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FESIM: A Modular Forest Economic Simulation Model

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Abstract

The Forest Economic Simulation Model (FESIM) is a dynamic, modular framework that generates integrated ecological and economic projections for forest enterprises under varying scenarios. Fully formulated in linear algebra, the model ensures complete reproducibility and interpretability. The use of a single forest inventory snapshot or management-plan data, augmented with (rudimentary) inputs such as area, yield tables, survival probabilities, and roundwood prices, enables FESIM to simulate the five-year time-step development of stand areas, roundwood volumes, carbon pools, and key economic metrics. The model integrates a Markov chain area-transition module with parameterised sigmoid functions to project growth and harvesting volumes, which are then fed into a valuation module employing multi-tier contribution-margin accounting. This approach captures profitability at successive cost allocation levels, ranging from stumpage values to net returns, thereby enabling the quantification of opportunity costs associated with conservation-oriented practices and policies. Optional submodules address habitat-tree retention and the dynamics of coarse woody debris, thereby establishing a direct linkage between biodiversity and carbon, on the one hand, and economic outputs, on the other. Crucially, the model overcomes the challenges of data-scarce regions by integrating pre-processed data, such as parameterised yield tables, effectively leveraging prior knowledge to ensure robust application.

Keywords: Forest economic simulation, Markov chain forest model, Yield table functions, Habitat tree retention, Coarse woody debris, Carbon accounting, Policy impact assessment

1 Introduction

Forestry in Central Europe is characterised by exceptionally long production cycles that unfold over periods far exceeding those found in most other land-use sectors, often spanning multiple human generations. These pose unique challenges for both economic analysis and policy advice. Silvicultural interventions implemented today may not reveal their ecological and economic consequences for several decades, and in some cases not before the next century [38, 62]. Consequently, neither managers nor policymakers can empirically evaluate the long-term implications of different management strategies within their own professional lifetimes. Instead, they must rely on simulation-based assessments to explore potential development pathways, quantify opportunity costs, and anticipate ecological and economic trade-offs. These challenges have become more pronounced as multifunctional forest management has expanded beyond roundwood production to encompass biodiversity conservation, climate mitigation, and other ecosystem services. Legal frameworks such as the Habitats Directive (92/43/EEC), Birds Directive (2009/147/EC) and Nature Restoration Regulation (2024/1991) or policy strategy documents of the European Commission such as the EU Biodiversity Strategy 2030 as part of the EU Green Deal impose additional constraints on forest use while simultaneously increasing the need for transparent economic assessments of policy impacts [9, 11–14]. Under these evolving conditions, simulation tools are indispensable for comparing alternative silvicultural options, typically benchmarked against business-as-usual management, and for generating dynamic projections of economic flows and stocks over time.

In recent decades, a wide variety of forest simulation tools have been developed to support strategic decision-making amid increasing ecological, economic, and policy uncertainty. The simulation tools differ significantly in their methodological foundations, data requirements, spatial resolution, and the degree to which they capture ecological

processes and economic outcomes. Despite substantial progress across these simulation tools, few existing tools provide an integrated, operationally feasible framework that simultaneously represents structural parameters relevant for biodiversity, such as habitat-tree retention or coarse woody debris (CWD), and delivers key economic indicators. When carbon accounting modules exist, they are often decoupled from roundwood valuation components, making it difficult to quantify the trade-offs among carbon sequestration, habitat-supporting structures, and roundwood revenues. Furthermore, although many contemporary simulators incorporate modular architectures, their applicability is often constrained by large data requirements or complex parameterisation, which impede the seamless addition of new components. Furthermore, a significant methodological barrier persists in the current simulation tool landscape: very few generic frameworks are entirely formalised using explicit mathematical notation, such as linear algebra, thereby ensuring theoretical transparency and universal reproducibility. Frequently, the underlying mechanistic logic of these simulators is inextricably embedded within bespoke, often sparsely documented source code. This reliance on computational implementation over mathematical description creates a ‘black box’ effect, rendering the internal dynamics difficult to independently verify and effectively inaccessible to a broader interdisciplinary audience.

The Forest Economic Simulation Model (FESIM) was developed to address these limitations by integrating ecological structural complexity with key economic metrics within a transparent, reproducible linear-algebra framework. FESIM operates based on a single inventory snapshot or input data from management plans, accompanied by pre-processed data for four core inputs: i.) stand area by tree species groups and age class, ii.) average rotation or final-utilisation times, with optional age-class survival probabilities, iii.) yield tables at a representative site-index level and iv.) average roundwood prices. This parsimonious design enables FESIM to generate deterministic projections of stand area, growing stock volume and yields, as well as

economic key figures such as contribution margins, liquidation values and present values across discrete five-year time steps, even in data-scarce regions, while ensuring that both its input and output variables remain intuitively interpretable for forestry practitioners and political decision-makers alike. Crucially, the overarching research objective of FESIM is not to undertake single-enterprise optimisation, but rather to assist policymakers in navigating frequently ambiguous policy drafts through scenario generation.

2 Model Structure and Conceptual Framework

2.1 System Boundaries and Temporal/Spatial Resolution

Given our primary research focus on large-scale dynamics, FESIM is designed to model aggregate structures at the enterprise, regional, or landscape level. Accordingly, it employs a Markov chain approach to capture system-wide transitions, deliberately shifting the focus away from the management or optimisation of individual stands. The system incorporates pooled-area data stratified by tree species group and age class. Temporally, FESIM simulates in discrete five-year time steps (matching typical European NFI/GHG cycles and aligning with standard intervention intervals in German forest management plans, which typically span 10 years) over a user-defined horizon. All forest growth metrics, harvest amounts, and conservation features (habitat-tree retention, coarse woody debris, carbon pools) are tracked up to roadside extraction; downstream processing and broader value-chain activities lie outside the system boundaries.

2.2 Architecture of FESIM

FESIM follows a modular processing pipeline that transforms static inventory data or data from forest management plans into dynamic projections of key ecological and economic figures. The simulation tool begins with a single-snapshot input

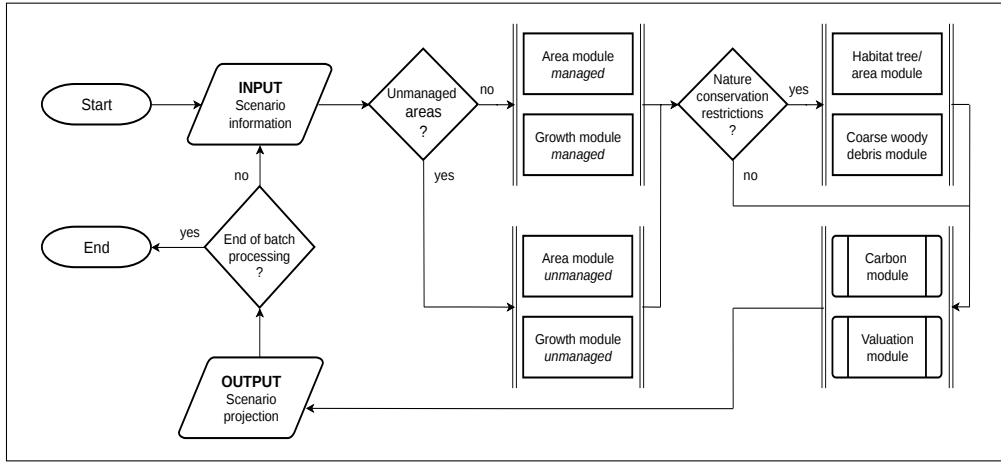


Figure 1 Flowchart of FESIM, according to ISO 5807 standard.

describing the forest area distribution by tree species group and age class, complemented by tree species group-specific survival probabilities, rotation lengths, and growth functions derived from yield tables at a representative site index (see Fig. 1). Within the core modules, the *Area Module* reallocates the area across tree species groups and age classes according to deterministic survival probabilities and a (pre-determined) harvesting regime. In contrast, the *Growth Module* applies difference equations to estimate per-hectare growing-stock volume, harvest amounts from thinning, diameter distributions, and mean height over time. Economic valuation is handled by the *Valuation Module*, which supports both net-price-multiplication approaches and diameter-specific net-revenue frameworks. Beyond the core modules, optional extension modules enable a more detailed representation of ecological structures that are relevant under nature conservation requirements and legislation: the *Habitat-Tree Module* and *Coarse Woody Debris (CWD) Module* integrate nature conservation-relevant structures, while the *Carbon Module* tracks aboveground, below-ground biomass and *CWD* carbon stocks across pools. The outputs consist of dynamic development describing both ecological and economic key figures, such as growing stock volume and increment, carbon stock and increment in all respective carbon pools,

habitat indicators such as *CWD* and *Habitat Trees*, harvesting amounts and economic metrics (e.g., contribution margins and NPV), thereby providing a comprehensive basis for scenario analysis and policy-oriented decision support.

2.3 Key Assumptions and Theoretical Underpinnings

FESIM is a deterministic, dynamic simulation model that combines a Markov chain framework (*Area Module*) with mathematical difference equations (*Growth Module*) to generate reproducible forest-development trajectories. Static inputs, such as yield tables and site indices, serve as boundary conditions, while state variables, including area, volume, and carbon pools, are updated dynamically at each time step. The simulation tool is deterministic, consistently producing identical outputs for identical inputs. It employs a Markov chain framework with fixed transition probabilities to reallocate area among age classes and tree species groups. With its forward-looking orientation, FESIM is calibrated to project future forest states and is primarily intended for scenario analysis and the evaluation of policy impacts, rather than purely descriptive or retrospective assessments.

3 Model Implementation

The predecessor of the FESIM simulation tool described in the present article was known as the ‘Strugholtz–Englert Simulation Model’ and was developed at the Department of Forest Economics at the Georg-August-University of Göttingen [62]. This simulation tool incorporated several elements from Möhring [31] and Smaltschinski [59], and was further developed at the Johann-Heinrich von Thünen-Institute of Forestry in the context of numerous research projects. Throughout its evolution, the simulation tool has primarily been used to evaluate the economic consequences of atypical management decisions, such as the implementation of usage restrictions under conservation regulations or policy drafts [6, 43, 44, 48–50] at an aggregate level.

In addition, it has been applied to quantify the additional costs associated with protection and restoration functions [10] or to assess the impacts and costs of various management decisions in the context of climate change [51]. The FESIM framework comprises a modular structure described in the following sections, beginning with the *Area Module*.

3.1 Area Module

The *Area Module* structures the forest aggregate, whether at the enterprise, regional, or national scale, by age class and tree species group. The *Area Module* is a Markov chain matrix model that incorporates transition probabilities based on calamity events and the final harvest. Furthermore, it facilitates the transition of age classes and tree species groups within the forest aggregate or enterprise.

The temporal development of the forest aggregate categorised by tree species groups and age classes is modelled as a discrete-time Markov chain. In this framework, system states represent combinations of tree species groups and age classes, with transitions governed by stochastic processes that account for natural disturbances and management factors. System states are represented by column vectors, with elements denoting the forested area for each (tree) species–age-class combination at the beginning of each time step. Transitions are governed by a column-stochastic matrix (i.e., column sums equal 1), where each column corresponds to a source age class and each row to a target age class (see eq. 4), thereby ensuring conservation of total forested area. Note that the simulation tool could equivalently be formulated using row vectors, in which case the transition matrix would be row-stochastic (i.e., row sums equal 1). In the Markov-chain *Area Module*, it is assumed that unstocked areas, such as gaps and open sites, are of constant extent and therefore fall outside the scope of the analysis. Every area removed from the stand, whether through natural disturbance or a (pre-planned) final harvest, is immediately regenerated.

The state of the system at discrete time step n is represented by a column vector $\mathbf{a}_n$, whose components correspond one-to-one to ordered pairs (s, k_s) , where $s \in \{1, \dots, S\}$ indexes tree species group and $k_s \in \{1, \dots, K_s\}$ denotes the k_s -th age class of tree species group s . Age classes are discrete and typically aligned with five-year inventory intervals, with K_s representing the tree species group-specific maximum age class (e.g., the maximum rotation period in managed forests or the maximum age in unmanaged forests, see appendix B.1.2). The total number of components of $\mathbf{a}_n$ is therefore

$$L = \sum_{s=1}^S K_s, \quad (1)$$

so that $\mathbf{a}_n \in \mathbb{R}^L$. To ensure computational tractability and interpretability, the vector is ordered in contiguous blocks by tree species group. Specifically, the linear index ℓ of the component associated with tree species group s and age class k_s is given by

$$\ell = \ell(s, k_s) = \left(\sum_{i=1}^{s-1} K_i \right) + k_s, \quad (2)$$

with $\ell \in \{1, \dots, L\}$. The transition matrix is of size $L \times L$, with rows and columns ordered consistently with the state vector.

Forest development over time is modelled using a column-stochastic transition matrix $\mathbf{T} \in \mathbb{R}^{L \times L}$. Each column of $\mathbf{T}$ sums to one, and all entries are non-negative probabilities, thereby conserving total forest area over time (i.e., assuming no net afforestation or deforestation). The matrix is time-homogeneous and Markovian: transitions depend only on the current state, not on past conditions.

$\mathbf{T}$ incorporates transition probabilities between age-classes and tree species groups (e.g., through regeneration or succession), shaped by natural disturbances and final harvests. The matrix $\mathbf{T}$ is structured in such a way that the sub-diagonal elements within each tree species-specific block represent age-class transitions. In contrast, the

first row of each block contains the default probabilities. This assumption holds only under the condition that the time step and the width of the age classes are identical; in the present article, both are 5 years. The off-diagonal blocks describe transitions between tree species groups (see eq. 4).

