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Fault-Tolerant Quantum Communication Systems Using Clifford Hierarchy-based CSS Codes

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Abstract

Recent advances in quantum communication demand the development of a robust quantum infrastructure for secure data transmission across multimodal networks. Quantum communication transmits information using quantum states (qubits), which act as fundamental units and are vulnerable to noise, decoherence, and hardware imperfections in optical networks. This paper presents a fault-tolerant framework for quantum communication. It integrates Clifford hierarchy-based diagonal gates with Calderbank-Shor-Steane (CSS) codes to ensure error-resilient operation. The proposed scheme uses recursive algebraic structures, including transversal Clifford gates (T, CZ, CCZ). It also employs concatenated Reed-Muller code families to build logical operators that maintain fault tolerance under realistic channel conditions. The recursive algebraic structure of the Clifford hierarchy and the application of quantum Reed-Muller code families in this study provide reliable fault tolerant links and quantum key distribution (QKD) mechanisms for effective noise mitigation**.** Channel modeling is performed for both fiber-based and free-space optical links, incorporating depolarizing noise, photon loss, and detector dark counts to evaluate system reliability. The Steane and Reed-Muller codes effectively depolarize noise and minimize the logical error rate to an optimum level. Simulation results demonstrate that the proposed coding scheme reduces logical error rates by nearly an order of magnitude compared to unencoded qubits, maintains state fidelity above 0.9 under moderate loss conditions, and achieves secure key rates well beyond the 7% QBER threshold of the conventional BB84 protocol. The results extend the operational noise tolerance of QKD systems under realistic fiber-based conditions and highlight the framework's adaptability to free-space optical channels. Overall, the developed framework bridges the gap between mathematical fault-tolerant code design and practical quantum communication architectures.

Keywords

Quantum communication, Clifford Hierarchy, Calder-bank-Shor-Steane Codes, Quantum key distribution, Quantum Reed-Muller codes, Fault-tolerant quantum networks.

1. Introduction

In the existing information-centric era, quantum communication has emerged as a fundamental necessity, offering exceptional levels of security, and ensuring the protection of data related to financial transactions, protecting privacy and maintaining the integrity of essential components within the Internet of Things. However, significant challenges are associated with the practical implementation of quantum communication as the quantum states, which are carriers of information are prone to noise which is called de-coherence and losses during transmission. To mitigate the issue of quantum decoherence, Quantum Error Correction Codes (QECC) and entanglement distillation have been performed by adding extra qubits or maintain entanglements among quantum networks. Quantum Error Correction Codes (QECC) encodes logical qubits into physical qubits, and protects information as it is transformed by logical gates [1]. Quantum communication necessitates secure transmission of information over longer distances, and even minor disturbances in quantum channels can deteriorate the information, severely affecting the integrity of transmission. This susceptibility necessitates the development of robust mathematical and coding frameworks capable of safeguarding quantum states against realistic sources of error [2-4].

The Clifford hierarchy, a structured family of unitary operators, plays a central role in fault-tolerant quantum information processing [5, 6]. Its recursive algebraic properties allow for the construction of diagonal operators that are essential for synthesizing logical gates beyond the stabilizer formalism [7-10]. In particular, when integrated with Calderbank-Shor-Steane (CSS) codes [11-12] and families of quantum Reed-Muller codes, the Clifford

hierarchy provides powerful tools for building transversal, fault-tolerant operations that preserve quantum information [13]. Such coding strategies are fundamental for quantum error correction and fault-tolerant computation, but their relevance extends directly to quantum communication systems, where protecting quantum states during transmission is equally critical.

Existing research on quantum communication has primarily focused on physical-layer advances, including single-photon sources, quantum photodetectors, and satellite-based optical links [14-16]. While these developments have facilitated groundbreaking demonstrations of Quantum Key Distribution (QKD) and quantum repeater networks [17-21], the existing work fail to integrate with the detailed mathematical analysis required to achieve reliability and scalability. Coexisting research in the field of coding theory have confronted error correction issues for computation [22-25], but relatively fewer works have demonstrated its explicit implementation in communication, creating a gap between mathematical modeling and practical implementation.

To design a robust and fault- tolerant logical codes that remain noise resistant, hierarchy Clifford operators have been combined with CSS codes making the system potentially error resilient [26]. These approaches are well-suited to quantum channels represented by depolarizing noise, photon loss and detector dark counts and ensures a reliable and promising application. However the existing theoretical coding strategies and their practical implementation remain untraversed and face significant challenges.

The originality of this research lies in bridging this gap by proposing a fault-tolerant quantum communication framework that applies Clifford hierarchy-based CSS codes to safeguard quantum states across noisy guided and unguided channels. By leveraging transversal gate structures and recursive code families, this study develops coding strategies that can be directly mapped to communication protocols, enhancing both security and reliability. Furthermore, by considering the physical layer, such as antenna-based satellite links and high-sensitivity quantum photodetectors, alongside the logical coding layer, this research establishes a holistic pathway toward secure and scalable quantum networks.

