regions where curvature approaches the effective field theory cutoff, and should not affect classical horizon properties or asymptotic observables in the weak-field regime.

It is important to emphasize that no explicit static or dynamical black-hole solutions are constructed in this work. The considerations above are qualitative and intended solely to indicate the potential scope of the effective description. A detailed analysis of black-hole geometries, including horizon structure, causal properties, and possible observational implications, lies beyond the scope of the present study and is left for future work.

7 Gravity, time, and effective asymmetry

7.1 Time symmetry in General Relativity

At the fundamental level, General Relativity is invariant under time reversal. The Einstein field equations,

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \tag{9}$$

are symmetric under the transformation $t \to -t$ when accompanied by the corresponding transformation of the matter fields. As a result, classical gravitational dynamics does not single out a preferred temporal direction, and both forward- and backward-time solutions are equally admissible.

In cosmological settings, the arrow of time is therefore not dictated by the gravitational equations themselves, but must be introduced through boundary conditions, matter properties, or coarse-graining assumptions.

7.2 Effective temporal asymmetry from geometric slip

In the present framework, an effective temporal asymmetry emerges at the level of cosmological solutions due to the geometric slip term proportional to $\gamma\dot{H}$. The modified Friedmann equation,

$$(1-\beta)H^2 - \gamma\dot{H} = \frac{1}{3}\left(\rho + \alpha a^{-6}\right), \tag{10}$$

dynamically correlates the expansion rate with its time derivative. Unlike the standard Friedmann equation, which is algebraic in H^2, Eq. (10) encodes a memory effect in the curvature sector.

For $\gamma > 0$, rapid variations of H are dynamically suppressed, leading to a smoothing of the evolution across high-curvature regimes. This mechanism introduces an effective irreversibility in the background dynamics, despite the fact that the underlying action remains local, covariant, and time-reversal invariant.

7.3 Emergent time variable and monotonicity

The presence of geometric memory allows for the definition of an intrinsic relational time variable $\tau(t)$ associated with the cosmological evolution. A convenient definition is given schematically by

$$\frac{d\tau}{dt} = \mathcal{F}(H, \dot{H}; \gamma), \tag{11}$$

where $\mathcal{F}$ is a positive-definite functional when $\gamma > 0$. Under these conditions, $\tau(t)$ is strictly monotonic along cosmological solutions, including across the nonsingular bounce.

In the limit $\gamma \to 0$, the slip term vanishes, $\mathcal{F}$ loses its preferred sign, and τ reduces to a trivial reparametrization of coordinate time. The time-reversal symmetric structure of General Relativity is then fully recovered.

7.4 Effective nature of the asymmetry and CPT safety

The temporal asymmetry described above is an emergent and effective property of the coarse-grained gravitational dynamics. It arises from curvature memory effects encoded in the effective field theory description and does not correspond to a fundamental breaking of time-reversal invariance at the level of the action.

In particular, no fundamental violation of CPT symmetry is implied. The microscopic symmetries of the theory remain intact, and the effective arrow of time disappears smoothly outside the regime where geometric memory effects are relevant. The present framework therefore provides an example in which gravity can give rise to an effective temporal ordering without modifying its fundamental symmetry structure.

8 Scope and limitations

This supplementary document provides technical support for the results presented in the main article and is not intended as a ultraviolet completion of gravity. The framework should be regarded as a controlled effective field theory description, valid within a finite regime of curvature and energy density below the EFT cutoff.

Within this regime, the qualitative features emphasized in the main text—in particular, the replacement of the initial cosmological singularity by a nonsingular bounce and the emergence of a monotonic relational time variable—are expected to be robust. By contrast, quantitative details of the dynamics may be sensitive to higher-order operators that have been neglected here and could become relevant near the cutoff scale. Such corrections are not expected to qualitatively alter the central conclusions, but they delimit the domain of validity and predictive power of the effective description.

9 Geometric slip and curvature memory

A distinctive feature of the effective framework considered in this work is the presence of a geometric slip term proportional to $\gamma\dot{H}$ in the cosmological equations of motion. This contribution does not correspond to an additional propagating degree of freedom, but rather encodes a memory effect in the curvature sector, modifying the way in which spacetime responds to changes in the expansion rate.

At the level of the homogeneous and isotropic background, the modified Friedmann equation can be written as

$$(1-\beta)H^2 - \gamma\dot{H} = \frac{1}{3}\left(\rho + \alpha a^{-6}\right), \tag{12}$$

where the $\gamma\dot{H}$ term couples the instantaneous expansion rate to its temporal variation. In standard General Relativity, the Friedmann equation is algebraic in H^2, and the evolution is fully local in time. By contrast, Eq. (12) introduces a dynamical relation between H and $\dot{H}$, effectively correlating nearby instants of cosmological evolution.

