# Supplementary Information: Quantitative Analysis of Transient Electron Compression — Glowing Ball Lightning Theory

Ruibo Li

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## 1. Debye Length Calculation

The Debye length is

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}}. \tag{1}$$

For $T_e = 1$–$2\,\mathrm{eV}$ and electron density $n_e = 10^{16}$–$10^{18}\,\mathrm{m^{-3}}$, we obtain the following representative values (using $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$, $e = 1.602 \times 10^{-19}\,\mathrm{C}$):

$$n_e = 10^{18}\,\mathrm{m^{-3}},\ T_e = 1\,\mathrm{eV} \Rightarrow \lambda_D \approx 7.43 \times 10^{-6}\,\mathrm{m}\ (7.43\ \mu\mathrm{m}), \tag{2}$$
$$n_e = 10^{18}\,\mathrm{m^{-3}},\ T_e = 2\,\mathrm{eV} \Rightarrow \lambda_D \approx 1.05 \times 10^{-5}\,\mathrm{m}\ (10.5\ \mu\mathrm{m}), \tag{3}$$
$$n_e = 10^{17}\,\mathrm{m^{-3}},\ T_e = 1\,\mathrm{eV} \Rightarrow \lambda_D \approx 2.35 \times 10^{-5}\,\mathrm{m}\ (23.5\ \mu\mathrm{m}), \tag{4}$$
$$n_e = 10^{17}\,\mathrm{m^{-3}},\ T_e = 2\,\mathrm{eV} \Rightarrow \lambda_D \approx 3.33 \times 10^{-5}\,\mathrm{m}\ (33.3\ \mu\mathrm{m}), \tag{5}$$
$$n_e = 10^{16}\,\mathrm{m^{-3}},\ T_e = 1\,\mathrm{eV} \Rightarrow \lambda_D \approx 7.43 \times 10^{-5}\,\mathrm{m}\ (74.3\ \mu\mathrm{m}), \tag{6}$$
$$n_e = 10^{16}\,\mathrm{m^{-3}},\ T_e = 2\,\mathrm{eV} \Rightarrow \lambda_D \approx 1.05 \times 10^{-4}\,\mathrm{m}\ (105.1\ \mu\mathrm{m}). \tag{7}$$

Therefore, for the adopted parameter range,

$$\lambda_D \sim 7 \times 10^{-6} \text{ to } 1 \times 10^{-4}\ \mathrm{m}$$

(i.e. roughly 7 $\mu$m up to $\sim$ 100 $\mu$m), depending on $n_e$ and $T_e$.

## 2. Plasma Beta Parameter

The plasma beta is defined as

$$\beta = \frac{2\mu_0 n_e k_B (T_e + T_i)}{B^2}, \tag{8}$$

and we take $T_i \approx T_e$, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$ and $B = 50\,\mu\mathrm{T}$ as before. Representative values:

$$n_e = 10^{18}\,\mathrm{m}^{-3},\ T_e = T_i = 1\,\mathrm{eV} \Rightarrow \beta \approx 3.22 \times 10^2, \tag{9}$$
$$n_e = 10^{18}\,\mathrm{m}^{-3},\ T_e = T_i = 2\,\mathrm{eV} \Rightarrow \beta \approx 6.44 \times 10^2, \tag{10}$$
$$n_e = 10^{17}\,\mathrm{m}^{-3},\ T_e = 1\,\mathrm{eV} \Rightarrow \beta \approx 3.22 \times 10^1, \tag{11}$$
$$n_e = 10^{16}\,\mathrm{m}^{-3},\ T_e = 1\,\mathrm{eV} \Rightarrow \beta \approx 3.22. \tag{12}$$

**Conclusion** Unlike the previous statement that assumed higher densities, here $\beta$ spans from order-unity up to several $10^2$ across the chosen density range. Thus:

- At the high end ($n_e \sim 10^{18}\,\mathrm{m}^{-3}$) $\beta \gg 1$ and thermal/kinetic pressure dominates magnetic pressure, so magnetic confinement is negligible.
- At the low end ($n_e \sim 10^{16}\,\mathrm{m}^{-3}$) $\beta \sim \mathcal{O}(1)$, meaning magnetic pressure and thermal pressure can be comparable; MHD effects may not be entirely negligible in that regime and should be checked case by case.