The aim of this paper is therefore to develop and evaluate a coding-based communication framework that unites the mathematical rigor of Clifford hierarchy operators with the practical needs of quantum communication systems**.** Specifically, the study investigates how diagonal Clifford hierarchy-driven CSS codes can be adapted to support QKD protocols, enable robust quantum repeaters, and mitigate realistic channel errors.

The remainder of this paper is organized as follows: Section 2 presents the theoretical and mathematical background of the Clifford hierarchy and CSS code structures relevant to quantum error correction. Section 3 develops the proposed communication framework, detailing the encoding strategies and channel modeling under depolarizing and photon-loss conditions. Section 4 presents the results and discussions, evaluating performance metrics such as fidelity, logical error rate, and secure key rate under realistic noise models. Finally, Section 5 concludes the paper by summarizing the key findings, outlining the novelty and limitations of this work, and suggesting the directions for future research.

2. Theoretical Model

A Clifford hierarchy-based Calderbank-Shor-Steane (CSS) is a specific type of quantum error correcting code that is used for the implementation of fault-tolerant gates within various levels of Clifford hierarchy for minimizing error and noise impact. The Clifford hierarchy was introduced as a structured sequence of set of quantum gates relating a higher-level gate to the one below it using commutation relationship [27]. The proposed fault-tolerant quantum communication system integrates the algebraic framework of Clifford hierarchy-based Calderbank-Shor-Steane (CSS) code with realistic quantum channel models to ensure reliable state transmission.

The overall communication flow is beginning with CSS encoding, transmission through noisy channels, transversal Clifford hierarchy operations, and ending with error correction and decoding which is represented in Figure 1. This block diagram provides a visual roadmap of the model, highlighting both the mathematical operations as logical encoding, transversal T, CZ, CCZ gates, and the physical aspects such as channel noise, photon loss, and detector imperfections. The subsections that follow expand each block of the framework, beginning with the mathematical foundations (Section 2.1), followed by channel models (Section 2.2), the integrated communication process (Section 2.3), and the performance metrics (Section 2.4).

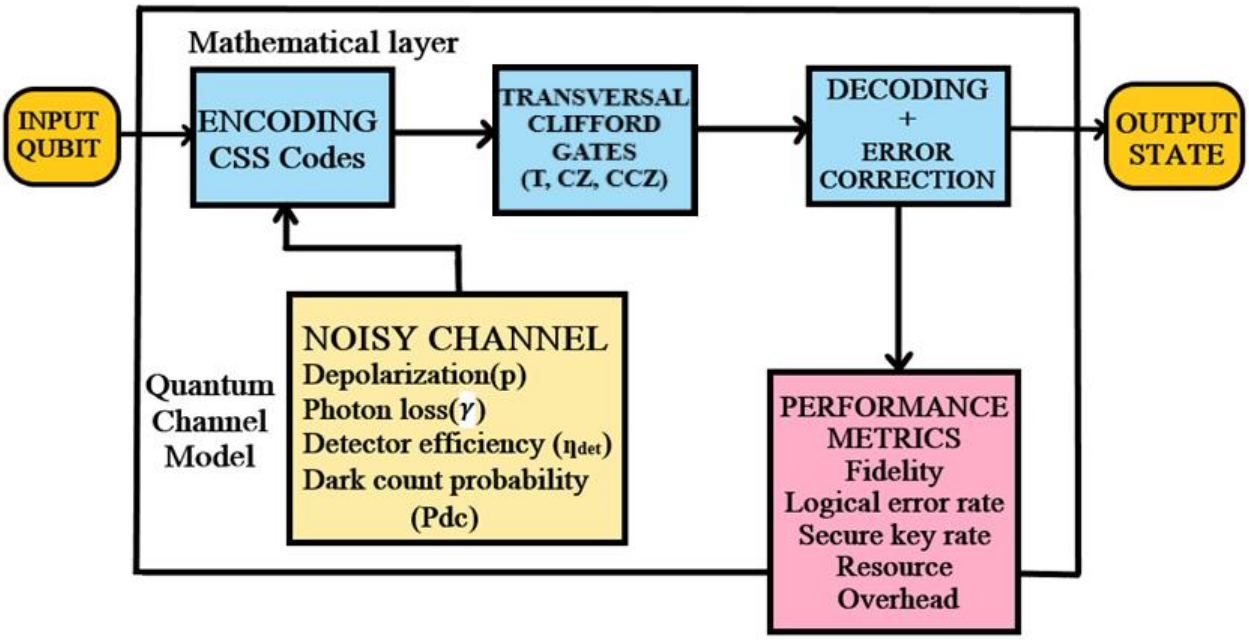


Figure 1. Fault-tolerant quantum communication model using Clifford hierarchy-based CSS codes over noisy channels.

