

Enacted technological pedagogical content knowledge of preservice mathematics teachers in addressing misconceptions in three dimensional geometry

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Abstract

Persistent misconceptions in three-dimensional (3D) geometry—such as misunderstandings of volume, surface area, nets, and spatial visualization—demand teachers’ ability to integrate content, pedagogy, and technology. This study examines how preservice mathematics teachers (PMTs) enact Technological Pedagogical Content Knowledge (TPACK) when addressing 3D geometry misconceptions through vignette-based tasks embedded in the Concrete–Representational–Abstract (CRA) model. Using a qualitative descriptive design, ten PMTs from an Indonesian university completed five validated vignette tasks representing common misconceptions. Data were analyzed thematically with a TPACK–CRA coding framework. Findings identified three levels of enactment. High-level responses demonstrated coherent CK–PK–TK integration, full CRA sequencing, and purposeful use of digital tools (GeoGebra s3D, AR, Cabri 3D). Moderate-level responses showed strong CK–PK reasoning but limited technology use and partial CRA application. Low-level responses reflected fragmented knowledge, procedural remediation, and minimal technology or CRA integration. High enactments highlighted transformative knowledge-in-use and pedagogical noticing, whereas moderate and low enactments revealed persistent challenges in linking technology to pedagogy and supporting conceptual change. The study contributes theoretically by embedding CRA within TPACK, offering a dynamic model of enacted instructional reasoning; methodologically by advancing vignette–CRA tasks as diagnostic and developmental tools; and practically by proposing differentiated pathways for preservice teacher education. Although limited by sample size and reliance on written responses, the study underscores the potential of vignette-based CRA tasks as scalable instruments to strengthen adaptive, technology-enhanced pedagogy in mathematics teacher education.

Introduction

While the TPACK framework highlights the integration of content, pedagogy, and technology, studies often emphasize perceptions or self-reported competence rather than enacted practice (Lim et al., 2024a; Saubern et al., 2020a). At the same time, CRA pedagogy has proven effective in scaffolding conceptual transitions in mathematics (Donovan & Fyfe, 2022; Ulusoy & Girit-Yildiz, 2024), yet its synergy with TPACK remains underexplored (Habiyaemye et al., 2023). Thus, this study advances the field by embedding CRA within vignette-based tasks to capture preservice teachers’ enacted instructional reasoning in addressing 3D geometry misconceptions (Oner, 2020; Scheiner & Montes, 2025).

One pressing challenge is addressing persistent misconceptions in three-dimensional (3D) geometry, including misunderstandings of volume, surface area, and nets, which emerge from tensions between intuitive perception and formal representation (Fujita et al., 2020a; Lee & Lee, 2021). Visualization technologies can strengthen spatial reasoning (Lu et al., 2025; Meroño et al., 2021), yet their effectiveness hinges on teachers’ ability to scaffold learning responsively (Kaye & Ogle, 2022). The Concrete–Representational–Abstract (CRA) model provides such scaffolding by guiding transitions from manipulatives to visual representations to abstraction (Donovan & Fyfe, 2022; S. Zhang et al., 2022), but

its synergy with TPACK enactment remains underexplored (Habiyaemye et al., 2023; Ulusoy & Girit-Yildiz, 2024).

Vignette-based approaches offer methodological promise in capturing preservice teachers' instructional reasoning in authentic yet controlled contexts (Oner, 2020; Scheiner & Montes, 2025). However, prior research has emphasized perceptions or noticing (Ortiz Colón et al., 2023a; Pareto & Willermark, 2019), rarely combining vignettes with CRA pedagogy. Moreover, enactment varies cross-nationally—technical in Germany, institutionally driven in South Korea, and enhanced through simulation in the U.S. (Backfisch et al., 2024; Kim et al., 2024a)—while in Indonesia, inequities in digital access and uneven preparation (Kusuma, 2021; Thohir et al., 2022) make context-specific investigations urgent.

This study therefore advances the field by: (1) shifting from self-perceptions to enacted TPACK in authentic misconception-driven tasks; (2) embedding CRA pedagogy within vignette methodology to trace reasoning from concrete to abstract; and (3) contributing empirical, context-sensitive insights from Indonesia to global discourse on adaptive, technology-enhanced pedagogy.

Research Gap and Novelty

Although the TPACK framework has been extensively applied in mathematics teacher education, much research still emphasizes self-perceptions and reported competencies rather than actual enactment in practice (Lim et al., 2024a; Saubern et al., 2020a). Such approaches offer insights into confidence and attitudes but provide limited evidence of how TPACK is mobilized when teachers confront authentic misconceptions (Backfisch et al., 2024; Runnalls & Hong, 2020a). This gap is especially critical in 3D geometry, where misconceptions about volume, surface area, and nets remain persistent, requiring the convergence of conceptual reasoning, spatial visualization, and purposeful technology integration (Kaye & Ogle, 2022; Lu et al., 2025; Meroño et al., 2021).

Existing studies often treat technology as an additive rather than integrative element, with tools like GeoGebra or AR used superficially rather than as transformative scaffolds (Backfisch et al., 2024; Valtonen et al., 2023). While benefits of visualization are noted (Özçakır & Çakıroğlu, 2022; Yildiz & Arpaci, 2024), few studies embed them within coherent pedagogical sequences addressing misconceptions. Consequently, systematic evidence of how preservice teachers enact TPACK in authentic, misconception-driven contexts remains scarce.

