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Hybrid, Ephemeris-Quality, Measurement-Free Estimation of the Potential 2024 YR₄ Lunar Impact

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Abstract

For perturbed two-body motion—such as small bodies in heliocentric orbits—the Cartesian state uncertainty alternates between near-Gaussian and strongly non-Gaussian regimes over the orbit. This behavior motivates hybrid estimators that adapt to the changing character of the uncertainty. Implementation of a hybrid filter hinges on the mechanisms that control the transitions between the two frameworks. In this paper, we introduce an unscented Kalman filter/particle filter (UKF/PF) hybrid estimation strategy that uses the normalized Euclidean distance to switch from the UKF (moment-based) to the PF (ensemble-based), and the Henze-Zirkler statistic to switch back, with dynamics sourced from ASSIST, an ephemeris-quality integrator. This autonomous switching technique requires only the abstractions propagated by the respective filters, enabling complete independence from measurement updates. We demonstrate the capability of the filter by propagating the state uncertainty of the asteroid 2024 YR₄ from its last reported orbit solution in November 2025 to its close approach of the Moon in December 2032. The hybrid UKF/PF accurately and efficiently predicts the lunar impact probability, velocity, angle, and potential cratering of 2024 YR₄, providing all the information necessary to assess the potential fallout of the ejecta created from such an impact event. The proposed hybrid approach effectively balances and integrates the complementary strengths of both filtering frameworks in a measurement-free environment.

Keywords: Asteroid 2024 YR₄; ASSIST; Henze-Zirkler statistic; Moment- vs. ensemble-based filter; Normalized Euclidean distance

1 Introduction

The problem of nonlinear state estimation centers on the representation and evolution of a general probability density, one that may require infinitely many parameters for exact specification, through a finite abstraction that approximates it with sufficient accuracy. In the field of orbit determination, the state of a space object (SO) is modeled as a random variable whose uncertainty is often sufficiently captured by a Gaussian distribution defined by its first and second central moments, the mean and covariance. This is because, although the dynamics of a SO are inherently nonlinear, measurement corrections from ground stations during the evolution serve to restore the diverging density back to Gaussianity. Moment-based estimation filters exploit this approximation by evolving and correcting only the mean and covariance of the SO. This approach is computationally efficient, making it well suited for real-time spacecraft navigation.

Alternatively, there are scenarios where the nonlinearity of the dynamics/measurement models and/or the infrequency of the corrections cause the mean and covariance alone to be a poor approximation of the state uncertainty. In these instances, state uncertainty may be more aptly represented by an ensemble of random members sampled from the initial state uncertainty and evolved subject to the dynamics of the SO. Ensemble-based estimation filters assign weights or density functions to each ensemble member and assimilate measurement corrections by updating the weights or density functions, resampling when the probability focuses to a minority of the particles. While this approach is more computationally expensive than the moment filter, the resulting estimated uncertainty is more accurate in these scenarios.

A hybrid estimation framework seeks to combine the complementary strengths of moment-based and ensemble-based approaches. In such a filter, the state uncertainty is initially represented by its mean and covariance, and a moment-based filter is used as long as this approximation remains valid. When the uncertainty departs sufficiently from Gaussianity, an ensemble of particles is generated and an ensemble-based filter is employed to represent the resulting non-Gaussian distribution. If the uncertainty subsequently returns to a near-Gaussian regime—either through the dynamics or following a measurement update—the filter transitions back to the moment-based representation. This adaptive switching enables the hybrid filter to maintain computational efficiency during near-Gaussian phases while preserving accuracy during strongly nonlinear, non-Gaussian phases. Figure 1 illustrates this concept for a two-body orbit.

Although the concept is simple in principle, its implementation is paradoxically hindered: transitioning from the moment filter to the ensemble filter requires higher-order information about the very state uncertainty that the moment filter approximates using only the first and second moments. While this presents an inherent limitation, several works have nevertheless proposed hybrid filters with switching mechanisms designed to infer these higher-order indicators with available knowledge. Raihan and Chakravorty [1] propose an unscented Kalman filter (UKF)-particle filter (PF) hybrid approach which uses the measurement model to trigger the switch between the two. When the measurements of the SO are available, the UKF [2] is employed, and when the SO is outside the field of view of the measurement sensor, the PF [3] is employed. Frei and Künsch [4] and Grooms and Robinson [5] each propose distinct variants of an ensemble Kalman filter (EnKF)/PF, a hybrid approach that combines the update mechanisms of both filters, thereby overcoming the limitations of the EnKF in representing non-Gaussian uncertainty [6] and the problem of particle degeneracy in the PF [7]. These hybrid approaches all depend on the availability of measurements to trigger the transition between the moment and ensemble filters. This dependency limits their effectiveness in measurement-sparse or measurement-free scenarios.

One such scenario is 2024 YR₄, an Apollo-type near-Earth asteroid approximately 60 meters in diameter [8]. At the time of writing, YR₄ has a 3.8% to 4.3% probability of impacting the Moon on 22 December 2032 [9]. Earlier analyses even suggested a non-negligible probability of impact with Earth, a scenario that would have carried far more serious consequences [10]. YR₄ is currently too distant from Earth to be observed by ground stations and will remain so until it comes within range in June 2028. The impact of such an asteroid on the lunar surface is expected to generate ejecta with potentially severe consequences for satellites operating in cislunar space, as well as for planned lunar infrastructure and surface operations [9]. Because YR₄ has faded from view after its 2025 apparition and will not be recoverable again

