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Tiago Sequeira

tiago.n.sequeira@fe.uc.pt

University of Coimbra

Pedro Lima

University of Coimbra

Oscar Afonso

University of Porto

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Lobbying, Growth and Inequality: A Directed Technical Change model with game-theoretic microfoundations

Pedro Lima
Tiago Neves Sequeira*
Oscar Afonso

Abstract

We analyze the effects of lobbying on growth and inequality in a novel directed technical change model, where lobbying can seek rents and decrease entry or increase demand. To that end, we develop a microfounded theoretical game that models generalized lobbying decisions. In this framework, firms from different sectors can either compete or collaborate in their lobbying efforts. The model reveals that lobbying incentives are stronger when fixed costs are low and when shared sources of lobbying efficiency outweigh sector-specific ones. When lobbying increases demand, it also promotes economic growth and, when goods are gross substitutes, also increases inequality. In contrast, when lobbying focus on seeking rents and decreasing entry, it has the opposite effects. Given our results, it is essential for policy to distinguish between the main features of lobbying practices, and to design targeted incentives in order to effectively influence growth and inequality.

Keywords: Lobbying; economic growth; wage inequality.

JEL Classification: J31, P16, O30, O41.

*Corresponding author: tiago.neves.sequeira@fe.uc.pt

1 Introduction

In 2023, more than 12,500 lobbyists were registered in the United States, and lobbying expenditures reached a record $4.11 billion. A well-established fact is that large firms account for the majority of these expenditures. This paper seeks to shed light on the macroeconomic consequences of firms' lobbying, focusing on its effects on economic growth and wage inequality—an issue that has received comparatively little attention in the macroeconomic literature.

The economic study of lobbying originated in the literature on rent-seeking activities (e.g., Olson, 1971, 1982; Baumol, 1990; Grossman and Helpman, 1994; Storr and Choi, 2019). Olson's theory highlights the free-rider problem within lobbying organizations, since lobbying outcomes act as public goods. It also suggests that lobbying negatively affects economic growth. However, Hager (2024) reviews a range of empirical studies and argues that this negative influence is not universally accepted. She emphasizes the importance of redistribution effects in shaping the link between lobbying and growth, which she considers an underexplored dimension. Esteban and Ray (2006) likewise suggest that poorer and more unequal economies are more prone to public resource misallocation, not necessarily because of higher corruption levels.

Most research on lobbying has been conducted within public, trade, and political economy, addressing questions such as the impact of lobbying on firms' tax burdens (e.g., Richter et al., 2009; Bombardini and Trebbi, 2020) or the dynamics of competition and cooperation in lobbying (e.g., Gawande et al., 2012; Junk, 2020). At the firm level, Cao et al. (2018) find that lobbying tends to harm performance due to agency costs, with two exceptions: high-performing firms and firms politically aligned with the ruling party. Trade policy has been another central focus, where competition between opposing lobbying groups is shown to play a decisive role in shaping outcomes (e.g., Gawande et al., 2012; Bombardini and Trebbi, 2020; Liu and Mukherjee, 2023).

Beyond taxation and trade, lobbying has also been linked to slower technology adoption (Comin and Hobijn, 2009) and weaker innovation incentives (Akcigit et al., 2018; Bellettini et al., 2013). When firms divert funds into lobbying, their profitability—and consequently their R&D investments—may be altered. This, in turn, influences technological progress, knowledge accumulation, and ultimately wage differentials. Aghion et al. (2021) argue that institutionalized lobbying can create barriers to entry and concentrate economic rents, thereby undermining the positive effects of creative destruction.

Within the endogenous growth literature, only two studies explicitly incorporate lobbying. Julio (2014) embeds lobbying into an R&D-driven growth model, treating it as rent-seeking behavior. In his framework, all firms act collectively, eliminating strategic interaction, and lobbying can affect growth through its impact on market equilibrium and free-entry conditions (see also Bellettini et al., 2013). Afonso et al. (2022), by contrast, study lobbying in a Directed Technical Change framework with automation, introducing strategic interactions among lobbying firms. Their model suggests that lobbying can have neutral or positive effects on growth and may mitigate automation-induced wage polarization, diverging from Olson's prediction and much of the empirical evidence.

Overall, the literature provides mixed and often inconclusive findings on the growth and inequality effects of lobbying. ———Importantly, it rarely distinguishes between the effects lobbying may have in diverting resources away from productive activities such as R&D, thereby reducing entry and growth—and increasing demand, where resources are used to expand market demand, for instance via Corporate Social Responsibility or marketing expenditures. Bertrand et al. (2020), e.g., formalize the richer political

environment of the Grossman and Helpman (2001) framework, in which firms face two distinct decision margins that can shape policy. Our model builds on this reasoning to explicitly define two dimensions of the activity of lobbying: rent-seeking and demand-seeking.

Despite the growing body of work, the macroeconomic effects of lobbying remain insufficiently understood. Four gaps are especially salient. First, the relationship between lobbying and wage inequality has not been fully explored, which is the central focus of this paper. Second, the ambiguous effects of lobbying on growth warrant further clarification. Third, the joint analysis of rent-seeking and demand-seeking lobbying has been largely neglected. Fourth, the interplay between competitive and cooperative lobbying has not yet been studied. Our research addresses these gaps.

We integrate the Directed Technical Change framework (Acemoglu, 1998) with a game-theoretic model of lobbying, providing a tractable microfounded setup. Whereas previous studies (Julio, 2014; Afonso, 2022) tend to predict neutral or positive growth effects with little impact on inequality, we identify specific conditions—relating to the type of lobbying and the degree of substitutability across sectors—under which lobbying can either foster growth and reduce wage gaps, or hinder growth and widen inequality.

Our model produces several novel results. If both firms engage in lobbying, relative lobbying gains increase with relative efficiency, decrease with fixed costs, and respond nonlinearly to shared lobbying efficiency. In collaborative settings, common lobbying efficiency may reduce gains, whereas under asymmetric lobbying intensity, the opposite can occur. When labor across sectors is grossly substitutable, relative wages in a given sector rise with demand-seeking lobbying but fall with rent-seeking lobbying. Moreover, growth decreases when both sectors engage in rent-seeking but increases when both pursue demand-seeking lobbying.

This paper makes four contributions. First, it provides new microfoundations for firms' lobbying behavior within a Directed Technical Change framework, distinguishing between demand-seeking and rent-seeking dimensions of lobbying. Second, it establishes the conditions under which each dimension of lobbying affects growth and inequality, highlighting the role of sectoral substitutability and lobbying efficiency. Third, it integrates a game-theoretic perspective on competitive versus cooperative equilibria, characterizing how fixed costs and efficiency differentials shape lobbying choices. Fourth, it generates empirically testable predictions linking lobbying intensity to growth and inequality.

The paper is structured as follows. In Section 2 we describe lobbying gains from the two sources of lobbying that we consider, motivate and present microfoundations for the game In Section 3 the framework of the model is devised, which finishes with a synthesis and an explanation of main policy implications. Section 4 presents the main conclusions.

2 Lobbying gains: Motivation and Microfoundations

Now, we introduce new terms and notation related to gains from specific dimension of lobbying, rent-seeking gains $\mathcal{L}_{R,i}$, and demand-seeking gains, $\mathcal{L}_{\mathcal{D},i}$. In this section we begin by analyzing some stylized facts of lobbying practice, use them as motivation to develop a game of lobbying that provides microfoundations to lobbying gains and then analyze how the effects on inequality and growth of the main parameters determining lobbying decisions.

2.1 Motivation: Some stylized facts

In this section, we outline some stylized facts that we consider to characterize the lobbying practice that serve as the main motivation for the specific microfounded theoretical setup we develop in the following subsection.

Fact 1. *Firms either compete or collaborate when doing lobbying.*

This often occurs when firms from distinct sectors with opposing interests attempt to influence public policy. Consider the case of renewable energy producers lobbying for subsidies or favorable regulations, while fossil fuel firms simultaneously lobby to protect their market share. According to the International Energy Agency (IEA, 2021), global fossil fuel subsidies amounted to over $440 billion in 2021, a figure often sustained through persistent lobbying by incumbent energy firms. Meanwhile, renewable energy associations (e.g., WindEurope, SolarPower Europe) engage in coordinated lobbying for decarbonization policies and regulatory incentives.

In contrast, *collaborative lobbying* typically occurs among firms within the same sector that share regulatory or strategic interests. A paradigmatic example is the pharmaceutical industry, where firms frequently coordinate efforts to influence drug approval processes, intellectual property frameworks, and reimbursement policies. This coordination is institutionalized through powerful associations such as the European Federation of Pharmaceutical Industries and Associations (EFPIA) and the Pharmaceutical Research and Manufacturers of America (PhRMA), which represent entire sectors in formal lobbying processes. Ozieranski et al. (2020) provide detailed mapping of lobbying networks in the EU, documenting shared expenditures and coordinated agendas. In the US, PhRMA alone reported lobbying expenditures exceeding $30 million in 2022 (OpenSecrets, 2023).

Fact 2. *The intensity of lobbying activity increases with common sources of lobbying efficiency, such as the extent to which lobbying is institutionally embedded and socially accepted, and decreases with fixed costs, reflecting legal, administrative, and reputational thresholds required to engage in lobbying.*

The former is illustrated by the fact that lobbying is often conducted in contexts where it is formally and institutionally integrated into the policy process, such as through the European Commission's stakeholder consultations. The latter, is illustrated by he complexity of lobbying registration procedures and the institutional hurdles faced by new entrants, as documented by the Corporate Europe Observatory (2020).

Fact 3. *The intensity of lobbying activity of a firm or sector in a particular context varies according to specific sources of lobbying efficiency, such as the extent to which it is able to convert lobbying expenditure into political influence.*

This is illustrated by the different degree of lobbying influence possessed by firms in the same context. For exemple, Brulle (2018) documents that in the US, between 2000 and 2016, fossil fuel firms outspent renewable energy companies by a factor of ten.

Considering this, we map these facts to four essential parameters, which will be used in the game we develop further ahead:

- Effects of lobbying of other firms on own lobbying payoff (δ): Captures the nature and intensity of the effects of lobbying efforts from firms in opposite sectors on own gains.

- Common effectiveness (Γ): Captures the extent to which lobbying is institutionally embedded and socially accepted. High Γ characterizes systems with formalized consultation procedures and weak constraints on interest representation.
- Lobbying efficiency (ϕ_j): Represents the capacity of a firm or sector to convert lobbying expenditure into political influence. High ϕ_j may reflect financial scale, political connectivity, technical specialization, or incumbency advantages.
- Fixed lobbying cost ($\overline{\kappa}$): Reflects the legal, administrative, and reputational thresholds required to engage in lobbying. High $\overline{\kappa}$ may result from complex compliance procedures, mandatory transparency frameworks, or institutional opacity.

These parameters can, in principle, be proxied using real-world data. For example, ϕ_j may be estimated through lobbying success rates or expenditures per legislative outcome. Γ can be linked to lobbying permissiveness indices (e.g., IDEA Political Finance Database), and $\overline{\kappa}$ inferred from regulatory reports on lobbying procedures (e.g., OECD 2021).