Vignette-based approaches provide promise by eliciting reasoning in realistic yet controlled dilemmas (Ivars et al., 2020; Oner, 2020; Scheiner & Montes, 2025), but most emphasize perception or reflection rather than enacted responses (Ortiz Colón et al., 2023a; Pareto & Willermark, 2019). Integration with the CRA sequence, despite its proven role in supporting conceptual transitions and durable learning (Donovan & Fyfe, 2022; Habiyaemye et al., 2023; Ulusoy & Girit-Yildiz, 2024; S. Zhang et al., 2022), remains underexplored.

Cross-national findings further reveal enactment's dependence on context: technical in Germany (Backfisch et al., 2024), institutionally driven in South Korea (Kim et al., 2024), and simulation-enhanced in the U.S. (Scheiner & Montes, 2025). In Indonesia, digital inequities and uneven preparation exacerbate challenges (Chang et al., 2025; Kusuma, 2021; Thohir et al., 2022), with limited evidence of how preservice teachers mobilize TPACK in practice (Njiku et al., 2021a; X. Yang & Kaiser, 2022).

This study addresses these gaps by: (1) moving from perceptions to enacted reasoning (Runnalls & Hong, 2020a; Saubern et al., 2020a); (2) integrating CRA pedagogy within vignette methodology (Donovan & Fyfe, 2022; Habiyaemye et al., 2023; S. Zhang et al., 2022); and (3) situating evidence from Indonesia within global discourses on adaptive, technology-enhanced pedagogy (Adriana Mendoza Mondragón, 2024a; Backfisch et al., 2024; Scheiner, 2023). Thus, vignette-based CRA tasks are positioned as diagnostic and developmental platforms that extend theoretical, methodological, and practical insights in mathematics teacher education (Açıkgül & Aslaner, 2020a; Habiyaemye et al., 2023; Wen & Shinas, 2021a).

This framework integrates Technological, Pedagogical, and Content Knowledge (TPACK) (Adriana Mendoza Mondragón, 2024a; M. Li et al., 2024a) with CRA scaffolding (Concrete → Representational → Abstract) (Donovan & Fyfe, 2022; S. Zhang et al., 2022) and vignette-based tasks (Oner, 2020; Scheiner & Montes, 2025) to address 3D geometry misconceptions. It highlights CRA as a theoretical enhancer of TPACK enactment (Fischer et al., 2022), vignette–CRA as a methodological diagnostic of authentic instructional reasoning (Hwang et al., 2020), and differentiated enactment profiles as a practical pathway for teacher education (Ortiz Colón et al., 2023b). By linking CK, PK, and TK adaptively, the model emphasizes that technology should function as a pedagogical scaffold rather than a technical add-on (Lim et al., 2024b; Saubern et al., 2020b).

Research Objectives and Questions

Building on these gaps, this study investigates how preservice mathematics teachers (PMTs) enact TPACK when responding to authentic 3D geometry misconceptions through vignette-based tasks embedded in the Concrete–Representational–Abstract (CRA) model. Unlike prior studies that emphasize perceptions or isolated lesson planning (Lim et al., 2024a; Saubern et al., 2020a), this research captures enacted instructional reasoning to provide practice-based evidence of TPACK-in-use. Embedding CRA into vignette design enables analysis of how PMTs scaffold learning across representational stages while leveraging digital tools such as GeoGebra 3D, AR, and Cabri 3D (Donovan & Fyfe, 2022; Lu et al., 2025; S. Zhang et al., 2022).

The study pursues four objectives: (1) identifying and classifying PMTs' enactment levels—high, moderate, and low—based on CK–PK–TK coherence (Adriana Mendoza Mondragón, 2024b; Wen & Shinas, 2021a); (2) examining how CRA pedagogy structures instructional design, particularly transitions from concrete to representational to abstract reasoning (Donovan & Fyfe, 2022; Ulusoy & Girit-Yildiz, 2024); (3) analyzing patterns of reasoning, technology use, and pedagogical responsiveness across

enactment profiles (Njiku et al., 2021a; Pareto & Willermark, 2019); and (4) providing context-sensitive recommendations for teacher education in Indonesia, while situating findings within international discourses on adaptive, technology-enhanced pedagogy (Backfisch et al., 2024; Kim et al., 2024a; Scheiner & Montes, 2025).

Accordingly, the study addresses three guiding questions:

RQ1

How do PMTs enact TPACK when addressing 3D geometry misconceptions through vignette-based CRA tasks?

RQ2

What distinguishes instructional reasoning, CRA integration, and technology use across enactment levels?

RQ3

How can vignette-based CRA tasks be leveraged to strengthen content–pedagogy–technology integration in preservice teacher education?

Theoretical Background

The TPACK framework integrates content, pedagogy, and technology, and is recognized as a key competence for 21st-century mathematics teachers (Adriana Mendoza Mondragón, 2024b; M. Li et al., 2024b). However, preservice teachers often reduce technology use to technical rather than pedagogical purposes (Backfisch et al., 2024; Bonafini & Lee, 2021). Collaborative design and reflection can strengthen enactment, but context and resource disparities remain challenges (Njiku et al., 2021a; Rocha, 2020).

