Supplementary information

Supplementary Table.1| X-ray characterisation of the diamond CRL. At-wavelength speckle-based metrology was used to quantify the phase-profile slopes of a laser-machined diamond CRL used in the achromat at 15, 20 and 25 keV. The average phase-gradient slope extracted from the speckle displacements was used to determine the experimental X-ray focal length F_{exp}, which is compared with the theoretical X-ray focal length F_{theory} calculated from the corresponding refractive index decrement δ. The percentage error remains below 1% across all energies, indicating only minimal deviations from the ideal parabolic profile.

Energy (keV)	Delta ($\times 10^{-6}$)	Slope of Grad x	Slope of Grad y	Avg slope	F_{exp}	F_{theory}	Error (%)
15	3.24	0.153	0.156	0.1545	11.65	11.57	0.7
20	1.82	0.086	0.088	0.0870	20.69	20.58	0.5
25	1.17	0.055	0.056	0.0555	32.43	32.16	0.8

At-wavelength speckle-based metrology technique[54,55] was used to characterise the phase profile errors of the diamond compound refractive lens. Characterisation was performed at the Beamline B16, Diamond Light Source, Didcot, United Kingdom. In this method, a fine random speckle pattern is placed upstream of the lens, and the resulting speckle images are recorded with and without the lens in the beam. By tracking the small lateral displacements of the speckles introduced by the lens, the slope of the optical phase profile can be determined. In this way, we measured the spatially resolved slope of the phase profile of a laser-machined parabolic diamond CRL. The experiment was performed at 15, 20, and 25 keV, and the focal length of the lenses at 3 different X-ray energies was determined, and the percentage error is calculated from the theoretical focal length at respective energies, shown in Supplementary Table. 1. The very minimal error could be due to a potential deviation from the parabolic phase profile due to fabrication errors.