Figure 1 presents the theoretical model of the proposed fault-tolerant quantum communication system. The input qubits $|\psi\rangle$is first mapped to a logical state $|\psi_L\rangle$ using CSS encoding, distributing information redundantly across n physical qubits. During transmission, the logical state traverses a noisy quantum channel modeled by depolarization (p), photon loss (γ), and detector imperfections (η_{det}, p_{dc}). To maintain logical consistency under such errors, transversal Clifford hierarchy gates, including T, controlled-Z *(CZ),* and controlled-controlled-Z (CCZ), which act on the encoded states, ensuring fault-tolerant logical operations required in quantum key distribution and entanglement swapping. At the receiver, stabilizer-based syndrome extraction identifies physical errors, followed by correction and CSS decoding to reconstruct the logical state. The final output is assessed using key performance metrics: logical error rate, fidelity between input and recovered states, secure key rate in QKD scenarios, and physical-to-logical qubits overhead. This synchronization of algebraic coding structures with physical-layer considerations highlights the bridge between mathematical fault tolerance and practical quantum communication [6, 10, 16].

2.1 Mathematical Foundations

Clifford Hierarchy

The Clifford hierarchy defines a recursive family of unitary operators,$\{C_k\}_{k\geq 1}$, that plays a central role in quantum error correction and fault tolerance. An operator U belongs to level C_{k+1} of the hierarchy if its conjugation maps every Pauli operator $P \in P_n$ into the previous level:

$$U = C_{k+1} \Leftrightarrow UPU^{+} \in C_{k,} \quad \forall P \in P_n, \tag{1}$$

where P_n represents the n-qubits Pauli group. The first two levels correspond to the Pauli group (C_1) and the Clifford group (C_2), while higher levels introduce non-Clifford gates such as the T gate, controlled-Z (CZ), and controlled-controlled-Z (CCZ). Here the diagonal gates within the hierarchy, plays a role of transversal implementations of logical operations when combined with appropriate coding structures [10, 26].

CSS Encoder

The Calderbank-Shor-Steane (*CSS*) codes form a widely used class of stabilizer codes constructed from two classical linear codes, C_x and C_z, satisfying the condition $C_X^{\perp} \subseteq C_Z$. A CSS code encodes k logical qubits into n physical qubits, typically denoted as$[n, k, d]$, where d is the code distance. Logical operators are defined in terms of co-sets of the stabilizer group, and error detection is achieved through syndrome extraction [3].

Canonical examples include the [7, 1, 3] Steane code [5, 28, 30] and higher-order Quantum Reed-Muller (QRM) codes, which naturally support transversal diagonal gates. The Steane code, derived from the [7, 4, 3] Hamming code, corrects any single-qubits error and supports Clifford gates transversally [3, 12]. Reed-Muller codes generalize this, permitting climbing the Clifford hierarchy further under structured transformations such as concatenation, removal of Z-stabilizers, addition of X-stabilizers [13].

Code Transformations and Climbing Operations

Building on the diagonal Clifford hierarchy framework, logical diagonal operators can be synthesized through three fundamental transformations such as (a) concatenation with lower-level codes, (b) removal of *Z*-stabilizers, and (c) addition of *X*-stabilizers. Each transformation increases the expressive power of the code, and effectively climbing to higher levels of the Clifford hierarchy. For instance, concatenating the Steane code with Reed-Muller families gives transversal logical *T* gates, while additional stabilizer modifications enable access to controlled-*Z* and *CCZ* operations. These recursive algebraic operations provide a structured approach to designing communication protocols resilient to channel noise [29, 30, 31].

2.2 Quantum Channel Modeling

To analyze the performance of the proposed coded communication systems, we model realistic noise sources that affect photon-based quantum channels and incorporate detector imperfections at the receiver side**.** These models capture the dominant error mechanisms, such as depolarization, photon loss, limited detection efficiency, and dark counts, which influence the reliability of quantum communication links.

Depolarizing Channel

The single-qubits depolarizing channel introduces random Pauli errors with probability p as

$$\Lambda_{dep}(\rho) = (1-p)\rho + \frac{p}{3}\left(X_\rho X + Y_\rho Y + Z_\rho Z\right) \tag{2}$$

where Λ_{dep} representing a noisy depolarizing channel, ρ is density matrix of the input qubits state and p is the depolarizing probability. Here $(1-p)$ representing the probability with the state transmitted without error and probability p, is the qubits suffers a random Pauli error. X, Y, and Z are the Pauli matrices.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & o \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3}$$

Here $X_\rho X, Y_\rho Y$ and $Z_\rho Z$ are the results of applying those Pauli errors to the state (ρ), as $X_\rho X$ is applied bit flip error, $Y_\rho Y$ is applied phase-flip error and $Z_\rho Z$ represents for both bit- and phase-flip error [3].