9.1 Interpretation as geometric memory

The slip term may be interpreted as a form of geometric memory: the present curvature dynamics depends not only on the instantaneous value of the Hubble parameter, but also on its recent rate of change. This behavior is analogous to dissipative or viscoelastic responses in effective macroscopic systems, where coarse-grained degrees of freedom retain information about past evolution.

Importantly, this memory effect is entirely geometric and classical. It arises from the structure of the effective action and does not rely on stochastic sources, quantum decoherence, or explicit time nonlocality. The underlying theory remains local and covariant; the apparent irreversibility emerges only at the level of effective background solutions.

9.2 Slip and effective dissipation

The presence of the $\gamma\dot{H}$ term introduces an effective damping mechanism in the high-curvature regime. To illustrate this point, consider rewriting Eq. (12) as

$$\dot{H} = \frac{1-\beta}{\gamma} H^2 - \frac{1}{3\gamma}\left(\rho + \alpha a^{-6}\right). \tag{13}$$

For $\gamma > 0$, rapid variations of the Hubble parameter are suppressed, leading to a smoother evolution across the high-curvature phase. This effect is responsible for the ultraviolet damping observed in the numerical solutions presented in the main text and plays a key role in stabilizing the cosmological dynamics near the bounce.

The damping induced by the slip term does not violate local causality or alter the propagation of perturbative modes. Instead, it modifies the background evolution in a way that reduces sensitivity to initial conditions and suppresses spurious oscillations, consistent with an effective field theory interpretation.

9.3 Slip and emergent temporal ordering

A further consequence of the geometric slip term is the emergence of a preferred temporal ordering in cosmological solutions. Because the sign of γ selects a direction of effective dissipation, the evolution equations admit a natural arrow of time at the level of solutions. This can be made explicit by defining a relational time variable $\tau(t)$ whose rate of change depends on the slip contribution,

$$\frac{d\tau}{dt} = \mathcal{F}(H, \dot{H}; \gamma), \tag{14}$$

with $\mathcal{F} > 0$ for $\gamma > 0$. In this case, $\tau(t)$ is strictly monotonic along the entire cosmological evolution, including through the bounce.

In the limit $\gamma \to 0$, the slip term vanishes and the equations recover the time-reversal symmetric structure of General Relativity. The emergent temporal ordering is therefore an effective property of the modified dynamics and not a fundamental violation of time-reversal or CPT symmetry.

9.4 Relation to effective field theory control

From the EFT perspective, the geometric slip term represents a leading-order correction in a derivative expansion that remains compatible with second-order equations of motion. Its coefficient γ controls the strength of curvature memory effects and must lie within the ghost-free and EFT-consistent domain discussed in the main text.

As long as the curvature scale remains below the EFT cutoff, the slip-induced memory and damping effects are reliably captured by the effective description. Higher-order operators may modify quantitative aspects of the evolution, but are not expected to qualitatively alter the role of geometric slip in generating nonsingular dynamics and a monotonic relational time variable.

10 Absence of ghost modes

A central consistency requirement of any effective modification of General Relativity is the absence of ghost-like degrees of freedom, i.e. modes with negative kinetic energy that would render the theory unstable at the quantum or classical level. In the present framework, this requirement is satisfied by construction.

The effective action introduced in Eq. (1) is engineered such that all field equations remain second order in time derivatives. As a consequence, the theory evades the Ostrogradsky instability that generically afflicts higher-derivative gravitational models [3]. No additional canonical momenta beyond those present in General Relativity are introduced, and the phase space of the theory remains finite.

At the level of linear perturbations around a smooth background, the spectrum consists of the standard massless spin–2 graviton and a single scalar curvature mode associated with the higher-curvature sector. The absence of ghost instabilities in this scalar sector is ensured by the positivity of the kinetic coefficient Q_s, as discussed in Sec. **??**. Explicitly, the conditions

$$\beta > 0, \qquad 3\alpha + \beta > 0, \tag{15}$$

guarantee that all propagating modes carry positive kinetic energy.

Importantly, the geometric slip term proportional to $\gamma\dot{H}$ does not introduce any additional propagating degrees of freedom. Its contribution is linear in time derivatives at the level of the background equations and does not modify the canonical structure of the perturbative action. As a result, the slip term affects the dynamics only through background evolution, inducing effective memory and damping effects without altering the number or nature of physical modes.

The theory should therefore be regarded as ghost-free within its domain of validity as an effective field theory. Possible instabilities associated with higher-order operators are expected to arise only beyond the EFT cutoff, where the present description ceases to apply and a more complete microscopic theory would be required.

11 Refences

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