The area distribution after n discrete time steps, denoted $\mathbf{a}_n$, is obtained by repeated application of the transition matrix $\mathbf{T}$ to the initial area distribution vector $\mathbf{a}_0$:

$$\mathbf{a}_n = \mathbf{T}^n \mathbf{a}_0, \quad (3)$$

where $\mathbf{T}^n$ denotes the n -fold matrix product of $\mathbf{T}$ with itself. The subscript notation $\mathbf{a}_n$ is used for time steps to avoid visual confusion with the matrix exponent $\mathbf{T}^n$, while preserving clarity in a model with hierarchical state indices. As a visual example of eq. 3, for one simulation step ($n = 1$) with the initial area distribution being a fully-regulated forest (i.e., all elements of $\mathbf{a}_0$ are 10), the area projection in 5 years (i.e., $\mathbf{a}_1$) is as follows:

$$\underbrace{\begin{bmatrix} 0 & 0.1 & 0.2 & 0.3 & 0.6 & 0 & 0.1 & 0.1 & 0.3 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0.1 & 0.1 & 0.2 & 0.4 & 0 & 0.3 & 0.3 & 0.7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ \hline 10 \\ 10 \\ 10 \\ 10 \end{bmatrix}}_{\mathbf{a}_0} = \underbrace{\begin{bmatrix} 17 \\ 10 \\ 8 \\ 7 \\ 5 \\ \hline 21 \\ 10 \\ 6 \\ 6 \end{bmatrix}}_{\mathbf{a}_1} \quad (4)$$

For irreducible and aperiodic stochastic matrices such as $\mathbf{T}$, repeated multiplication leads to convergence to a unique stationary distribution as $n \rightarrow \infty$. Specifically, $\mathbf{T}^n$ converges to a matrix in which every column equals the normalised eigenvector associated with the dominant eigenvalue 1, normalised such that its l^1 -norm (i.e., the sum of all vector entries) equals the total forest area. This result assumes area conservation—i.e., no net afforestation or deforestation. Consequently, the long-term area distribution stabilises and becomes independent of the initial configuration $\mathbf{a}_0$, provided total area is conserved. In a forestry context, this stationary state represents a long-term equilibrium structure of the forest aggregate or enterprise, one shaped by disturbance regimes, management practices, and ecological dynamics, but invariant to initial conditions.

The transition matrix $\mathbf{T}$ is constructed as a product of constituent matrices, each representing a key ecological or management driver:

$$\mathbf{T} = \mathbf{W} \mathbf{C} \mathbf{E} \mathbf{H} \quad (5)$$

where:

- H** Hazard matrix captures the effects of natural disturbances (e.g., windthrow, fire, or insect calamities)
- E** Final harvest matrix represents scheduled final felling operations
- C** Cyclical shift-down matrix advances all age classes by one time step
- W** Species conversion matrix models transitions between tree species groups (e.g., through regeneration or succession)

Each constituent matrix is of dimension $L \times L$ (see eq. 1 and 2) and exhibits a block structure. The appendix sections A.1.1, A.1.2 and A.1.3 provide a detailed description of the individual submatrices that constitute the transition matrix $\mathbf{T}$.

3.2 Growth Module

The *Growth Module* consolidates various growth-related metrics specified in standardised yield tables, reflecting the common practice in Germany/ Central Europe of simulating forest growth and utilisation (from thinning) [27, 40]. FESIM implements yield tables via parameterised functions (see Appendix B.2). The growth-related metrics on which this module provides information are mean height (**hg**) development, growing stock volume (**gsv**) development, total stock volume (**tsv**) development, total volume increment (**tvi**) as the first derivative of **tsv**, and the harvested volume from thinning (**thv**). All simulated state variables are stored as vectors $\in \mathbb{R}^{K_s}$ in order to document the development of the projections and to enable their representation as time series. FESIM utilises different yield tables and associated growth functions, such as the Schober [56] tables for moderate and strong thinning from below, the *North-west German Forest Research Institute* (NW-FVA) tables for staggered thinning from above [1, 2, 37], or the un-thinned control-plot based growth functions by Pretzsch et al. [41] for unmanaged forests.

It is important to note that yield tables, which present the growth dynamics of pure stands in tabular form, serve as models for the interrelated metrics of growth and utilisation (from thinning) [27]. These tables assume even-aged pure stands growing on identical or comparable sites, characterised by homogeneous full stocking (stocking degree of 1.0, i.e., 100% crown closure or complete site occupancy with no significant gaps between trees, relative to the reference stock from the yield table) under analogous silvicultural treatments [27]. In Germany, it is customary, for instance, in forest planning to subdivide mixed stands into so-called ‘calculated pure stands’.

The following section refers to a single tree species group at a time, with S sub-vectors, each comprising K_s elements, whose lengths sum to L (see eq. 1). The growth-related metrics provided by this module are computed sequentially using the matrix model described below. It should be noted, however, that in the *Area Module* (and the

corresponding block matrix, see eq. 3 and eq. 4), all tree species groups are processed in parallel, each being represented by its respective tree species group index.

The core growth-related metrics are modelled as state variables of a deterministic process. Stand development is represented tree species wise, by a sequence of state vectors $(\mathbf{v}_{s,0}, \mathbf{v}_{s,1}, \dots, \mathbf{v}_{s,N})$, where $\mathbf{v}_{s,n} \in \mathbb{R}^{K_s}$ denotes the state vector for tree species group s at simulation time step n containing K_s age-classes as components (e.g., $\mathbf{v}_{s,n} = \mathbf{gsv}_{s,n}$ when referring to growing stock). The initial state $\mathbf{v}_{s,0}$ can be initialised using, e.g., data from the National Forest Inventory (NFI), the Forest Accountancy Data Network (FAN) or other data sources, for example, from forest management plans. Subsequent states are computed deterministically: $\mathbf{v}_{s,n+1} = f(\mathbf{v}_{s,n})$.

Silvicultural prescriptions aim to steer stand structure toward a long-term target. This target is expressed as a vector of desired stocking degrees, denoted $\mathbf{sdg}_{s,n}^{\text{yt}}$, where each component corresponds to a given tree species–age class and is typically set to 1.0, representing full stocking according to a reference yield table (denoted in the superscript as ‘yt’). The observed (or scenario-specific) stocking degree vector $\mathbf{sdg}_{s,n}^{\text{sc}} \in \mathbb{R}^{K_s}$ is derived from the ratio of the current growing stock volume $\mathbf{gsv}_{s,n}^{\text{sc}}$ to the target growing stock volume $\mathbf{gsv}_{s,n}^{\text{yt}}$ from the reference yield table. In line with [26], the stocking degree derived from growing stock volume closely corresponds to the commonly used measure based on basal area. Formally,

$$\mathbf{sdg}_{s,n}^{\text{sc}} = (\mathbf{GSV}_{s,n}^{\text{yt}})^{-1} \mathbf{gsv}_{s,n}^{\text{sc}}, \quad (6)$$

where $\mathbf{gsv}_{s,n}^{\text{yt}}$ is the vector of target growing stock volumes specified by the reference yield table, and $\mathbf{GSV}_{s,n}^{\text{yt}} := \text{diag}(\mathbf{gsv}_{s,n}^{\text{yt}})$ is the corresponding diagonal matrix $\in \mathbb{R}^{K_s \times K_s}$. Since all entries of $\mathbf{gsv}_{s,n}^{\text{yt}}$ are strictly positive, $\mathbf{GSV}_{s,n}^{\text{yt}}$ is invertible. By construction, a component of $(\mathbf{sdg}_{s,n}^{\text{sc}})_{k_s} = 1.0$ indicates that the current growing stock in age-class k_s matches the yield-table expectation.

The vectors of observed stocking degrees $\mathbf{sdg}_{s,n}^{\text{sc}}$ and mean stand heights $\mathbf{hg}_{s,n}^{\text{sc}}$ are used to compute the vector of increment reduction factors $\mathbf{irf}_{s,n}^{\text{sc}}$. The increment reduction function (see eq. B45) is based on the principle that both very low and very high stocking degrees (e.g., 0.3 or 1.7) limit increment, whereas increment is optimal at a stand density near 1.0. This function, $\text{irf}(\mathbf{sdg}_{s,n}^{\text{sc}}, \mathbf{hg}_{s,n}^{\text{sc}})$, is applied element-wise to the input vectors:

$$\mathbf{irf}_{s,n}^{\text{sc}} = \mathbf{SH}_s \text{irf}(\mathbf{sdg}_{s,n}^{\text{sc}}, \mathbf{hg}_{s,n}^{\text{sc}}), \quad (7)$$

where $\mathbf{SH}_s \in \mathbb{R}^{K_s \times K_s}$ is a modified shift down matrix that advances each age class by one time step (i.e., element i moves to position $i + 1$), while the oldest element is removed and the first element is retained (see eq. 8). This arrangement is required because stock and flow variables must be summed and multiplied. It follows the standard convention used in yield tables, in which flow variables are listed one position (row) below the related stock variables. The resulting vector $\mathbf{irf}_{s,n}^{\text{sc}}$ is then used to construct the diagonal matrix $\mathbf{IRF}_{s,n}^{\text{sc}} := \text{diag}(\mathbf{irf}_{s,n}^{\text{sc}})$. Specifically the modified shift down Matrix $\mathbf{SH}_s$, is given by:

$$\mathbf{SH}_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad (8)$$

To project stand development, the observed growing stock volume vector $\mathbf{gsv}_{s,n}^{\text{sc}}$ is first advanced by one age class using the shift operator $\mathbf{SH}_s$, aligning it with the corresponding elements (age-classes) of the total volume increment $\mathbf{tvi}_{s,n}^{\text{sc}}$ from the yield table. The growing stock after increment, denoted $\mathbf{sai}_{s,n}^{\text{sc}}$, is then computed as:

$$\mathbf{sai}_{s,n}^{\text{sc}} = \mathbf{SH}_s \mathbf{gsv}_{s,n}^{\text{sc}} + \mathbf{IRF}_{s,n}^{\text{sc}} \mathbf{tvi}_{s,n}^{\text{sc}}, \quad (9)$$

where $\mathbf{tvi}_{s,n}^{\text{yt}}$ is the vector of total volume increments from the reference yield table for tree species group s at time step n .

The simulated increment under the scenario is given directly by the second term:

$$\mathbf{tvi}_{s,n}^{\text{sc}} = \mathbf{IRF}_{s,n}^{\text{sc}} \mathbf{tvi}_{s,n}^{\text{yt}}. \quad (10)$$

Harvested volume from thinning, $\mathbf{thv}_{s,n}^{\text{sc}}$, is derived as the excess of $\mathbf{sai}_{s,n}^{\text{sc}}$ over the target growing stock $\mathbf{gsv}_{s,n}^{\text{yt}}$, with negative values truncated to zero (since extraction cannot be negative). This is expressed as

$$\mathbf{thv}_{s,n}^{\text{sc}} = \frac{|\mathbf{sai}_{s,n}^{\text{sc}} - \mathbf{gsv}_{s,n}^{\text{yt}}| + (\mathbf{sai}_{s,n}^{\text{sc}} - \mathbf{gsv}_{s,n}^{\text{yt}})}{2}, \quad (11)$$

where the absolute value is applied element-wise.

Finally, the growing stock volume for the next time step $n + 1$ is updated as

$$\mathbf{gsv}_{s,n+1}^{\text{sc}} = \mathbf{sai}_{s,n}^{\text{sc}} - \mathbf{thv}_{s,n}^{\text{sc}}. \quad (12)$$

This updated vector serves as the initial state for the subsequent simulation step (resuming at eq. 6), closing the recursive loop of the (managed) *Growth module*. The unmanaged *Growth module* is provided in Appendix A.2.1.

3.3 Valuation Module

The *Valuation Module* enables economic evaluation by translating physical aggregated enterprise-level data, projected by the *Growth Module* and *Area Module*, into monetary values. The results from our FESIM projections can be combined with external datasets (e.g., the Forest Accountancy Data Network [15]) to compute key economic indicators, including contribution margins, stumpage values, net present

value ('NPV'), and land expectation value ('LEV'). The valuation process comprises three methodological steps: price modelling, revenue aggregation, and comprehensive cost accounting (detailed mathematical derivations are provided in the appendix [A.3.1](#), [A.3.2](#) and [A.3.3](#)).

First, the module calculates net roundwood prices (stumpage prices) by linking tree species group-specific diameters at breast height ('DBH') with key exogenous parameters, including the harvesting system, (regional) cost levels, and roundwood quality classes (see eq. [A12](#)). The underlying reference assortments for Germany are aligned with the German RVR framework (Rahmenvereinbarung für den Rohholzhandel, detailed in tab. [C11](#) in the appendix). However, the simulation tool's flexible architecture allows for seamless adaptation to the respective roundwood grading guidelines and assortment standards of any other country or region.

Second, to accurately estimate roundwood revenues, the module disaggregates the simulated area state vectors (eq. [A13](#)) into three distinct utilisation types based on age-class-specific failure probabilities: unplanned calamity harvests ('salvage logging', eq. [A16](#)), pre-planned final harvests (eq. [A18](#)), and intermediate thinning operations. By multiplying the respective extracted volumes with the net roundwood prices—and applying designated price markdown factors for salvage roundwood, the simulation tool derives the aggregate stumpage value (eq. [A21](#)).

Finally, the module evaluates economic performance through a step-wise cost-accounting methodology (see sec. [A.3.3](#)). While the current implementation applies the established German contribution margin framework ('Deckungsbeitragsrechnung'), the system's modular architecture permits the seamless integration of any alternative, country-specific financial accounting system.