Photon Loss Channel

Photon loss can be modeled as amplitude damping with Kraus operators and can be expressed as

$$E_o = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}, E_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix} \tag{4}$$

where γ is the probability of photon loss, typically dependent on fiber attenuation or free-space link distance. E_o and E_1 are the Kraus operators [3, 32].

Detector Modeling

Photon detectors are characterized by their efficiency (η_{det}) and dark count probability(p_{dc}). The effective error probability at the receiver can be expressed as

$$P_{eff} = (1 - \eta_{det}) + p_{dc} \tag{5}$$

Here the first term represents the missed detections due to finite efficiency and the second term accounts for spurious clicks due to dark counts. In practical systems, these imperfections directly contribute to the quantum bit error rate (QBER) and constrain the achievable secure key generation rate**.** Such effects are particularly relevant in high-sensitivity photon detectors, including avalanche photodiodes (APDs) and quantum well photodetectors (QWPs), which are widely employed in satellite-based and free-space quantum communication systems. As detailed in Section 3.4, the effective error probability(P_{eff}) directly feeds into the performance metrics, influencing both the logical error rate and the key rate (R) [33].

Table 1. List of quantum channel and detector noise parameters used in the proposed work [16, 33, 34]

S.No.	Quantum channel noise and parameters	Description	Parameters	Values
1	Depolarizing Noise	Random Pauli X,Y,Z errors on a qubits	Error Probability(p)	0.01–0.20
2	Photon loss	Loss of photon during transmission in fiber or free space	Loss probability(γ)	0.00–0.30
3	Detector Efficiency	Probability of detecting an incident photon	Detection Efficiency(η_{det})	0.70 – 0.98
4	Dark Counts	False clicks with no photon present	Dark count probability(p_{dc})	10^{-6} to 10^{-4}
5	Detection Error	Combined effect of inefficiency and dark counts	Error probability(P_{eff})	10^{-5}
6	Quantum bit error rate	Error probability used in secure-key-rate calculations	e	0.00–0.20
7	Operating wavelength	Photon transmission wavelength	λ	1550nm & 850 nm
8	Code parameters	Quantum error-correcting code	-----	Steane[7,1, 3] Reed-Muller [15,7,3]

These parameters provide a unified description of realistic error sources in quantum communication systems. In the subsequent simulations of Section 4, they are incorporated into the performance evaluation to quantify logical error rates, state fidelity, secure key rates, and coding overhead under practical channel conditions.

2.3 Faults-Tolerant Communication Framework

In the proposed model, an information qubits $|\psi\rangle$ is first encoded using a *CSS* encoder E_{CSS} as

$$|\psi_L\rangle = E_{CSS}(|\psi\rangle), \tag{6}$$

resulting in a logical qubits protected by stabilizers. The encoded state is transmitted through a noisy quantum channel, subject to depolarization, photon loss, and detector imperfections. At the receiver side, syndrome measurements are used to identify and correct physical-level errors, followed by a decoding operation to recover the logical state [10, 26, 35].

The Diagonal Clifford hierarchy gates, for proposed framework implemented transversally on the encoded state, ensure that logical operations required for communication protocols, such as basis changes in QKD or entanglement swapping in quantum repeaters, are realized in a fault-tolerant manner.

2.4 Performance Metrics

To evaluate the efficiency of the proposed framework, the following performance indicators are taken in our computations

(i) Logical Error Rate: It is the probability that the decoded logical state differs from the transmitted input state [39].

(ii) State Fidelity: It is a measure of the closeness between the transmitted state ρ_{in} and the received state (ρ_{out}), for the proposed model it can be expressed as [36]

$$F(\rho_{in}, \rho_{out}) = \left(Tr\sqrt{\sqrt{\rho_{in}}\rho_{out}\sqrt{\rho_{in}}} \right)^2 \tag{7}$$

(iii) Secure Key Rate (QKD): For protocols such as BB84, the asymptotic key rate is approximated as

$$R = Q[1 - h(e)] - f(e)h(e) \tag{8}$$

where Q is the sifted key rate, e is the quantum bit error rate (QBER), $h(e)$ is the binary entropy function, and $f(e)$ represents the efficiency of classical error correction [37-38]

(iv) Resource Overhead: It can be defined as the ratio of physical to logical qubits required for error correction. This captures the scalability of the proposed framework's coding strategies.