Geometry misconceptions—particularly in 3D contexts such as volume, surface area, and nets—persist across students due to weak spatial reasoning (Fujita et al., 2020b; Lee & Lee, 2021). Tools such as GeoGebra 3D and Augmented Reality enhance visualization (Meroño et al., 2021; Vahey et al., 2020), yet require pedagogical scaffolding to be effective (Lim et al., 2024a). Without structured support, preservice teachers default to procedural responses (Runnalls & Hong, 2020a; Saubern et al., 2020a).

The CRA model (Concrete–Representational–Abstract) provides such scaffolding, guiding learners through manipulatives, visualizations, and abstract reasoning (Donovan & Fyfe, 2022; J. Zhang et al., 2021). It has proven effective in diverse mathematics topics, including geometry misconceptions (Habiyaemye et al., 2023). Integrating CRA with TPACK links technological affordances to pedagogical reasoning (Flores et al., 2023; Kim, 2020).

Vignette-based tasks situate teachers in authentic scenarios that reveal enacted instructional reasoning (Oner, 2020; Scheiner et al., 2024). They have been used to assess noticing, decision-making, and integration of CK–PK–TK (Lim et al., 2024a; K.-L. Yang et al., 2023). Yet, prior research focuses more on perception than actual enactment, leaving a gap for studies that examine how preservice teachers mobilize TPACK through CRA in practice.

This study therefore builds on recent scholarship by combining TPACK, CRA, and vignette-based tasks into an integrative framework (Açıkgül & Aslaner, 2020b; Wen & Shinas, 2021b), offering a novel lens to analyze preservice mathematics teachers' reasoning in addressing 3D geometry misconceptions.

Methodology

Research Design

This study employed a qualitative descriptive design using vignette-based tasks to capture preservice mathematics teachers' (PMTs) enacted instructional reasoning when addressing common 3D geometry misconceptions. Vignettes are recognized as effective tools for situating participants in authentic yet controlled scenarios, thereby revealing teachers' decision-making processes in contexts approximating classroom practice (Ivars et al., 2020; K.-L. Yang et al., 2023). To deepen pedagogical responses, the **Concrete–Representational–Abstract (CRA)** sequence was embedded in the vignette framework, supporting analysis of how PMTs scaffold transitions from manipulatives to visual models and abstraction (Donovan & Fyfe, 2022; S. Zhang et al., 2022).

The choice of design rests on three considerations. First, it prioritizes practice-based evidence of TPACK enactment rather than relying on self-reports, which often misrepresent enacted competence (Lim et al., 2024a; Saubern et al., 2020a). Second, it responds to calls for diagnostic and developmental approaches that capture teachers' reasoning in response to authentic student misconceptions (Oner, 2020; Scheiner & Montes, 2025). Third, it enables examination of technology-supported CRA scaffolding in simulated yet ecologically valid settings, bridging reflection and practice (Habiyaremye et al., 2023; Njiku et al., 2021a).

Participants

The study involved ten preservice mathematics teachers enrolled in the final year of a professional education program at a public university in Indonesia. All participants had completed coursework in geometry and instructional technology, ensuring sufficient exposure to CK, PK, and TK prior to participation. A purposive sampling strategy was used to select participants with relevant academic backgrounds, consistent with best practices in qualitative teacher education research (Njiku et al., 2021a; X. Yang & Kaiser, 2022).

While the sample size ($n = 10$) is relatively small, this is appropriate for vignette-based qualitative studies that prioritize depth of reasoning over statistical generalization (Runnalls & Hong, 2020a). Data

saturation was achieved as no new themes emerged after repeated analysis cycles, a standard criterion in qualitative research (Braun & Clarke, 2006).

Instrument Development and Vignette Design

The study employed five vignette tasks representing persistent 3D geometry misconceptions widely documented in prior research, including: confusion between volume and surface area (Kaye & Ogle, 2022), misinterpretation of geometric nets (Patahuddin et al., 2020), misclassification of solids such as prisms and pyramids (Habiyaemye et al., 2023), errors with diagonals and perspectives (Özçakır & Çakıroğlu, 2022), and transformation-related misconceptions in rotation or shadow visualization (Rossano et al., 2020).

Vignette construction followed a three-stage process. First, misconceptions were identified through curriculum analysis and prior studies (Deng et al., 2023; Runnalls & Hong, 2020a). Second, drafts were developed embedding CRA scaffolding and explicit prompts for technology use (Kim, 2020). Third, expert validation was conducted by three mathematics education specialists to ensure accuracy, cognitive appropriateness, and curricular alignment (S. Li et al., 2022; Patahuddin et al., 2020).

This process secured content validity and ecological authenticity, aligning with established standards for vignette-based research in teacher education (Oner, 2020; Scheiner, 2023).

Procedure

Participants received the vignettes in written form and were asked to: (1) identify the underlying student misconception, (2) propose a step-by-step instructional strategy integrating CK, PK, and TK, and (3) justify the role of technology pedagogically. Tasks were completed individually within a two-week period to allow reflective reasoning, consistent with best practices in vignette-based TPACK research (Scheiner & Montes, 2025; K.-L. Yang et al., 2023). Written responses were collected and, when necessary, clarified through follow-up correspondence, ensuring accuracy and interpretive depth (Oner, 2020; Pareto & Willermark, 2019).