Under the applied German framework, the previously calculated aggregate stumpage value serves as the initial gross margin ('Contribution Margin I' or 'DB I'). To derive the subsequent silvicultural margin ('DB II'), the module deducts stand

establishment and tending expenses. These encompass pre-commercial thinning (eq. A26) alongside regeneration costs, which are explicitly categorised into post-harvest species retention (eq. A22), species transition (eq. A23), and post-disturbance recovery (eq. A24). By progressively deducting product-group-specific, area-level, and general administrative overheads, the simulation tool facilitates a holistic assessment of the forest enterprise’s long-term profitability, ultimately yielding the net return (‘Contribution Margin V’; see tab. A1)

3.4 Coarse Woody Debris Module

The *Coarse Woody Debris* (‘CWD’) or *Deadwood Module* estimates the accumulation and decay of CWD within the FESIM framework. CWD is relevant to the simulation tool framework because nature conservation regulations often define target quantities, as it constitutes an essential habitat for various saproxylic organisms/species [28]. Moreover, CWD is one of the five carbon pools (along with aboveground biomass, belowground biomass, litter, and soil) explicitly accounted for in GHG inventories and IPCC guidelines [21]. The module balances CWD stocks against decay dynamics, enables specification of target CWD levels, adjusts harvest removals accordingly, and assesses associated opportunity costs. In the following example, as well as in the current implementation of FESIM, the decay rates are based on data from Rock et al. [46]. However, any other set of decay constants could, in principle, be applied. The integration into FESIM and the discussion of different decay constants and models are provided in Regelman et al. [43]. The following text describes the matrix formulation, using the Rock et al. [46] decay rates for the tree species group ‘beech’ as an example.

The model operates over $N + 1$ time steps, corresponding to N discrete five-year simulation intervals, indexed by $n = 0, 1, \dots, N$. CWD dynamics are represented by a decay matrix $\mathbf{D} \in \mathbb{R}^{(N+1) \times (N+1)}$, which maps a recruitment vector $\mathbf{r} \in \mathbb{R}^{N+1}$ to the

resulting CWD stock vector $\mathbf{v} \in \mathbb{R}^{N+1}$:

$$\mathbf{v} = \mathbf{D} \mathbf{r}. \quad (13)$$

The matrix $\mathbf{D}$ is a lower triangular Toeplitz matrix, where entry d_{ij} represents the fraction of CWD recruited in step j that remains at the end of step i ($i \geq j$). In index notation, this relationship could be expressed as:

$$v_i = \sum_{j=0}^i d_{ij} r_j, \quad i = 0, 1, \dots, N, \quad (14)$$

where $d_{ij} = 0$ for $i < j$. Formally,

$$d_{ij} = \begin{cases} \lambda_{i-j+1}, & i \geq j, \\ 0, & i < j, \end{cases} \quad \forall i, j \in \{0, 1, \dots, N\}, \quad (15)$$

with $\lambda_1 = 1$ (CWD recruited in step j is fully present at the end of the same step), and λ_n decreasing for $n > 1$. An illustrative example is provided for tree species group ‘beech’, where the values follow Rock et al. [46]:

$$\boldsymbol{\lambda} = (1.00, 0.72, 0.51, 0.37, 0.26, 0.19).$$

For a 5-step example ($N + 1 = 6$), the decay matrix is:

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.72 & 1 & 0 & 0 & 0 & 0 \\ 0.51 & 0.72 & 1 & 0 & 0 & 0 \\ 0.37 & 0.51 & 0.72 & 1 & 0 & 0 \\ 0.26 & 0.37 & 0.51 & 0.72 & 1 & 0 \\ 0.19 & 0.26 & 0.37 & 0.51 & 0.72 & 1 \end{bmatrix}. \quad (16)$$

Because $\mathbf{D}$ is a lower triangular Toeplitz matrix with non-zero diagonal entries (all diagonal entries are $\lambda_1 = 1$), it is invertible. This enables two operational modes:

3.4.1 Forward calculation

Given recruitment $\mathbf{r}$ (e.g., $22 \text{ m}^3 \text{ ha}^{-1}$, from the NFI at $n = 0$ and a periodic recruitment rate of $3.5 \text{ m}^3 \text{ ha}^{-1}$), compute CWD stock as $\mathbf{v} = \mathbf{D} \mathbf{r}$:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.72 & 1 & 0 & 0 & 0 & 0 \\ 0.51 & 0.72 & 1 & 0 & 0 & 0 \\ 0.37 & 0.51 & 0.72 & 1 & 0 & 0 \\ 0.26 & 0.37 & 0.51 & 0.72 & 1 & 0 \\ 0.19 & 0.26 & 0.37 & 0.51 & 0.72 & 1 \end{bmatrix}}_{\mathbf{D}} \underbrace{\begin{bmatrix} 22 \\ 3.5 \\ 3.5 \\ 3.5 \\ 3.5 \\ 3.5 \end{bmatrix}}_{\mathbf{r}} = \underbrace{\begin{bmatrix} 22 \\ 19.2 \\ 17.3 \\ 15.8 \\ 14.8 \\ 14.1 \end{bmatrix}}_{\mathbf{v}} \quad (17)$$

As shown by the resulting CWD stock vector $\mathbf{v}$, the CWD stock declines over time from an initial value of $22 \text{ m}^3 \text{ ha}^{-1}$ to $14.1 \text{ m}^3 \text{ ha}^{-1}$. If a nature-conservation requirement exists to maintain the initial stock (opening balance) of $22 \text{ m}^3 \text{ ha}^{-1}$ indefinitely, it is evident that the periodic recruitment rate of $3.5 \text{ m}^3 \text{ ha}^{-1}$ is insufficient to sustain this target level indefinitely. Should the required annual or periodic recruitment volumes

need to be determined for a given target stock $\tilde{\mathbf{v}}$, the decay matrix $\mathbf{D}$ should be inverted and multiplied by the prescribed target stock vector $\tilde{\mathbf{v}}$. The procedure for this backward calculation is outlined in the next section.

3.4.2 Backward calculation

To determine the required periodic recruitment that sustains a prescribed target stock $\tilde{\mathbf{v}}$, one inverts the decay matrix and computes $\mathbf{r} = \mathbf{D}^{-1} \tilde{\mathbf{v}}$. For instance, to maintain a constant CWD stock of $22 \text{ m}^3 \text{ ha}^{-1}$ across all time steps:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -0.72 & 1 & 0 & 0 & 0 & 0 \\ 0 & -0.72 & 1 & 0 & 0 & 0 \\ 0 & 0 & -0.72 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.72 & 1 & 0 \\ 0 & 0 & 0 & 0 & -0.72 & 1 \end{bmatrix}}_{\mathbf{D}^{-1}} \underbrace{\begin{bmatrix} 22 \\ 22 \\ 22 \\ 22 \\ 22 \\ 22 \end{bmatrix}}_{\tilde{\mathbf{v}}} = \underbrace{\begin{bmatrix} 22 \\ 6.3 \\ 6.3 \\ 6.3 \\ 6.3 \\ 6.3 \end{bmatrix}}_{\mathbf{r}} \quad (18)$$

As shown in eq. 17 and eq. 18, a higher periodic recruitment rate of $6.3 \text{ m}^3 \text{ ha}^{-1}$ is required to maintain the initial opening balance of CWD stock of $22 \text{ m}^3 \text{ ha}^{-1}$ indefinitely over time. The inverse decay matrix $\mathbf{D}^{-1}$ is also Toeplitz, with the constant value -0.72 on the first subdiagonal. In managed scenarios, the backward mode determines the harvest reduction required to meet predefined CWD targets: the required recruitment, $\mathbf{r}$, is subtracted from the simulated harvest at each time step. In unmanaged scenarios, the forward mode applies: natural mortality contributes to $\mathbf{r}$, and the resulting CWD stock $\mathbf{v}$ is computed via $\mathbf{v} = \mathbf{D} \mathbf{r}$.

3.5 Habitat Tree Module

This module implements a modeling approach for habitat tree conservation measures. Unlike trees in conventional roundwood production, which are typically harvested upon reaching their economic maturity, habitat trees are intentionally retained within the stand to complete their full biological life cycle, including senescence and eventual decay. They are distinguished from trees for standard roundwood production by the presence or potential development of crucial microhabitats, such as cavities, crown deadwood, and structural anomalies. Consequently, these trees are critical for maintaining forest biodiversity by providing essential nesting sites, food resources, and shelter for a diverse range of species. Because these retained trees are permanently withdrawn from conventional roundwood production, the primary objective of this module is to account for the resulting reduction in actively managed operational forest area. Specifically, the module calculates this effectively unproductive area as a function of specific nature conservation mandates (namely, the required number of retained trees per hectare) and the cumulative crown projection area of these designated trees. By mathematically subtracting this habitat tree crown projection area from the total operational area, the module facilitates the estimation of the resulting opportunity costs relative to a baseline reference scenario of unconstrained roundwood production. To this end, the following section presents a mathematical framework for quantifying spatial area requirements by integrating tree species-group-specific parameters, age-class distributions, and diameter metrics. The simulation tool computes the required habitat tree area per hectare for tree species group s and age class k_s , ensuring compliance with conservation regulations. It follows the concepts described in Rosenkranz and Seintsch [48] and further developed in Regelman et al. [43] and Brinkord et al. [6].

The simulation tool calculates the habitat tree area proportion per hectare, denoted c_{s,k_s}^{ht} , for tree species group s and age class k_s , based on the diameter at breast

height ('DBH'), dg_{s,k_s}^{ht} . The superscript 'ht' denotes quantities related to habitat trees, distinguishing them from corresponding quantities in scenarios without habitat tree requirements. The area proportion is computed using a tree species group-specific function $f_{s,k_s}(dg_{s,k_s}^{\text{ht}})$ by Nagel [35] also used by Brinkord et al. [6], parameterised by empirically calibrated coefficients a_s , b_s , c_s , and d_s .

The habitat tree area proportion per hectare is given by:

$$c_{s,k_s}^{\text{ht}} = \kappa (a_s + b_s dg_{s,k_s}^{\text{ht}})^2 \left(1 - \exp \left(- \left(\frac{dg_{s,k_s}^{\text{ht}}}{c_s} \right)^{d_s} \right) \right)^2, \quad (19)$$

where $\kappa = \frac{\pi}{4 \cdot 10^4}$ is a scaling factor converting diameter-based crown width to hectare-normalised proportions (assuming DBH in cm and area in m^2 , hence the factor 10^4 to convert to hectares). The term $(a_s + b_s dg_{s,k_s}^{\text{ht}})^2$ models the quadratic relationship between diameter and area, modulated by a sigmoid-like function $\left(1 - \exp \left(- \left(\frac{dg_{s,k_s}^{\text{ht}}}{c_s} \right)^{d_s} \right) \right)^2$, which ensures that c_{s,k_s}^{ht} approaches a maximum as dg_{s,k_s}^{ht} exceeds the inflection point defined by $c_s \neq 0$ and shape parameter d_s . Here, dg_{s,k_s}^{ht} is assumed to be a scalar (e.g., mean DBH of age-class k_s) or the elements of the vector $\mathbf{dg}_s^{\text{ht}}$. Eq. 19 results in s row vectors of $1 \times k_s$ elements.

To determine the required number of habitat trees per hectare $\eta_{s,n}^{\text{ht}}$ for each tree species group s and each simulation step n , the simulation tool incorporates a target number τ_s^{ht} (e.g., 5 trees per hectare) for each tree species group s and corrects this number into a specifically required number $\eta_{s,n}^{\text{ht}}$ for each age class (e.g., 7 trees per hectare) depending on the ratio of total area of all age classes $ta_{s,n}$ to the suitable area of age classes $sa_{s,n}$ of the enterprise or forest complex:

$$\eta_{s,n}^{\text{ht}} = \tau_s^{\text{ht}} \cdot \frac{ta_{s,n}}{sa_{s,n}} = \tau_s^{\text{ht}} \cdot \frac{\sum_{k_s=1}^{K_s} a_{s,n,k_s}}{\sum_{k=k_m}^{K_s} a_{s,n,k_s}}, \quad (20)$$

where:

- τ_s^{ht} : target number of habitat trees per hectare for tree species group s , as required by regulation for the total area of an enterprise (e.g., 5 trees per hectare),
- $ta_{s,n} = \sum_{k_s=1}^{K_s} a_{s,n,k_s}$: total area for tree species group s at step n ,
- $sa_{s,n} = \sum_{k=k_m}^{K_s} a_{s,n,k_s}$: suitable area of age classes above k_m , where it is assumed that the denominator $sa_{s,n} > 0$ is strictly positive,
- k_m : minimum suitable age class (e.g., $k_m = \lceil 70/5 \rceil + 1 = 15$ for 5-year age intervals, corresponding to trees aged above 70 years),
- a_{s,n,k_s} : Area of the s -th tree species group, for age-class k_s and simulation step n .

The ratio $\frac{ta_{s,n}}{sa_{s,n}}$ adjusts the target density to account for the proportion of suitable age classes. Eq. 20 results in s column vectors of $n \times 1$ elements.

The required area $\varphi_{s,n,k_s}^{\text{ht}}$ of total habitat trees, is computed as the outer (dyadic) product of the area proportion c_{s,k_s}^{ht} and the required number $\eta_{s,n}^{\text{ht}}$:

$$\varphi_{s,n,k_s}^{\text{ht}} = \eta_{s,n}^{\text{ht}} c_{s,k_s}^{\text{ht}}, \quad (21)$$

resulting in s different $n \times k_s$ matrices for tree species group s , simulation steps n , and age classes k_s . This operation combines the required number of habitat trees $\eta_{s,n}^{\text{ht}}$ per-hectare (varying over n) with the area proportion c_{s,k_s}^{ht} per-hectare (varying over k_s). The simulation tool facilitates compliance with conservation regulations by optimising habitat tree retention during harvesting cycles. This approach provides a quantifiable

basis for balancing roundwood production with conservation restrictions.

$$\underbrace{\begin{bmatrix} \eta_{s1} \\ \eta_{s2} \\ \eta_{s3} \\ \eta_{s4} \\ \eta_{s5} \end{bmatrix}}_{\eta_{s,n}^{\text{ht}} (5 \times 1)} \otimes \underbrace{\begin{bmatrix} c_{s1} & c_{s2} & c_{s3} & c_{s4} \end{bmatrix}}_{c_{s,k_s}^{\text{ht}} (1 \times 4)} = \underbrace{\begin{bmatrix} \eta_{s1} \cdot c_{s1} & \eta_{s1} \cdot c_{s2} & \eta_{s1} \cdot c_{s3} & \eta_{s1} \cdot c_{s4} \\ \eta_{s2} \cdot c_{s1} & \eta_{s2} \cdot c_{s2} & \eta_{s2} \cdot c_{s3} & \eta_{s2} \cdot c_{s4} \\ \eta_{s3} \cdot c_{s1} & \eta_{s3} \cdot c_{s2} & \eta_{s3} \cdot c_{s3} & \eta_{s3} \cdot c_{s4} \\ \eta_{s4} \cdot c_{s1} & \eta_{s4} \cdot c_{s2} & \eta_{s4} \cdot c_{s3} & \eta_{s4} \cdot c_{s4} \\ \eta_{s5} \cdot c_{s1} & \eta_{s5} \cdot c_{s2} & \eta_{s5} \cdot c_{s3} & \eta_{s5} \cdot c_{s4} \end{bmatrix}}_{\varphi_{s,n,k_s}^{\text{ht}} (5 \times 4)}$$

As a result, the obtained s different $n \times k_s$ matrices provide the required quantities per hectare to achieve the prescribed habitat tree retention targets within the forest enterprise or complex. This information must be used to revise the corresponding harvest quantities of the remaining stand derived from the *Area Module* and *Growth Module*. For the purposes of this analysis, we adopt the simplifying normative assumption that crown projection area is equivalent to the corresponding productive forest area (i.e., 1 m² of habitat tree crown occupies 1 m² of productive forest area).