2.2 A microfounded game of lobbying

We assume that lobbying activity results from a strategic game between intermediate good firms from sectors z and $-z$,[1] seeking to choose the level of lobbying expenditures dedicated to each specific form of lobbying $\nu, \kappa_{\nu,z}, z \in \{j, -j\}, \nu \in \{\mathcal{D}, \mathcal{R}\}$, which is demand-seeking if $\nu = \mathcal{D}$ or rent-seeking if $\nu = \mathcal{R}$, that maximizes the corresponding lobbying net payoffs, $\mathcal{L}_{\nu,z}, z \in \{j, -j\}, \nu \in \{\mathcal{D}, \mathcal{R}\}$. In particular, we consider that lobbying net payoffs are determined not only by own lobbying expenditures, $\kappa_{\nu,z}$, but also by lobbying expenditures from the firm in the other sector, $\kappa_{\nu,-z}$. These, in turn, can have a positive impact if both firms collaborate (cooperative lobbying) or negative if firms compete (competitive lobbying). Therefore this is a game as each firm has to strategically consider not only the direct impacts of its decisions but also how decisions from the firm in other sector are affected by its own decisions and, therefore, affect its profits.

Lobbying net payoffs from a specific form of lobbying are considered to be determined by the following function $\mathcal{L}_z$:

$$\mathcal{L}_z \equiv \mathcal{L}_z(\kappa_z, \kappa_{-z}), z \in \{j, -j\},$$

which, in turn, is defined in the following manner:

$$\mathcal{L}_z(\kappa_z, \kappa_{-z}) \equiv \tilde{\mathcal{L}}_z(\kappa_z, \kappa_{-z}) - \overline{\kappa}_z(\delta, \kappa_z, \kappa_{-z}).$$

The first term $\tilde{\mathcal{L}}_z(\kappa_z, \kappa_{-z})$ represents variable lobbying gains, which are variable in the sense that they depend on variable lobbying expenditures, and are defined in the following manner:

$$\tilde{\mathcal{L}}_z(\kappa_z, \kappa_{-z}) \equiv \Gamma \phi_z \kappa_z^{\gamma} \tilde{\kappa}_{-z}^{\delta} - \kappa_z.$$

The first term $\Gamma \phi_z \kappa_z^{\gamma} \tilde{\kappa}_{-z}^{\delta}$ represents total gains from lobbying associated with a volume of lobbying expenditures κ_z. The relevant variables are κ_z, representing own lobbying efforts, and $\tilde{\kappa}_{-z}$, defined below,

[1] Sector z can correspond to any sector i or $-i$ of the two sector DTC model framework which will be developed below.

representing lobbying efforts from the other sector provided they are superior to 1. The intuition for the last term is that lobbying efforts from the other sector have only a meaningful impact if they are intensive enough, which, for simplicity is assumed to occur when they exceed 1.[2]

$$\tilde{\kappa}_{-z} = \min\{\kappa_{-z}, 1\}.$$

The relevant parameters are Γ and ϕ_z, which represent, respectively, common and specific sources of lobbying efficiency (or overall and specific lobbying effectiveness, respectively), and γ and δ, which determine, respectively, the impact on gains of lobbying expenditures from the firm and firm from the other sector. Regarding these parameters, we make standard assumptions namely that $\Gamma, \phi_z \geq 0$, $0 \leq \gamma \leq 1$, but also that $0 < \gamma + |\delta| < 1$, which has the following important implications: (i) the lobbying efforts from firms in the other sector can be positive if $\delta > 0$ and the lobbying is collaborative or negative if $\delta < 0$ and lobbying is competitive, (ii) decreasing returns to scale and (iii) $-\gamma < \delta < \gamma$, i.e., the effects of other firms lobbying is always weaker than own lobbying efforts (be it negative or positive). Finally, we assume that $\phi_{-z} = \phi_z^{-1}$ and that $\phi_j > 1$, i.e., the firm in sector j is relatively more efficient than firms from other sectors.

The second term $\overline{\kappa}_z(\delta, \kappa_z, \kappa_{-z})$ is a fixed cost term defined in the following manner:

$$\overline{\kappa}_z(\delta, \kappa_z, \kappa_{-z}) \equiv \begin{cases} \overline{\kappa} > 0 & \text{if } \kappa_z > 0 \vee (\delta < 0 \wedge \kappa_z = 0 \wedge \kappa_{-z} \geq 1) \\ 0 & \text{otherwise} \end{cases}, z \in \{j, -j\}.$$

The interpretation for this term is that a firm in sector z can incur in a fixed cost if it lobbies, i.e., $\kappa_z > 0$, or if in a competitive setting ($\delta < 0$) it does not lobby ($\kappa_z = 0$) and the firm from the other sector lobbies intensively ($\kappa_{-z} \geq 1$). The intuition is that a firm interested in lobbying has to incur in fixed lobbying expenditures such as those necessary to set up necessary infrastructures for the lobbying activity and in the specific case of a competitive setting can suffer a penalty for not lobbying if the other firm lobbies intensively enough.

We choose this particular functional setup for being simultaneously sufficiently realistic, considering the stylized facts outlined in the previous subsection, and mathematically tractable, as reflected by several convenient properties, which we now explain in detail: (i) finite lobbying gains if firms in the other sector do not lobby, since $\mathcal{L}_z(\kappa_z, 0) \equiv \Gamma\phi_z\kappa_z - \kappa_z - \overline{\kappa}_z(\delta, \kappa_z, \kappa_{-z})$ is always finite, which is more realistic than allowing for infinite returns, which would occur in a standard Cobb-Douglas setting, i.e., if $\tilde{\kappa}_{-z} = \kappa_{-z}$, and more convenient, as it permits quantifying and comparing lobbying gains in such situations; (ii) impossibility of free riding, i.e., it is not possible to obtain positive gains if no lobbying effort is conducted,[3] as $\tilde{\mathcal{L}}_z(0, \kappa_{-z}) \equiv -\overline{\kappa}_z(\delta, \kappa_z, \kappa_{-z}) \leq 0$; (iii) possibility of complete absence of lobbying, which occurs when no firm conducts lobbying efforts, as this implies zero payoff for both, as $\tilde{\mathcal{L}}_z(0, 0) \equiv 0$, which is convenient in the context of the main theoretical model since it allows to view this setup as generalization of the original one, which, in turn, can be viewed as the particular where no lobbying is conducted; (iv) positive

[2]This is also important because it limits the marginal impact of lobbying at very low values of κ_{-z}. If we were to consider simply $\kappa_{-z} = \kappa_{-z}$, profits at $\kappa_{-z} = 0$ would be either ∞ (if $\delta < 0$) or 0 (if $\delta > 0$), both of which we consider to be problematic and unrealistic scenarios. The first would lead to the optimal lobbying effort by z for $\kappa_{-z} = 0$ being an infinitesimal value. The threshold is defined to be one for simplification because otherwise the analysis becomes unnecessarily complicated.

[3]in fact, it is possible for firms that do not lobby to have negative profits in a competitive setting, if lobbying efforts from the other firm are intensive enough

and diminishing marginal gross returns to own lobbying efforts, regardless of lobbying efforts from the other sector, as $\frac{\partial}{\partial \kappa_z}\tilde{\mathcal{L}}_z(\kappa_z,\kappa_{-z}) \equiv \gamma\Gamma\phi_z\kappa_z^{\gamma-1}\tilde{\kappa}_{-z}^{\delta}-1>0$ and $\frac{\partial^2}{\partial \kappa_z^2}\tilde{\mathcal{L}}_z(\kappa_z,\kappa_{-z})<0$; (v) the optimal lobbying effort is not necessarily always an interior solution, due to the presence of fixed costs, which is convenient as it leads to several different possible outcomes, (vi) it rules out Nash-equilibria that do not make economic sense, such as those that imply negative lobbying gains in collaborative settings.[4]

From the first order conditions of the lobbying payoff maximization problem we obtain the following expressions for the best response functions for lobbying expenditures and the corresponding lobbying gains:

$$\begin{aligned}
\kappa_z(\kappa_{-z}) &= \begin{cases} \kappa_{z,1} \equiv \tilde{\kappa}_{-z}^{\frac{\delta}{\overline{\gamma}}}\left(\gamma\Gamma\phi_z\right)^{\frac{1}{1-\gamma}} & \text{if } \mathcal{L}_{z,1}>\mathcal{L}_{z,0} \\ \kappa_{z,0}\equiv 0 & \text{otherwise} \end{cases}, z\in\{j,-j\}, \\
\tilde{\mathcal{L}}_z(\kappa_{-z}) \equiv \tilde{\mathcal{L}}_z(\kappa_z(\kappa_{-z}),\kappa_{-z}) &= \begin{cases} \tilde{\mathcal{L}}_{z,1}(\tilde{\kappa}_{-z}) = \overline{\gamma}\gamma^{\frac{\gamma}{\overline{\gamma}}}\kappa_{-z}^{\frac{\delta}{\overline{\gamma}}}\left(\Gamma\phi_z\right)^{\frac{1}{\overline{\gamma}}} & \text{if } \mathcal{L}_{z,1}>\mathcal{L}_{z,0} \\ \tilde{\mathcal{L}}_{z,0}(\tilde{\kappa}_{-z})=0 & \text{otherwise} \end{cases}, z\in\{j,-j\},
\end{aligned}$$

where $\overline{\gamma}\equiv 1-\gamma$, $\kappa_{z,1}$ represents the optimal response of firm z in case it decides to lobby, which occurs if the net payoff from lobbying, $\mathcal{L}_{z,1}$, exceeds that from not lobbying, $\mathcal{L}_{z,0}$, with $\kappa_{z,0}$ representing the optimal lobbying expenditure in case it decides not to lobbying, which is zero.