## 3. Quasi-neutrality and Surface Charge Limit

Surface charge and potential of a sphere of radius $R$:

$$Q = 4\pi\varepsilon_0 RV. \tag{13}$$

For $R = 0.1\,\mathrm{m}$ and a modest potential $V = 5\,\mathrm{V}$:

$$Q \approx 5.6 \times 10^{-11}\,\mathrm{C},$$

which is $\ll 10^{-10}\,\mathrm{C}$. Even with the larger Debye lengths in the updated low-density regime, the macroscopic object remains globally quasi-neutral: the net charge is negligible compared with the total number of electrons present.

## 4. Density Scan: Debye Length and Plasma Beta

For range $n_e = 10^{16}$–$10^{18}\,\mathrm{m}^{-3}$:

Hence as $n_e$ decreases from $10^{18}$ to $10^{16}\,\mathrm{m}^{-3}$, $\lambda_D$ increases by a factor of 10 (from $\sim 7 \times 10^{-6}\,\mathrm{m}$ to $\sim 7 \times 10^{-5}\,\mathrm{m}$), and $\beta$ decreases by a factor of $10^2$ (e.g. $\sim 3 \times 10^2 \to\sim 3$ for $T_e = 1\,\mathrm{eV}$).

## 5. Dust Combustion Rate

Dust mass-loss rate remains written as:

$$\frac{dm_d}{dt} = -K\,n_{O_2}\,\exp\!\left(-\frac{E_a}{RT}\right), \tag{14}$$

or surface-based:

$$\frac{dm_d}{dt} = -k_0\,A_{\mathrm{surf}}\,C_{O_2}\,\exp\!\left(-\frac{E_a}{RT}\right). \tag{15}$$

## 6. Total Chemical Energy

With chemical energy

$$Q_{\text{chem}} \approx 2 \times 10^7 \text{ J/kg},$$

and dust mass $m_d = 3 \times 10^{-5}$–$4 \times 10^{-4}$ kg,

$$E_{\text{chem}} = m_d Q_{\text{chem}} \approx 600\text{–}8000 \, \text{J}.$$

## 7. Energy Balance and Radiated Power

Steady-state energy balance:

$$\eta_{\text{chem}} \, \dot{m}_d Q_{\text{chem}} = P_{\text{rad}} + P_{\text{cond}} + P_{\text{conv}}. \tag{16}$$

Alternatively, average radiated power estimate:

$$P_{\text{rad}} \approx \frac{\eta \, m_d Q_{\text{chem}}}{t_{\text{life}}}. \tag{17}$$

Using typical $m_d$, $\eta$ and $t_{\text{life}}$ values yields

$$P_{\text{rad}} \approx 5\text{–}15 \text{ W},$$

unchanged as this depends primarily on dust mass, chemistry and lifetime rather than the background electron density in the range considered.

## 8. Radiative Transfer and Visible Efficiency

Total visible efficiency:

$$\eta_{\text{vis}} = \frac{P_{\text{vis}}}{P_{\text{tot}}} \approx 0.2\text{–}0.4.$$

Justification remains the same: a combination of (i) the fraction of blackbody continuum that falls in the visible for the relevant temperatures, (ii) strong atomic lines in the visible (Si I, Fe I, Ca I), and (iii) dust/particle grey-body emission. Note that changes in plasma density may affect line excitation rates and collisional radiative transfer; at the lower densities ($n_e \lesssim 10^{16}$–$10^{17} \, \text{m}^{-3}$) collisional excitation is reduced, which can slightly modify the partition between continuum and line emission.

## 9. Recombination Emission Timescale

Radiative recombination time remains:

$$\tau_{\text{rec}} \sim 10^{-8}\text{–}10^{-6} \, \text{s},$$

so recombination processes still correspond to sub-microsecond to microsecond emission and cannot account for sustained second-scale luminosity by themselves.

## 10. Reynolds Number

$$\mathrm{Re} = \frac{\rho U R}{\mu_{\mathrm{eff}}}. \tag{18}$$

Typical choices:

$$\rho \approx 1.2\ \mathrm{kg/m^3}, \quad U = 0.1\text{–}1\ \mathrm{m/s}, \quad R = 0.1\ \mathrm{m}, \quad \mu_{\mathrm{eff}} = 10^{-5}\text{–}10^{-3}\ \mathrm{Pa \cdot s}.$$

This gives

$$\mathrm{Re} \sim 10^1\text{–}10^4,$$

indicating transitional to turbulent flow in the observed range. These hydrodynamic estimates are essentially independent of $n_e$ in the given plasma/dust regime.