Data Analysis

Data were analyzed through thematic analysis (Braun & Clarke, 2006) using a TPACK–CRA coding framework. The process began with open coding, where individual statements were segmented and labeled according to indicators of CK, PK, TK, and CRA (Donovan & Fyfe, 2022; S. Zhang et al., 2022). Next, axial coding was conducted to identify relationships between codes, particularly how CRA scaffolding intersected with technology use and pedagogical noticing (Pareto & Willermark, 2019). Finally, codes were consolidated into three enactment categories—High, Moderate, and Low—based on coherence of CK–PK–TK integration, CRA sequencing, and technology justification (Ortiz Colón et al., 2023a; Wen & Shinas, 2021a). To enhance traceability and reliability, illustrative excerpts were

systematically mapped to each category, and discrepancies between coders were resolved through iterative discussions until consensus was reached (X. Li & Sang, 2023).

Reliability and Trustworthiness

To ensure rigor and trustworthiness, several strategies were applied. Triangulation across the five vignettes was conducted to identify consistent reasoning patterns (Lemon & Hayes, 2020). Member checking allowed participants to review researcher summaries for accuracy (Schafer & Phillippi, 2025), while peer debriefing involved two external experts to minimize bias (Bingham, 2023). In addition, inter-rater reliability was established as two coders independently analyzed 30% of the data, reaching a Cohen’s kappa of 0.87, considered near-perfect agreement (X. Li & Sang, 2023). Although kappa may be sensitive to prevalence effects (Cibulka & Strube, 2021), iterative calibration and consensus discussions mitigated this limitation.

Ethical Considerations

Ethical clearance was obtained from the university’s institutional review board. Informed consent was secured from all participants, emphasizing voluntary participation, confidentiality, and the right to withdraw. All identifying information was anonymized in accordance with ethical standards for teacher education research (Wang & Wang, 2023).

Results

The analysis of preservice mathematics teachers’ (PMTs) responses to vignette-based CRA tasks revealed three levels of TPACK enactment: High, Moderate, and Low. Each level differed in terms of CK–PK–TK integration, CRA sequencing, use of technology, and instructional responsiveness. The heatmap illustrates the comparative variation of TPACK–CRA enactment profiles across participants.

Classification of TPACK Enactment

The table summarizes the classification of TPACK–CRA enactment levels along with key characteristics and illustrative excerpts.

Table 1

Classification of Preservice Teachers' TPACK Enactment in Responding to 3D Geometry Misconceptions

Level	Participants	Instructional Characteristics	CRA–TPACK Integration	Illustrative Excerpts	Supporting Literature
High	Lia, Silvi, Rini, Billa, Hamida, Nuril, Noufal	- Accurate diagnosis of misconceptions.- Complete CRA sequencing (C→R→A).- Purposeful and pedagogically justified use of technology (GeoGebra 3D, AR, Cabri 3D).- Adaptive and student-centered strategies.	CK, PK, and TK fully integrated. Strong pedagogical noticing.	<i>"I begin with unit cubes, then use GeoGebra 3D to show how cubes form the prism, and finally generalize the formula."</i> (Lia) <i>"Using AR, students can rotate the solid themselves to see the diagonal vs. edge."</i> (Silvi)	Donovan & Fyfe (2022); Zhang et al. (2022); Lu et al. (2025); Klepsch & Seufert (2020); Scheiner (2023)
Moderate	Julva, Ratna	- Correctly identify misconceptions.- Strong CK–PK alignment.- Partial or underdeveloped CRA application.- Limited or superficial technology use.	CK and PK strong, TK weak or unjustified. CRA partial.	<i>"I use cardboard nets so students can fold them into cubes."</i> (Julva) <i>"I ask groups to solve net problems by drawing their own nets."</i> (Ratna)	Saritepeci (2022); Koyunkaya & Dede (2024); Backfisch et al. (2024)
Low	Santi	- Fragmented, procedural reasoning.- Reliance on memorization.- No CRA sequencing.- Technology mentioned nominally.	Minimal CK, PK, TK integration. Procedural remediation only.	<i>"Students should memorize the surface area formula and practice it."</i> (Santi) <i>"I might show them a short video of the cube."</i>	Farrugia & Busuttil (2021); Valtonen et al. (2023)

Narrative Analysis

High-Level Enactment

Participants in this category demonstrated transformative TPACK integration. Technology was employed not as an add-on but as a pedagogical scaffold to mediate conceptual transitions. Their responses illustrated complete CRA progression and strong instructional noticing. This aligns with findings that

structured use of CRA with digital tools enhances durable conceptual understanding (Donovan & Fyfe, 2022; Lu et al., 2025; S. Zhang et al., 2022).

Moderate-Level Enactment

Moderate-level participants showed solid content–pedagogy reasoning but weak technology integration. CRA application was partial, often limited to physical representation without digital or abstract extension. These findings mirror broader challenges where preservice teachers acknowledge technology but fail to embed it meaningfully (Koyunkaya & Dede, 2024; Saritepeci, 2022).

Low-Level Enactment

Low-level enactment revealed fragmented instructional reasoning, procedural approaches, and superficial use of technology. Misconceptions were left unaddressed at the conceptual level. This pattern echoes prior studies showing that underprepared preservice teachers often default to rote formulas rather than scaffolding conceptual change (Farrugia & Busuttil, 2021; Valtonen et al., 2023).