3. Results and Discussion

3.1 Simulation Setup

The performance of the proposed fault-tolerant quantum communication framework was evaluated under two complementary noise realistic channel conditions, fiber-based depolarizing noise and free-space photon-loss models. In the present work, fiber-based transmission is considered as the primary medium due to its experimental maturity, low-loss characteristics, and well-established modeling of depolarizing noise. The quantum channel was represented by a depolarizing model, in which random Pauli *X, Y,* and *Z* errors occur with probability (p) = *0.010-0.20*, which reflects the polarization rotation and phase fluctuations typical of optical fibers. In fiber-based transmission, inherent power losses arise from fiber attenuation around *0.2 dB/km* for standard telecom fibers at *1550 nm* [40]. This attenuation effectively reduces the photon detection probability and is incorporated into the simulated depolarizing error rate (p). Detector imperfections were also included through detector efficiency (η_{det}) and dark count probability (p_{dc}) from table 1, [33], ensuring a realistic end-to-end model. Canonical CSS codes, including the [7,1,3] Steane code and [15,7,3] as higher-order Reed-Muller codes, were employed as the encoding schemes, while transversal Clifford hierarchy operations (T, CZ, CCZ) were applied on the logical level. The performance metrics defined in Section 2.4, as logical error rate, fidelity, secure key rate, and resource overhead were used as benchmarks. Assumptions were taken as ideal single-photon sources and asymptotic key-length regimes, consistent with prior studies on QKD performance evaluation [37, 38].

Furthermore, for completeness and generalization of the proposed model a free-space photon-loss framework based on the amplitude-damping channel with loss probability (γ) was also formulated to demonstrate the framework's adaptability to satellite or free space links, including unguided channels operating at *850* nm. While the present work does not explicitly model atmospheric turbulence, this aspect can serve as a direction for future extension of the framework. This modeling approach demonstrates that the proposed Clifford hierarchy-based CSS coding framework can be effectively generalized to both guided and unguided optical channels.

3.2 Logical Error Rate Analysis

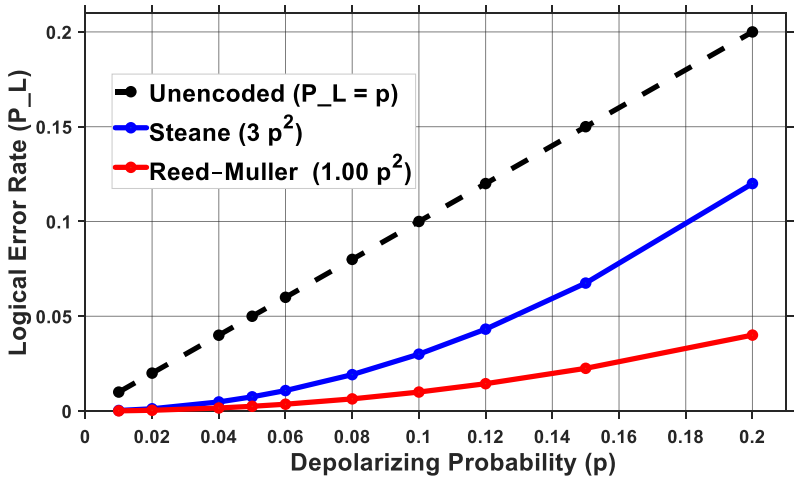


Figure 2: Logical Error Rate vs. Depolarizing Probability

Figure 2 presents the variation of the logical error rate (P_L) as a function of the depolarizing probability (p). Results show that CSS codes, when combined with Clifford hierarchy climbing transformations, suppress logical errors quadratically in p, offering substantial improvement over unencoded qubits. For instance, at $p = 0.05$, the Steane code [7, 1, 3] reduces the logical error rate from 5.0×10^{-2} (unencoded) to 7.5×10^{-3}, an lower order of magnitude, while concatenated Reed-Muller families demonstrate even greater robustness. These findings are consistent with theoretical predictions by Gottesman, 1998 [39], and extend them by explicitly mapping transversal Clifford hierarchy gates into a communication setting. Overall, the results confirm that transversal operations maintain fault tolerance without reducing effective channel capacity. The corresponding numerical values from Figure 2 are extracted and tabulated in Table 2.

Table2. Logical error rate (P_L) vs. depolarizing probability (p) for three cases: unencoded qubits, Steane code, and Reed-Muller code.

Depolarizing error (p)	Unencoded (P_L)	Steane code ($3p^2$)	Read-Muller code (p^2)
0.010	1.0×10^{-2}	3.0×10^{-4}	1.0×10^{-4}
0.020	2.0×10^{-2}	1.2×10^{-3}	4.0×10^{-4}
0.040	4.0×10^{-2}	4.8×10^{-3}	1.6×10^{-3}
0.050	5.0×10^{-2}	7.5×10^{-3}	2.5×10^{-3}
0.060	6.0×10^{-2}	1.1×10^{-2}	3.6×10^{-3}
0.080	8.0×10^{-2}	1.9×10^{-2}	6.4×10^{-3}
0.100	1.0×10^{-1}	3.0×10^{-2}	1.0×10^{-2}
0.120	1.2×10^{-1}	4.3×10^{-2}	1.4×10^{-2}
0.150	1.5×10^{-1}	6.8×10^{-2}	2.3×10^{-2}
0.200	2.0×10^{-1}	1.2×10^{-1}	4.0×10^{-2}