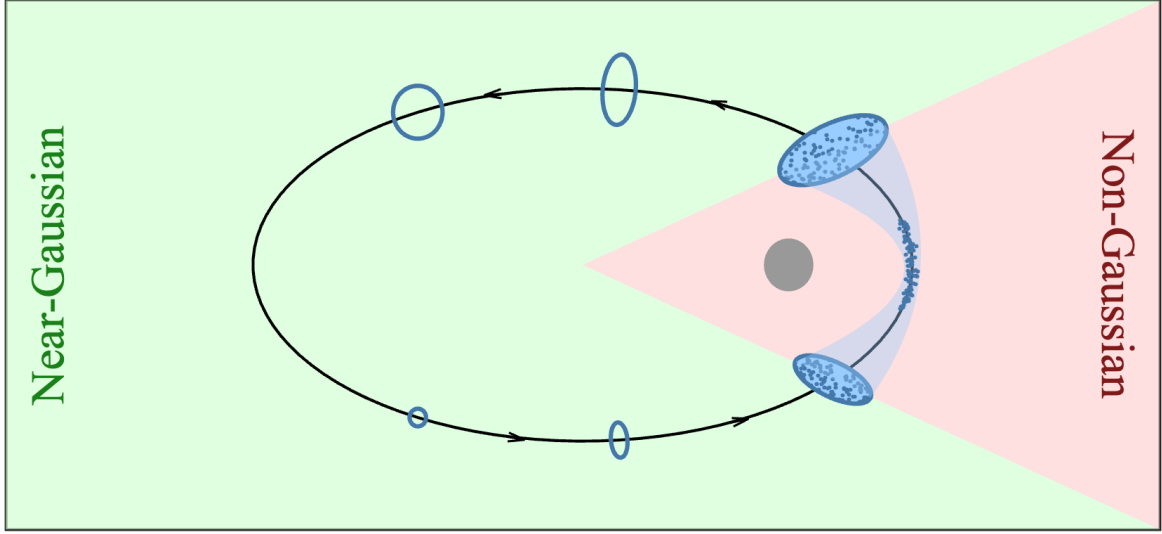


Fig. 1 Schematic illustrating the hybrid filtering strategy for a two-body orbit. The state uncertainty is propagated with a moment filter while it remains near-Gaussian. When the uncertainty becomes non-Gaussian, an ensemble is sampled and an ensemble filter is applied. Once the uncertainty again approaches a near-Gaussian distribution, the filter transitions back to the moment-based propagation.

until 2028, any preventative mitigation concepts formulated in the interim must rely on orbit solutions propagated over a multi-year measurement-free interval. Although the asteroid will be observable again roughly four years before the 2032 Earth–Moon encounter, the associated state uncertainty during this gap still plays a central role in sizing and assessing DART-like mitigation strategies [11].

YR₄ is especially suitable for the hybrid estimation approach proposed due to its dynamics model. YR₄ orbits the Sun in a perturbed two-body trajectory, a model demonstrated by Flegel and Bennett [12, 13] to cause Cartesian state uncertainty to oscillate between approximately Gaussian near apoapsis and highly non-Gaussian near periapsis. This oscillation between the two regimes theoretically ensures that the combined strengths of the hybrid filter will be leveraged. In addition, Flegel and Bennett found a strong relationship in the two-body problem between a measure of skew derived from the unscented transform (UT) sigma points [2, 14] propagated by the UKF and a consistent multivariate normality test. Using the mapping they derive, it is possible to predict when state uncertainty has sufficiently diverged from Gaussianity from the UKF alone. This capability enables hybrid filtering for two-body trajectories.

In this paper, we present a new UKF/PF hybrid estimation approach that utilizes the Gaussianity derived by Flegel and Bennett for the autonomous filter switching mechanism. Because this mechanism is independent of the measurement model, and because the filter employs an ephemeris-quality integrator for prediction, it is a perfect candidate for the state uncertainty propagation of YR₄. To demonstrate this, we use the novel UKF/PF to predict the lunar impact likelihood and statistics of YR₄ and compare the results to a large MC and a UKF. In Sec. 2, we delve further into the mapping derived by Flegel and Bennett for the two-body problem. In Sec. 3, we outline the steps required to attain ephemeris-quality initial conditions of YR₄. In Sec. 4, we list the uncertainty propagation methods to be compared in this paper. In Sec. 5, we explain the accurate calculation of the predicted lunar impact locations using the lunar Digital Elevation Model [15]. In Sec. 6, we present the results of the estimation of the YR₄ lunar impact parameters as well as the magnitude of the resulting cratering and ejecta. Finally, in Sec. 7, we comment on the implications of the results achieved by our novel UKF/PF.

2 Gaussianity in the Two-Body Problem

Consider the state estimation of a general system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{w}, \quad \mathbf{y} = \mathbf{h}(\mathbf{x}, t) + \mathbf{v}, \quad (1)$$

where $\mathbf{x} \in \mathbb{R}^d$ is a realization of the d -dimensional random variable $\mathbf{X}$, $\mathbf{f}$ is the dynamics model, $\mathbf{w}$ is the process noise, $\mathbf{y}$ is the measurement, $\mathbf{h}$ is the measurement model, and $\mathbf{v}$ is the measurement noise. If $\mathbf{f}$ and $\mathbf{h}$ are linear, and $\mathbf{w}$ and $\mathbf{v}$ are Gaussian zero-mean white noise, then the probability density of $\mathbf{X}$ remains Gaussian globally. If $\mathbf{f}$ or $\mathbf{h}$ are nonlinear, then generally speaking, $\mathbf{X}$ will diverge from Gaussianity over time.

Measuring Gaussianity is nontrivial for multivariate random variables. Monte Carlo (MC)-based statistical tests find the likelihood that a large ensemble was sampled from a Gaussian distribution [16–19]. However, since the primary challenge in the hybrid filter is the inference of Gaussianity from a moment filter abstraction, statical tests like these cannot be employed. In an attempt to find an alternative to these MC tests, Flegel and Bennett used a large set of trajectories from low-Earth orbit (LEO) to geosynchronous orbit (GEO) to relate the Henze-Zirkler (HZ) statistic [20], a consistent, ensemble-based measure of Gaussianity, to the normalized Euclidean distance (NED), a skew metric derived from the UT weighted sigma points. Because the UT weighted sigma points are propagated by the UKF, a moment filter¹, this provides our proposed hybrid filter a mechanism for switching from moments to ensemble without reliance on a measurement model. We now define these two measures of Gaussianity.

2.1 Henze-Zirkler Statistic

The HZ statistic measures the distance between an observed distribution and a hypothesized Gaussian distribution. The observed distribution is represented by a random sample $\mathcal{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$, where n is the sample size, and the hypothesized Gaussian distribution has first and second moments equal to the sample mean and covariance

$$\bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^n \mathbf{x}_i \quad \text{and} \quad S = \frac{1}{n} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}})^\top (\mathbf{x}_i - \bar{\mathbf{x}}). \quad (2)$$

Then, the HZ statistic is defined as

$$\begin{aligned} \text{HZ} = & \left[\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n \exp \left(-\frac{h^2}{2} D(\mathbf{x}_i, \mathbf{x}_j) \right) \right] - \left[2(1+h^2)^{-\frac{d}{2}} \right. \\ & \left. \times \sum_{i=1}^n \exp \left(-\frac{h^2}{2(1+h^2)} D(\mathbf{x}_i, \bar{\mathbf{x}}) \right) \right] + \left[n(1+2h^2)^{-\frac{d}{2}} \right], \end{aligned} \quad (3)$$

where $D(\mathbf{x}, \mathbf{y}) = (\mathbf{x} - \mathbf{y})^\top S^{-1}(\mathbf{x} - \mathbf{y})$ is the squared Mahalanobis distance, and h is a smoothing parameter, defined as

$$h = \frac{1}{\sqrt{2}} \left(\frac{n(2d+1)}{4} \right)^{\frac{1}{d+4}}. \quad (4)$$

¹In this paper, we consider the UKF a moment filter. Although the UKF does march a deterministically-sampled ensemble of weighted sigma points, the ensemble is too small to perform ensemble-based statistical tests, and the mean and covariance are reconstructed at measurement intervals when observations are assimilated.