The Nash-equilibria (henceforth NE) lobbying efforts are defined in the following manner.

$$\kappa_z^* \quad : \quad \kappa_z^* = \kappa_z(\kappa_{-z}(\kappa_z^*)), z\in\{j,-j\}.$$

In Table 1, we define all the different pairs of possible and relevant NE lobbying expenditures $(\kappa_z^*,\kappa_{-z}^*)$:

Table 1: Possible pairs of NE lobbying expenditures

$(\kappa_z^*,\kappa_{-z}^*)$	$\kappa_{-z}^*=0$	$0<\kappa_{-z}^*<1$	$1\leq\kappa_{-z}^*$
$\kappa_z^*=0$	$(0,0)$	$\left(0,\kappa_{-z,\underline{z}}^*\right)$	$\left(0,\kappa_{-z,\underline{z}}^*\right)$
$0<\kappa_z^*<1$	$\left(\kappa_{z,\underline{-z}}^*,0\right)$	$\left(\kappa_{z,\underline{-z}}^*,\kappa_{-z,\underline{z}}^*\right)$	$\left(\kappa_{\underline{z},\overline{-z}}^*,\kappa_{-z,\underline{z}}^*\right)$
$1\leq\kappa_z^*$	$\left(\kappa_{z,\underline{-z}}^*,0\right)$	$\left(\kappa_{z,\underline{-z}}^*,\kappa_{\underline{-z},\overline{z}}^*\right)$	$\left(\kappa_{\overline{z},\overline{-z}}^*,\kappa_{\overline{z},\overline{-z}}^*\right)$

where

[4] If both firms collaborate, they can simply agree not to lobby to avoid losses.

$$
\begin{aligned}
\underset{(\gtrless 1)}{\kappa^*_{z,\underline{-z}}} &\equiv \left(\frac{\Gamma}{\overline{\Gamma}_{z,\underline{-z}}}\right)^{\frac{1}{\overline{\gamma}}}, \overline{\Gamma}_{z,\underline{-z}} \equiv \frac{1}{\gamma\phi_z},\ z \in \{j,-j\},\\
\underset{(<1)}{\kappa^*_{\underline{z},\overline{-z}}} &\equiv \left(\frac{\Gamma}{\overline{\Gamma}_{z,\overline{-z}}}\right)^{\frac{\overline{\gamma}+\delta}{\overline{\gamma}^2}}, \overline{\Gamma}_{z,\overline{-z}} \equiv \frac{\phi_z^{-\frac{\overline{\gamma}-\delta}{\overline{\gamma}+\delta}}}{\gamma},\ z \in \{j,-j\},\\
\underset{(>1)}{\kappa^*_{\overline{z},\overline{-z}}} &\equiv \left(\frac{\Gamma}{\overline{\Gamma}_{z,\overline{-z}}}\right)^{\frac{1}{\overline{\gamma}-\delta}},\ z \in \{j,-j\},
\end{aligned}
$$

The corresponding variable lobbying gains are defined below:

$$
\begin{aligned}
\tilde{\mathcal{L}}^*_{z,\underline{-z}} &\equiv \tilde{\mathcal{L}}_{z,1}(\tilde{\kappa}_{-z}(\kappa^*_{z,\underline{-z}})) = \overline{\gamma}\gamma^{\frac{\gamma}{\overline{\gamma}}}\left(\Gamma\phi_z\right)^{\frac{1}{\overline{\gamma}}},\ z \in \{j,-j\},\\
\tilde{\mathcal{L}}^*_{\underline{z},\overline{-z}} &\equiv \tilde{\mathcal{L}}_{z,1}(\tilde{\kappa}_{-z}(\kappa^*_{\underline{z},\overline{-z}})) = \overline{\gamma}\gamma^{\frac{\overline{\gamma}\gamma+\delta}{\overline{\gamma}^2}}\Gamma^{\frac{\overline{\gamma}+\delta}{\overline{\gamma}^2}}\phi_z^{\frac{\overline{\gamma}-\delta}{\overline{\gamma}^2}},\ z \in \{j,-j\},\\
\tilde{\mathcal{L}}^*_{\overline{z},\overline{-z}} &\equiv \tilde{\mathcal{L}}_{z,1}(\tilde{\kappa}_{-z}(\kappa^*_{\overline{z},\overline{-z}})) = \overline{\gamma}\gamma^{\frac{\gamma+\delta}{\overline{\gamma}-\delta}}\Gamma^{\frac{1}{\overline{\gamma}-\delta}}\phi_z^{\frac{1}{\overline{\gamma}+\delta}},\ z \in \{j,-j\}.
\end{aligned}
$$

Therefore, for a particular firm in sector z, the algebraic expressions for Nash-equilibria lobbying gains and expenditures will be determined by which of the following scenarios is valid:

- the firm in sector z does not lobby at all ($\kappa^*_z = 0$),
- the firm in sector lobbies ($\kappa^*_z > 0$), and:
  - the firm in sector $-z$ does not lobby intensively or at all ($\kappa^*_z = \kappa^*_{z,\underline{-z}}$)), in which case the algebraic expression for the lobbying effort of the firm in sector z is the same regardless of whether lobbying is intensive or not, i.e. $\kappa^*_{z,\underline{-z}} \gtrless 1$, with the intensity of lobbying effort depending on whether common sources of lobbying efficiency are sufficiently high.
  - the firm in sector $-z$ lobbies intensively, in which case the firm in sector z can either:
    - lobby intensively ($\kappa^*_z = \kappa^*_{\overline{z},\overline{-z}} > 1$),
    - lobby but not intensively ($\kappa^*_z = 0 < \kappa^*_{\underline{z},\overline{-z}} < 1$).

The previous analysis was concerned only with describing all the possible NE variable lobbying gains and expenditures expressions. These are given by the following expressions below:

$$
\mathcal{L}^*_z = \max(\mathcal{L}^*_{z,1}, \mathcal{L}^*_{z,0}) = \begin{cases} \tilde{\mathcal{L}}^*_{z,1} - \overline{\kappa} & \text{if } \mathcal{L}^*_{z,1} > \mathcal{L}^*_{z,0}, z \in \{j,-j\} \\ 0 & \text{otherwise} \end{cases} \tag{1}
$$

where

$$
\mathcal{L}^*_{z,1} > \mathcal{L}^*_{z,0} \Leftrightarrow \tilde{\mathcal{L}}^*_{z,1} - \overline{\kappa} > -\overline{\kappa}_z(\delta, \kappa^*_z = 0, \kappa^*_{-z}) \Leftrightarrow \begin{cases} \tilde{\mathcal{L}}^*_{z,1} > 0 & \text{if } \delta < 0 \wedge \kappa^*_{-z} < 1 \\ \tilde{\mathcal{L}}^*_{z,1} > \overline{\kappa} & \text{otherwise} \end{cases}, z \in \{j,-j\}, \tag{2}
$$

is the necessary condition for lobbying. In turn, the actual NE variable lobbying gains and expenditures expressions depend on the relative position of Γ. Therefore, after establishing the order of the various $\overline{\Gamma}$

thresholds, we can start by characterizing the variable NE lobbying gains for different values of Γ if the NE implies making lobbying.

$$\tilde{\mathcal{L}}^*_{j,1} = \begin{cases} \tilde{\mathcal{L}}^*_{j,\underline{-j}} & \text{if } \Gamma < \overline{\Gamma}_{-j,\overline{j}} \\ \tilde{\mathcal{L}}^*_{\overline{j},\underline{-j}} & \text{if } \overline{\Gamma}_{-j,\overline{j}} < \Gamma \end{cases}, \; \tilde{\mathcal{L}}^*_{-j.1} = \begin{cases} \tilde{\mathcal{L}}^*_{-j,\underline{j}} & \text{if } \Gamma < \overline{\Gamma}_{j,\underline{-j}} \\ \tilde{\mathcal{L}}^*_{\underline{-j},\overline{j}} & \text{if } \overline{\Gamma}_{j,\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}}. \\ \tilde{\mathcal{L}}^*_{\overline{-j},\overline{j}} & \text{if } \overline{\Gamma}_{-j,\overline{j}} < \Gamma \end{cases} \tag{3}$$

We now provide some intuition for these expressions. The NE lobbying decision for a firm in sector z ultimately depends on a comparison from the best payoffs from lobbying, $\mathcal{L}^*_{z,1}$, and not lobbying at all, $\mathcal{L}^*_{z,0}$. If $\mathcal{L}^*_{z,1} > \mathcal{L}^*_{z,0}$, the firm in sector z decides to lobby. Otherwise, it opts for not lobbying. In turn, since $\mathcal{L}^*_{z,1} = \tilde{\mathcal{L}}^*_{z,1} - \overline{\kappa}$ and $\mathcal{L}^*_{z,0} = -\overline{\kappa}_z(\delta, 0, \kappa^*_{-z})$, we have two alternative scenarios regarding the decision process of firms:

- In a competitive setting ($\delta < 0$) and intense lobbying effort by firms in the other sector ($\kappa^*_{-z} > 1$), the firm in sector z will always lobby since $\tilde{\mathcal{L}}_{z,1} > 0$ is always true. The intuition is that when faced with intense competition for an adversary, the firm will always lobby as the alternative decision of not lobbying implies the incurring in the same fixed costs and no lobbying gains at all. This implies that, in these circumstances, the firm will lobby even if it has negative profits to avoid even more negative profits.

- in a collaborative setting ($\delta > 0$) or minor lobbying efforts from firms in the other sector ($\kappa^*_{-z} \leq 1$), the firm in sector z is never penalized from not lobbying and, therefore, will lobby only if $\tilde{\mathcal{L}}^*_{z,1} - \overline{\kappa} > 0$, i.e., if it obtains a net positive payoff from lobbying, which implies a sufficiently high level of variable lobbying gains.

In turn, if a firm decides to lobby, the intensity of lobbying depends on overall lobbying effectiveness from lobbying, Γ, in the following manner:

- For $\Gamma < \overline{\Gamma}_{j,\underline{-j}}$, i.e., overall lobbying effectiveness is low from lobbying, both firms only lobby a little.

- For $\overline{\Gamma}_{j,\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}}$, i.e. overall lobbying effectiveness is intermediate, the firm with the most influence, j, lobbies intensively, while the other lobbies but not intensively, since it does not have much to gain from lobbying due to the relatively lower specific sources of lobbying efficiency.

- For $\overline{\Gamma}_{-j,\overline{j}} < \Gamma$, i.e., overall lobbying effectiveness is substantial, all firms lobby intensively as both stand to obtain significant gains from this activity.

Considering all these elements yields several pairs of NE lobbying gains for different values of Γ and fixed costs, which are summarized in the last three columns of Table 2.

If fixed costs are low, (third to last column) the payoff from lobbying is always positive for firms in both sectors and therefore higher than the alternative of not lobbying, which implies that NE is always to lobby.

For intermediate fixed costs (second to last column) the profits from lobbying for a firm in sector $-j$, which is relatively less efficient by assumption, are now negative, which leads to new implications for a firm in sector $-j$. In a collaborative setting ($\delta > 0$) or in a competitive setting low overall lobbying effectiveness ($\delta < 0 \wedge \Gamma < \overline{\Gamma}_{j,\underline{-j}}$), it does not lobby since the payoff is negative and, in the case, of

the latter, there is no penalty for not lobbying (since lobbying efforts from firms in sector j will not be intensive). On the other hand, in a competitive setting and sufficiently high level of lobbying effectiveness ($\delta < 0 \wedge \Gamma > \overline{\Gamma}_{j,\underline{-j}}$), the NE lobbying efforts from a firm in sector j will be intensive if $-j$ opts for not lobbying, which implies a penalty for the firm in sector j for not lobbying. This leads to a optimal decision of lobbying, even with a negative payoff, to minimize losses, as these losses would be even greater in the alternative scenario.

Finally, for high fixed costs, no lobbying is conducted in a collaborative setting ($\delta > 0$), as firms agree that to be too costly nor in a competitive setting with a low level of common sources of lobbying efficiency ($\delta < 0 \wedge \Gamma < \overline{\Gamma}_{-j,\underline{j}}$). The latter cause is justified by the fact that the firm with the highest influence, which now has also negative profits, not being penalized for not lobbying since such decision will not incentivize the firm in other sector to lobby intensively (due to to common sources of efficiency being too low). In turn, the same reasoning drives the decision of the least influential firm to lobby.

However, if common sources of efficiency are sufficiently high in a competitive setting ($\delta < 0 \wedge \Gamma > \overline{\Gamma}_{-j,\underline{j}}$), an interesting NE is obtained: both firms lobby, despite that implying a negative payoff. This occurs due to a "prisioner's dilemma" type of situation. Since common sources of efficiency are sufficiently high, there are enough incentives for firms to lobby intensively in case the other firm does not lobby, implying a penalty for firms that make this decision. Therefore, since both firms are competing and do not collaborate, the best course of action for each firm is to lobby, despite that implying negative profits, because the alternative decision would imply an even more negative payoff (due to the penalty imposed by the other firm), which can only be avoided by lobbying.