## 11. Turbulent Diffusion Coefficient

Mixing-length scaling:

$$D_{\mathrm{turb}} \sim u' \ell'. \tag{19}$$

With $u' = 0.01\text{–}0.1\,\mathrm{m/s}$ and $\ell' = 0.01\text{–}0.1\,\mathrm{m}$:

$$D_{\mathrm{turb}} \sim 10^{-4}\text{–}10^{-2}\,\mathrm{m^2/s}.$$

Typical values used in the main text ($10^{-3}$–$10^{-2}\,\mathrm{m^2/s}$) remain justified.

## 12. Radiative/Conductive Fading Timescale

Characteristic fading time from diffusion scaling:

$$\tau_{\mathrm{fade}} = \frac{R^2}{D_{\mathrm{turb}}}.$$

With $R = 0.1\,\mathrm{m}$,

$$\tau_{\mathrm{fade}} \approx 1\text{–}10\,\mathrm{s},$$

consistent with observed durations. Again, this hydrodynamic timescale is not sensitively dependent on the electron density in the present range.

## 13. Runaway Ignition Condition

Runaway ignition condition:

$$\dot{Q}_{\mathrm{chem}} > \dot{Q}_{\mathrm{rad}} + \dot{Q}_{\mathrm{cond}}.$$

Local dust pockets with high dust mass density (example threshold used in the main text)

$$\rho_{\mathrm{dust}} \gtrsim 0.1\ \mathrm{kg/m^3}$$

are capable of producing chemical power exceeding conductive and radiative losses and thus can trigger rapid thermal runaway. The density of the ambient plasma affects cooling and ion-enhanced chemistry, but the primary runaway criterion remains energetic balance dominated by local dust loading and mixing.

## 14. Spectral Synthesis Components

Total emissivity model:

$$I_\lambda = I_\lambda^{\mathrm{BB}}(T) + \sum_{\mathrm{lines}} I_{\lambda,i}^{\mathrm{atomic}} + I_\lambda^{\mathrm{dust}}.$$

Atomic line strengths are constrained by the Saha-Boltzmann relations:

$$\frac{n_{i+1}}{n_i} = \frac{2}{n_e}\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{k_B T}\right), \tag{20}$$

so changing $n_e$ modifies ionization balance: at the lower end of the updated range ($n_e \sim 10^{16}\,\mathrm{m}^{-3}$) the Saha ratio shifts compared with $n_e \sim 10^{18}\,\mathrm{m}^{-3}$, tending to favor a higher degree of ionization for the same temperature (via the explicit $1/n_e$ factor), which can influence line vs continuum strength. Detailed line intensity modeling should therefore use the updated $n_e$ when computing level populations.

## 15. Coulomb Explosion Energy

Coulomb energy of a charged object (sphere of radius $R$):

$$E_{\mathrm{Coulomb}} = \frac{Q^2}{4\pi\varepsilon_0 R}. \tag{21}$$

Using the surface-charge bound $Q = 4\pi\varepsilon_0 RV$ (with $V$ of order a few volts) yields $Q \lesssim 10^{-10}\,\mathrm{C}$ for $R \sim 0.1\,\mathrm{m}$, hence

$$E_{\mathrm{Coulomb}} \ll 10^{-6}\,\mathrm{J}.$$

Significant Coulomb explosion energies (Joule-scale) would require $Q \sim 10^{-5}\,\mathrm{C}$, far above macroscopic quasi-neutral limits. Note: the increased Debye length at lower $n_e$ (up to $\sim 10^{-4}\,\mathrm{m}$) implies somewhat weaker local screening over small scales, but it does not raise the global net charge enough to permit Coulomb explosion energies comparable to the chemical or mechanical energy scales discussed above.