3.6 Carbon Module

The *Carbon Module* enables the quantitative assessment of forest carbon pools and the fluxes among them. It simulates the dynamics of three of the five IPCC carbon pools, aboveground biomass, belowground biomass, and coarse woody debris ('CWD'), in accordance with national and international guidelines Pretzsch [40], Röhling et al. [47], as implemented in Brinkord et al. [7], Regelmann et al. [44]. The mineral soil, organic soil and litter carbon pools are omitted. Downstream processes and interactions are deliberately omitted, including the harvested wood products ('HWP') pool, substitution effects in material and energy use, and off-site emissions (e.g., from processing, transport, or end-of-life of wood products). On-site emissions from harvesting

and forwarding operations are also neglected, as their contribution to total carbon fluxes is negligible ($< 1\%$ of annual carbon uptake) when excluding emissions from downstream roundwood logistics [see 22, 23].

All carbon stocks are expressed in terms of their carbon dioxide (CO_2) equivalents, following established conventions in greenhouse gas ('GHG') accounting and policy reporting. The module structure is aligned with the official LULUCF ('Land Use, Land-Use Change and Forestry') carbon pool definitions [45, 65] and incorporates model-specific adaptations where necessary. Only CO_2 is quantified; other GHGs such as CH_4 and N_2O are excluded from consideration.

Tree species group-specific biomass expansion factors for aboveground biomass are derived following the approach of Pretzsch [40], adjusted to maintain consistency with the German GHG inventory methodology [36, 45, 47, 63]. These factors, provided in the Appendix, convert stand-level growing stock volumes and other volume-related growth metrics to carbon dioxide mass. The carbon dioxide amounts of the aboveground biomass are calculated as follows:

$$\omega_{s,n}^{\text{agb}} = \sum_{k_s=1}^{K_s} (gsv_{s,n,k_s} \beta_{s,k_s}^{\text{agb}}) \rho_s \chi^{\text{C}} \chi^{\text{CO}_2} \quad (22)$$

where $\omega_{s,n}^{\text{agb}}$ is the quantity of carbon dioxide for tree species group s and time-step n , summed over age-class k_s , gsv_{s,n,k_s} is the growing stock volume ($\text{m}^3 \text{ ha}^{-1}$), $\beta_{s,k_s}^{\text{agb}}$ is the aboveground biomass-expansion factor converting growing stock volume to aboveground dry biomass, ρ_s is the tree species group-specific roundwood density, whereas χ^{C} and χ^{CO_2} denote the carbon content factor and the carbon dioxide factor, respectively. The belowground biomass is calculated as the expansion of the (aboveground) growing stock volume as follows:

$$\omega_{s,n}^{\text{bgb}} = \sum_{k_s=1}^{K_s} (gsv_{s,n,k_s} \beta_{s,k_s}^{\text{bgb}}) \rho_s \chi^{\text{C}} \chi^{\text{CO}_2} = \omega_{s,n}^{\text{agb}} \frac{\sum_{k_s=1}^{K_s} (gsv_{s,n,k_s} \beta_{s,k_s}^{\text{bgb}})}{\sum_{k_s=1}^{K_s} (gsv_{s,n,k_s} \beta_{s,k_s}^{\text{agb}})} \quad (23)$$

where $\omega_{s,n}^{\text{bgb}}$ is the quantity of carbon dioxide for tree species group s and time step n , computed by summing over age classes k_s , as in $\omega_{s,n}^{\text{agb}}$. The remaining factors are identical to those used for aboveground biomass, except that a different biomass expansion factor $\beta_{s,k_s}^{\text{bgb}}$ is applied. As for the *cwd* pool, the conversion of the CWD stock $\mathbf{w}$ (see eq. 13 and 14) to carbon dioxide $\omega_{s,n}^{\text{cwd}}$ is calculated as follows:

$$\omega_{s,n}^{\text{cwd}} = w_{s,n}^{\text{cwd}} \rho_s \chi^{\text{C}} \chi^{\text{CO}_2} \quad (24)$$

where $\omega_{s,n}^{\text{cwd}}$ is the quantity of carbon dioxide of *cwd* for tree species group s and simulation step n . The calculation of $\omega_{s,n}^{\text{cwd}}$ is performed similarly to $\omega_{s,n}^{\text{agb}}$ (see eq. 22) and $\omega_{s,n}^{\text{bgb}}$ (see eq. 23) with the difference that a biomass expansion factor is omitted.

4 Discussion, Limitations and Research Outlook

4.1 Strengths and Limitations

The main strength of FESIM lies in its ability to generate integrated ecological-economic projections with low data input requirements. Unlike large-scale matrix models such as EFISCEN or EFDm, which typically require consecutive inventory cycles to estimate empirical transition probabilities (in FESIM, we derive these from our functions; see app. B.1), FESIM operates on a single inventory snapshot supplemented by a few key parameters. This parsimonious design substantially lowers the barrier to application for comprehensive policy impact assessments. Consequently, it proves particularly valuable for quantifying the economic implications of environmental regulations in data-scarce regions or across highly fragmented ownership structures, thereby providing robust, evidence-based support for regional and national forest policy formulation. The simulation tool’s modular architecture and reliance on widely available yield tables make it accessible and easy to adapt, without requiring extensive

calibration or proprietary data. Furthermore, FESIM bridges the gap between complex ecological simulators and purely economic models by coupling detailed growth, yield and decay dynamics with comprehensive monetary valuation. The deterministic formulation (with fixed transition rules and yield functions) ensures the transparency and reproducibility of results, which are valuable for communicating scenario outcomes to stakeholders and policymakers.

However, the simplified structure of FESIM entails certain limitations. The use of yield tables within the *Growth Module*, while practical, inherently assumes even-aged, homogeneous stand conditions. This indicates that the simulation tool is less well-suited to uneven-aged or structurally complex forests unless appropriate yield data or adjustments are incorporated. An additional constraint arising from this reliance on yield tables is the rigid coupling of growth projections with predefined thinning regimes. While this limits the flexibility to simulate alternative silvicultural interventions, the limitation can be partially mitigated by incorporating increment-reduction functions (see app. B.3). The Markov chain approach in the *Area Module*, which assumes transitions depend only on the current state and not on prior history, is a standard simplification in long-term planning. Consequently, path-dependent processes, as the case may be, are not explicitly represented. The simulation tool currently operates at an aggregate enterprise-class level, lacking fine-grained spatial detail; it projects areas by age class rather than individual stands or spatially explicit locations. This limits its ability to capture within-stand variability or landscape spatial dynamics, which more detailed simulators (e.g., individual-based or spatially explicit simulation tools) could address. Additionally, FESIM operates as a deterministic projection tool based on constant transition rates. It does not natively account for random variability in growth, mortality, or market fluctuations; therefore, uncertainty must be examined via scenario analysis rather than intrinsic model stochasticity. Finally, the economic

valuation focuses primarily on marketable outputs, such as roundwood yields and associated costs. Non-market ecosystem services, aside from carbon storage, are not yet integrated into the economic module, representing a clear avenue for future simulation tool enhancement.

4.2 Methodological Reflections

Methodologically, FESIM builds upon a rich tradition of forest simulation models, positioning itself among area-based matrix models with notable extensions. Following the classification by Sallnäs [53], FESIM is placed into the third group of forest matrix models, which comprises area-transition models operating on aggregated age-class distributions. Early exemplars of this group (as cited by Sallnäs [53]) include simulation tools by Hool [20] and Vaux [66], as well as the deterministic Markov chain models developed by Kouba [24] and Kouba [25].

Fundamentally, later prominent works such as EFISCEN [54], EFDM [39, 42], and the original simulation tool developed by Sallnäs [53] all belong to this same overarching class. They share a highly similar structural foundation, operating deterministically by applying transition fractions (often referred to as transition ‘probabilities’) to simulation tool the temporal development of forest areas across aggregated, discrete states, such as age and volume classes. Furthermore, the simulation of management interventions, including thinnings and final fellings, is an established mechanism embedded across all these simulation tools to reflect planned activities and varied silvicultural practices. EFISCEN has been extensively and successfully employed for European-scale forest resource projections, assessing biomass availability, carbon sequestration, and policy impacts [32, 33, 64]. EFDM similarly functions as an area-based Markov model designed to integrate national-level information [39]. To estimate their transition fractions, both EFISCEN and EFDM typically rely on national forest inventory

(NFI) data, which structurally necessitates two consecutive inventories conducted at the same sample points using identical survey methodologies.

Compared to these contemporary large-scale simulation tools, FESIM exhibits both foundational commonalities and distinct divergences. While FESIM adopts the proven deterministic matrix approach for area dynamics, it distinguishes itself by tightly coupling this framework with stand-level growth curve modelling and a dedicated economic evaluation. While both EFISCEN and EFDM are powerful tools for conducting national-to-international roundwood yield projections and policy impact analyses, their primary focus remains firmly on biophysical forest development. Consequently, evaluating economic outcomes generally requires external post-processing. In contrast, the integrated design of FESIM facilitates the immediate assessment of policy scenarios, such as altered harvesting intensities or conservation set-asides, within a single simulation framework. This enables direct evaluation of not only resource volumes and carbon dynamics but also key economic metrics, such as contribution margins and net present values. Furthermore, the explicit, integrated calculation of biodiversity-oriented components, such as the retention of habitat trees and coarse woody debris, alongside these economic indicators further distinguishes FESIM from established large-scale biophysical simulation tools.

4.3 Scientific Novelty

The core scientific contribution of FESIM lies in its comprehensive formulation within the transparent and universally accessible framework of linear algebra. Unlike many established forestry simulators, which are available only as proprietary code, spreadsheet implementations, or sparingly documented algorithms, FESIM expresses all ecological and economic processes through explicitly defined vectors and matrices. This mathematical notation substantially improves reproducibility, facilitates review, and enables

straightforward extension or coupling with other (matrix-based) modelling frameworks. Building on this robust mathematical foundation, the following paragraphs provide a detailed examination of FESIM’s core architecture to elucidate the specific methodological innovations embedded in each component.

A central innovation is the area-module, which employs a block-structured tree species (group)-age-class transition matrix. Whereas most operational forest inventory and planning models treat tree species or species groups as independent strata, thereby precluding realistic transitions among tree species after disturbance or deliberate conversion, FESIM enables dynamic transitions among tree species groups within the same spatial unit. This capability is essential for evaluating long-term management strategies under climate change or shifting economic objectives.

FESIM further extends the range of silvicultural interventions that can be represented. By incorporating a comprehensive set of yield tables spanning the full spectrum from moderate thinning from below (‘mäßige Niederdurchforstung’) and staggered thinning from above (‘gestaffelte Hochdurchforstung’) to completely unthinned or unmanaged stands, the simulation tool allows users to explore the entire gradient of feasible management intensity and its consequences for stand structure, carbon storage, and economic key figures. Importantly, the implementation of a increment reduction functions (see app. [B.3](#)) enables dynamic transitions between different yield tables, thinning intensities and intervention timings, providing the flexibility to simulate adaptive silvicultural pathways that deviate from standard yield table trajectories.

Another significant advance is the explicit, dynamic simulation of ecological structural elements that are increasingly mandated by certification schemes and conservation-oriented legislation. Habitat trees and CWD are tracked as dedicated state variables. Notably, CWD pools can be computed both forward (via mortality and

decay) and backwards (to initialise realistic CWD goals from legislation). This bidirectional capability greatly enhances the simulation tool's utility for both prospective scenario analysis and retrospective impact assessment.

In the domain of carbon accounting, FESIM quantifies storage across three of the five IPCC-compliant compartments (aboveground biomass, belowground biomass, and CWD), also accounting for their interactions, and links these pools directly to economic valuation modules. The simulation tool thus permits the evaluation of 'carbon farming', including the simulation of payment schemes operating within, for example, the voluntary carbon market. By endogenising both the biophysical dynamics and the economic side of carbon storage, FESIM enables a consistent analysis of trade-offs and synergies among roundwood production, biodiversity conservation, and climate change mitigation.

In terms of practical implications, FESIM's lightweight input requirements and modular design make it highly adaptable. Students and researchers can readily use it to explore scenarios such as policy interventions (e.g., harvest restrictions or carbon pricing) or climate-induced changes in survival probabilities and growth rates. The simulation tool's deterministic nature, often a limitation in purely ecological terms, here serves to make scenario outcomes straightforward to interpret and compare, without conflating them with stochastic variability. Moreover, the closed-form growth functions used in FESIM (derived from standard yield tables and established growth models) ensure that the outputs remain consistent with traditional forestry expectations, lending credibility and transparency to the projections.

Collectively, these innovations position FESIM as a versatile, transparent, and policy-relevant tool that bridges the longstanding divide between detailed process understanding, operational forest management, and emerging demands for multi-objective, ecosystem services-oriented forest management assessment.