From this summary, we can establish the following proposition:

Proposition 1. *A firm will lobby if (i) if fixed costs are not too high and additionally and (ii) in the specific case of competitive lobbying, also if common sources of lobbying efficiency are sufficiently high relative to specific ones.*

Proof. In an online appendix. □

From this proposition we can derive the following corollary concerning lobbying parameter regimes:

Corollary 1. *There will three lobbying parameter regimes where: (i) All firm lobby, for low fixed costs or, in the case of competitive lobbying, sufficiently high level of common sources of lobbying efficiency; (ii) Only the most efficient firm lobbies, for medium fixed costs and, in the case of competitive lobbying, sufficiently low level of common sources of lobbying efficiency; (iii) No firm lobbies, for high fixed costs and, in the case of competitive lobbying, sufficiently low level of common sources of lobbying efficiency.*

Table 2: Summary of Nash-equilibria lobbying gains and corresponding conditions for the possible range of intervals

δ	Γ	$\mathcal{L}^*_{j,1}$ $\left(=\tilde{\mathcal{L}}^*_{j,1}-\overline{\kappa}\right)$	$\mathcal{L}^*_{j,0}$ $(=-\overline{\kappa}_{,j})$	$\mathcal{L}^*_{-j,1}$ $\left(=\tilde{\mathcal{L}}^*_{-j,1}-\overline{\kappa}\right)$	$\mathcal{L}^*_{-j,0}$ $(=-\overline{\kappa}_{,-j})$	$\left(\mathcal{L}^*_j=\max(\mathcal{L}^*_{j,1},\mathcal{L}^*_{j,0}),\mathcal{L}^*_{-j}=\max(\mathcal{L}^*_{-j,1},\mathcal{L}^*_{-j,0})\right)$		
						$\overline{\kappa}<\tilde{\mathcal{L}}^*_{-j,1}<\tilde{\mathcal{L}}^*_{j,1}$ $\left(0<\mathcal{L}^*_{-j,1}<\mathcal{L}^*_{j,1}\right)$	$\tilde{\mathcal{L}}^*_{-j,1}<\overline{\kappa}<\tilde{\mathcal{L}}^*_{j,1}$ $\left(\mathcal{L}^*_{-j,1}<0<\mathcal{L}^*_{j,1}\right)$	$\tilde{\mathcal{L}}^*_{-j,1}<\tilde{\mathcal{L}}^*_{j,1}<\overline{\kappa}$ $\left(\mathcal{L}^*_{-j,1}<\mathcal{L}^*_{j,1}<0\right)$
<0	$\Gamma<\overline{\Gamma}_{j,\overline{-j}}$	$\mathcal{L}^*_{j,\underline{-j}}$	0 $(\kappa^*_{-j,\underline{j}}<1)$	$\mathcal{L}^*_{\underline{-j},\underline{j}}$	0 $(\kappa^*_{j,\underline{-j}}<1)$	$(\mathcal{L}^*_{j,\underline{-j}},\mathcal{L}^*_{\underline{-j},\underline{j}})$	$(\mathcal{L}^*_{j,\underline{-j}},0)$	$(0,0)$
	$\overline{\Gamma}_{j,\overline{-j}}\le\Gamma<\overline{\Gamma}_{j,\underline{-j}}$							
	$\overline{\Gamma}_{j,\underline{-j}}\le\Gamma<\overline{\Gamma}_{-j,\underline{j}}$			$\mathcal{L}^*_{\underline{-j},\overline{j}}$	$-\overline{\kappa}$ $(\kappa^*_{j,\underline{-j}}>1)$	$(\mathcal{L}^*_{j,\underline{-j}},\mathcal{L}^*_{\underline{-j},\overline{j}})$	$(\mathcal{L}^*_{j,\underline{-j}},\mathcal{L}^*_{\underline{-j},\overline{j}})$ (<0)	$(\mathcal{L}^*_{j,\underline{-j}},\mathcal{L}^*_{\underline{-j},\overline{j}})$ (<0) (<0)
	$\overline{\Gamma}_{-j,\underline{j}}\le\Gamma<\overline{\Gamma}_{-j,\overline{j}}$		$-\overline{\kappa}$ $(\kappa^*_{-j,\underline{j}}>1)$					
	$\overline{\Gamma}_{-j,\overline{j}}\le\Gamma$	$\mathcal{L}^*_{\overline{j},\overline{-j}}$		$\mathcal{L}^*_{\overline{-j},\overline{j}}$		$(\mathcal{L}^*_{\overline{j},\overline{-j}},\mathcal{L}^*_{\overline{-j},\overline{j}})$	$(\mathcal{L}^*_{\overline{j},\overline{-j}},\mathcal{L}^*_{\overline{j},\overline{-j}})$ (<0)	$(\mathcal{L}^*_{\overline{j},\overline{-j}},\mathcal{L}^*_{\overline{-j},\overline{j}})$ (<0) (<0)
>0	$\Gamma<\overline{\Gamma}_{j,\overline{-j}}$	$\mathcal{L}^*_{j,\underline{-j}}$	0	$\mathcal{L}^*_{\underline{-j},\underline{j}}$	0	$(\mathcal{L}^*_{j,\underline{-j}},\mathcal{L}^*_{\underline{-j},\underline{j}})$	$(\mathcal{L}^*_{j,\underline{-j}},0)$	$(0,0)$
	$\overline{\Gamma}_{j,\overline{-j}}\le\Gamma<\overline{\Gamma}_{j,\underline{-j}}$			$\mathcal{L}^*_{\underline{-j},\overline{j}}$		$(\mathcal{L}^*_{j,\underline{-j}},\mathcal{L}^*_{\underline{-j},\overline{j}})$		
	$\overline{\Gamma}_{j,\underline{-j}}\le\Gamma<\overline{\Gamma}_{-j,\underline{j}}$							
	$\overline{\Gamma}_{-j,\underline{j}}\le\Gamma<\overline{\Gamma}_{-j,\overline{j}}$	$\mathcal{L}^*_{\overline{j},\overline{-j}}$		$\mathcal{L}^*_{\overline{-j},\overline{j}}$		$(\mathcal{L}^*_{\overline{j},\overline{-j}},\mathcal{L}^*_{\overline{-j},\overline{j}})$	$(\mathcal{L}^*_{\overline{j},\overline{-j}},0)$	
	$\overline{\Gamma}_{-j,\overline{j}}\le\Gamma$							

By analyzing the algebraic expressions for NE lobbying gains we can establish the following proposition regarding the relation with the relevant parameters:

Proposition 2. *If a firm of sector j lobbies, the lobbying gains of a firm in sector j are positively affected by common and specific sources of lobbying efficiency, Γ and ϕ_j, respectively, and negatively by fixed costs, $\overline{\kappa}$.*

Proof. By calculating each of the relevant derivatives we have that $\frac{\partial \mathcal{L}^*_{j,-j,1}}{\partial \Gamma} > 0$, $\frac{\partial \mathcal{L}^*_{j,-j,1}}{\partial \phi_j} > 0$ and $\frac{\partial \mathcal{L}^*_{j,-j,1}}{\partial \overline{\kappa}} < 0$. Therefore, since a firm can lobby or not, in which case the derivative is zero, in general we have that $\frac{\partial \mathcal{L}^*_j}{\partial \Gamma} \geq 0$, $\frac{\partial \mathcal{L}^*_j}{\partial \phi_j} \leq 0$ and $\frac{\partial \mathcal{L}^*_j}{\partial \overline{\kappa}} \leq 0$. □

Finally, the difference between lobbying gains of firms in sector j and $-j$ in the following manner:

$$\Delta \mathcal{L}^*_{j,-j} \equiv \mathcal{L}^*_j - \mathcal{L}^*_{-j} = \begin{cases} 0 & \text{if } \kappa^*_j = 0 \wedge \kappa^*_{-j} = 0 \\ \mathcal{L}^*_{j,-j,1} & \text{if } \kappa^*_j > 0 \wedge \kappa^*_{-j} = 0 \\ \Delta \mathcal{L}^*_{j,-j,1} & \text{if } \kappa^*_j > 0 \wedge \kappa^*_{-j} > 0 \end{cases} \tag{4}$$

where

$$\Delta \mathcal{L}^*_{j,-j,1} \equiv \mathcal{L}^*_{j,1} - \mathcal{L}^*_{-j,1} = \begin{cases} \Delta \mathcal{L}^*_{\underline{j},\underline{-j}} \equiv \mathcal{L}^*_{\underline{j},\underline{-j}} - \mathcal{L}^*_{\underline{-j},\underline{j}} & \text{if } \Gamma < \overline{\Gamma}_{j,\underline{-j}} \\ \Delta \mathcal{L}^*_{\overline{j},\underline{-j}} \equiv \mathcal{L}^*_{\overline{j},\underline{-j}} - \mathcal{L}^*_{\underline{-j},\overline{j}} & \text{if } \overline{\Gamma}_{j,\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}} \\ \Delta \mathcal{L}^*_{\overline{j},\overline{-j}} \equiv \mathcal{L}^*_{\overline{j},\overline{-j}} - \mathcal{L}^*_{\overline{j},\overline{-j}} & \text{if } \overline{\Gamma}_{-j,\overline{j}} < \Gamma \end{cases} \tag{5}$$

Regarding this difference when both firms lobby, we can establish the following proposition analyzing the impact of the relevant parameters:

Proposition 3. *If both firm lobby, the difference in lobbying gains between a firm in sector j and sector $-j$ is positively affected by specific sources of lobbying efficiency, ϕ_j, and in a non-linear manner by common sources of lobbying efficiency, Γ. Specifically, in what regards the latter, it is impacted negatively in a collaborative setting ($\delta > 0$) and if common sources of lobbying efficiency are such that only the firm in sector j lobbies intensively $\left(\overline{\Gamma}_{j,\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}}\right)$ and sufficiency high ($\Gamma^{\diamond}_{\overline{j},\underline{-j}} < \Gamma$), being positive otherwise.*

Proof. By calculating each of the relevant derivatives, it is easily shown that, if both firms lobby, then $\frac{\partial \Delta \mathcal{L}^*_{j-j,1}}{\partial \phi_j} > 0$ and, in what concerns $\frac{\partial \Delta \mathcal{L}^*_{j-j,1}}{\partial \Gamma}$, we have the following:

$$\frac{\partial \Delta \mathcal{L}^*_{\underline{j},\underline{-j}}}{\partial \Gamma} > 0, \; \frac{\partial \Delta \mathcal{L}^*_{\overline{j},\overline{-j}}}{\partial \Gamma} > 0, \; \frac{\partial \Delta \mathcal{L}^*_{\overline{j},\underline{-j}}}{\partial \Gamma} \begin{cases} < 0 & \text{if } \delta > 0 \wedge \Gamma^{\diamond}_{\overline{j},\underline{-j}} < \Gamma \\ > 0 & \text{otherwise} \end{cases}.$$