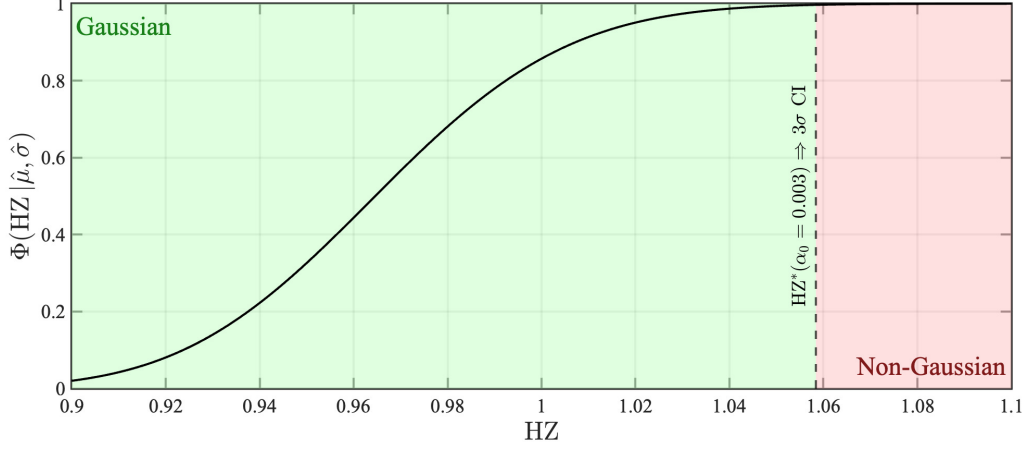


Fig. 2 The lognormal CDF $\Phi(\text{HZ} | \hat{\mu}, \hat{\sigma})$ with $n = 1000$ and $d = 6$. The GVB is at $\text{HZ}^*(\alpha_0 = 0.003) \approx 1.06$.

The random variable HZ is approximately lognormal distributed; i.e., it has a lognormal CDF $\Phi(\text{HZ} | \hat{\mu}, \hat{\sigma})$, with mean and variance

$$\hat{\mu} = \log \sqrt{\frac{\mu^4}{\sigma^2 + \mu^2}} \quad \text{and} \quad \hat{\sigma}^2 = \sqrt{\log \left(\frac{\sigma^2 + \mu^2}{\mu^2} \right)}, \quad (5)$$

where

$$\begin{aligned} \mu &= 1 - \ell^{-\frac{d}{2}} \left(1 + \frac{dh^2}{\ell} + \frac{d(d+2)h^4}{2\ell^2} \right), \\ \sigma^2 &= 2(1 + 4h^2)^{-\frac{d}{2}} + 2\ell^{-d} \left(1 + \frac{2dh^4}{\ell^2} + \frac{3d(d+2)h^8}{4\ell^4} \right) - 4w^{-\frac{d}{2}} \left(1 + \frac{3dh^4}{2w} + \frac{d(d+2)h^8}{2w^2} \right), \end{aligned} \quad (6)$$

$\ell = 1 + 2h^2$, and $w = (1 + h^2)(1 + 3h^2)$. We define the Gaussian validity boundary (GVB) as the measure value where uncertainty transitions from being considered Gaussian to non-Gaussian, given the parameters n , d , and a significance level α_0 . The HZ-GVB, defined as HZ^* , can be found using the critical $(1 - \alpha_0)$ -quantile, approximated as

$$\text{HZ}^*(\alpha_0) = \frac{\mu}{\sqrt{1 + \frac{\sigma^2}{\mu^2}}} \exp \left(\Phi^{-1}(1 - \alpha_0 | 0, 1) \sqrt{\log \left(\frac{1 + \sigma^2}{\mu^2} \right)} \right). \quad (7)$$

Eq. (7) may then be used to test the null hypothesis H_0 that the random variable is Gaussian distributed given the test statistic from Eq. (3):

$$\begin{aligned} \text{HZ}^*(\alpha_0) < \text{HZ} &\Rightarrow H_0 \text{ should be rejected,} \\ \text{HZ}^*(\alpha_0) \geq \text{HZ} &\Rightarrow H_0 \text{ cannot be rejected.} \end{aligned} \quad (8)$$

In summary, given a large random sample, Eqs. (3)–(8) are used to compute a consistent measure of Gaussianity, compare it against a boundary value based on a specified confidence interval, and either reject or fail to reject the hypothesis of Gaussianity. Figure 2 displays $\Phi(\text{HZ} | \hat{\mu}, \hat{\sigma})$ with $n = 1000$ and $d = 6$; the CDF is partitioned into the regimes of Gaussianity for $\alpha_0 = 0.003$, which corresponds to a 99.7%, or 3σ , confidence interval (CI). For the remainder of the paper, HZ^* assumes $\alpha_0 = 0.003$.

2.2 Normalized Euclidean Distance

NED is a measure of skew computed from the UT weighted sigma points. The UT is a statistical linearization wherein a probability distribution with mean $\boldsymbol{\mu}$ and covariance Σ is represented by a set of sigma points $\mathcal{Z} = \{\mathbf{z}_0, \dots, \mathbf{z}_{2d}\}$ and weights $\mathcal{W} = \{W_0, \dots, W_{2d}\}$. For this paper, we use the scaled UT (SUT), a modification of the UT derived by Julier [21] that improves the numerical stability and flexibility of the transformation, defined as

$$\begin{aligned} \mathcal{Z} &= \left\{ \begin{array}{l} \mathbf{z}_0 = \boldsymbol{\mu} \\ \mathbf{z}_j = \boldsymbol{\mu} + \left(\sqrt{(d+\lambda)\Sigma} \right)_j \\ \mathbf{z}_{j+d} = \boldsymbol{\mu} - \left(\sqrt{(d+\lambda)\Sigma} \right)_j \end{array} \right\} \in \mathbb{R}^{(2d+1) \times d} \quad \text{and} \\ \mathcal{W} &= \left\{ \begin{array}{l} W_0^{(m)} = \frac{\lambda}{d+\lambda}, \quad W_0^{(c)} = \frac{\lambda}{d+\lambda} + (1 - \alpha^2 + \beta) \\ W_j^{(m)} = W_j^{(c)} = \frac{1}{2(d+\lambda)}, \quad \text{for } j = 1, \dots, d, \\ W_{j+d}^{(m)} = W_{j+d}^{(c)} = \frac{1}{2(d+\lambda)}, \quad \text{for } j = 1, \dots, d, \end{array} \right\} \in \mathbb{R}^{(2d+1) \times 2}, \end{aligned} \quad (9)$$

where $(\sqrt{\cdot})_j$ is the j^{th} row or column of the matrix square root, $\lambda = \alpha^2(d + \kappa) - d$, α is a positive scaling parameter that controls the spread of the sigma points, κ is a secondary scaling parameter often set to zero, and β is used to incorporate prior knowledge of the distribution, where $\beta = 2$ is optimal for an initially Gaussian distribution. These $2d + 1$ weighted sigma points are defined such that

$$\sum_{j=0}^{2d} W_j^{(m)} \mathbf{z}_j = \boldsymbol{\mu} \quad \text{and} \quad \sum_{j=0}^{2d} W_j^{(c)} (\mathbf{z}_j - \boldsymbol{\mu})^\top (\mathbf{z}_j - \boldsymbol{\mu}) = \Sigma. \quad (10)$$