## 16. Magnetic pinch (j×B) — formulas and numerical estimates

The magnetic hoop (pinch) pressure produced by an axial current $I$ in a cylindrical plasma of radius $r$ can be expressed by the magnetic pressure

$$p_{\mathrm{mag}} = \frac{B^2}{2\mu_0}, \qquad B_\theta(r) = \frac{\mu_0 I}{2\pi r}, \tag{22}$$

hence

$$p_{\mathrm{mag}} = \frac{\mu_0 I^2}{8\pi^2 r^2}. \tag{23}$$

Compare this with the thermal pressure

$$p_{\mathrm{th}} = n\,k_B T \;=\; n\,(T_{\mathrm{eV}}\,e), \tag{24}$$

(where $T_{\mathrm{eV}}$ is electron temperature in eV and $e$ is the elementary charge). The current required to make magnetic pressure equal to thermal pressure is therefore

$$I_{\mathrm{crit}} = \sqrt{\frac{8\pi^2 r^2 p_{\mathrm{th}}}{\mu_0}}. \tag{25}$$

**Numerical examples.** Using $T = 1$ eV (i.e. $k_B T = 1 \times e$ J) and typical plasma densities $n = 10^{18}$–$10^{20}$ m$^{-3}$, we obtain:

| $n$ | $r = 1.0 \times 10^{-2}\,$m (1 cm) | $r = 1.0 \times 10^{-3}\,$m (1 mm) |
|---|---|---|
| $10^{18}$ m$^{-3}$ | $I_{\text{crit}} \approx 3.17 \times 10^{1}$ A | $I_{\text{crit}} \approx 3.17 \times 10^{0}$ A |
| $10^{19}$ m$^{-3}$ | $I_{\text{crit}} \approx 1.00 \times 10^{2}$ A | $I_{\text{crit}} \approx 1.00 \times 10^{1}$ A |
| $10^{20}$ m$^{-3}$ | $I_{\text{crit}} \approx 3.17 \times 10^{2}$ A | $I_{\text{crit}} \approx 3.17 \times 10^{1}$ A |

Thus for mm–cm scale plasma columns the pinch-relevant currents are in the ampere-to-hundreds-of-ampere range — certainly accessible transiently in discharge events. For reference, the azimuthal magnetic field at the plasma edge for $I = 100$ A and $r = 1$ cm is

$$B_\theta = \frac{\mu_0 I}{2\pi r} \approx 2.0 \times 10^{-3}\ \text{T} \quad (= 2\ \text{mT}),$$

and for $I \approx 3.17 \times 10^2$ A one obtains $B_\theta \sim 6.3$ mT.

**Current density and electron drift velocity.** For a cylindrical cross-section $A = \pi r^2$ the current density $j = I/A$ and electron drift velocity $v_d$ (assuming electrons carry the current) are

$$j = \frac{I}{\pi r^2}, \qquad v_d = \frac{I}{en\pi r^2}.$$

Example (for $n = 10^{19}$ m$^{-3}$, $r = 1\,$cm, $I \approx 100\,$A):

$$j \approx 3.2 \times 10^5\ \text{A/m}^2, \quad v_d \approx 2.0 \times 10^5\ \text{m/s},$$

while the electron thermal speed at 1 eV is

$$v_{\text{th}} = \sqrt{\frac{2eT_{\text{eV}}}{m_e}} \approx 5.9 \times 10^5\ \text{m/s},$$

so the required drift velocity is comparable to, but below, $v_{\text{th}}$. This shows that the necessary currents can be sustained without requiring super-thermal drift velocities.

**Physical implication.** If $I \gtrsim I_{\text{crit}}$ the inward $j \times B$ force is comparable to internal pressure gradients and can cause radial compression (pinch), enhancing local density and temperature — a positive feedback for ionization and heating.

## 17. Electrostatic acceleration followed by collisional ionization

Electrons accelerated by an electrostatic field $E$ gain energy $eE\ell$ across a distance $\ell$. The mean free path for electron–neutral collisions at neutral number density $n_0$ and cross-section $\sigma$ is

$$\lambda_{\text{mfp}} = \frac{1}{n_0 \sigma}. \tag{26}$$

To enable ionization an electron must gain at least the neutral ionization potential $\chi$ (typically $\chi \sim 10$–$20$ eV for common species). A minimal field estimate is therefore

$$E_{\min} \sim \frac{\chi}{e\lambda_{\text{mfp}}}. \tag{27}$$

**Numerical estimates.** Take representative ionization energy $\chi \approx 15$ eV. For atmospheric neutral density $n_0 \approx 2.5 \times 10^{25}$ m$^{-3}$ and a collisional cross-section $\sigma \sim 10^{-19}$ m$^2$,

$$\lambda_{\mathrm{mfp}} \approx \frac{1}{2.5 \times 10^{25} \times 10^{-19}} \approx 4 \times 10^{-7} \text{ m},$$

giving

$$E_{\min} \approx \frac{15 \text{ V}}{4 \times 10^{-7} \text{ m}} \approx 3.8 \times 10^7 \text{ V/m}.$$