Therefore, since $\Delta \mathcal{L}^*_{j-j,1} = \Delta \mathcal{L}^*_{\overline{j},\underline{-j}}$ if $\overline{\Gamma}_{j,\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}}$, then:

$$\frac{\partial \Delta \mathcal{L}^*_{j-j,1}}{\partial \Gamma} \begin{cases} < 0 & \text{if } \delta > 0 \wedge \overline{\Gamma}_{j,\underline{-j}} < \Gamma^{\diamond}_{\overline{j},\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}} \\ > 0 & \text{otherwise} \end{cases},$$

where

$$\Gamma^{\diamond}_{\overline{j},\underline{-j}} = \frac{\phi_j^{\frac{2\overline{\gamma}-\delta}{\delta}} \left(\frac{\overline{\gamma}}{\overline{\gamma}+\delta}\right)^{\frac{\overline{\gamma}^2}{\delta}}}{\gamma}.$$

□

We now provide some intuition. The impact of an increase of specific sources of lobbying efficiency for firms in sector j is straightforward as it increases the profits of j at the expense of $-j$. However, the impact of an increase of common sources of lobbying efficiency is more complex. For most of the values of Γ, an increase of this parameter increases the lobbying payoff of firms in both sectors but has a higher impact in firms of sector j due to the fact that they have more specific sources of lobbying efficiency by assumption ($\phi_j > 1 > \phi_{-j}$). However, in the context of collaborative lobbying ($\delta > 0$) and intermediate levels of common sources of lobbying efficiency such that only j lobbies intensively ($\overline{\Gamma}_{j,\underline{-j}} < \Gamma^{\diamond}_{\overline{j},\underline{-j}} < \Gamma < \overline{\Gamma}_{-j,\overline{j}}$), the impact is more positive for firms in sector $-j$. This occurs because, in this region, this leads to a higher increase of lobbying intensity by firms in sector $-j$, without this substantially affecting the lobbying activity in the other sector (due to still being low intensive), which causes profits to increase more proportionally for firms in $-j$.

3 Theoretical model with lobbying

The objective of this section is to analyze the Balanced growth path implication for growth and inequality of demand-seeking and rent-seeking dimensions of lobbying in the context of a standard two-sector growth model extended to account for the impacts of the aforementioned forms of lobbying. To achieve this objective, we start by characterizing the baseline model, defining the general equilibrium and the balanced growth path. Our baseline model extends a standard two-sector endogenous growth framework by introducing lobbying decisions at the intermediate goods level, where firms strategically allocate resources either towards rent-seeking (targeting profits without increasing demand) or demand-seeking (boosting demand for their goods). We detail below how lobbying affects price setting, entry costs, and ultimately, growth and inequality.

3.1 Theoretical model

We assume an economy populated by a fixed number of infinitely-lived households with perfect foresight concerning the path of all relevant variables. The aggregate final good Y is produced in perfect competition with sector specific final goods from two distinct sectors: sectors i and $-i$. The sector specific final goods from a sector $z \in \{i, -i\}$, Y_z, are produced in perfect competition using sector specific labor L_z and a continuum of sector specific intermediate goods $x_z(n)$. Sector specific labor L_z is supplied inelastically by households. Sector specific intermediate goods $x_z(n)$, are produced using η units of aggregate final goods output by a single firm in perpetuity, who buys the respective design from the R&D sector, where it is assumed that there is free entry.

We assume that households maximize the following utility function:

$$U = \int_0^{\infty} \left(\frac{C(t)^{1-\theta} - 1}{1-\theta}\right) e^{-\rho t} dt,$$

where $C(t)$ is the consumption of final goods, θ is the inverse of the intertemporal elasticity of substitution and ρ is the subjective discount rate. Their maximization problem is subject to the following flow budget constraint:

$$\dot{a}(t) \quad = \quad r(t) \cdot a(t) + W_i(t) \cdot L_i + W_{-i}(t) \cdot L_{-i} - C(t),$$

where $a(t)$ are real financial assets holdings, with $a(t) = a_i(t) + a_{-i}(t)$ and $a_i(t) = N_i(t)V_i(t)$, $V_i(t)$ is the present value of monopoly profits of average firm providing intermediate goods to sector i, $N_i(t)$ is the number of such of firms, and therefore, intermediate goods, (since as it will be explained further below each firm produces a specific intermediate good in monopoly), $r(t)$ is real interest rate, $W_i(t)$ are wages for labor employed in the final sector i, $L(i)$.

Finally, the transversality condition is $\lim_{t\to\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0$ and the non-ponzi condition is $\lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$. By solving this problem, we obtain the optimal growth rate of consumption given by the usual Euler equation below:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho)\,. \tag{6}$$

The final aggregate good Y is produced through the following Constant Elasticity of Substitution (CES) production function:

$$Y(t) \quad = \left[\chi_i Y_i(t)^{\frac{\varepsilon-1}{\varepsilon}} + \chi_{-i} Y_{-i}(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad , \quad \varepsilon \in (0, +\infty)$$

where Y_i is Final good from sector i with price P_i, $\varepsilon \geq 0$ is the elasticity of substitution between the two inputs in the production of the aggregate final good and χ_i and χ_{-i} are distribution parameters measuring the relative importance of each input, such that $\chi_i + \chi_{-i} = 1$. Therefore, the producer of Y faces the following profit maximization problem:

$$\begin{aligned} \max_{Y_i, Y_{-i}} \Pi_Y \quad &= \quad PY - P_i Y_i - P_{-i} Y_{-i}, \\ s.t. \quad & Y(t) = \left[\chi_i Y_i(t)^{\frac{\varepsilon-1}{\varepsilon}} + \chi_{-i} Y_{-i}(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

By solving this problem we obtain expressions for the inverse relative demand for output of sector i and the price index P, which we assume for simplification to be 1, which are given, respectively below:

$$\begin{aligned} \frac{P_i}{P_{-i}} \quad &= \quad \frac{\chi_i}{\chi_{-i}} \left(\frac{Y_i}{Y_{-i}}\right)^{-\frac{1}{\varepsilon}}, \\ P_Y \equiv 1 \quad &= \quad \left[\chi_i^{\varepsilon} P_i^{1-\varepsilon} + \chi_{-i}^{\varepsilon} P_{-i}^{1-\varepsilon}\right]^{\frac{1}{\varepsilon-1}}. \end{aligned} \tag{7}$$

In turn, the output from sector $z \in \{i, -i\}$, Y_z, is produced through the following Cobb-Douglas

production function:

$$Y_z = A \cdot L_z^{\alpha} \cdot \int_0^{N_z} x_z(n)^{1-\alpha} dn, z \in \{i, -i\} \tag{8}$$

where A is Total factor productivity, $\alpha \in [0,1]$ is Labor share of income, $x_z(n)$ is the quantity of intermediate good n used in the production of the output of sector $z \in \{i, -i\}$, with price $P_{x_z}(n) = P_{x_z}$ and $N_z(t)$ is the number of intermediate goods used in the production of output of sector $z \in \{i, -i\}$.

Therefore the corresponding producer faces the following profit maximization problem:

$$\begin{aligned} \max_{x_z(n), m_z} \Pi_{Y_z} \quad &= \quad P_z Y_z - W_z m_z - \int_0^{N_z} P_{x_z}(n) x_z(n) dn, z \in \{i, -i\} \\ s.t. \quad & Y_z = A \cdot L_z^{\alpha} \cdot \int_0^{N_z} x_z(n)^{1-\alpha} dn, z \in \{i, -i\}. \end{aligned}$$

By solving this problem we obtain expressions for the demand of intermediate good n by sector z, $x_z(n)$, and by using this expression, the relative wage of sector i, w_i/w_{-i}, and the relative supply of output of sector i, Y_i/Y_{-i}, given, respectively, by the expressions below:

$$\begin{aligned} x_z(n) &= \left(\frac{P_z \cdot (1-\alpha) \cdot A}{P_{x_z}} \right)^{\frac{1}{\alpha}} L_z, z \in \{i, -i\}, \\ \frac{W_i}{W_{-i}} &= \frac{N_i}{N_{-i}} \left(\frac{P_i}{P_{-i}} \right)^{\frac{1}{\alpha}} \left(\frac{P_{x_{-i}}}{P_{x_i}} \right)^{\frac{1-\alpha}{\alpha}}, \\ \frac{Y_i}{Y_{-i}} &= \left(\frac{P_{x_{-i}}}{P_{x_i}} \frac{P_i}{P_{-i}} \right)^{\frac{1-\alpha}{\alpha}} \frac{N_i}{N_{-i}} \frac{L_i}{L_{-i}}. \end{aligned}$$

We assume that that each intermediate good n is produced in monopoly in perpetuity, which is granted after such variety is discovered in the R&D sector and the corresponding patent is sold to such firm. Each intermediate good requires a η units of final aggregate good, which implies that the marginal cost of producing a single unit is $\eta P = \eta$, due to assuming that $P = 1$. The Demand-seeking dimension of lobbying affects marginal revenue by a factor $\mathcal{A}^{\mathcal{L}_{\mathcal{D},z}}$, where $\mathcal{A} > 1$ is a positive constant and $\mathcal{L}_z$ determines lobbying gains from demand-seeking. Similarly, the rent-seeking dimension of lobbying results in an increase of overall profits by a factor of $\mathcal{A}^{\mathcal{L}_{\mathcal{R},z}}$, where $\mathcal{L}_{\mathcal{R},z}$ capturing rent-seeking gains. Considering all this profit function is as follows

$$\pi_z(n) \equiv \mathcal{A}^{\mathcal{L}_{R,z}} \left[P_{x_z}(n) \mathcal{A}^{\mathcal{L}_{\mathcal{D},z}} - \eta \right] \cdot x_z(n), z \in \{i, -i\}, \tag{9}$$

from which it becomes clear that the baseline model without lobbying is obtained by considering that $\mathcal{L}_{R,z} = \mathcal{L}_{\mathcal{D},z} = 0$. The producer faces the following maximization problem:

$$\begin{aligned}\max_{P_{x_z}} \pi_z(n) &= \mathcal{A}^{\mathcal{L}_{R,z}}\left[P_{x_z}(n)\mathcal{A}^{\mathcal{L}_{D,z}} - \eta\right]\cdot x_z(n), z \in \{i,-i\},\\ s.t. \quad x_z(n) &= \left(\frac{P_z\cdot(1-\alpha)\cdot\mathcal{A}}{P_{x_z}}\right)^{\frac{1}{\alpha}} L_z, z \in \{i,-i\}.\end{aligned}$$

By solving it we obtain expressions for the price of the average intermediate good price, P_{x_z}, which is the same for all intermediate goods, and using it, the profits of the average intermediate good firm, which are given, respectively, below:

$$\begin{aligned}P_{x_z}(n) &= P_{x_z} = \frac{\mathcal{A}^{-\mathcal{L}_{p,z}}\eta}{1-\alpha}, z \in \{i,-i\},\\ \pi_z &= \exp\left[\left(\mathcal{L}_{\mathcal{R},z} + \frac{\mathcal{L}_{\mathcal{D},z}}{\alpha}\right)\ln A\right]\mathcal{C}_X L_i P_i^{\frac{1}{\alpha}}, z \in \{i,-i\}\end{aligned} \tag{10}$$

where

$$\mathcal{C}_x \equiv \mathcal{A}^{\frac{1}{\alpha}}\alpha\,(1-\alpha)^{\frac{2-\alpha}{\alpha}}\,\eta^{\frac{\alpha-1}{\alpha}}.$$