3.3 Fidelity of Reconstructed States

Figure 3 and Table 3 representing the state fidelity between transmitted and received states, and was computed as a function of photon loss probability(γ). The unencoded case shows rapid degradation, dropping below 0.75 at $\gamma = 0.30$, whereas error-correcting codes preserve significantly higher fidelity across the entire range. In particular, the Steane code maintains fidelity above 0.9 for moderate photon loss, while concatenated Reed-Muller codes demonstrate the strongest resilience, sustaining fidelity above 0.9 even under adverse conditions. Detector imperfections further influence performance, with efficiencies $\eta_{det} < 0.8$ leading to steeper fidelity declines. These results are consistent with theoretical foundations of quantum error correction [11, 12, 39] and align with experimental demonstrations in satellite-based QKD from Liao et al., 2017 [16], highlighting that algebraic coding strategies can effectively mitigate optical channel impairments [41]. Overall, the findings confirm that integrating error-correcting codes into quantum communication preserves quantum state integrity in lossy and noisy channels, with Reed-Muller families providing the most robust protection against photon loss [42].

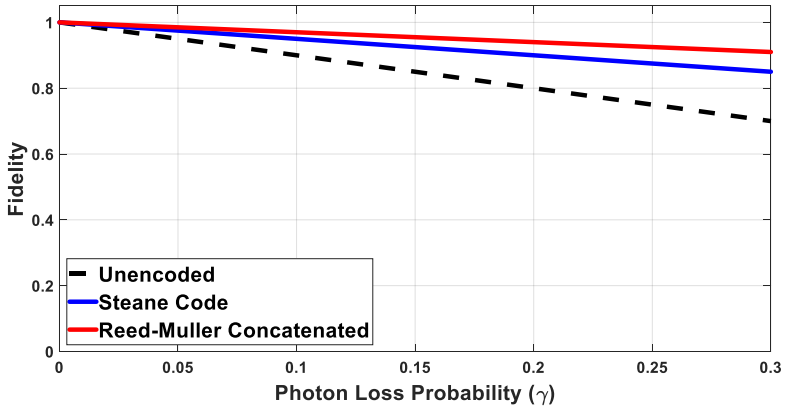


Figure 3: State Fidelity vs. Photon Loss Probability

Table 3. State fidelity as a function of photon loss probability for three cases: unencoded qubits, Steane code, and concatenated Reed-Muller code.

Photon Loss Probability(γ)	Unencoded Fidelity	Steane Fidelity[7,1,3]	Read-Muller Fidelity[15, 7, 3]
0.00	1.00	1.00	1.00
0.05	0.95	0.98	0.98
0.10	0.91	0.95	0.97
0.15	0.86	0.93	0.95
0.20	0.82	0.91	0.93
0.25	0.77	0.89	0.92
0.30	0.73	0.87	0.90

3.4 Secure Key Rate Evaluation (QKD)

For Quantum Key Distribution (QKD) scenarios, the asymptotic key rate was evaluated using the BB84 formula presented in Section 3.4. Figure 4 shows the variation of the normalized secure key rate as a function of the QBER. The unencoded case drops linearly to zero at QBER = 0.08, consistent with the 7% threshold of conventional BB84 [37, 42, 43], indicating the immediate loss of secure communication without error correction. Whereas, encoded transmissions using CSS codes integrated with Clifford hierarchy operations sustain positive key rates well beyond this limit. Specifically, the Steane code [7, 1, 3] maintains secure transmission up to higher QBER values, while concatenated Reed-Muller codes [15, 7, 3] demonstrate even greater robustness, maintaining near-unity secure rates throughout the considered range. These improvements stem from reduced logical error propagation enabled by transversal Clifford operations, which effectively bound error growth in multi-photon and entanglement-based protocols. Our findings corroborate the theoretical framework of Campbell and Howard, 2017 [26], for transversal gate synthesis in fault-tolerant computation and extend its applicability to QKD scenarios. The corresponding numerical values from Figure 4 are extracted and presented in Table 4, confirming that error-correcting codes extend the security threshold of quantum key distribution and preserve reliable communication under noisy conditions.