Cross-Case Patterns

Analysis across the three enactment levels revealed five overarching patterns. First, the depth of integration ranged from seamless CK–PK–TK synergy at the high level to CK–PK dominance with weak TK at the moderate, and fragmented domains at the low (Wu, 2013). Second, CRA sequencing was decisive: complete CRA enabled conceptual remediation, partial CRA offered limited scaffolding, and absence reinforced proceduralism (Donovan & Fyfe, 2022; S. Zhang et al., 2022). Third, the role of technology proved transformative only when pedagogically justified, otherwise remaining ornamental or absent (Backfisch et al., 2024; Lu et al., 2025). Fourth, instructional responsiveness characterized by strong noticing and adaptive strategies distinguished high-level enactment from generic or absent responsiveness at moderate and low levels (Kilic & Dogan, 2022; Scheiner, 2023). Finally, professional development needs varied: advanced evaluative training for high-level PMTs, structured TK–PK integration for moderate, and foundational scaffolding for low (Açıkgül & Aslaner, 2020b; Njiku et al., 2021a).

Discussion

This study examined preservice mathematics teachers' (PMTs) enactment of TPACK through vignette-based CRA tasks in addressing 3D geometry misconceptions. The findings revealed three levels of enactment—High, Moderate, and Low—each reflecting different degrees of CK–PK–TK integration, CRA sequencing, and technology use. In this section, we critically discuss these findings in relation to existing scholarship and highlight their implications for theory, methodology, and practice.

High-Level Enactment: Transformative Knowledge-in-Use

Participants in the high-level group enacted coherent CK–PK–TK integration, supported by complete CRA sequencing and purposeful use of digital tools such as GeoGebra 3D and AR. Their responses

demonstrated pedagogical noticing (Kilic & Dogan, 2022; Scheiner, 2023), as they diagnosed specific misconceptions and adapted strategies accordingly.

These findings resonate with prior research showing that technology is most effective when embedded within structured pedagogical models (Donovan & Fyfe, 2022; S. Zhang et al., 2022). For example, Lia's strategy of moving from manipulatives → GeoGebra → abstract formulas aligns with evidence that CRA-supported visualization fosters durable conceptual change (Klepsch & Seufert, 2020; Lu et al., 2025). Similarly, Silvi's use of AR to distinguish diagonals from edges reflects how embodied, interactive representations expand students' spatial reasoning repertoires (Özçakır & Çakıroğlu, 2022).

Importantly, these enactments support arguments that TPACK is not a static trait but enacted knowledge-in-use, dynamically mobilized in response to instructional contexts (Ortiz Colón et al., 2023a; Pareto & Willermark, 2019). By demonstrating adaptive integration, high-level participants exemplify the type of professional competence required for teaching complex geometry topics.

Moderate-Level Enactment: Pedagogical Strengths but Technological Gaps

Moderate-level participants displayed solid CK–PK reasoning but weak or superficial TK integration. Their reliance on physical or collaborative strategies (e.g., cardboard nets, group problem-solving) reflects strong pedagogical awareness but limited ability to embed technology in conceptually meaningful ways.

This finding confirms earlier studies reporting that preservice teachers often acknowledge technology but struggle to justify its pedagogical role, leading to additive rather than transformative integration (Backfisch et al., 2024; Koyunkaya & Dede, 2024; Saritepeci, 2022). Similar patterns were documented by Lim et al. (2024a), who found that teachers recognized the affordances of GeoGebra but defaulted to traditional strategies when uncertain about pedagogical alignment.

Thus, moderate-level enactment represents a transition zone: preservice teachers have developed pedagogical reasoning but require structured training to link TK and PK explicitly. This aligns with Açıkgül & Aslaner (2020), who demonstrated that microteaching with integrated technology can strengthen this connection.

Low-Level Enactment: Fragmented Knowledge and Proceduralism

The low-level case revealed fragmented CK–PK–TK integration, reliance on rote memorization, and nominal use of technology, echoing prior findings that novice teachers often default to formula-driven remediation that leaves misconceptions unresolved (Farrugia & Busuttil, 2021; Valtonen et al., 2023). This reflects the broader challenge in teacher education where technology is treated as an add-on rather than a pedagogical scaffold (Backfisch et al., 2024; Koh, 2020). Research consistently shows that

preservice teachers struggle to enact TPACK meaningfully, with PCK frequently dominating TPK and resulting in fragmented knowledge (Karchmer-Klein & Konishi, 2023; Silva et al., 2021; Valtonen et al., 2023).

Addressing this requires explicit training in technology–pedagogy integration (Lim et al., 2024a; Nugraheni & Srisawasdi, 2025), enhanced digital literacy aligned with international standards (Baek & Sung, 2021), and innovative practicum placements that connect theory with practice (Lim et al., 2024a). Emerging approaches highlight the potential of AI to explore preservice teachers’ perceptions (EğİN et al., 2025) and equity-oriented digital pedagogies to promote transformative practice (Herring Watson & Wake, 2025). While the challenges remain significant, they also create opportunities to strengthen TPACK enactment and foster adaptive, conceptually focused mathematics teacher education.

CRA Sequencing and Instructional Responsiveness

A cross-cutting theme is the role of CRA sequencing in shaping enactment quality. High-level participants effectively mobilized CRA to scaffold conceptual change, while moderate-level participants applied CRA partially, and low-level responses omitted it altogether. These differences confirm findings by Donovan & Fyfe, (2022) and S. Zhang et al. (2022) that CRA enhances conceptual durability when fully enacted.

Moreover, the integration of CRA with technology amplified instructional responsiveness, enabling participants to address misconceptions adaptively. This supports Habiyaemye et al. (2023), who emphasized that representational sequencing combined with digital tools promotes both noticing and responsive teaching. The present study extends this claim by showing that CRA can serve as a diagnostic marker of TPACK enactment quality in preservice teachers.