4.4 Research Outlook

Although FESIM already provides a coherent and operational framework for integrated ecological–economic forest-enterprise simulation, several scientifically valuable extensions can be envisioned to broaden its scope further.

A natural progression would be the transformation of FESIM into a combined enterprise- and sector-level simulation tool. By aggregating multiple forest enterprises (each parameterised with its own management objectives, cost structures, and constraints) and introducing regional or national roundwood supply functions, the framework could be used to analyse market feedback, policy-induced shifts in harvesting behaviour, and economy-wide carbon-sequestration potentials.

The current area module remains spatially implicit, treating enterprises as aggregated and independent units without neighbourhood interactions. Extending it to a spatially explicit raster- or vector-based representation would enable the simulation of edge effects, landscape-scale habitat connectivity, and spatially targeted conservation measures – features increasingly demanded in biodiversity-oriented forest planning and landscape-level policy evaluation.

The growth module presently relies on static yield tables or manual site-index adjustments. Although the current implementation already affords considerable methodological flexibility, enabling dynamic transitions among different yield tables, management intensities, and intervention timings via the incorporated increment-reduction function, the underlying baseline growth trajectories remain fundamentally static. A key objective for future development is the systematic integration of NFI data to move beyond these static assumptions. By calibrating climate-sensitive growth modifiers against repeated NFI measurements and tree-ring chronologies, future versions of FESIM could explicitly account for progressive ‘site drift’. Such an empirical

enhancement would allow the model to represent species-specific vulnerabilities to climate change—such as drought stress, late frost, or heat extremes within its existing flexible architecture.

In the economic module, roundwood prices are currently treated as exogenous parameters. Introducing price elasticities of supply and demand, possibly through a partial-equilibrium sub-model, would allow endogenous price formation and the projection of price-dependent harvesting volumes. This extension could be essential for a more credible assessment of policy instruments such as carbon pricing, harvest quotas, or subsidies for biodiversity-enhancing practices. Furthermore, it could assess the impact of calamity-induced market surpluses on roundwood prices.

The CWD pool could be refined by disaggregating it into standing and lying fractions and further stratifying it by decomposition classes analogous to those used in the NFI. Coupling decay rates to projected temperature and moisture regimes would improve both the accuracy of carbon accounting and the reliability of habitat-quality indicators linked to saproxylic biodiversity.

Although FESIM already tracks the aboveground and belowground biomass and CWD pools, it currently omits the litter and organic and mineral soil carbon compartments. Coupling the framework to an established soil-carbon model would close this gap, enabling complete Tier-3 IPCC-compliant reporting and more robust evaluation of long-term carbon-sequestration services under alternative management and climate scenarios.

Finally, while the deterministic structure of FESIM enhances transparency and facilitates scenario comparison, many real-world decisions are made under considerable uncertainty. Introducing Monte Carlo or quasi-stochastic disturbance regimes (windthrow, bark-beetle outbreaks, wildfire) and price volatility would permit quantification of Value-at-Risk, option values for adaptive management, and the robustness of strategies across plausible futures.

Pursuing these extensions would position FESIM as an even more versatile platform capable of addressing emerging research and policy questions at the interface of sustainable forest management, climate-change mitigation, and biodiversity conservation under deep uncertainty.

5 Conclusions

FESIM has been developed as a flexible and comprehensive forest simulation tool that integrates core aspects of forest dynamics and economics into a single, scenario-driven toolkit. Addressing the longstanding challenge of integrating ecological complexity with economic analysis, FESIM enables users to assess how current silvicultural decisions unfold over decades of forest development and affect key economic metrics. By grounding the entire simulation tool within a generic linear algebra framework, FESIM achieves a high degree of mathematical transparency and methodological traceability. Rather than relying on opaque algorithms or complex proprietary code, the model integrates the Markov area transition approach, parametric growth functions, and multi-tier economic valuation exclusively through explicit matrix and vector operations. Ultimately, the FESIM framework empowers forestry stakeholders and policymakers to navigate the complex trade-offs inherent in contemporary forest management. By systematically quantifying and visualising the competing demands of roundwood production, carbon storage, and biodiversity conservation, the simulation tool facilitates robust, evidence-based decision-making aligned with multifaceted environmental and economic policy targets.

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Appendix A Detailed Matrix Formulations

A.1 Area Module

A.1.1 Final harvest and hazard matrix

The hazard matrix $\mathbf{H}$ and the final harvest matrix $\mathbf{E}$ model the retention and failure dynamics of age classes within each tree species group. Both are stochastic matrices of dimension $L \times L$, where $L = \sum_{s=1}^S K_s$, with S denoting the number of tree species groups and K_s the number of age classes for tree species group s . Each matrix consists of $S \times S$ blocks, where the diagonal blocks $\mathbf{H}_{s,s}$ or $\mathbf{E}_{s,s}$ (of size $K_s \times K_s$) govern transitions within tree species groups s , and all off-diagonal blocks $\mathbf{H}_{s,s'}$ or $\mathbf{E}_{s,s'}$ for $s \neq s'$ are zero matrices of size $K_s \times K_{s'}$.

The diagonal blocks $\mathbf{H}_{s,s}$ (or $\mathbf{E}_{s,s}$) are defined as:

$$(\mathbf{H}_{s,s})_{ij} = \begin{cases} 1 - hz_{s,j} & \text{if } i = j, i = 1, \dots, K_s - 1, \\ hz_{s,K_s} & \text{if } i = K_s, j = 1, \dots, K_s, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A1})$$

where $1 - hz_{s,j} \in [0, 1]$ is the transition probability (additive complementary probability to the failure probability) of the tree species group s and age class j advancing one age class under hazard conditions (e.g., surviving without disturbance), whereas $hz_{s,j} \in [0, 1]$ is the failure probability. For the final harvesting matrix $\mathbf{E}$, the diagonal blocks are defined analogously, with entries $1 - fh_{s,j} \in [0, 1]$ representing the probability of advancing on age class j , and fh_{s,K_s} the failure probability (e.g., due to replanting after harvesting). All off-diagonal blocks are zero.

For illustration, consider a system with $S = 2$ tree species groups, each with $K_s = 5$ for species 1 and $K_s = 4$ for species 2 different age classes. Let the transition probabilities for $\mathbf{H}$ be $hz_{1,1} = 1$, $hz_{1,2} = 0.9$, $hz_{1,3} = 0.8$, $hz_{1,4} = 0.7$, $hz_{1,5} = 1$,

$hz_{2,1} = 1, hz_{2,2} = 0.8, hz_{2,3} = 0.7, hz_{2,4} = 1$. The hazard matrix $\mathbf{H}$ is:

$$\mathbf{H} = \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.2 & 0.3 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.3 & 1 \end{array} \right]. \quad (\text{A2})$$

The final harvesting matrix $\mathbf{E}$ has an identical structure, with transition probabilities $1 - fh_{s,i}$ replacing $1 - hz_{s,i}$. In a system with $S = 2$ tree species groups, each with $K_s = 5$ for species 1 and $K_s = 4$ for species 2 different age classes, let the transition probabilities for $\mathbf{E}$ be $fh_{1,1} = 1, fh_{1,2} = 0.9, fh_{1,3} = 0.8, fh_{1,4} = 0.7, fh_{1,5} = 1, fh_{2,1} = 1, fh_{2,2} = 0.8, fh_{2,3} = 0.8, fh_{2,4} = 1$. The final harvesting matrix $\mathbf{E}$ is:

$$\mathbf{E} = \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.2 & 0.3 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 1 \end{array} \right]. \quad (\text{A3})$$

Both matrices are stochastic, with each column summing to 1. Please note that, in the context of unmanaged forests, the end-use matrix must not utilise minimum, median, or maximum rotation periods. Instead, the characteristic maximum age of the respective tree species group must be applied (see Appendix B.1.2 for more information).

A.1.2 Cyclical shift-down Matrix

The cyclic shift-down matrix $\mathbf{C}$ models the progression of age classes for each tree species group in a forest aggregate or enterprise. Like $\mathbf{T}$ it is a block-diagonal matrix of dimension $L \times L$ (see eq. 1 and 2), with S denoting the number of tree species group and K_s the number of age classes for tree species group s . The matrix is defined as follows:

$$\mathbf{C} = \text{diag}(\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_S), \quad (\text{A4})$$

where $\mathbf{C}_s$ is the $K_s \times K_s$ cyclic shift matrix for tree species group s , with elements:

$$(\mathbf{C}_s)_{ij} = \begin{cases} 1 & \text{if } i = j + 1, \quad j = 1, \dots, K_s - 1, \\ 1 & \text{if } j = K_s \text{ and } i = 1, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A5})$$

All off-diagonal blocks $\mathbf{C}_{s,s'}$ for $s \neq s'$ are zero matrices of size $K_s \times K_{s'}$. Each diagonal block $\mathbf{C}_s$ shifts the age classes of tree species groups s forward by one position (from age class j to $j + 1$), with the final age class ($j = K_s$) cycling back to the first age class ($j = 1$), representing, for example, reforestation after harvesting.

For illustration, consider a system with $S = 2$ tree species groups, each with $K_s = 5$ for species 1 and $K_s = 4$ for species 2 different age classes. The matrix $\mathbf{C}$ is:

$$\mathbf{C} = \left[\begin{array}{ccccc|cccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]. \quad (\text{A6})$$

This structure ensures that $\mathbf{C}$ is a stochastic matrix, with each column summing to 1.

A.1.3 Species transition matrix

The species transition matrix $\mathbf{W}$ models the dynamics of tree species group transitions in our model framework. It is a stochastic matrix of dimension $L \times L$ (see eq. 1 and 2). The matrix $\mathbf{W}$ consists of $S \times S$ blocks, where each block $\mathbf{W}_{s,s'}$ is of size $K_s \times K_{s'}$. The diagonal blocks $\mathbf{W}_{s,s}$ govern the age progression within the tree species group s , while specific off-diagonal blocks $\mathbf{W}_{s,s'}$ for $s \neq s'$ model transitions between tree species groups at the first age class.

The diagonal blocks $\mathbf{W}_{s,s}$ are defined as:

$$(\mathbf{W}_{s,s})_{ij} = \begin{cases} w_{s,s} & \text{if } i = 1, j = 1, \\ 1 & \text{if } i = j, i = 2, \dots, K_s, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A7})$$

where $w_{s,s} \in [0, 1]$ is the probability of remaining in the first age class of tree species groups s . The off-diagonal blocks $\mathbf{W}_{s,s'}$ for $s \neq s'$ are defined as:

$$(\mathbf{W}_{s,s'})_{ij} = \begin{cases} w_{s,s'} & \text{if } i = 1, j = 1, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A8})$$

where $w_{s,s'} \in [0, 1]$ is the probability of transitioning from the first age class of tree species group s to the first age class of tree species group s' . To ensure that $\mathbf{W}$ is stochastic, the probabilities satisfy $\sum_{s'=1}^S w_{s,s'} = 1$ for each tree species group s .

For illustration, consider a system with $S = 2$ tree species groups, each with $K_s = 5$ for species 1 and $K_s = 4$ for species 2 different age classes. Let the transition

probabilities be $w_{1,1} = 0.6$, $w_{1,2} = 0.3$, $w_{2,1} = 0.4$, and $w_{2,2} = 0.7$. The matrix $\mathbf{W}$ is:

$$\mathbf{W} = \left[\begin{array}{ccccc|ccccc} 0.6 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0.4 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]. \quad (\text{A9})$$

This structure ensures that $\mathbf{W}$ is a stochastic matrix, with each column summing to 1.

A.2 Growth Module

A.2.1 Unmanaged forests

The unmanaged forest variant of the *Growth Module* constitutes a special case of the *Growth Module* with a few minor variations. As with managed forests, the discrete values of parameterised sigmoidal functions are used here (see appendix B.2). For the unmanaged forest *Growth Module* variant, unthinned/unmanaged yield tables (known as ‘A-grade’) are used, as the growth functions are derived from unthinned control plots and sample stands according to Pretzsch et al. [41]. As in the managed forest *Growth Module*, the unmanaged variant aims to steer stand structure toward the long-term maximum density or stocking. This maximum density or stocking is expressed as a vector of maximum stocking degrees, denoted $\mathbf{sdg}_{s,n}^{\text{yt}}$, where each component corresponds to a given tree species–age class, representing full stocking according to

a reference unmanaged yield table. Equations 6, 7, 8, 9, and 10 are the same as in the managed *Growth Module*. Only the two equations 11 and 12 differ, as there is no planned thinning in unmanaged stands. Instead, there is density-dependent mortality. The exact mechanism is described in Regelmann et al. [44]; here is a summary of the formula for density-dependent mortality $\mathbf{ddm}_{s,n}^{\text{sc}}$ and the update to $\mathbf{gsv}_{s,n+1}^{\text{sc}}$:

$$\mathbf{ddm}_{s,n}^{\text{sc}} = \frac{|\mathbf{sai}_{s,n}^{\text{sc}} - \mathbf{gsv}_{s,n}^{\text{yt}}| + (\mathbf{sai}_{s,n}^{\text{sc}} - \mathbf{gsv}_{s,n}^{\text{yt}})}{2}. \quad (\text{A10})$$

Like the thinning in the managed forest *Growth Module* in eq. 11, the absolute value of eq. A10 is applied element-wise. The update to the subsequent simulation period of the growing stock volumes $\mathbf{gsv}_{s,n+1}^{\text{sc}}$ is performed analogously to eq. 12 as follows:

$$\mathbf{gsv}_{s,n+1}^{\text{sc}} = \mathbf{sai}_{s,n}^{\text{sc}} - \mathbf{ddm}_{s,n}^{\text{sc}}. \quad (\text{A11})$$

This updated vector serves as the initial state for the subsequent simulation step (resuming at eq. 6, 7, 8, 9 and 10), closing the recursive loop of the unmanaged forest *Growth Module*.