A statistical linearization is utilized in filtering techniques like the UKF because, generally, it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function [22]. The scaling parameters used in this paper are $\{\alpha, \beta, \kappa\} = \{0.8, 2.0, 0.0\}$.

To introduce the NED, we consider the UT of a probability distribution with mean $\boldsymbol{\mu}^{(k-1)}$ and covariance $\Sigma^{(k-1)}$ at $t = t^{(k-1)}$. Initially, the center sigma point $\mathbf{z}_0$ is defined such that

$$\begin{aligned} \mathbf{z}_0^{(k-1)} &= \boldsymbol{\mu}^{(k-1)} \\ &= \sum_{j=0}^{2d} W_j^{(m)} \mathbf{z}_j^{(k-1)}. \end{aligned} \quad (11)$$

Now, consider that $\mathcal{Z}$ is propagated to $t = t^{(k)}$ via a transformation function $\mathbf{f}$:

$$\mathbf{z}_j^{(k)} = \mathbf{f}(\mathbf{z}_j^{(k-1)}, t^{(k-1)}) \quad \text{for } j = 0, \dots, 2d. \quad (12)$$

If $\mathbf{f}$ is nonlinear, then generally,

$$\mathbf{z}_0^{(k)} \neq \sum_{j=0}^{2d} W_j^{(m)} \mathbf{z}_j^{(k)}. \quad (13)$$

A nonzero distance between the center sigma point and the weighted mean indicates a skewed uncertainty, where skew is a measure of the asymmetry of a probability distribution captured by the third central

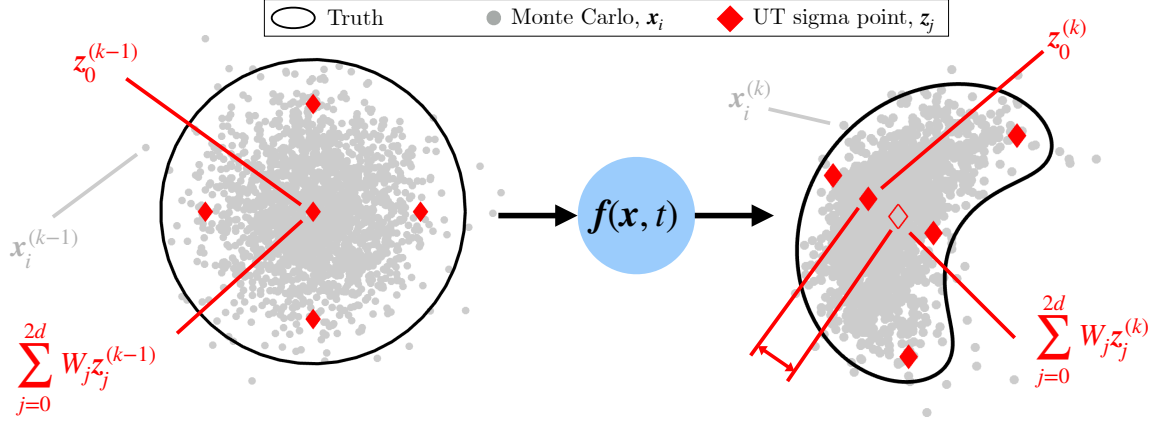


Fig. 3 A two-dimensional truth distribution, MC random sample, and UT sigma point set transformed via a nonlinear function. The nonlinearity of the transformation causes the initially Gaussian uncertainty to become non-Gaussian.

moment. The distance between the center sigma point and the weighted mean normalized by the spread of the sigma points, i.e., the NED, is defined as

$$\text{NED} = \left\| L^{-1} \left(z_0 - \sum_{j=0}^{2d} W_j^{(m)} z_j \right) \right\|, \quad (14)$$

where $\|\cdot\|$ is the Euclidean norm operator and L^{-1} is the inverse lower triangular of the weighted covariance of the sigma points. Figure 3 provides a two-dimensional schematic of a nonlinear transformation inducing skew in an initially Gaussian uncertainty.

Although the NED provides some information on the Gaussianity of a distribution, it is not subject to a hypothesis test. Therefore, it alone cannot provide the likelihood that a distribution is Gaussian. In their paper, Flegel and Bennett set out to determine if there was a relationship between the HZ statistic and the NED for the two-body problem. Using hundreds of trajectories at different altitudes, they marched large MC samples and UT weighted sigma point sets in tandem and tracked the respective Gaussianity measures over time. They found with strong certainty that a $\text{NED}^* = 0.05$ corresponds to the HZ-GVB for the two-body problem². This conclusion will serve as the mechanism for autonomously switching from the UKF to the PF in our hybrid filter.

3 Ephemeris-quality Uncertainty Propagation

To predict the lunar impact of YR₄ with ephemeris quality requires an accurate initial state and uncertainty, as well as an ephemeris-quality integrator to propagate that state and uncertainty forward in time. In this section, we outline the steps taken to achieve both.

3.1 Initial State and Uncertainty

The initial state of YR₄ is obtained from the open-source Jet Propulsion Laboratory (JPL) Small-Body Database (SBDB) [23], a tool that provides data related to asteroids like YR₄. Although the last observation of YR₄ was on 2025-May-11, the SBDB maps the orbit of YR₄ to the latest standard epoch from the Minor Planet Center (2025-Nov-21 00:00:00 Barycentric Dynamical Time [TDB]). Specifically, the SBDB provides the heliocentric orbital elements $[q \ e \ i \ \Omega \ \omega \ t_p]^\top$, where q is the perihelion distance, e is

²In the Flegel and Bennett paper, NED is measured on the position subspace of the entire state.