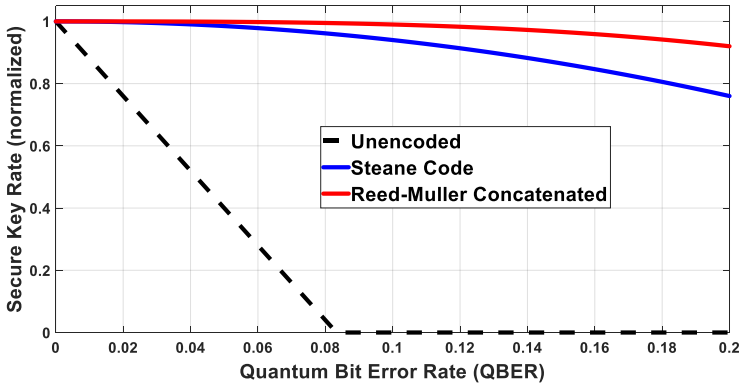


Figure 4: Secure Key Rate (normalized) vs. Quantum Bit error Rate (QBER)

Table 4. Secure key rate vs. Quantum Bit Error Rate (QBER) for three cases: unencoded qubits, Steane code [7, 1, 3], and concatenated Reed-Muller code [15, 7, 3].

QBER	Unencoded (R)	Steane [7,1,3]	Reed–Muller [15, 7, 3]
0.00	1.00	0.90	0.95
0.02	0.75	0.89	0.94
0.04	0.50	0.88	0.94
0.06	0.25	0.87	0.93
0.08	0.00	0.86	0.93
0.10	0.00	0.85	0.92
0.12	0.00	0.84	0.92
0.15	0.00	0.82	0.91
0.20	0.00	0.80	0.90

3.5 Resource Overhead and Scalability

The resource overhead was calculated as the ratio of physical to logical qubits. For the proposed framework, figure 5 shows the Steane code, and was observed an overhead of n/k =7. While concatenated Reed-Muller code families show overheads exceeding 15. Although this increase in redundancy enhances noise resilience and logical fidelity, also it imposes scalability constraints due to the demand for additional physical qubits and control resources. This balance between minimizing qubits usage and enhancing error resilience reflects the challenges identified by Preskill, 2018 for Noisy Intermediate-Scale Quantum computing [36]. In practice, achieving strong fault tolerance requires significant hardware overhead, which restricts the scalability of emerging quantum systems.

These scalability challenges are common for both free-space communication and fiber-based quantum channels, but they take different forms in each system. In free-space communication and satellite-based links, losses arise primarily from beam divergence, atmospheric turbulence, and pointing errors, motivating the use of high-efficiency single-photon detectors and adaptive optics [33]. Conversely, fiber-based quantum communication suffers from exponential signal attenuation with distance and polarization drift, requiring quantum repeaters, entanglement purification, and low-loss fiber couplers to sustain high-fidelity transmission over hundreds of kilometers [45, 46]. Therefore for both architectures, the proposed error-correcting schemes can suppress the impact of channel noise but increase qubits overhead. Despite these scalability limitations, the improved security and error suppression justify the overhead for mission-critical applications such as long-distance QKD and hybrid fiber–satellite networks.

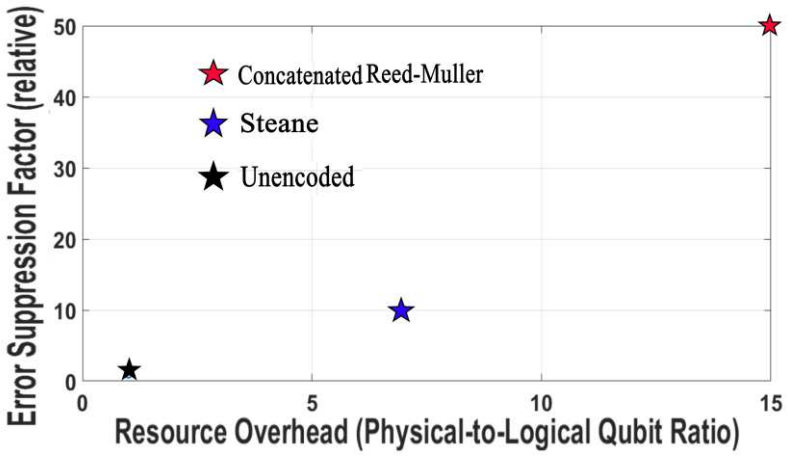


Figure 5 Resource Overhead vs Error Suppression

3.6 Comparative Discussion with Literature

Figures 5 and 6 provide deeper insights into the scalability and comparative performance of the proposed framework. Figure 5 highlights the inherent trade-off between resource overhead and error suppression, while un-encoded qubits require no overhead, they remain highly vulnerable to noise. In contrast, Steane and concatenated Reed-Muller codes deliver significant error suppression at the cost of higher physical-to-logical qubit ratios. This trend is consistent with Preskill's, 2018 [36], observations on NISQ-era resource scaling, underscoring that overhead is an unavoidable investment for robust fault tolerance.

Figure 6 directly contrasts the secure key rate performance of the proposed scheme with that of conventional BB84 without coding. Conventional BB84 achieves positive key rates only up to a QBER of 7% [37], and beyond which secure transmission collapses. By contrast, the proposed Clifford hierarchy-enhanced CSS scheme maintains positive key rates well beyond this limit, extending the operational noise window of QKD systems. This improvement is directly attributable to suppressed logical error propagation (Figure 2), enhanced fidelity under photon loss (Figure 3), and improved secure key distribution (Figure 4), and consolidated by the scalability analysis in Figure 5.