Contextual and Global Implications

The findings affirm that TPACK enactment is context-dependent. In Germany, integration remains largely technical (Backfisch et al., 2024); in South Korea, enactment is shaped by institutional training cultures (Kim et al., 2024a); and in the U.S., vignette and simulation approaches have proven effective (Scheiner & Montes, 2025). In Indonesia, however, persistent disparities in digital resources and uneven professional preparation (Chang et al., 2025; Kusuma, 2021; Thohir et al., 2022) constrain enactment.

Within this context, vignette-based CRA tasks offer a low-cost, scalable, and reflective intervention for teacher education. By eliciting authentic reasoning in simulated scenarios, they provide both diagnostic evidence of current readiness and developmental opportunities for growth (Njiku et al., 2021b; Runnalls & Hong, 2020b). Thus, the study contributes to global discourse on scenario-driven, technology-enhanced pedagogy, while offering contextually grounded insights for Indonesia.

Theoretical Contributions

This study contributes to scholarship in three principal ways. First, it integrates TPACK and CRA, extending TPACK theory by embedding CRA as a sequencing scaffold that provides a dynamic account of enacted instructional reasoning (Donovan & Fyfe, 2022; Habiyaemye et al., 2023). Second, it shifts the focus from perception to enactment, moving beyond self-reports (Lim et al., 2024a; Saubern et al., 2020a) by capturing authentic instructional responses in misconception-driven contexts. Third, it expands vignette methodology, showing that when combined with CRA, vignettes illuminate not only teacher noticing but also the quality of representational reasoning (Oner, 2020; Scheiner & Montes, 2025).

Practical Implications

The differentiated enactment profiles imply distinct professional development trajectories. High-level PMTs require advanced reflection and evaluative practices to refine adaptive expertise (Scheiner, 2023). Moderate-level PMTs need structured support for principled TK–PK integration through activities such as technology-embedded microteaching (Açıkgül & Aslaner, 2020a; Rocha, 2020). Low-level PMTs demand reinforcement of foundational CK–PK–TK coherence and explicit modeling of CRA-based instruction (Koh, 2020; Valtonen et al., 2023). From these emerge three design principles: embedding vignette-based CRA tasks as authentic assessments, aligning technology use with CRA progression to ensure conceptual scaffolding, and fostering pedagogical noticing through structured reflection on student misconceptions (Donovan & Fyfe, 2022; Scheiner, 2023).

Beyond theoretical and methodological implications, the findings carry policy-level significance for mathematics teacher education in Indonesia. The enactment gaps identified suggest the need for institutionalized professional development programs that explicitly integrate CRA and technology into the curriculum, supported by equitable digital infrastructure (Kusuma, 2021; Thohir et al., 2022). Universities and teacher education providers should embed vignette-based CRA tasks as part of standard assessment and training modules, ensuring preservice teachers move beyond procedural remediation towards adaptive, technology-enhanced pedagogy (Habiyaemye et al., 2023; Njiku et al., 2021a). At the national level, these insights can inform policy directions to align teacher preparation standards with the Merdeka Belajar agenda, emphasizing adaptive use of digital tools for conceptual learning in mathematics classrooms (Adriana Mendoza Mondragón, 2024b; Chang et al., 2025).

Conclusion

This study examined how preservice mathematics teachers (PMTs) enact TPACK in addressing 3D geometry misconceptions through vignette-based tasks embedded in the Concrete–Representational–Abstract (CRA) model. Analysis revealed three profiles: High, with coherent CK–PK–TK integration, full CRA sequencing, and transformative technology use (GeoGebra 3D, AR, Cabri 3D); Moderate, with strong CK–PK but superficial TK integration and partial CRA; and Low, with fragmented reasoning, procedural remediation, and minimal CRA or pedagogical noticing (Backfisch et al., 2024; Donovan & Fyfe, 2022; Habiyaemye et al., 2023).

The study contributes in three ways. Theoretically, it extends TPACK by embedding CRA as a structuring principle for enacted instructional reasoning (Donovan & Fyfe, 2022; Habiyaemye et al., 2023). Methodologically, it advances vignette research by showing the diagnostic and developmental potential of vignette–CRA integration (Oner, 2020; Scheiner & Montes, 2025). Practically, it proposes differentiated professional development: reflective evaluation for high-level PMTs, structured TK–PK integration for moderate, and foundational CRA-informed scaffolding for low (Açıkgül & Aslaner, 2020b; Rocha, 2020).

Limitations include the small sample ($n = 10$) and reliance on written responses, which restrict ecological validity despite strong inter-rater reliability ($\kappa = 0.87$) (X. Li & Sang, 2023). Future research should expand samples across contexts (K.-L. Yang et al., 2023), adopt longitudinal designs (Njiku et al., 2021c), employ multimodal data (Ivars et al., 2020), and explore emerging tools such as AI and immersive technologies (Mishra et al., 2023).

Ultimately, the findings underscore that TPACK enactment is an adaptive competence, requiring deliberate CK–PK–TK integration through CRA and enacted with sensitivity to misconceptions—offering a scalable model for mathematics teacher education globally, and particularly in contexts with digital inequities such as Indonesia (Chang et al., 2025; Kusuma, 2021).