A.3 Valuation Module

A.3.1 Pricing model

Building on the framework of Fuchs et al. [17], it converts tree species group-specific diameter at breast height (DBH)—provided by the *Growth Module*—into net roundwood prices. These prices are derived based on five key exogenous parameter inputs: roundwood quality class $rqc_{s,n}^{\text{sc}}$, harvesting system $hs_{s,n}^{\text{sc}}$, cost level $hcl_{s,n}^{\text{sc}}$, and reference assortment price $asp_{s,n}^{\text{sc}}$ and tree species group-specific diameter at breast height provided by the vector $\mathbf{dg}_{s,n}^{\text{sc}} \in \mathbb{R}^{K_s}$ in the FESIM framework for scenario ‘sc’ tree species group s , simulation time step n and age-class k_s . Please note that a different

$\mathbf{dg}_{s,n}^{\text{sc}}$ is applied to the outgoing stock (thinning amounts) compared to the remaining stock (final harvest and calamity harvest).

$$\mathbf{nrp}_{s,n}^{\text{sc}} = \text{nrp}(\mathbf{dg}_{s,n}^{\text{sc}}, hs_{s,n}^{\text{sc}}, rqc_{s,n}^{\text{sc}}, hcl_{s,n}^{\text{sc}}, asp_{s,n}^{\text{sc}}). \quad (\text{A12})$$

The $\text{nrp}()$ function results in net roundwood prices i.e., stumpage prices (net of harvesting and forwarding costs) $\mathbf{nrp}_{s,n}^{\text{sc}} \in \mathbb{R}^{K_s}$. Reference assortments follow the German RVR system (*Rahmenvereinbarung für den Rohholzhandel in Deutschland* [52]), with tree species group-specific assignments as specified in tab. C11 in the appendix.

A.3.2 Aggregation and post-processing

To enable tree species group-specific processing, the state vectors from the *Area Module* $\mathbf{a}_n \in \mathbb{R}^L$ (see sec. 3.1 and eq. 3) are represented as a continuous block vector composed of tree species group-wise sub-vectors $\mathbf{a}_{s,n} \in \mathbb{R}^{K_s}$. Specifically,

$$\mathbf{a}_n = \begin{bmatrix} \mathbf{a}_{1,n} \\ \mathbf{a}_{2,n} \\ \vdots \\ \mathbf{a}_{S,n} \end{bmatrix}, \quad \text{with } \mathbf{a}_{s,n} \in \mathbb{R}^{K_s}. \quad (\text{A13})$$

Each sub-vector $\mathbf{a}_{s,n}$ contains the age-class elements k_s of tree species group s at time step n , i.e.,

$$\mathbf{a}_{s,n} = \begin{bmatrix} a_{s,n,1} \\ a_{s,n,2} \\ \vdots \\ a_{s,n,K_s} \end{bmatrix}. \quad (\text{A14})$$

Hence, the full state vector $\mathbf{a}_n$ can be expressed as the column-wise concatenation of all tree species group-specific state vectors,

$$\mathbf{a}_n = \text{col}(\mathbf{a}_{1,n}, \dots, \mathbf{a}_{S,n}), \quad (\text{A15})$$

ensuring a contiguous and tree species group-ordered block structure consistent with the ordering of the transition matrix.

The two vectors $\mathbf{hz}_{s,n} \in \mathbb{R}^{K_s}$ and $\mathbf{fh}_{s,n} \in \mathbb{R}^{K_s}$ contain the failure probabilities for tree species group s across age-class elements $k_s = 1, \dots, K_s$. Since the FESIM framework assigns distinct prices to different utilisation types—(unplanned) calamity harvesting, (planned) final harvesting, and thinning—it is necessary to extract intermediate results for each type separately. To this end, FESIM leverages the failure probabilities as follows: the probability vectors $\mathbf{hz}_{s,n}$ and $\mathbf{fh}_{s,n}$ are multiplied element-wise (please note that we use ‘ $\odot$ ’ as the Hadamard operator in the following section) with the state vector of area projections $\mathbf{a}_{s,n-1}$ from the previous time step (for scenario sc , tree species group s , and time step n). The calculation of the area fractions $\mathbf{a}_{s,n}^{\text{hz}}$ exiting the system due to calamity events is as follows:

$$\mathbf{a}_{s,n}^{\text{hz}} = \mathbf{hz}_{s,n} \odot \mathbf{a}_{s,n-1}. \quad (\text{A16})$$

The stumpage value $\mathbf{sv}_{s,n}^{\text{hz}}$ for the salvage roundwood is then calculated by multiplying the hazard area $\mathbf{a}_{s,n}^{\text{hz}}$ element-wise with the growing stock volume $\mathbf{gsv}_{s,n}$ and the net roundwood prices $\mathbf{nrp}_{s,n}$, as well as applying a calamity-related price markdown factor $\text{pmf}_{s,n}$ as following:

$$\mathbf{sv}_{s,n}^{\text{hz}} = \mathbf{a}_{s,n}^{\text{hz}} \odot \mathbf{gsv}_{s,n} \odot \mathbf{nrp}_{s,n} \cdot \text{pmf}_{s,n}. \quad (\text{A17})$$

The remaining area $\mathbf{a}_{s,n}^{\text{fh}}$, which is already affected by (unplanned) calamity-induced salvage utilisation, is then used for (pre-planned) final harvesting:

$$\mathbf{a}_{s,n}^{\text{fh}} = [\mathbf{a}_{s,n-1} - (\mathbf{hz}_{s,n} \odot \mathbf{a}_{s,n-1})] \odot \mathbf{fh}_{s,n}. \quad (\text{A18})$$

Subsequently the stumpage value $\mathbf{sv}_{s,n}^{\text{fh}}$ for pre-planned final harvesting is derived by:

$$\mathbf{sv}_{s,n}^{\text{fh}} = \mathbf{a}_{s,n}^{\text{fh}} \odot \mathbf{gsv}_{s,n} \odot \mathbf{nrp}_{s,n}. \quad (\text{A19})$$

Thinning operations are applied across the entire projected area. Consequently, the stumpage value is calculated as follows:

$$\mathbf{sv}_{s,n}^{\text{th}} = \mathbf{a}_{s,n} \odot \mathbf{thv}_{s,n} \odot \mathbf{nrp}_{s,n}. \quad (\text{A20})$$

Finally, we derive our total stumpage value $\mathbf{sv}_{s,n}^{\text{ttl}}$ as a sum of the revenues from calamity-induced hazards, pre-planned final harvesting and thinning:

$$\mathbf{sv}_{s,n}^{\text{ttl}} = \mathbf{sv}_{s,n}^{\text{hz}} + \mathbf{sv}_{s,n}^{\text{fh}} + \mathbf{sv}_{s,n}^{\text{th}}. \quad (\text{A21})$$

The stumpage value vector, denoted as $\mathbf{sv}_{s,n}^{\text{ttl}} \in \mathbb{R}^{K_s}$, approximately corresponds to the *Contribution Margin I* (see [A.3.3](#)). It should be noted that the stumpage value and *Contribution Margin I* are not identical. The latter excludes, e.g., the entrepreneurial profit (and any return on the entrepreneur's own resources) and therefore represents only a proxy variable (or surrogate measure) for the true stumpage value.

To derive the regeneration costs, please note that in the first age class ($k_s = 1$), immediate regeneration is assumed, whereas at age 20 ($k_s = 4$), pre-commercial thinning is conducted. Regeneration costs vary with the cause of stand removal and

with whether tree species group composition is retained or altered. Three cases are distinguished:

- i) post-final harvest regeneration with tree species group retention (see eq. A22),
- ii) post-final harvest regeneration with tree species group change (see eq. A23), and
- iii) post-disturbance regeneration after hazard events (see eq. A24).

The species transition matrix $\mathbf{W}^{\text{ord}}$ is used to separate cases (i) and (ii), as its diagonal elements represent retention probabilities and off-diagonal elements capture transition probabilities to alternative tree species groups. To calculate the tree species group specific planting costs $\mathbf{cp}_s$, the corresponding formulations are given as follows:

$$\mathbf{cp}_s^{\text{fh,ret}} = [(\mathbf{W}^{\text{ord}} \odot \mathbf{I}) \mathbf{G}_{s,\ell}^{\text{cp}} \mathbf{a}_n^{\text{fh}}] \cdot \text{cpu}_s^{\text{fh,ret}}, \quad (\text{A22})$$

$$\mathbf{cp}_s^{\text{fh,chan}} = [(\mathbf{W}^{\text{ord}} - (\mathbf{W}^{\text{ord}} \odot \mathbf{I})) \mathbf{G}_{s,\ell}^{\text{cp}} \mathbf{a}_n^{\text{fh}}] \cdot \text{cpu}_s^{\text{fh,chan}}, \quad (\text{A23})$$

$$\mathbf{cp}_s^{\text{hz}} = [\mathbf{W}^{\text{ord}} \mathbf{G}_{s,\ell}^{\text{cp}} \mathbf{a}_n^{\text{hz}}] \cdot \text{cpu}_s^{\text{hz}}. \quad (\text{A24})$$

Here, $\mathbf{a}_n^{\text{en}}$ and $\mathbf{a}_n^{\text{hz}}$ denote the state vectors of areas affected by (pre-planned) final harvest and calamity, respectively. $\mathbf{G}_{s,\ell}^{\text{cp}} \in \mathbb{R}^{S \times L}$ represents the tree species group- and age-class-specific aggregation matrix, whereas $\mathbf{W}^{\text{ord}} \in \mathbb{R}^{S \times S}$ is the ordinary species transition matrix (different then in sec. 3.1), and $\mathbf{I} \in \mathbb{R}^{S \times S}$ is the identity matrix ensuring that only transitions with or without tree species group change are considered, whereas cpu_s are the tree species group-specific unit planting costs. $\mathbf{G}_{s,\ell}^{\text{cp}} \in \mathbb{R}^{S \times L}$ as an example with $S = 3$, $K_s = 4$ and $L = 12$, is structured as follows:

$$\mathbf{G}_{s,\ell}^{\text{cp}} = \left[\begin{array}{cccc|cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right]. \quad (\text{A25})$$

To calculate the pre-commercial thinning costs $\mathbf{cth}_s$, the respective cost component must be extracted, e.g., for the second age class of each tree species group and multiplied accordingly:

$$\mathbf{cth}_s = [\mathbf{G}_{s,\ell}^{\text{cth}} \mathbf{a}_n] \cdot ctu_s \quad (\text{A26})$$

where $\mathbf{G}_{s,\ell}^{\text{cth}}$ denotes the aggregation-matrix of the thinning costs. As an example for pre-commercial thinning at age-class 2, with $S = 3$, $K_s = 4$ and $L = 12$, $\mathbf{G}_{s,\ell}^{\text{cth}}$ is structured as follows:

$$\mathbf{G}_{s,\ell}^{\text{cth}} = \left[\begin{array}{cccc|cccc|cccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array} \right]. \quad (\text{A27})$$

Subsequently, the planting costs $\mathbf{cp}_s$ and pre-commercial thinning costs $\mathbf{cth}_s$ subtracted from the stumpage values $\mathbf{sv}_s$ to derive the silvicultural contribution margin ($\mathbf{scm}_s$; corresponding to *Contribution Margin II*).

A.3.3 Economic valuation

The valuation module of FESIM employs *contribution margin analysis* (‘Deckungsbeitragsrechnung’), a widely established cost accounting method in German-speaking forestry (the DACH region: Germany, Austria, and Switzerland) [57]. This approach decomposes enterprise revenues and costs into successive stages of contribution margins (DB/CM I–V) [60]. Each stage allocates additional cost categories to roundwood revenues, thereby providing a stepwise assessment of profitability that is particularly suited to forestry enterprises with long production horizons and heterogeneous cost structures [3, 60].

Although the terminology of ‘Deckungsbeitrag I–V’ is specific to the German-speaking context [57], it corresponds conceptually to internationally recognised

measures such as *contribution margin*, *gross margin*, *segment margin*, and *net return*. For example, DB I (*erntekostenfreier Holzerlös*) represents roundwood revenue net of variable harvesting costs [57] and roughly corresponds to the internationally used concept of *stumpage value* (see page 44). It is imperative to acknowledge that DB I and *stumpage value* should not be regarded as identical, as they do not incorporate the entrepreneur's profit, among other discrepancies. However, they frequently exhibit a comparable magnitude. Subsequent contribution margins (DB II–IV) gradually incorporate silvicultural fixed costs, product-group-specific costs, and area-level overheads. Finally, DB V (*Reinertrag*) corresponds to the *net return* of the forest enterprise after deduction of administrative overheads [60].

This stepwise valuation enables the model to reflect both the short-term profitability of roundwood harvesting operations (via DB I) and the long-term economic sustainability of forest management (via DB V) [60]. For international readers, the system can thus be understood as a structured application of contribution margin accounting adapted to the institutional and cost environment of Central European forestry.

Table A1 Stages of contribution margin accounting in FESIM's Valuation Module and their international equivalents.

German term (DB)	Forestry-specific interpretation	International equivalent
DB I (<i>erntekostenfreier Holzerlös</i>)	Timber revenues minus variable harvesting costs (felling, extraction, transportation).	Contribution margin / gross margin; often expressed as <i>stumpage value</i> .
DB II (<i>waldbaulicher Deckungsbeitrag</i>)	DB I minus silvicultural fixed costs (regeneration, planting, tending).	Contribution margin after product-specific fixed costs; <i>net returns after silvicultural costs</i> .
DB III	DB II minus product-group-specific fixed costs (e.g., road construction, infrastructure attributable to product groups).	Segment margin (after allocation of group-specific fixed costs).
DB IV	DB III minus area-level fixed costs (e.g., divisional overheads, machinery pools).	Division margin / operating margin.
DB V (<i>Reinertrag</i>)	DB IV minus administrative and general fixed costs (overheads); net operating surplus of the enterprise.	Net return / net operating income (before taxes and financing).