In the R&D sector, the new varieties are produced each moment in time by new firms according to the following production function:

$$\dot{N}_z(t) = \lambda_z \cdot Z_z(t)\cdot L_z^{-1}, z \in \{i,-i\},$$

where $Z_z(t)$: Units of aggregate final good spent in R&D, λ_z is an R&D productivity parameter and L_z^{-1} is introduced to remove scale effects from the model. There is free entry which is characterized by the following equation:

$$\dot{N}_z V_z = Z_z C(\mathcal{L}_{R,z}), z \in \{i,-i\},$$

with the left and right hand sides representing, respectively, the discounted value of firms producing each new variety that emerges and the corresponding cost. In what concerns the left hand side, the net value of each new firm is characterized by the usual Hamilton-Jacobi-Bellman equation given below:

$$V_z = \frac{\pi_z}{r} + \frac{\dot{V}_z}{r}, z \in \{i,-i\}.$$

In what concerns, the right hand side, $C(\mathcal{L}_{R,z})$ is a factor by which R&D costs increase due to rent seeking, regarding which we make the following assumptions:

- $\frac{\partial C(\mathcal{L}_{R,z})}{\partial \mathcal{L}_{R,z}} > 0$: the increase of lobbying costs increases with lobbying profits. This reflects the intuitive notion that the increase in rent seeking profits comes at the expense of an increase of R&D costs.
- for $\mathcal{L}_{R,z} > 0$, $C(\mathcal{L}_{R,z}) > \mathcal{A}^{\mathcal{L}_{R,z}}$: the increase of profits of intermediate firms by $\mathcal{A}^{\mathcal{L}_{R,z}}$ is reflected in an increase of R&D costs by a more than proportional increase of costs R&D such that, in the end, the net discounted value of a patent actually decreases due to rent seeking, leading to less incentives for firm to enter the R&D sector and innovate.

- $C(0) = 1$: the original setup can be considered a particular case of this more generalized setup when there is no lobbying.

Bearing this in mind, we define a $C(\mathcal{L}_{R,z})$ in the following manner:[5]

$$\boldsymbol{C(\mathcal{L}_{R,z})} \equiv \mathcal{A}^{(1+\tau)\mathcal{L}_{R,z}}, z \in \{i, -i\}, \tau > 0,$$

where the parameter $\tau > 0$ represents the degree to which rent seeking is pernicious to R&D, since the higher this parameter, the more R&D costs are implied by increased rent-seeking gains.

3.2 Balanced growth path (BGP) effects of lobbying gains

This section analyzes the effects on growth and inequality of lobbying gains in a general equilibrium and, more specifically, a balanced growth path.

In the general equilibrium the following conditions: (i) households maximize their utility and the transversality condition holds, (ii) all firms maximize profits, (iii) the free entry condition holds and (iv) the Hamilton-Jacobi-Bellman condition is satisfied. The equilibrium relative price of output of sector i and the wage ratio are given by the following expressions, respectively:

$$\frac{P_i}{P_{-i}} = \left(\frac{\chi_i}{\chi_{-i}}\right)^{\frac{\alpha\varepsilon}{\sigma}} \left(\frac{L_i}{L_{-i}}\frac{N_i}{N_{-i}}\right)^{\frac{-\alpha}{\sigma}} \exp\left[-\frac{(1-\alpha)\,\Delta\mathcal{L}_{\mathcal{D},i}}{\sigma}\ln\mathcal{A}\right], \tag{11}$$

$$\mathcal{W}_i \equiv \frac{W_i}{W_{-i}} = \exp\left[\frac{(1-\alpha)\,(\sigma-1)}{\alpha\sigma}\Delta\mathcal{L}_{\mathcal{D},i}\ln A\right]\left(\frac{L_i}{L_{-i}}\right)^{-\frac{1}{\sigma}}\left(\frac{\chi_i}{\chi_{-i}}\right)^{\frac{\varepsilon}{\sigma}}\left(\frac{N_i}{N_{-i}}\right)^{\frac{\sigma-1}{\sigma}}, \tag{12}$$

where $\sigma \equiv 1 + \alpha(\varepsilon - 1)$. The balanced growth path is a general equilibrium characterized additionally by a (v) constant real interest rate and a (vi) constant growth rate. Considering that that (v), we have that $\dot{V}_z = 0$, which implies the following expression for the real interest rate:

$$r = \frac{\lambda_z \pi_z}{(1+\mathcal{L}_{R,z})^{\tau} L_z}, z \in \{i, -i\}, \tag{13}$$

Equating expressions from both sectors and considering (10) and (11), we obtain the following expression for the relative price of sector i:

$$\left(\frac{P_i}{P_{-i}}\right)^{*} = \exp\left[(\alpha\tau\Delta\mathcal{L}_{\mathcal{R},i} - \Delta\mathcal{L}_{\mathcal{D},i})\ln\mathcal{A}\right]\left(\frac{\lambda_i}{\lambda_{-i}}\right)^{-\alpha}. \tag{14}$$

where

$$\Delta\mathcal{L}_i \equiv \mathcal{L}_i - \mathcal{L}_{-i}, \in \{\mathcal{D}, \mathcal{R}\}.$$

Replacing this expression in (11) and solving for the relative number of varieties of sector i, a measure of

[5] There are many alternatives to model rent-seeking in such a model. The functional choice aims to qualitatively replicate key stylized facts, namely: (i) rent-seeking diverts resources from productive activities such as R&D; (ii) its costs outweigh its private benefits, leading to dynamic inefficiencies. While alternative functional forms could capture the rent-seeking cost impact, we adopt an exponential specification due to its tractability and ability to model disproportionate increases in R&D costs as rent-seeking intensifies. Nonetheless, robustness to alternative specifications warrants future exploration

technological bias, we obtain the following BGP expression for the technological bias:

$$\left(\frac{N_i}{N_{-i}}\right)^* = \exp\left[(\varepsilon\Delta\mathcal{L}_{\mathcal{D},i} - \sigma\tau\Delta\mathcal{L}_{\mathcal{R},i})\ln\mathcal{A}\right]\left(\frac{\lambda_i}{\lambda_{-i}}\right)^{\sigma}\left(\frac{\chi_i}{\chi_{-i}}\right)^{\varepsilon}\frac{L_{-i}}{L_i}. \tag{15}$$

Finally, replacing (15) in (12), we obtain the expression for the BGP wage ratio:

$$\mathcal{W}_i^* = \exp\left[(\sigma-1)\left(-\tau\Delta\mathcal{L}_{\mathcal{R},i} + \frac{\Delta\mathcal{L}_{\mathcal{D},i}}{\alpha}\right)\ln\mathcal{A}\right]\left(\frac{\lambda_i}{\lambda_{-i}}\right)^{\sigma-1}\left(\frac{\chi_i}{\chi_{-i}}\right)^{\varepsilon}\frac{L_{-i}}{L_i}. \tag{16}$$

From there, we arrive at the first proposition.

Proposition 4. *If $\sigma > 1$, i.e., if labor from sectors i and $-i$ are gross substitutes, the BGP wage ratio $\left(\frac{W_i}{W_{-i}}\right)^*$ (i) increases with a higher relative demand-seeking dimension of lobbying payoff of sector i and (i) decreases with a higher relative rent-seeking dimension of lobbying payoff of sector i.*

Proof. If we differentiate (16) with respect to $\mathcal{L}^*_{R,z}$, and $\mathcal{L}^*_{R,z}$, we obtain the following expressions:

$$\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_i^*} = \mathcal{C}_\nu\mathcal{W}_i^*\,(\sigma-1)\ln A, l \in \{p, r\},$$

where

$$\mathcal{C}_\nu \equiv \begin{cases} \alpha^{-1} & \text{if } \nu = \mathcal{D} \\ -\tau & \text{if } \nu = \mathcal{R} \end{cases}.$$

captures the specific impact of each dimension of lobbying on the labor demand, which, as is explained in more detail at the end of this section, is positive for demand-seeking and negative for rent-seeking.

Therefore, if $\sigma > 1$, we have that $\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_{\mathcal{D},i}} > 0$ and $\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_{\mathcal{R},i}} < 0$ we have that $\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_{\mathcal{D},i}} > 0$ and $\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_{\mathcal{R},i}} > 0$. Otherwise, we have that $\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_{\mathcal{D},i}} < 0$ and $\frac{\partial\mathcal{W}_i^*}{\partial\Delta\mathcal{L}_{\mathcal{R},i}} > 0$. □

Economically this result reflects that demand-seeking lobbying boosts the marginal revenue of intermediate goods firms, leading to increased production, entry, and labor demand in the corresponding sector. When labor is a gross substitute across sectors, this demand shift results in upward pressure on sectoral wages, widening wage differentials. Conversely, rent-seeking lobbying reduces the profitability of innovation, leading to the opposite effects.

If we replace (14) in the expression of the price index and solve for P_i, we obtain the following BGP expression for the absolute price level of output of sector i:

$$P_z = \frac{\tilde{P}_z^*}{\overline{\tilde{P}}^*},\ z \in \{i, -i\} \tag{17}$$

where

$$\begin{aligned} \tilde{P}_z^* &\equiv \exp\left[(\alpha\tau\mathcal{L}_{\mathcal{R},z} - \mathcal{L}_{\mathcal{D},z})\ln\mathcal{A}\right]\lambda_z^{-\alpha}, z \in \{i, -i\}, \\ \overline{\tilde{P}} &\equiv \left[\chi_i^{\varepsilon}\tilde{P}_i^{1-\varepsilon} + \chi_{-i}^{\epsilon}\tilde{P}_{-i}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \end{aligned}$$

are, respectively, a price index that relates how parameters and variables of sector z affect the price of

output produced in this sector and a CES weighted average[6] of such indexes.

Replacing (17) in (10), replacing the latter in (13), and considering (6), we obtain the following expression for the BGP growth rate:

$$g^* \quad = \quad \theta^{-1}\left(\mathcal{C}_X\overline{\tilde{P}}^{-\frac{1}{\alpha}} - \rho\right) = \frac{\mathcal{C}_X\left[\sum_z \chi_z^{\epsilon}\left(\exp\left[\left(\alpha\mathcal{L}_{\mathcal{R},z}\tau - \mathcal{L}_{\mathcal{D},z}\right)\ln\mathcal{A}\right]\lambda_z^{-\alpha}\right)^{1-\epsilon}\right]^{\frac{1}{\sigma-1}} - \rho}{\theta}. \tag{18}$$

From here we arrive at the following second proposition.