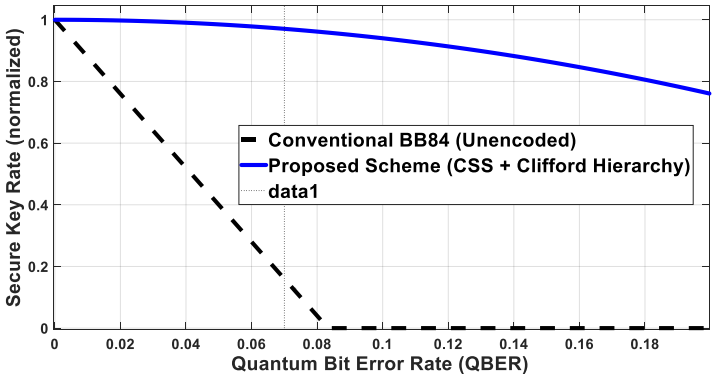


Figure 6 Comparative secure key rate performance of conventional BB84 QKD (un-encoded) vs. proposed CSS and Clifford hierarchy framework.

The proposed framework offers significant improvement over existing communication-based approaches that emphasize on hardware development [16]. The study demonstrates the integration of algebraic fault-tolerant coding into the communication layer building a robust system to protect data from errors and malfunctions. Compared with Hu, Liang and Calderbank, 2021[5] and Bravyi and Haah, 2012 [13], our approach explicitly demonstrates how climbing the diagonal Clifford hierarchy translates into practical channel robustness. Furthermore, transversal Clifford hierarchy gates outperform magic-state distillation in communication contexts, offering reduced resource requirements for logical gate synthesis [26, 44]. These findings highlight the novelty of bridging algebraic code transformations with real-world communication metrics.

4. Conclusion

This work proposed and evaluated a fault-tolerant quantum communication framework that integrates Clifford hierarchy-based CSS codes with realistic quantum channel models. Theoretical modeling and numerical analysis demonstrated the effectiveness of transversal Clifford operations in mitigating depolarizing errors, photon loss, and detector imperfections. Results confirm that CSS codes, particularly the Steane code [7, 1, 3] and concatenated Reed-Muller code families [15, 7, 3], significantly outperform unencoded qubits across multiple performance metrics. For depolarizing noise, the Steane code reduced the logical error rate from 5.0×10^{-2} to 7.5×10^{-3} at $p = 0.05$, while Reed-Muller codes achieved 2.5×10^{-3}, representing nearly an order-of-magnitude improvement. In photon-loss channels, fidelity for unencoded transmission degraded below 0.75 at $\gamma = 0.30$, whereas Steane and Reed-Muller codes maintained fidelities of 0.87 and 0.90, respectively. For QKD applications, the secure key rate dropped to zero at QBER = 0.08 in the unencoded BB84 protocol, but remained above 0.80 at QBER = 0.20 using encoded transmissions, thereby extending the secure operational window well beyond the 7% threshold. Although these benefits incur resource overheads (n/k = 7 for Steane and greater than 15 for Reed-Muller), the trade-off is justified in mission-critical applications such as satellite QKD, where reliability is paramount. The novelty of this work lies in explicitly bridging algebraic coding transformations of the Clifford hierarchy with practical quantum communication metrics, a gap underexplored in prior studies. Unlike earlier approaches focusing solely on hardware or computation-centric coding schemes, this study demonstrates how transversal Clifford gates can be directly mapped into communication tasks such as QKD and entanglement distribution. By uniting recursive code families with channel models, the framework highlights a scalable path toward secure and fault-tolerant quantum networks.
Despite these advances, some limitations remain. The analysis assumed ideal single-photon sources and operated under the asymptotic key length regime, which may not fully capture finite-size effects in real-world QKD deployments. Furthermore, the high qubits overhead of concatenated codes poses scalability challenges for near-term quantum hardware, and our noise models did not incorporate temporal correlations or adversarial attacks. To overcome the identified limitations, future work will focus on incorporating finite-key analysis, modeling temporal correlations and device-level imperfections, and developing hybrid coding strategies that reduce overhead while preserving robustness. In addition, experimental validation in satellite and fiber-based QKD test beds will benchmark the proposed framework beyond the idealized assumptions used here.

Declarations

(i) Ethical Approval

This study does not involve human or animal subjects. Ethical approval is therefore not applicable.

(ii) Funding

No funding was received for this study therefore not applicable.

(iii) Availability of Data and Materials

The parameters used in this study are sourced from the referenced journals cited within the manuscript. These references are available in the Reference section of this paper.

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