the eccentricity, i is the inclination with respect to the ecliptic plane, Ω is the longitude of the ascending node, ω is the argument of perihelion, and t_p is the time of perihelion passage. ASSIST, the ephemeris-quality integrator used herein (see Sec. 3.2), requires the initial conditions of test particles to be expressed in a Cartesian coordinate frame centered at the Solar System Barycenter (SSB), with angular components referenced to the equatorial plane, and specified by the state vector $[\mathbf{r}_A \ \mathbf{v}_A]^\top$. We first convert the SBDB orbital elements to Cartesian coordinates centered at the Sun, relative to the ecliptic plane, defined as $[\mathbf{r} \ \mathbf{v}]^\top$, at time t (in Julian Date [JD]) [24]:

$$\begin{bmatrix} q \\ e \\ i \\ \Omega \\ \omega \\ t_p \end{bmatrix} \iff \begin{cases} a = \frac{q}{1-e}, & M = \sqrt{\frac{\mu}{a^3}}(t - t_p) \\ & = E - e \sin E \\ f = 2 \tan^{-1} \left\{ \sqrt{\frac{1+e}{1-e}} \tan \left(\frac{E}{2} \right) \right\} \\ \gamma = \tan^{-1} \left(\frac{e \sin f}{1 + e \cos f} \right) \\ r = \frac{a(1-e^2)}{1 + e \cos f}, \quad v = \sqrt{\frac{2\mu}{a(r-1)}} \end{cases} \iff \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} r [\cos(\omega + f)\hat{\mathbf{e}}_1 + \sin(\omega + f)\hat{\mathbf{e}}_2] \\ v [\sin(\gamma - \omega + f)\hat{\mathbf{e}}_1 + \cos(\gamma - \omega + f)\hat{\mathbf{e}}_2] \end{bmatrix}, \quad (15)$$

where a is the semi-major axis, M is the mean anomaly, μ is the gravitational parameter, E is the eccentric anomaly, f is the true anomaly, γ is the flight path angle, $r = |\mathbf{r}|$, $v = |\mathbf{v}|$, $\hat{\mathbf{e}}_1 = [\cos \Omega \ \sin \Omega \ 0]^\top$ and $\hat{\mathbf{e}}_2 = [-\cos i \sin \Omega \ \cos i \cos \Omega \ \sin i]^\top$. Next, we rotate the position and velocity to the equatorial frame using the obliquity angle ε and shift by the Cartesian position and velocity of the Sun centered at the SSB relative to the equatorial plane $[\mathbf{r}_\odot \ \mathbf{v}_\odot]^\top$:

$$\begin{bmatrix} \mathbf{r}_A \\ \mathbf{v}_A \end{bmatrix} = \begin{bmatrix} R_\varepsilon & \mathbf{0}^{3 \times 3} \\ \mathbf{0}^{3 \times 3} & R_\varepsilon \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} \mathbf{r}_\odot \\ \mathbf{v}_\odot \end{bmatrix} \quad \text{where} \quad R_\varepsilon = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & -\sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{bmatrix}, \quad (16)$$

and $\mathbf{0}^{d \times d}$ is a $(d \times d)$ zero matrix. The state of YR₄, provided in Tab. A1 in both forms corresponding to Eqs. (15) and (16), is now expressed in the proper format for integration with ASSIST, with length and time given in astronomical units (AU) and days, respectively.

An accurate initial state alone is insufficient for ensuring ephemeris-quality uncertainty propagation; we also establish the initial uncertainty with high precision. For this, we source the SBDB, which provides the complete covariance matrix of the heliocentric orbital elements of YR₄ at the initial epoch of 2025-Nov-21 00:00:00. To get the covariance matrix into ASSIST form, we sample a large ensemble ($n = 1\text{E}6$) from the initial state and covariance, convert each member using Eqs. (15) and (16), then reconstruct the sample mean and covariance using Eq. (2). The full covariance matrix used for the YR₄ uncertainty propagation can be found in Tab. A1. We confirm the accuracy of this numerical conversion by checking the diagonal values of the converted covariance matrix with those provided by JPL Horizons³. With an accurate initial state and covariance established, we now turn to the method of integration.

3.2 ASSIST: The Ephemeris-quality Integrator

The ASSIST software package is an ephemeris-quality extension of the N -body integrator REBOUND [25, 26]. Rather than numerically integrating the state of each body in the system as REBOUND does, ASSIST uses ephemeris files for the masses, positions, and velocities of the Sun, Moon, planets, and 16

³JPL Horizons only provides the diagonal components of the Cartesian covariance matrix, thus it cannot be used for sourcing the complete initial covariance of YR₄.

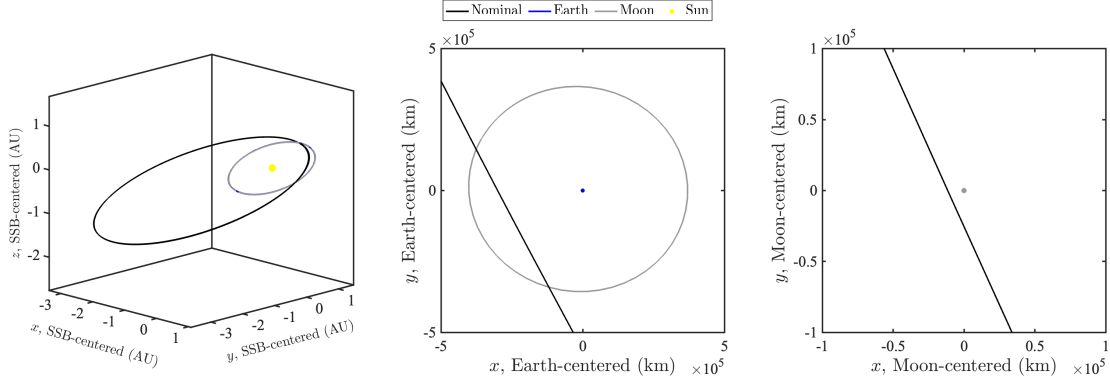


Fig. 4 YR4 trajectory in (*left*) SSB-centered, (*center*) Earth-centered, and (*right*) Moon-centered frames, respectively. The Sun radius is not to scale, but the Earth and Moon radii are.

massive asteroids included in the simulation environment. In addition to the direct Newtonian gravitational forces from the bodies listed, we elect to use the following forces available in ASSIST: Earth’s J_2 through J_5 zonal harmonics, the J_2 zonal harmonic of the Sun, and the Einstein-Infeld-Hoffmann general relativistic treatment for the Sun and planets [27]. ASSIST employs the IAS15 integrator [28, 29], a 15th-order predictor-corrector integrator that is highly efficient for N -body integration. Due to its accuracy, computational efficiency, and open-source availability, ASSIST is exceptionally well suited for achieving ephemeris-quality uncertainty propagation of YR₄.