In addition to advancing theoretical and methodological discourse, the study offers practical and policy-oriented implications for mathematics teacher education in Indonesia and beyond. Embedding vignette–CRA tasks as scalable diagnostic tools within teacher preparation curricula can strengthen adaptive TPACK enactment. By addressing both micro-level instructional reasoning and macro-level program design, this approach bridges the gap between research, practice, and policy.

Limitations and Future Research

This study is limited by its small sample size ($n = 10$) from a single Indonesian program and reliance on written vignette responses, which, while valuable for capturing reflective reasoning, may not fully represent classroom enactments. Although reliability was supported by high inter-rater agreement ($\kappa = 0.87$) and peer debriefing, the interpretive nature of qualitative coding may still influence categorization. These factors constrain the transferability and ecological validity of findings.

Future research should therefore adopt larger and more diverse samples across institutions and cultural contexts, employ multimodal data collection (e.g., observations, microteaching, video analysis) to triangulate vignette evidence, and consider longitudinal designs to trace developmental trajectories of CRA–TPACK enactment. Moreover, exploring the integration of emerging technologies such as AI, VR, and immersive visualization into CRA-informed instruction may offer new insights into strengthening adaptive, technology-enhanced pedagogy in mathematics teacher education.

Declarations

STATEMENTS AND DECLARATIONS

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Approval Statement

Ethical approval for this study was granted by the Institutional Review Board of the Faculty of Teacher Training and Education, Universitas Siliwangi, under the authority of the Dean, Dr. Nani Ratnaningsih, M.Pd. The research protocol was reviewed and approved in accordance with the ethical guidelines and regulations for educational research involving human participants established by the Faculty of Teacher Training and Education, Universitas Siliwangi.

Accordance with Ethical Standards

All methods employed in this study were performed in strict accordance with the ethical guidelines and regulations of the Faculty of Teacher Training and Education Ethics Committee, Universitas Siliwangi, under the authority of the Dean of the Faculty of Teacher Training and Education, Dr. Nani Ratnaningsih, M.Pd., which approved the research protocol. The study was conducted in full compliance with the approved procedures governing research involving human participants.

Consent to Participate

Written informed consent was obtained from all participants prior to their participation in this study. Participants were fully informed about the purpose of the research, the procedures involved, the voluntary nature of participation, their right to withdraw at any time without penalty, and the measures taken to ensure confidentiality and anonymity. Participation was entirely voluntary, and no identifying information was collected or disclosed.

Consent to Publish Statement

Not applicable

Data Availability Statement

The datasets used and/or analysed during the current study are available from the corresponding author upon reasonable request.

Author Contributions

Dedi Muhtadi conceptualized the study, supervised the analysis, and led the writing of the manuscript. Yohanes Leonardus Sukestiyarno and Isti Hidayah contributed to the development of the theoretical framework, validation, and refinement of the findings. Amin Suyitno contributed to reviewing, editing, and aligning the manuscript with current research discourse. All authors have read and approved the final manuscript.