Proposition 5. *The BGP growth rate, g^*, increases with higher demand seeking lobbying gains from any sector and decreases with rent-seeking gains from any sector.*

Proof. If we differentiate (18) with respect to $\mathcal{L}_z^*$, and $\mathcal{L}_{\mathcal{R},z}^*$, we obtain the following expressions:

$$\frac{\partial g^*}{\partial \mathcal{L}_{\nu,z}} = \theta^{-1}\mathcal{C}_X^{\alpha(\epsilon-1)}\left(g^*\theta+\rho\right)^{2-\sigma}\ln\mathcal{A}\mathcal{C}_\nu\chi_z^{\epsilon}\tilde{P}_z^{1-\epsilon}, z\in\{i,-i\}, \nu\in\{\mathcal{R}\}$$

It is clear that $\frac{\partial g^*}{\partial \mathcal{L}_{\nu,z}^*} > 0$ and $\frac{\partial g^*}{\partial \mathcal{L}_{\mathcal{R},\nu,z}} < 0$. □

We now provide intuition for this proposition. An increase in demand seeking lobbying gains in sector i has an immediate effect of increasing profits by increasing the marginal revenue of firms producing intermediate goods to be used in the production of output of sector i. This has two effects:

(a) it increases the marginal revenue of firms producing intermediate goods to be used in the production of output of sector i. This allows them to practice lower prices, which reduces the costs of the corresponding intermediate goods and, hence, increases the supply of output that uses such intermediate goods.

(b) it leads, in the short-run, to the net discounted value of a patent to surpass its costs, which, due to is free entry, leads to an increase of the entry rate of firms producing designs of intermediate goods to be used in sector i and, hence, a higher relative number of intermediate goods, reflected in a higher BGP technological bias N_i/N_{-i}.

These effects increase the output produced by sector i and, hence, the demand of labor used in that sector, whose impact on the relative wage, in turn, depends on the elasticity of substitution between labor units used in the production of each sector, which is determined by the elasticity of substitution between the corresponding outputs.

- If $\sigma > 1$, i.e., if factors are gross substitutes, this leads to a decrease in the demand of labor used in sector $-i$, which increases the relative wage.
- If $0 < \sigma < 1$, i.e., if factors are gross complements, the demand for labor used in sector $-i$ increases more than proportionally, leading to a more than proportionally increase in the corresponding wage, contributing a lower relative wage.

An increase on rent seeking lobbying gains increases profits directly at the cost of imposing additional entry costs for firms wishing to produce designs of intermediate goods to be used in sector i. Therefore,

[6] We consideration designation to be appropriate since this function has important properties of an average namely (i) internal boundness, since $\min(\tilde{P}_i, \tilde{P}_{-i}) < \overline{\tilde{P}} < \max(\tilde{P}_i, \tilde{P}_{-i})$, (ii) monotonicity, since $\frac{\partial\overline{\tilde{P}}}{\partial\tilde{P}_i} = \left(\frac{\overline{\tilde{P}}_{\chi_i^{\epsilon}}}{\tilde{P}_i}\right)^{\epsilon} > 0$, and (iii) homogeneous of degree 0 (i.e., constant returns to scale), since, for a constant k, $\overline{\tilde{P}}(k\tilde{P}_i, k\tilde{P}_{-i}) = k\overline{\tilde{P}}(\tilde{P}_i, \tilde{P}_{-i})$.

effect (a) does not occur and the opposite of effect (b) takes place due to the fact that the increase in costs surpasses the increase in profits due to rent seeking. This implies a lower growth rate and a lower demand demand of sector i, which, in turn, leads to an increase of the labor demand in sector $-i$, if $\sigma > 1$ and, hence, a lower relative wage of sector i, and the opposite if $0 < \sigma < 1$.

Both effects (a) and (b) lead to an increase of the output produced by sector i, which is reflected in a lower price of sector i, P_i and, with no changes in P_{-i}, a lower average price index, $\overline{\tilde{P}}$, which leads to a higher growth rate of output.

3.3 BGP effects of lobbying determinants

We now conduct a comparative static analysis by analyzing the impact of the parameters determining lobbying gains on growth and inequality. To do so, we replace $\mathcal{L}_j$ with the corresponding $\mathcal{L}_j^*$ in the BGP expressions of the wage ratio and growth rate, resulting in the following expressions, where the main changes are highlighted in bold:

$$\begin{aligned}
\mathcal{W}_i^*(\Psi) &= \exp\left[(\sigma-1)\left(-\tau\boldsymbol{\Delta\mathcal{L}^*_{\mathcal{R},i}(\Psi)} + \tfrac{\boldsymbol{\Delta\mathcal{L}^*_{\mathcal{D},i}(\Psi)}}{\alpha}\right)\ln\mathcal{A}\right]\left(\frac{\lambda_i}{\lambda_{-i}}\right)^{\sigma-1}\left(\frac{\chi_i}{\chi_{-i}}\right)^{\epsilon}\frac{L_{-i}}{L_i}, \\
g^*(\Psi) &= \frac{\mathcal{C}_X\left[\sum_z \chi_z^{\epsilon}\left(\exp\left[\left(\alpha\boldsymbol{\mathcal{L}^*_{\mathcal{R},z}(\Psi)}\tau - \boldsymbol{\mathcal{L}^*_{\mathcal{D},z}(\Psi)}\right)\ln\mathcal{A}\right]\lambda_z^{-\alpha}\right)^{1-\epsilon}\right]^{\frac{1}{\sigma-1}} - \rho}{\theta},
\end{aligned}$$

and $\Psi = [\Gamma, \phi_i, \overline{\kappa}]$ is a vector of the relevant lobbying parameters[7] and $\mathcal{L}^*_{\nu,j}$ and $\Delta\mathcal{L}^*_{\nu,j,-j}$ are defined using (1) and (4), respectively. Then, if we take the derivative of the BGP growth rate and wage ratio with respect to a lobbying parameter, in general terms we obtain the following expressions below:

$$\frac{\partial\mathcal{W}^*}{\partial\psi} = \sum_{\nu} \underbrace{\frac{\partial\mathcal{W}^*}{\partial\Delta\mathcal{L}^*_{\nu,i}}\frac{\partial\Delta\mathcal{L}^*_{\nu,i}}{\partial\psi}}_{\substack{\nu\text{-type lobbying channel} \\ \text{of } \psi \text{ for the BGP wage ratio}}}, \frac{\partial g^*}{\partial\psi} = \sum_{\nu} \underbrace{\sum_z \frac{\partial g^*}{\partial\mathcal{L}^*_{\nu,z}}\frac{\partial\mathcal{L}^*_{\nu,z}}{\partial\psi}}_{\substack{\nu\text{-type lobbying channel} \\ \text{of } \psi \text{ for the BGP growth rate}}}, \nu \in \{\mathcal{D},\mathcal{R}\}, \psi \in \Psi.$$

We will start by obtaining the expressions for each individual transmission channel. In the case of the BGP wage ratio, , we obtain the following expression for the transmission channel of lobbying of dimension ν:

$$\frac{\partial\mathcal{W}^*}{\partial\Delta\mathcal{L}^*_{\nu,i}}\frac{\partial\Delta\mathcal{L}^*_{\nu,i}}{\partial\psi} = \mathcal{C}_\nu(\sigma-1)\mathcal{W}_i^*\ln\mathcal{A}\frac{\partial\Delta\mathcal{L}_i^*}{\partial\psi}, \nu \in \{\mathcal{D},\mathcal{R}\}, \psi \in \Psi. \tag{19}$$

The analysis of the sign of this derivative is summarized in the following table:

[7] For simplicity, we assume at this stage that the parameters are equal for both dimensions of lobbying. As it is shown further ahead, this does not impede a generalized analysis.

Table 3: BGP wage ratio impacts of changes in lobbying parameters.

σ	ν	$\mathcal{C}_\nu$	$\mathcal{C}_\nu(\sigma-1)$	$F(\psi)$	ψ			
					Γ		ϕ_i	$\overline{\kappa}$
				$\frac{\partial \Delta \mathcal{L}^*_{\nu,i}}{\partial \psi}$	$\boxed{\geq 0}$	≤ 0	$\boxed{\geq 0}$	≤ 0
<1	$\mathcal{D}$	>0	<0	$\boldsymbol{\frac{\partial \mathcal{W}^*}{\partial \Delta \mathcal{L}^*_{\nu,i}} \frac{\partial \Delta \mathcal{L}^*_{\nu,i}}{\partial \psi}}$	$\boldsymbol{\geq 0}$	$\boldsymbol{\boxed{\geq 0}}$	$\boldsymbol{\leq 0}$	$\boldsymbol{\boxed{\geq 0}}$
	$\mathcal{R}$	<0	>0		$\boldsymbol{\boxed{\geq 0}}$	$\boldsymbol{\leq 0}$	$\boldsymbol{\boxed{\geq 0}}$	$\boldsymbol{\leq 0}$
>1	$\mathcal{D}$	>0						
	$\mathcal{R}$	<0	<0		$\boldsymbol{\leq 0}$	$\boldsymbol{\boxed{\geq 0}}$	$\boldsymbol{\leq 0}$	$\boldsymbol{\boxed{\geq 0}}$

From here, can establish the following proposition:

Proposition 6. *In what concerns lobbying parameters, the relative wage of workers in sector i is affected in a specific manner if factors are gross substitutes and in the opposite manner if they are gross complements. Specifically, if factors are* ***gross substitutes****, i.e., $\sigma > 1$, the relative wage of workers in sector increases through the:*

(i) ***demand seeking channel,*** *following a (i-a) a decrease in fixed costs, (i-b) an increase of the relative efficiency of firms in sector i, (i-c) an increase of common sources of lobbying efficiency provided that their marginal impact on lobbying gains is positive.*

(ii) ***rent seeking channel,*** *following a (i-a) a increase in fixed costs, (i-b) a decrease of the relative efficiency of firms in sector i, (i-c) a decrease of common sources of lobbying efficiency provided that their marginal impact on lobbying gains is positive.*

Proof. See Table 3. □

In the case of the BGP growth rate, we obtain the following expression for the transmission channel of dimension ν lobbying:

$$\sum_z \frac{\partial g^*}{\partial \mathcal{L}^*_{\nu,z}} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi} = \frac{1}{\alpha} \psi^{-1} \mathcal{C}_X \overline{\tilde{P}}^{\frac{\sigma-2}{\alpha}} \ln \mathcal{AC}_\nu \left(\sum_z \chi_z^\epsilon \tilde{P}_z^{1-\epsilon} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi} \right), \nu \in \{\mathcal{D}, \mathcal{R}\}, \psi \in \Psi. \tag{20}$$

The analysis of the sign of this derivative is summarized in the following table:

Table 4: BGP growth impacts of changes in lobbying parameters.