According to the SBDB, the close approach is predicted to occur on 2032-Dec-22 at 15:10 TDB. We integrate with ASSIST from the initial epoch of 2025-Nov-21 00:00:00 through 2032-Dec-23 00:00:00, for a total of 2589 days. We employ ASSIST’s built-in integration routine, `reb.simulation.step`, which uses adaptive time-stepping to maintain accuracy through the close approach, with a minimum step size of 5 seconds and a maximum step size of 0.5 days⁴. The resulting nominal trajectory can be found in Fig. 4 in the SSB-centered, Earth-centered, and Moon centered-frames, respectively. To assess the accuracy of the integration, we source JPL Horizons [30], an online system that provides ephemerides for Solar System objects, including YR₄. Through the Horizons API, we can specify the epoch, origin, and reference plane to obtain a direct comparison with the ASSIST integration. However, because ASSIST uses an adaptive time-stepping scheme, the integrated trajectory has irregular time intervals. Rather than manually querying the Horizons API at every epoch along the trajectory, we employ `pull_horizons` [31], a GitHub repository created by the authors that uses the `curl` package to efficiently retrieve Horizons data at irregular calendar dates, yielding an ephemeris identical in format to the official Horizons output. Using the generated ephemeris, we compute the position and velocity error over time, as shown in Fig. 5. The center date in each subfigure corresponds to the first close approach of YR₄ with the Earth-Moon system (although this approach is expected to occur at a distance several orders of magnitude greater than the 2032 approach). Throughout the entire integration, the position and velocity error remain below 0.5 km and 1E-7 km/s, respectively. Given that these maximum errors lie well below the initial state uncertainty (as demonstrated in Sec. 3.1), we conclude that ASSIST achieves the level of numerical precision required for ephemeris-quality prediction. With an accurate integration scheme, we begin with the filter comparison.

4 Filter Comparison

The purpose of this analysis is to demonstrate the efficacy of a hybrid filter applied to the uncertainty propagation of a perturbed two-body trajectory. The hybrid filter will be compared against its two constituent

⁴The minimum step size maintains accuracy and stability during close approaches to the Moon, while the maximum step size prevents the integrated trajectory from becoming overly rough elsewhere along the orbit.

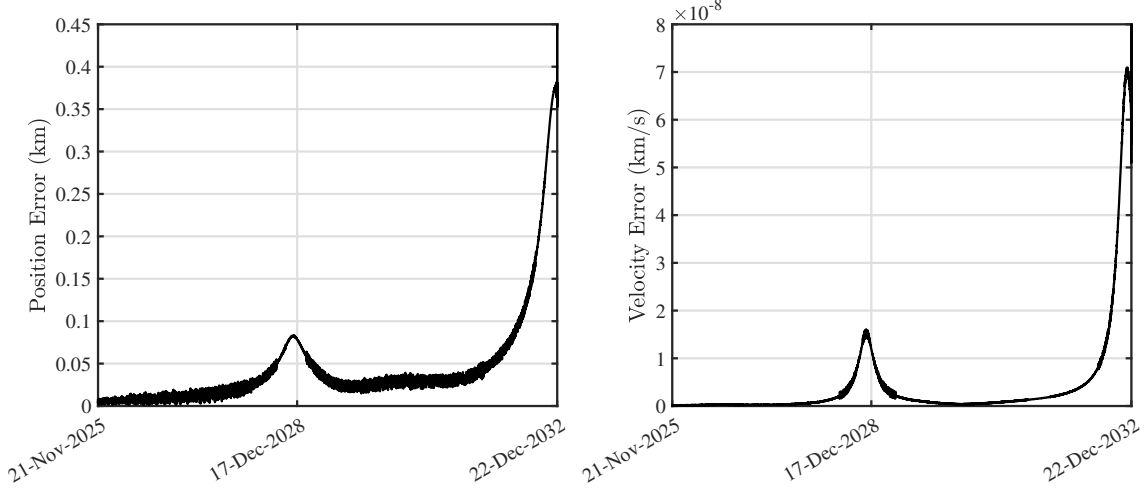


Fig. 5 (left) Position and (right) velocity error of the ASSIST-propagated trajectory compared with Horizons. The central epoch on the time axis corresponds to the final close approach preceding the 2032 encounter.

components: a large MC (used as the truth model), and a UKF. Because the hybrid filter leverages the strengths of both the moment and ensemble filters, its performance will be evaluated on accuracy, assessed through the estimated lunar impact statistics, and efficiency, quantified by computational runtime. We now define the parameters of each of the filter compared.

4.1 MC

A large MC is used as a benchmark for both accuracy and efficiency. An ensemble of 5,000 members is sampled from the initial state and covariance of YR₄ and propagated with ASSIST through 2032-Dec-23 00:00:00 TDB. During the propagation, the distance to the surface of the Moon (assuming a spherical Moon with radius 1,737.4 km) is monitored to determine if a lunar impact has occurred. Figure 6 displays the MC trajectories in the Earth-centered frame, Moon-centered frame, and distance to the Moon’s surface on the date of the 2032 close approach. According to the nominal trajectory, 15:10 is the time of close approach on 2032-Dec-22, the same time predicted by the SBDB.

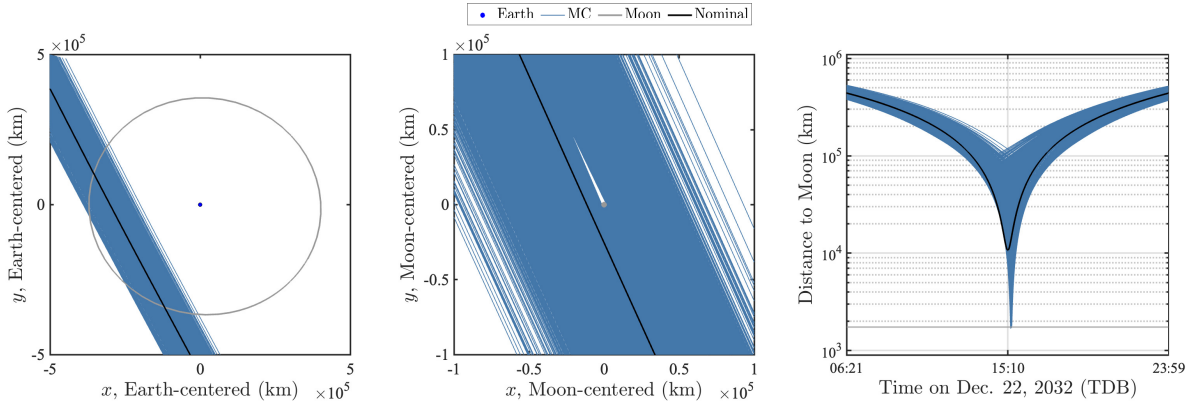


Fig. 6 MC trajectories in the (left) Earth-centered and (center) Moon-centered frames, respectively. (right) The MC and nominal trajectories displayed as distances to the Moon’s surface (assuming a spherical Moon with a radius of 1,737.4 km), with the center tick indicating the close approach time.