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Figures

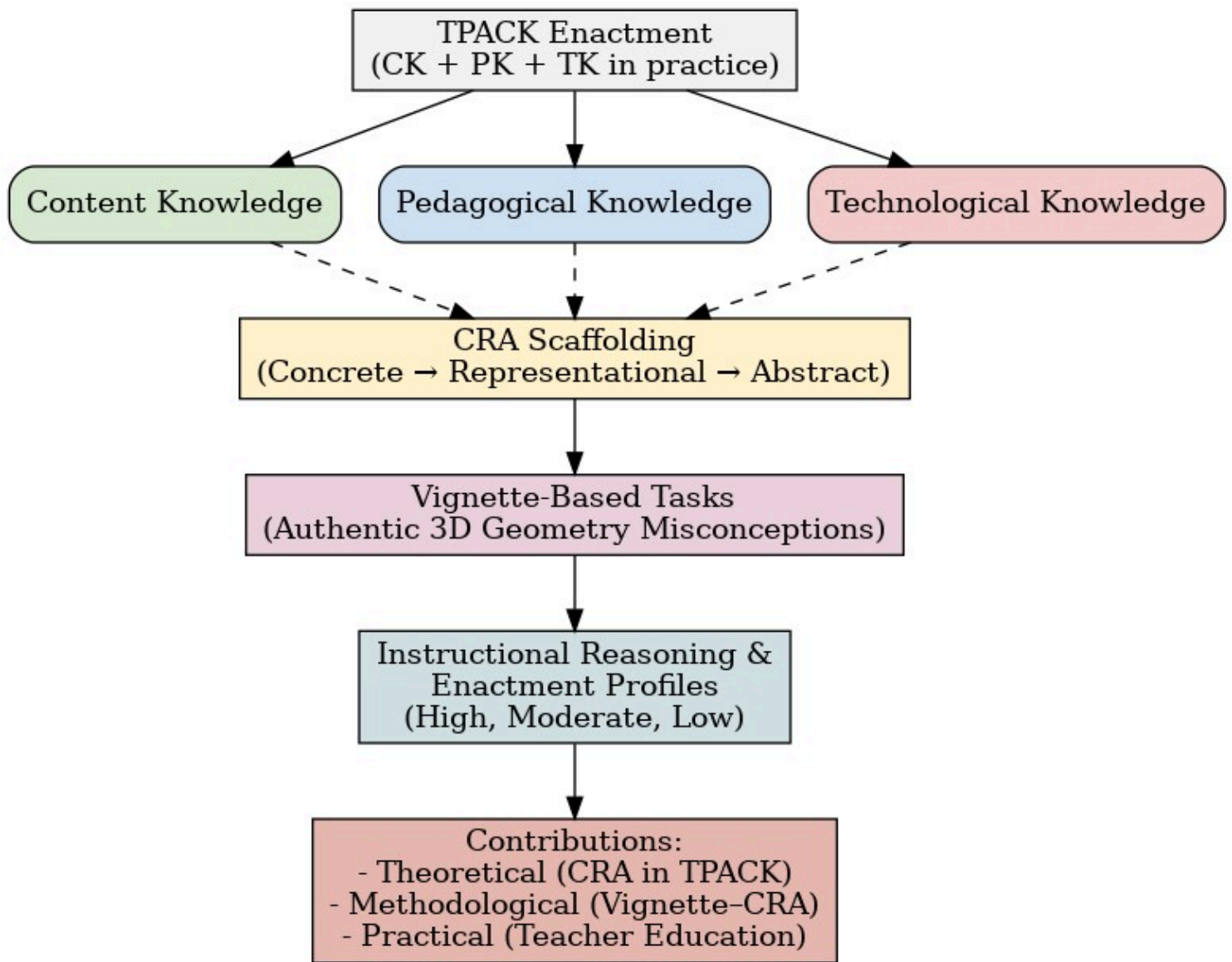


Figure 1

Conceptual Framework (TPACK-CRA-Vignette)

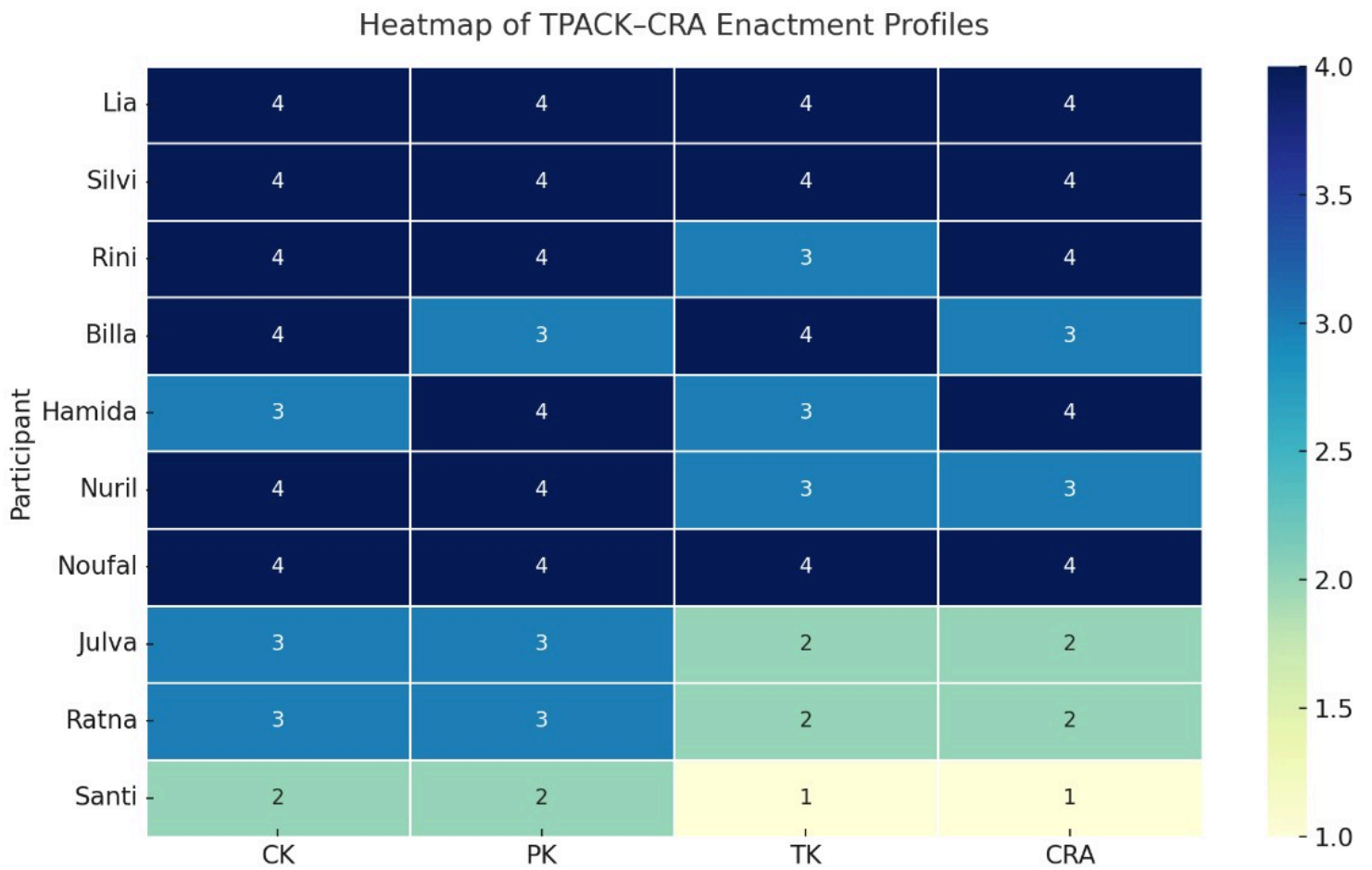


Figure 2

Heatmap of TPACK-CRA Enactment Profiles