Appendix B Underlying Functions

B.1 Area module

In this section, we describe the functions used to model default probabilities arising from calamities and (pre-planned) final harvesting. These functions, based on the Weibull, Log-Norm and modified power form of the beta distribution, provide a framework for quantifying failure probabilities for specific tree species group s under Representative Concentration Pathway (RCP) scenarios r .

- For **most tree species groups** and calamity-related hazards, the Weibull distribution is utilised due to its flexibility in representing a wide range of failure behaviours.
- For **pine tree species group**, the log-normal distribution is a more appropriate fit, given its right-skewed nature and better compatibility with the data compared to the Weibull distribution.
- For **final harvesting** (e.g., pre-planned harvesting), the modified power form of the beta distribution provides a suitable approximation for pre-planned final harvesting, given that not all of the stand is to be removed simultaneously. This should model current silvicultural strategies, in which the aim is to achieve an uneven-aged age-class structure by implementing extended regeneration phases while retaining the old stand.

B.1.1 Calamity induced hazard functions

Equation (B28) presents the hazard function $hz_{rs}(t)$ used for all tree species groups except the pine group. The input variable t denotes stand age (in years). The parameters $\lambda_{rs} > 0$ and $k_{rs} > 0$ represent the scale and shape parameters of a Weibull-type hazard model, calibrated under RCP emission scenario r and for tree species group

s. Calibration is based on empirical disturbance data derived from the pan-European ICP-Forests dataset, which was evaluated and parameterised by Brandl et al. [5] using bioclimatic co-variables from the WorldClim 1.4 dataset [19], following the methodological framework of Fuchs et al. [16]. The selected climate co-variables include: annual mean temperature, maximum temperature of the warmest month, minimum temperature of the coldest month, annual precipitation, and precipitation of the warmest quarter.

$$hz_{rs}(t) = \begin{cases} 1 - \exp \left[\left(\frac{t-w-t_0}{\lambda_{rs}} \right)^{k_{rs}} - \left(\frac{t-t_0}{\lambda_{rs}} \right)^{k_{rs}} \right], & \text{if } t - w - t_0 \geq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{B28})$$

where $w = 5$ (age-class width, in years) and $t_0 = 20$ (failure-free period, in years) are fixed constants. This formulation enables tree species group- and scenario-specific projections of disturbance risk, reflecting the influence of varying climatic conditions across different RCP pathways.

For the pine species group, which exhibits right-skewed failure behaviour, a log-normal hazard model was found to provide a better fit [5]. The corresponding hazard function $hz_{rs}(t)$ is given in Equation (B29):

$$hz_{rs}(t) = \begin{cases} \frac{\text{erf}(u_1(t)) - \text{erf}(u_2(t))}{1 - \text{erf}(u_2(t))}, & \text{if } t - w - t_0 \geq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (\text{B29})$$

where

$$u_1(t) = \frac{\ln(t - t_0) - \mu_{rs}}{\sigma_{rs}\sqrt{2}}, \quad u_2(t) = \frac{\ln(t - w - t_0) - \mu_{rs}}{\sigma_{rs}\sqrt{2}}, \quad \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-v^2} dv.$$

The constants $w = 5$ (age-class width, in years) and $t_0 = 20$ (failure-free period, in years) are fixed across all tree species groups and scenarios. The function $\text{erf}(x)$ denotes the Gaussian error function with the integration variable v . The input variable t denotes stand age (in years). The parameters μ_{rs} and σ_{rs} represent the mean and standard deviation of the underlying log-normal distribution, calibrated for RCP scenario r and tree species group s (= pine). Like in the Weibull-type function, the calibration is based on empirical IPC disturbance data from Brandl et al. [5] and bioclimatic variables from the WorldClim 1.4 dataset [19].

B.1.2 Final harvesting functions

Mathematically, the instantaneous failure rate derived from a parametric distribution can be regarded as a hazard rate, regardless of whether the underlying process is due to a disturbance/calamity or a planned final harvest. However, to avoid semantic confusion—since the term *hazard* is commonly associated with disturbance-induced failure—we hereafter refer to the corresponding failure function for planned final harvesting as the *final harvest probability* $fh_{rs}(t)$. The final harvest probability $fh_{rs}(t)$ is modelled using a modified power form of the beta distribution. This functional form is designed to represent gradual or staggered final harvesting, reflecting contemporary silvicultural practice in Central Europe, where clear-cutting has largely been abandoned. A key advantage of this approach is that it can be fully parameterised using only three intuitive descriptive statistics: the minimum harvest age $\min_{rs}$, the median harvest age $\tilde{x}_{rs}$, and the maximum harvest age $\max_{rs}$. From these, the shape parameter α_{rs} is uniquely determined, eliminating the need for additional assumptions—such as boundary slopes or extra quantiles—and avoiding unnecessary degrees of freedom.

The values $\min_{rs}$, $\tilde{x}_{rs}$, and $\max_{rs}$ are typically derived from national silvicultural guidelines or expert assessments, such as those developed for the WEHAM (‘Waldentwicklungs- und Holzaufkommensmodellierung’ in english: ‘forest development and timber resource modelling’) scenario framework [4]. The final harvesting

probability under management scenario r and for tree species group s is defined as:

$$fh_{rs}(t) = \begin{cases} \frac{u_1(t)^{\alpha_{rs}} - u_2(t)^{\alpha_{rs}}}{1 - u_2(t)^{\alpha_{rs}}}, & \text{if } \min_{rs} \leq t \leq \max_{rs}, \\ 0, & \text{if } t < \min_{rs}, \\ 1, & \text{if } t > \max_{rs}, \end{cases} \quad (\text{B30})$$

where

$$u_1(t) = \frac{\max_{rs} - t}{\max_{rs} - \min_{rs}}, \quad u_2(t) = \frac{\max_{rs} - (t - w)}{\max_{rs} - \min_{rs}}, \quad \alpha_{rs} = \frac{\ln(0.5)}{\ln\left(\frac{\max_{rs} - \tilde{x}_{rs}}{\max_{rs} - \min_{rs}}\right)}.$$

This formulation (eq. B30) ensures bounded support on $[\min_{rs}, \max_{rs}]$, smooth and continuous probability and density functions, and no probability mass outside the feasible age range. It requires only three readily available inputs, is numerically stable (relying solely on powers and logarithms), and offers flexible tail behaviour through the parameter α_{rs} . Rooted in the principle of maximum entropy, this approach has been widely used in PERT (Program Evaluation and Review Technique) and Monte Carlo applications. It provides a robust alternative to standard, piecewise-linear, or spline-based methods—particularly when data are limited but policy-relevant harvesting schedules must be specified.

B.2 Growth Module

The *Growth Module* (see sec. 3.2) of FESIM relies on parameterised yield tables to allow for smooth interpolation and extrapolation of growth data. This functional representation enables the derivation of stand-level growth trajectories beyond the range of empirical observations—a capability essential for long-term projections and scenario analyses. However, reliable extrapolation requires growth functions that adequately

capture the asymptotic and sigmoidal dynamics characteristic of forest stand development [8, 59]. To this end, FESIM employs three well-established functions: the Sloboda, Hossfeld IV, and Chapman–Richards functions, all of which exhibit flexible sigmoidal shapes capable of representing a wide range of site indices and growth conditions [8].

Each function is defined by two primary inputs: time t , representing stand age or age class, and site index θ , which characterises site productivity. This functional form enables the generation of all yield table metrics using only a small set of parameters. The first derivative with respect to time, $\partial f / \partial t$, yields the current annual increment—a key quantity for growth modelling. Moreover, the partial derivative with respect to site index, $\partial f / \partial \theta$, can be used to assess how growth responds to changes in site productivity, such as those induced by climate change or long-term shifts in site quality. All three functions can also be inverted with respect to θ , thereby allowing the site index to be estimated from observed age-height combinations.

These functions are used to model core yield table metrics, including mean height, top height, growing stock volume, and total volume production. Additional metrics, such as current annual increment, are derived directly from the time derivatives of the functions. Growth-related metrics are parameterised in the same way as the height models, following Staupendahl [61]. The specific parameter values are provided in the Appendix. The functional forms and their analytical derivatives are presented below.

B.2.1 Sloboda function

The Sloboda function is defined for each combination of growth-related metric m (e.g., top height, growing stock volume) and tree species group s as

$$f_{s,m}(t, \theta) = k^{a_{s,m}} \left(\frac{\theta}{k^{a_{s,m}}} \right)^{y_{s,m}(t)}. \quad (\text{B31})$$

where t denotes stand age, θ is the site index, $k = 65$ is a scaling constant (from the original publication according to Sloboda [58]), and $a_{s,m}$, $b_{s,m}$, $c_{s,m}$ are parameters

specific to growth-related metric m and tree species group s . The exponent $y_{s,m}(t)$ is given by

$$y_{s,m}(t) = \exp \left[\frac{b_{s,m}}{(c_{s,m} - 1) t^{c_{s,m}-1}} - \frac{b_{s,m}}{(c_{s,m} - 1) t_\theta^{c_{s,m}-1}} \right], \quad (\text{B32})$$

with t_θ denoting the reference age at which the site index is defined.

The current annual increment is obtained as the partial derivative of $f_{s,m}$ with respect to time t :

$$\frac{\partial f_{s,m}(t, \theta)}{\partial t} = -b_{s,m} t^{-c_{s,m}} f_{s,m}(t, \theta) \ln \left(\frac{f_{s,m}(t, \theta)}{k^{a_{s,m}}} \right). \quad (\text{B33})$$

To assess the sensitivity of yield to changes in site productivity (e.g., due to climate change), the partial derivative with respect to site index is

$$\frac{\partial f_{s,m}(t, \theta)}{\partial \theta} = y_{s,m}(t) \left(\frac{\theta}{k^{a_{s,m}}} \right)^{y_{s,m}(t)-1} \cdot \frac{1}{k^{a_{s,m}}}, \quad (\text{B34})$$

$$= y_{s,m}(t) \frac{f_{s,m}(t, \theta)}{\theta}. \quad (\text{B35})$$

This expression shows that the sensitivity of f to changes in θ is proportional to both the function value itself and the exponent $y_{s,m}(t, \theta)$.

The Sloboda function is strictly monotonic with respect to the site index $\theta > 0$ for positive parameters $a_{s,m}, b_{s,m}, c_{s,m}$. Consequently, it can be inverted to express θ as a function of observed values $f_{s,m}^{\text{obs}}$ and age t (see Smaltschinski [59]).

Solving for θ , we obtain

$$\theta = k^{a_{s,m}} \left(\frac{f_{s,m}^{\text{obs}}}{k^{a_{s,m}}} \right)^{1/y_{s,m}(t)}. \quad (\text{B36})$$

This invertibility allows direct site index estimation θ , where $f_{s,m}^{\text{obs}}$ denotes the observed value of a growth-related metric (e.g., growing stock volume) at stand age t , to assign a site index based on empirical data.

B.2.2 Hossfeld IV function

The Hossfeld IV function is defined as

$$f_{s,m}(t, \theta) = \theta \left(\frac{t}{t_\theta} \right)^{b_{s,m}} \frac{a_{s,m} + t_\theta^{b_{s,m}}}{a_{s,m} + t^{b_{s,m}}}, \quad (\text{B37})$$

where t denotes stand age, θ is the site index (i.e., the predicted value of the growth-related metric at reference age t_θ), and $a_{s,m}$, $b_{s,m}$ are parameters specific to growth-related metric m and tree species group s . The reference age t_θ is fixed by convention (e.g., 100 years for conifers in Central Europe) and defines the age at which the site index θ is standardised.

The partial derivative with respect to stand age t is given by

$$\frac{\partial f_{s,m}(t, \theta)}{\partial t} = \theta b_{s,m} t^{b_{s,m}-1} \frac{a_{s,m} (a_{s,m} + t_\theta^{b_{s,m}})}{(a_{s,m} + t^{b_{s,m}})^2}, \quad (\text{B38})$$

and the partial derivative with respect to the site index θ is

$$\frac{\partial f_{s,m}(t, \theta)}{\partial \theta} = \left(\frac{t}{t_\theta} \right)^{b_{s,m}} \frac{a_{s,m} + t_\theta^{b_{s,m}}}{a_{s,m} + t^{b_{s,m}}}. \quad (\text{B39})$$

Because the function is linear in $\theta > 0$ for positive parameters, it can be explicitly inverted to estimate the site index from an observed value $f_{s,m}^{\text{obs}}$ of the growth-related metric and stand age t :

$$\theta = \frac{f_{s,m}^{\text{obs}} (a_{s,m} + t^{b_{s,m}})}{(t/t_\theta)^{b_{s,m}} (a_{s,m} + t_\theta^{b_{s,m}})}. \quad (\text{B40})$$

This provides a direct method for deriving site index from empirical measurements of stand growth.

B.2.3 Chapman–Richards-type function

The Chapman–Richards-type function from Pretzsch et al. [41] is defined as

$$f_{s,m}(t, \theta) = a_{s,m} \theta^{b_{s,m}} \left(1 - \exp \left(- (c_{s,m} t)^{d_{s,m}} \right) \right), \quad (\text{B41})$$

where t denotes stand age, θ is the site index, and $a_{s,m}$, $b_{s,m}$, $c_{s,m}$, $d_{s,m}$ are parameters specific to growth-related metric m and tree species group s .

The partial derivatives with respect to time t and site index θ are given by

$$\frac{\partial f_{s,m}(t, \theta)}{\partial t} = a_{s,m} \theta^{b_{s,m}} d_{s,m} c_{s,m} (c_{s,m} t)^{d_{s,m}-1} \exp \left(- (c_{s,m} t)^{d_{s,m}} \right), \quad (\text{B42})$$

$$\frac{\partial f_{s,m}(t, \theta)}{\partial \theta} = b_{s,m} \frac{f_{s,m}(t, \theta)}{\theta}. \quad (\text{B43})$$

For positive $b_{s,m}$, the function is strictly monotonic in $\theta > 0$ and can be inverted to estimate the site index from an observed value $f_{s,m}^{\text{obs}}$ of the growth-related metric:

$$\theta = \left(\frac{f_{s,m}^{\text{obs}}}{a_{s,m} \left(1 - \exp(-(c_{s,m} t)^{d_{s,m}}) \right)} \right)^{1/b_{s,m}}. \quad (\text{B44})$$

These functions, with their asymptotic and sigmoidal properties, provide a robust framework for modeling forest growth dynamics of growth-related metrics across a range of site qualities and time scales, facilitating both interpolation within empirical data and cautious extrapolation beyond it.