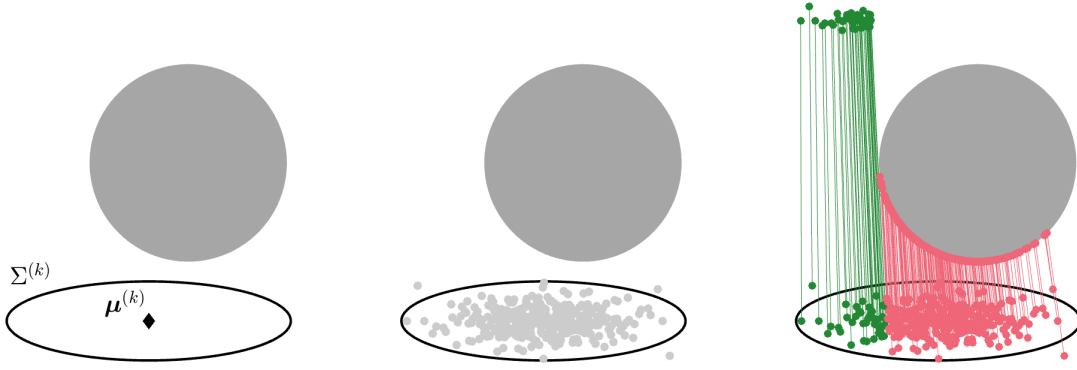


Fig. 7 Schematic illustrating the UKF lunar impact prediction procedure. (*left*) From the close approach mean and covariance at $t = t^{(k)}$, (*center*) the MC ensemble is sampled and (*right*) propagated through the impact. The red members are those that impact the Moon and the green members are those that do not.

4.2 UKF

Since the UKF propagates only $2d + 1$ weighted sigma points, an additional step beyond the standard implementation is required to predict the lunar impact of YR₄. We initialize $\{\mathcal{Z}, \mathcal{W}\}$ with the initial state and covariance from Sec. 3.1, then propagate $\mathcal{Z}$ with ASSIST to the close-approach epoch, 2032-Dec-22 15:10:00 TDB. Here, we generate an ensemble of 1,000 members⁵ from the mean and covariance, assuming a Gaussian distribution, reconstructed from the propagated sigma points using Eq. (10). We then propagate the ensemble to 2032-Dec-23 00:00:00 TDB, tracking the ensemble members in the same manner as done in Sec. 4.1. This procedure is illustrated in Fig. 7. When measuring the computational runtime of the UKF, we include both the time taken to propagate the sigma points to close approach and the time taken to propagate the MC sample through the impact.

4.3 UKF/PF with NED/HZ Switching

The novel UKF/PF we propose uses the NED-GVB to transition from the UKF to the PF and the HZ-GVB to transition back, achieving hybrid, measurement-free prediction that is both accurate and efficient. To begin, $\{\mathcal{Z}, \mathcal{W}\}$ are initialized from the initial state and covariance of YR₄ $\{\mu^{(0)}, \Sigma^{(0)}\}$ using Eq. (9). The UKF/PF then propagates $\mathcal{Z}$ with ASSIST, monitoring the NED at each time step using Eq. (14). When $\text{NED} > \text{NED}^*$, an ensemble of 1,000 members with equal weights $\{\mathcal{X}, \mathcal{W}_{\text{PF}}\}$ is initialized from the mean and covariance reconstructed using Eq. (10). $\mathcal{X}$ is propagated with ASSIST, and at certain time steps, the HZ statistic is computed using Eq. (3). Initially, the value of HZ is likely to be below the HZ-GVB, as $\mathcal{X}$ is sampled from a Gaussian distribution. However, as the uncertainty propagates and becomes non-Gaussian, HZ will increase until it exceeds HZ*. If after this point, $\text{HZ} < \text{HZ}^*$, the PF transitions back to the UKF by reconstructing the mean and covariance using Eq. (2). Then $\{\mathcal{Z}, \mathcal{W}\}$ are re-initialized, and the process repeats until the final epoch is reached. We force the UKF/PF to stay in PF mode after passing the close approach epoch, allowing the lunar impact statistics to be accurately captured. A state transition diagram of the proposed UKF/PF with NED/HZ switching is provided in Fig. 8.

For the NED-GVB, we use the value calculated by Flegel and Bennett: $\text{NED}^* = 0.05$. Selecting a HZ-GVB is more nuanced, as in PF mode the hybrid filter provides a more accurate approximation of the state uncertainty, thereby reducing the likelihood of divergence. If accuracy is prioritized over efficiency, then the statistically recommended value of HZ* determined by Eq. (7) should be used ($\text{HZ}^* \approx 1.06$ corresponds to the HZ-GVB when $\{n, d, \alpha_0\} = \{1000, 6, 0.003\}$). However, if efficiency is prioritized, then

⁵We use 1,000 members to ensure comparable computational effort to the PF, since the UKF/PF, described in Sec. 4.3, also employs 1,000 members while in PF mode.

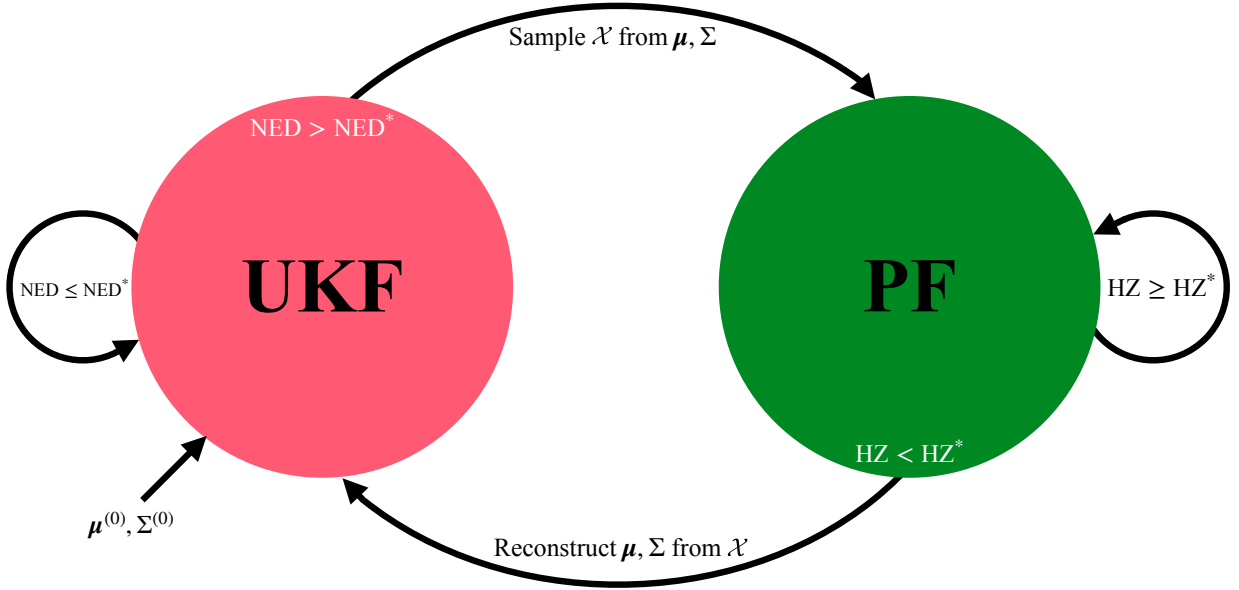


Fig. 8 A state transition diagram of the UKF/PF with NED/HZ switching. When transitioning from the PF to the UKF, the HZ value must be crossing HZ^* from above, indicating that the ensemble was non-Gaussian, then returned to the near-Gaussian.