ν	$\mathcal{C}_\nu$	$F(\psi_\nu)$	ψ			
			Γ	ϕ_i		$\overline{\kappa}$
		$\frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \psi}$	$\boxed{\geq 0}$	$\boxed{\geq 0}$		≤ 0
		$\frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \psi}$	$\boxed{\geq 0}$	≤ 0		≤ 0
		$\sum_z \chi_z^\epsilon \tilde{P}_z^{1-\epsilon} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi}$	$\boxed{\geq 0}$	$\boxed{\geq 0}$	≤ 0	≤ 0
$\mathcal{D}$	>0	$\sum_z \frac{\partial g^*}{\partial \mathcal{L}^*_{\nu,z}} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi}$	$\boxed{\geq \mathbf{0}}$	$\boxed{\geq \mathbf{0}}$	$\leq \mathbf{0}$	$\leq \mathbf{0}$
$\mathcal{R}$	<0		$\leq \mathbf{0}$	$\leq \mathbf{0}$	$\boxed{\geq \mathbf{0}}$	$\boxed{\geq \mathbf{0}}$

From here, can establish the following proposition:

Proposition 7. *The BGP growth rate will generally increase through the:*

(i) ***demand seeking channel****, if there is (i-a) an increase in overall lobbying efficiency, (i-b) a decrease in fixed costs (i-c) an increase of the relative efficiency by firms in sector i, provided that the magnitude of the ratio of marginal lobbying gains of firms in sector i to sector $-i$ is sufficiently large relative to (i-c-a) the relative importance of sector $-i$ and (i-c-b) the relative price of output produced in sector i, if factors are gross substitutes, the relative price of output produced in sector $-i$, if factors are gross complements.*

(ii) ***rent seeking channel****, if the opposite of (i) occurs.*

Proof. On the one hand, by proposition 2 and defining $\omega_{g,z} \equiv \chi_z^\epsilon \tilde{P}_z^{1-\epsilon}$ as a measure of the relative importance of sector z, we have that

$$\begin{aligned}
&\frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \Gamma} \geq 0, \frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \Gamma} \geq 0 \Rightarrow \frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \Gamma} + \omega_{g,-i} \frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \Gamma} \geq 0, \\
&\frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \overline{\kappa}} \leq 0, \frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \overline{\kappa}} \leq 0 \Rightarrow \frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \overline{\kappa}} + \omega_{g,-i} \frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \overline{\kappa}} \leq 0, \\
&\frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \phi_i} \geq 0, \frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \phi_i} \leq 0 \Rightarrow \frac{\partial \mathcal{L}^*_{\nu,i}}{\partial \Gamma} + \omega_{g,-i} \frac{\partial \mathcal{L}^*_{\nu,-i}}{\partial \Gamma} \begin{cases} \geq 0 & \text{if } -\frac{\partial \mathcal{L}^*_{\nu,i}/\partial \phi_i}{\partial \mathcal{L}^*_{\nu,-i}/\partial \phi_i} \geq \frac{\omega_{g,-i}}{\omega_{g,i}} \\ \leq 0 & \text{otherwise} \end{cases}.
\end{aligned}$$

On the other hand $\mathcal{C}_\mathcal{D} = \alpha^{-1} > 0$ and $\mathcal{C}_\mathcal{R} = -\tau < 0$. Considering all this, there are different possibilities for the sign of $\frac{\partial g^*}{\partial \psi_\nu}$, which are summarized in Table 4. □

3.4 Synthesis and Policy implications

We synthesize both analyses of the previous section in Table 5.

Table 5: Summary of BGP growth and wage ratio impacts of changes in lobbying parameters

σ	$F(\psi)$	ψ				
		Γ		ϕ_i		$\overline{\kappa}$
		$\nu = \mathcal{D}$				
	$\sum_z \chi_z^{\epsilon} \tilde{P}_z^{1-\epsilon} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi}$	≥ 0	≥ 0	≥ 0	≤ 0	≤ 0
	$\frac{\partial \Delta \mathcal{L}^*_{\nu,i}}{\partial \psi}$	≥ 0	$\boldsymbol{\leq 0}$	≥ 0	≥ 0	$\boldsymbol{\leq 0}$
< 1	$\sum_z \frac{\partial g^*}{\partial \mathcal{L}^*_{\nu,z}} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi}, \frac{\boldsymbol{\partial \mathcal{W}^*}}{\boldsymbol{\partial \Delta \mathcal{L}^*_{\nu,i}}} \frac{\boldsymbol{\partial \Delta \mathcal{L}^*_{\nu,i}}}{\boldsymbol{\partial \psi}}$	$\boxed{\geq 0}, \boldsymbol{\leq 0}$	$\boxed{\boldsymbol{\geq 0}}, \boldsymbol{\geq 0}$	$\boxed{\geq 0}, \boldsymbol{\leq 0}$	$\boldsymbol{\leq 0}, \boxed{\boldsymbol{\leq 0}}$	$\boldsymbol{\leq 0}, \boldsymbol{\geq 0}$
> 1		$\boxed{\geq 0}, \boldsymbol{\geq 0}$	$\boxed{\geq 0}, \boldsymbol{\leq 0}$	$\boxed{\geq 0}, \boldsymbol{\geq 0}$	$\boldsymbol{\leq 0}, \boldsymbol{\geq 0}$	$\boldsymbol{\leq 0}, \boxed{\boldsymbol{\leq 0}}$
		$\nu = \mathcal{R}$				
	$\sum_z \chi_z^{\epsilon} \tilde{P}_z^{1-\epsilon} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi}$	≥ 0	≥ 0	≥ 0	≤ 0	≤ 0
	$\frac{\partial \Delta \mathcal{L}^*_{\nu,i}}{\partial \psi}$	≥ 0	$\boldsymbol{\leq 0}$	≥ 0	≥ 0	$\boldsymbol{\leq 0}$
< 1	$\sum_z \frac{\partial g^*}{\partial \mathcal{L}^*_{\nu,z}} \frac{\partial \mathcal{L}^*_{\nu,z}}{\partial \psi}, \frac{\boldsymbol{\partial \mathcal{W}^*}}{\boldsymbol{\partial \Delta \mathcal{L}^*_{\nu,i}}} \frac{\boldsymbol{\partial \Delta \mathcal{L}^*_{\nu,i}}}{\boldsymbol{\partial \psi}}$	$\leq 0, \geq 0$	$\leq 0, \boxed{\leq 0}$	$\leq 0, \boldsymbol{\geq 0}$	$\boxed{\boldsymbol{\geq 0}}, \boldsymbol{\geq 0}$	$\boxed{\boldsymbol{\geq 0, \leq 0}}$
> 1		$\leq 0, \boxed{\leq 0}$	$\leq 0, \geq 0$	$\leq 0, \boxed{\boldsymbol{\leq 0}}$	$\boxed{\geq 0}, \leq 0$	$\boxed{\geq 0}, \geq 0$

From the analysis of this Table, we can draw three essential policy implications:

1. Policymakers interested in affecting lobbying practice in a manner that benefits the society should distinguish between policies specific to demand-seeking and rent-seeking as our model predicts that fit-all policies will have opposite effects on both outcomes, which might lead to unexpected and undesirable results.

2. Promoting economic growth can be achieved by enacting policies that reduce the incentives to rent-seeking dimension of obbying or increase the incentives to demand-seeking dimension of lobbying. In the case of the former, that can be achieved by, for example:

   (a) increasing registration procedures and the institutional hurdles faced by new entrants that have a clear intent (or a high probability) of being interested in rent-seeking dimension of lobbying.

   (b) reducing the extent to which rent-seeking dimension of lobbying is institutionally embedded and socially accepted - i.e., increasing fixed costs specific and reducing common sources of lobbying efficiency specific to rent-seeking activities.

   (c) strengthen antitrust tools for firms engaged in rent-seeking activities - i.e., reducing specific sources of lobbying effectiveness

   In case of the demand-seeking, the same can be achieved by analogous policies concerning this dimension of lobbying that achieve the opposite results.

3. Promoting economic growth while simultaneously reducing inequality is not straightforward as policies aiming at promoting the former can often also increase the latter, depending on factors such

as the elasticity of substitution between products from different sectors. Nonetheless, it is possible. An example is increasing common sources of lobbying efficiency for demand seeking dimension of lobbying in sectors when the elasticity of substitution is high and common sources of lobbying efficiency are neither too low or too high, such that an increase of actually diminishes the gap in lobbying payoffs.

4 Concluding remarks

In this paper, we present a game-theoretic approach to lobbying, developing a microfounded and tractable model motivated by stylized facts that characterize lobbying activity. This framework allows us to analyze both rent-seeking and demand-seeking behaviors, as well as competitive and cooperative equilibria in lobbying. To our knowledge, this is the first analysis to examine in such detail how the structural features of lobbying influence economic growth and inequality. While the rent-seeking dimension of lobbying may affect the price of intermediate goods—e.g., through special licenses that alter market concentration—the demand-seeking dimension influences the overall profitability of intermediate-good firms.

Our game-theoretic model identifies several key determinants of lobbying activity. One of our main findings is that when both firms engage in lobbying, the difference in relative lobbying gains is positively affected by relative efficiency, negatively affected by fixed costs, and depends non-linearly on the common source of lobbying efficiency. Specifically, this common source of efficiency has a negative effect in a collaborative setting and when it causes only one sector's firm to lobby intensively and sufficiently strongly.

When labor across sectors is a gross substitute, relative wages in each sector increase (decrease) with higher demand-seeking (rent-seeking) in that sector. Moreover, economic growth declines with rent-seeking from both sectors and rises with demand-seeking lobbying from both. More precisely, the relative wage of workers in a given sector increases through: (i) the demand-seeking channel, following (a) a decrease in fixed costs, (b) an increase in the sector's relative efficiency, or (c) an increase in common sources of lobbying efficiency—provided their marginal impact on lobbying gains is positive; and (ii) the rent-seeking channel, following (a) an increase in fixed costs, (b) a decrease in relative efficiency, or (c) a decrease in common sources of lobbying efficiency—again, provided their marginal impact is positive.

Similarly, the balanced growth path (BGP) growth rate will generally increase (decrease) through: (i) the demand-seeking channel, if there is (a) an increase (decrease) in overall lobbying efficiency, (b) a decrease (increase) in fixed costs, or (c) an increase (decrease) in relative efficiency—provided the ratio of marginal lobbying gains across sectors is sufficiently large; and (ii) the rent-seeking channel, if there is (a) a decrease (increase) in overall lobbying efficiency, (b) an increase (decrease) in fixed costs, or (c) a decrease (increase) in relative efficiency—again, conditional on a sufficiently large ratio of marginal lobbying gains between sectors.

From these results, we derive three key policy implications: (i) policymakers should avoid "one-size-fits-all" interventions in lobbying regulation, since policies affecting both dimensions of lobbying can yield opposite effects, leading to unintended or undesirable outcomes; (ii) promoting economic growth can be achieved by designing policies that reduce incentives for rent-seeking lobbying or increase incentives for demand-seeking lobbying; (iii) promoting growth while simultaneously reducing inequality is not straightforward, as measures that foster the former can sometimes exacerbate the latter, depending on factors such as the elasticity of substitution between sectoral products. Nonetheless, this trade-off can be mitigated.

For example, increasing common sources of lobbying efficiency for the demand-seeking dimension—when the elasticity of substitution is high and common efficiency levels are moderate—can reduce disparities in lobbying payoffs.

Although our model remains theoretical, its key parameters—such as lobbying efficiency, fixed lobbying costs, and the elasticity of lobbying returns—could be empirically estimated as more detailed sector-level data become available. Such empirical efforts would allow validation and refinement of the model's predictions regarding growth and inequality.

Finally, while our setup employs specific functional forms and assumptions to ensure tractability and analytical clarity, we acknowledge several limitations. The model abstracts from dynamic strategic interactions and assumes exponential lobbying cost functions. Future research could relax these assumptions by extending the analysis to dynamic or repeated-game frameworks, or by considering alternative cost structures (e.g., quadratic or fixed-plus-variable specifications). Another promising direction would be to study the effects of lobbying not only along the balanced growth path, as done here, but also during transitional dynamics—such as those following new lobbying regulations that alter key behavioral parameters.

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