B.3 Increment reduction function

The increment reduction function used to compute the vector of increment reduction factors $\mathbf{irf}_{s,n}^{\text{sc}}$ (see eq. 7) follows the empirically derived formulation of Lemm [29], also applied by Smaltschinski [59] and Strugholtz [62]. For tree species group s and

time-step n , the reduction factor for an individual age class k_s is given by

$$\text{irf}_{s,n,k_s}^{\text{sc}} = \exp \left[\varphi(hg_{s,n,k_s}^{\text{sc}}) \ln(sdg_{s,n,k_s}^{\text{sc}}) + (1 - sdg_{s,n,k_s}^{\text{sc}}) \psi(hg_{s,n,k_s}^{\text{sc}}) \right], \quad (\text{B45})$$

where $sdg_{s,n,k_s}^{\text{sc}}$ and hg_{s,n,k_s}^{sc} denote the stocking degree and mean height under scenario ‘sc’. The tree species (group)-specific functions $\varphi(hg_{s,n,k_s}^{\text{sc}})$ and $\psi(hg_{s,n,k_s}^{\text{sc}})$ are quadratic polynomials with height as independent variable:

$$\varphi(hg_{s,n,k_s}^{\text{sc}}) = a_s (hg_{s,n,k_s}^{\text{sc}})^2 + b_s (hg_{s,n,k_s}^{\text{sc}}) + c_s, \quad (\text{B46})$$

$$\psi(hg_{s,n,k_s}^{\text{sc}}) = d_s (hg_{s,n,k_s}^{\text{sc}})^2 + e_s (hg_{s,n,k_s}^{\text{sc}}) + f_s, \quad (\text{B47})$$

with parameters $a_s, b_s, c_s, d_s, e_s, f_s$ calibrated for each tree species group s . Please note that index notation is used, as the increment reduction function and its corresponding vectors are applied element-wise. Alternatively, the vectorised notation (applied element-wise using Hadamard operator $\odot$) for each s and n is:

$$\mathbf{irf}_{s,n}^{\text{sc}} = \exp \left[\varphi_s(\mathbf{hg}_{s,n}^{\text{sc}}) \odot \ln(\mathbf{sdg}_{s,n}^{\text{sc}}) + (\mathbf{1} - \mathbf{sdg}_{s,n}^{\text{sc}}) \odot \psi_s(\mathbf{hg}_{s,n}^{\text{sc}}) \right], \quad (\text{B48})$$

Appendix C Table of parameters

Table C2 Description of indices used in the model

Index	Range	Description
i	row index	Matrix row index
j	column index	Matrix column index
n	$n = 1, 2, \dots, N$	Time step (current time step up to N)
s	$s = 1, 2, \dots, S$	Tree species group
k_s	$k_s = 1, 2, \dots, K_s$	Age class of tree species group s (up to K_s)

Table C3 Parameters of climate-sensitive hazard functions under RCP 8.5 (MPI-ESM-LR scenario).

Tree species group	Distribution	Scale / Mean	Shape / SD	Emission scenario	GCM
Spruce	Weibull	0.2188	1.2933	RCP 8.5	MPI-ESM-LR
Pine	Lognormal	4.8345	1.7895	RCP 8.5	MPI-ESM-LR
Douglas fir	Weibull	0.3681	1.0569	RCP 8.5	MPI-ESM-LR
Beech	Weibull	0.6074	1.1081	RCP 8.5	MPI-ESM-LR
Oak	Weibull	0.5858	1.0591	RCP 8.5	MPI-ESM-LR
Fir	Weibull	0.6982	1.2718	RCP 8.5	MPI-ESM-LR

Note: For Weibull distributions, the scale parameter s_{120} and shape parameter α are provided. For lognormal distributions (Scots pine), Mean and Standard deviation of log-diameter) are given instead. All parameters are parameterised under the emission scenario RCP 8.5 using the global climate model (GCM) MPI-ESM-LR climate model with the datasets of Hijmans et al. [19], Brandl et al. [5] according to the methodology of Fuchs et al. [16].

Table C4 Final harvesting ranges according to the WEHAM control variables [4]

Tree species group	Minimum	Median	Maximum
Beech	130	160	250
Oak	160	200	400
Alh	100	130	200
Aln	60	80	150
Spruce	80	110	160
Fir	100	120	200
Douglas fir	80	120	200
Pine	110	140	200
Larch	120	130	160

Table C5 Parameter values of the growing stock volume used in FESIM for selected tree species groups

Tree species group	Function	a	b	c	Source
Beech	Sloboda	$1.9732 \exp(-0.0615 \text{ } yc)$	1.5439	$1.2320 \exp(-0.0471 \text{ } yc)$	Albert et al. [1]
Oak	Sloboda	$1.5700 \exp(-0.0100 \text{ } yc)$	0.6119	$0.8796 \exp(-0.0118 \text{ } yc)$	Albert et al. [1]
Alh	Sloboda	$2.3858 \exp(-0.1739 \text{ } yc)$	4.1949	$1.8546 \exp(-0.1259 \text{ } yc)$	Nagel [34]
Aln	Hossfeld IV	$1034.1522 \exp(0.2402 \text{ } yc)$	$2.0370 + 0.0184 \text{ } yc$	—	Lockow [30]
Spruce	Sloboda	$1.8480 \exp(-0.0529 \text{ } yc)$	0.4865	$0.8718 \exp(-0.0726 \text{ } yc)$	Albert et al. [1]
Fir	Sloboda	$1.6762 \exp(-0.0208 \text{ } yc)$	0.3497	$0.6099 \exp(-0.0250 \text{ } yc)$	Hausser [18]
Douglas fir	Sloboda	$-0.0492 \exp(-1.4016 \text{ } yc)$	-13.4542	$2.1362 \exp(-0.0191 \text{ } yc)$	Albert et al. [1]
Pine	Sloboda	$1.8957 \exp(-0.0436 \text{ } yc)$	0.5682	$1.0791 \exp(-0.0217 \text{ } yc)$	Albert et al. [1]
Larch	Hossfeld IV	$475.3500 \exp(0.7446 \text{ } yc)$	$1.5214 + 0.2125 \text{ } yc$	—	Schober [55]

Note: yc denotes yield class. For Hossfeld-IV functions, the parameter c is not required and therefore marked with an em-dash. The values are post-parameterised and may differ from the discrete individual values of the original sources.

Table C6 Parameter values of the total volume production used in FESIM for selected tree species groups

Tree species group	Function	a	b	c	Source
Beech	Sloboda	$-9.0281 \exp(0.3447 \text{ } yc)$	-0.6386	$1.6802 \exp(0.0244 \text{ } yc)$	Albert et al.
Oak	Sloboda	$2.4884 \exp(-0.0479 \text{ } yc)$	0.6560	$1.1850 \exp(-0.0343 \text{ } yc)$	Albert et al.
Alh	Sloboda	$2.6677 \exp(-0.1254 \text{ } yc)$	2.5428	$1.6332 \exp(-0.0764 \text{ } yc)$	Nagel [34]
Aln	Hossfeld IV	$1071.1905 \exp(0.2359 \text{ } yc)$	$1.9112 + 0.0484 \text{ } yc$	—	Lockow [30]
Spruce	Sloboda	$2.2453 \exp(-0.0399 \text{ } yc)$	0.6988	$1.0828 \exp(-0.0380 \text{ } yc)$	Albert et al.
Fir	Sloboda	$1.9284 \exp(-0.0233 \text{ } yc)$	0.2202	$0.5718 \exp(-0.0295 \text{ } yc)$	Hausser [18]
Douglas fir	Sloboda	$-15.2811 \exp(0.2044 \text{ } yc)$	-0.2791	$1.6723 \exp(0.0149 \text{ } yc)$	Albert et al.
Pine	Sloboda	$1.8885 \exp(-0.0276 \text{ } yc)$	0.4296	$0.8485 \exp(-0.0248 \text{ } yc)$	Albert et al.
Larch	Sloboda	$1.9892 \exp(-0.0634 \text{ } yc)$	1.1016	$1.1855 \exp(-0.0483 \text{ } yc)$	Schober [55]

Note: yc denotes yield class. For Hossfeld-IV functions, the parameter c is not required and therefore marked with an em-dash. The values are post-parameterised and may differ from the discrete individual values of the original sources.

Table C7 Parameter values of the mean height used in FESIM for selected tree species groups

Tree species group	Function	a	b	c	Source
Beech	Sloboda	$-3.7245 \exp(0.4125 \text{ } yc)$	-4.1039	$2.0800 \exp(0.0282 \text{ } yc)$	Albert et al.
Oak	Hossfeld IV	$126.6371 \exp(0.2983 \text{ } yc)$	$1.1854 + 0.0347 \text{ } yc$	—	Albert et al.
Alh	Hossfeld IV	$39.2619 \exp(0.1523 \text{ } yc)$	$0.9833 + 0.0055 \text{ } yc$	—	Nagel [34]
Aln	Hossfeld IV	$67.6539 \exp(0.0321 \text{ } yc)$	$1.3485 + 0.0072 \text{ } yc$	—	Lockow [30]
Spruce	Sloboda	$-1.7727 \exp(0.4183 \text{ } yc)$	-3.7095	$2.0265 \exp(0.0176 \text{ } yc)$	Albert et al.
Fir	Hossfeld IV	$1054.9132 \exp(0.7248 \text{ } yc)$	$1.6310 + 0.1369 \text{ } yc$	—	Hausser [18]
Douglas fir	Sloboda	$-4.7854 \exp(0.4847 \text{ } yc)$	-0.5515	$1.7633 \exp(0.0497 \text{ } yc)$	Albert et al.
Pine	Hossfeld IV	$203.3731 \exp(0.1887 \text{ } yc)$	$1.3616 + 0.0363 \text{ } yc$	—	Albert et al.
Larch	Hossfeld IV	$180.1492 \exp(0.1526 \text{ } yc)$	$1.3803 + 0.0287 \text{ } yc$	—	Schober [55]

Note: yc denotes yield class. For Hossfeld-IV functions, the parameter c is not required and therefore marked with an em-dash. The values are post-parameterised and may differ from the discrete individual values of the original sources.

Table C8 Parameter values of the mean diameter of the growing stock volume used in FESIM for selected tree species groups

Tree species group	Function	a	b	c	Source
Beech	Sloboda	$-2.2643 \exp(0.4051 y_c)$	-0.2957	$1.2337 \exp(0.0542 y_c)$	Albert et al. [1]
Oak	Sloboda	$-2.2633 \exp(0.1523 y_c)$	-0.1422	$1.1497 \exp(0.0126 y_c)$	Albert et al. [1]
Alh	Sloboda	$1.1625 \exp(0.0110 y_c)$	0.4116	$0.8476 \exp(0.0436 y_c)$	Nagel [34]
Aln	Hossfeld IV	$176.4570 \exp(-0.1239 y_c)$	$1.3877 - 0.0033 y_c$	—	Lockow [30]
Spruce	Sloboda	$10.3496 \exp(-0.3403 y_c)$	0.1516	$1.5060 \exp(-0.0911 y_c)$	Albert et al. [1]
Fir	Sloboda	$1.2748 \exp(-0.0065 y_c)$	0.1204	$0.6267 \exp(-0.0066 y_c)$	Hausser [18]
Douglas fir	Sloboda	$-1.9828 \exp(0.8472 y_c)$	-0.1409	$1.1699 \exp(0.1327 y_c)$	Albert et al. [1]
Pine	Sloboda	$0.4006 \exp(-0.0471 y_c)$	-207.6943	$2.4868 \exp(-0.0199 y_c)$	Albert et al. [1]
Larch	Hossfeld IV	$146.8153 \exp(0.0914 y_c)$	$0.9521 - 0.0008 y_c$	—	Schober [55]

Note: y_c denotes yield class. For Hossfeld-IV functions, the parameter c is not required and therefore marked with an em-dash. The values are post-parameterised and may differ from the discrete individual values of the original sources.

Table C9 Parameters of the increment reduction function used in FESIM,

Tree species group	a	b	c	d	e	f	Source
Beech	-0.0003	0.0421	1.7152	-0.0003	0.0290	2.0711	Smaltschinski [59]
Oak	-0.0018	0.0726	2.3750	-0.0027	0.1074	2.6444	Smaltschinski [59]
Spruce	-0.0007	0.0422	1.1929	-0.0008	0.0267	1.6888	Smaltschinski [59]
Pine	-0.0001	0.0329	1.7907	-0.0002	0.0250	2.1015	Smaltschinski [59]

Note: The increment reduction function is described in sec. B.3 in eq. B45 eq. B48

Table C10 Parameters for eq. 19 by tree species groups [35].

Tree species group	a	b	c	d
Beech	2.0837	0.1500	5.7292	1.3341
Oak	2.6618	0.1152	8.3381	1.4083
Spruce	1.2644	0.1072	0.000001	1.0000
Pine	1.2783	0.11388	8.70522	1.33944

Table C11 Reference assortments according to the German RVR system used in the valuation approach by Fuchs et al. [17].

Tree species group	Reference Assortment
Oak	B4
Beech	B4
Birch (<i>/Aln</i>)	B4
Ash (<i>/Alh</i>)	B4
Spruce (<i>/Fir</i>)	B2b
Douglas fir	B2b
Pine	B2b
Larch	B2b

Note: The table lists the reference roundwood assortments, which define standard high-value sawlog categories used for valuation. These assortments represent industrially usable stemwood of quality class B (good to excellent quality, exogenous parameter) with mid-diameters of 40–49 cm (class 4) for broadleaves and 25–29 cm (class 2b) for conifers

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