This is a large field; however, if the neutral density is reduced (e.g. partially preheated or locally rarefied), for $n_0 \sim 10^{23}$ m$^{-3}$ one finds $\lambda_{\mathrm{mfp}} \sim 10^{-4}$ m and

$$E_{\min} \sim \frac{15}{10^{-4}} \text{ V/m} \sim 1.5 \times 10^5 \text{ V/m},$$

which is much more attainable for transient microdischarges or local field enhancements.

**Ionization frequency.** The collisional ionization rate per electron may be approximated by

$$\nu_{\mathrm{ion}} \approx n_0 \, \sigma_{\mathrm{ion}} \, v_e,$$

where $v_e$ is the electron speed. For $v_e \sim 10^6$ m/s and $n_0 = 10^{23}$ m$^{-3}$, $\sigma_{\mathrm{ion}} = 10^{-20}$ m$^2$,

$$\nu_{\mathrm{ion}} \sim 10^{23} \times 10^{-20} \times 10^6 = 10^9 \text{ s}^{-1},$$

indicating that once electrons exceed ionization energy they can produce rapid avalanche ionization on nanosecond–microsecond timescales (depending on local density and cross-sections).

**Net picture.** Electrostatic acceleration is therefore an effective route to seed ionization provided either (i) the local neutral density is reduced (longer $\lambda_{\mathrm{mfp}}$), or (ii) very high local fields are generated (field enhancement near sharp structures, edges, or in pinched geometry). Combined with collisions and secondary electron emission, this can lead to local ionization avalanches.

## 18. Resonant energy concentration in confined geometries (EM / cavity resonances)

Energy stored in a resonant mode is related to input power $P_{\mathrm{in}}$, resonator quality factor $Q$, and angular frequency $\omega$:

$$U = \frac{Q P_{\mathrm{in}}}{\omega}. \tag{28}$$

If this energy is confined within a mode volume $V_{\mathrm{mode}}$, the rms electric field amplitude is

$$E_{\mathrm{rms}} = \sqrt{\frac{2U}{\varepsilon_0 V_{\mathrm{mode}}}}. \tag{29}$$

High $Q$ and small $V_{\mathrm{mode}}$ therefore concentrate field energy and can produce very large local $E$-fields capable of direct ionization or strong acceleration of charges.

**Numerical example.** Take $P_{\text{in}} = 10$ W, $Q = 100$ and $f = 10$ kHz ($\omega = 2\pi f$):

$$U = \frac{100 \times 10}{2\pi \times 10^4} \approx 1.59 \times 10^{-2} \text{ J}.$$

If this energy is confined to $V_{\text{mode}} = 10^{-6}$ m$^3$ (a small cavity or localized hot spot),

$$u = \frac{U}{V_{\text{mode}}} \approx 1.59 \times 10^4 \text{ J/m}^3,$$

and

$$E_{\text{rms}} \approx \sqrt{\frac{2 \times 1.59 \times 10^{-2}}{\varepsilon_0 \times 10^{-6}}} \approx 6.0 \times 10^7 \text{ V/m}.$$

Fields of this magnitude are sufficient to accelerate electrons to ionization energies over sub-micron distances and therefore can trigger ionization and subsequent plasma formation. Even with lower $P_{\text{in}}$ or smaller $Q$ the field is enhanced by the ratio $Q/\sqrt{V_{\text{mode}}}$.

**Remarks on realism.** The achievable stored energy and field depend critically on:

- the actual $Q$-factor (loss mechanisms — dielectric, ohmic, radiative),
- the true mode volume (geometric confinement and field localization),
- coupling efficiency of the input power into the resonant mode.

Nevertheless, the scaling law ($U \propto QP_{\text{in}}/\omega$) shows that resonant amplification is a viable route to create very large local fields in small volumes, and hence to trigger ionization or drive nonlinear processes.