a higher value of HZ^* may be employed without sacrificing much accuracy. This expectation is validated in Sec. 6.

In the current form of the UKF/PF, the PF ensemble is initialized with equal weights, as is standard for the PF. This is because the HZ statistical test is not applicable to unequally weighted samples. Nonetheless, the weights remain essential for performing measurement updates outside the scope of this paper, should observations of YR_4 become available when it returns within optical range in 2028. Following any such update, a different Gaussianity metric would need to be employed in place of HZ. A medium sample size is adopted in PF mode because computing the HZ statistic at each step is computationally expensive, making larger ensembles impractical. As demonstrated in Sec. 6, even with a medium sample size, the UKF/PF achieves high accuracy. Even with the medium sample size employed in PF mode, computing the HZ statistic at each time step is costly as well as unnecessary when the value is far from HZ^* . To mitigate this cost, we adaptively adjust the number of time steps between evaluations of Eq. (3), defined as n_{HZ} , based on its distance from HZ^* using a saturating hyperbolic function [32]:

$$n_{HZ} = \max \left(n_{\min}, \left\lceil \frac{n_{\max}}{1 + 1/|HZ - HZ^*|} \right\rceil \right), \quad (17)$$

where $n_{\min}$ and $n_{\max}$ are the minimum and maximum number of time steps between HZ evaluations, respectively. Eq. (17) ensures that the time step at which the HZ value crosses the HZ-GVB is captured without requiring evaluation at every time step. For this paper, we use $n_{\min} = 1$ and $n_{\max} = 10$.

5 Determining Lunar Impact Locations via the Lunar Digital Elevation Model

To meaningfully present the lunar impact locations predicted by the respective filters, we convert them from the ASSIST frame to lunar latitude and longitude using the lunar Mean Earth (ME) axis system

[33], a body-fixed reference system. During the ASSIST propagation, the distance from each ensemble member to the lunar surface is continuously monitored, assuming a spherical Moon with radius 1,737.4 km. If this distance falls below the surface threshold, the corresponding state is logged, and integration for that member is terminated. Because the minimum integration step size in ASSIST is set to 5 seconds, the recorded impact state may not coincide precisely with the true moment of impact. Given YR₄'s relative velocity of approximately 14 km/s at close approach, this temporal resolution can introduce a spatial error of up to 70 km in the impact location, which subsequently affects the computed lunar latitude and longitude. To mitigate this error and improve spacial accuracy, interpolation of the elevation profile determined with the Lunar Reconnaissance Orbiter (LRO) lunar Digital Elevation Model (DEM) [15] along the asteroid's trajectory is employed to refine the estimated impact locations.

To enable interpolation of the lunar impact, both the final and penultimate Cartesian states and times integrated by ASSIST are recorded. To convert these state vectors to the lunar ME frame, we translate them by the Cartesian state of the Moon in the inertial SSB-centered, equatorial-aligned frame $[\mathbf{r}_\zeta \ \mathbf{v}_\zeta]^\top$, then convert to the ecliptic-aligned inertial frame, and finally rotate the resulting vector dependent on the time of impact. The complete transformation is given by the following equation:

$$\begin{bmatrix} \mathbf{r}_{\text{ME}} \\ \mathbf{v}_{\text{ME}} \end{bmatrix} = \begin{bmatrix} R_\theta & \mathbf{0}^{3 \times 3} \\ \mathbf{0}^{3 \times 3} & R_\theta \end{bmatrix} \begin{bmatrix} R_\epsilon^{-1} & \mathbf{0}^{3 \times 3} \\ \mathbf{0}^{3 \times 3} & R_\epsilon^{-1} \end{bmatrix} \left(\begin{bmatrix} \mathbf{r}_A \\ \mathbf{v}_A \end{bmatrix} - \begin{bmatrix} \mathbf{r}_\zeta \\ \mathbf{v}_\zeta \end{bmatrix} \right) \quad \text{where} \quad R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} R_\zeta, \quad (18)$$

where $\theta = -\omega_\zeta \Delta t$, with $\omega_\zeta = 0.22997$ rad/day being the rotation rate of the Moon and Δt being the difference in time of the impact relative to the close approach epoch 2032-Dec-22 15:10:00 TDB, expressed in days. R_ζ is the rotation matrix from the J2000 frame to the lunar ME frame at the close approach, computed via Spice [34].

With the penultimate and final state vectors in the lunar ME frame, we linearly interpolate 70 points between these states along the asteroid's trajectory, considering a constant velocity, and find the state that closest intersects the Moon according to the lunar DEM. This refinement enables determination of the impact location and time with a spatial resolution of at most 1 km and a temporal accuracy better than 0.1 seconds. We then calculate the lunar latitude and longitude of each impact location using the following equations:

$$\text{latitude} = \sin^{-1} \left(\frac{z_{\text{ME}}}{|\mathbf{r}_{\text{ME}}|} \right) \quad \text{and} \quad \text{longitude} = \tan^{-1} \left(\frac{y_{\text{ME}}}{x_{\text{ME}}} \right). \quad (19)$$

Having established an intelligent method for determining lunar impact locations, we now present the results of the filter comparison.

6 Results

We now present the predicted lunar impact estimates obtained from the filters described in Sec. 4. First, we summarize the statistical properties of the impact parameters predicted by each filter. Then, we estimate the corresponding crater dimensions and assess the resulting cislunar fallout from the expected ejecta.

6.1 Lunar Impact Parameters

Using the conversion explained in Sec. 5, we resolve the lunar impact locations for each of the filters into latitude and longitude. The impact corridor as seen from Earth is plotted using an orthographic projection generated via MATLAB, displayed in Fig. 9. The results are consistent with those of Wiegert et al. [9], providing confidence to proceed with the quantitative analysis. The accuracy of the estimated lunar impact is evaluated based on three parameters: impact probability, impact velocity, and impact

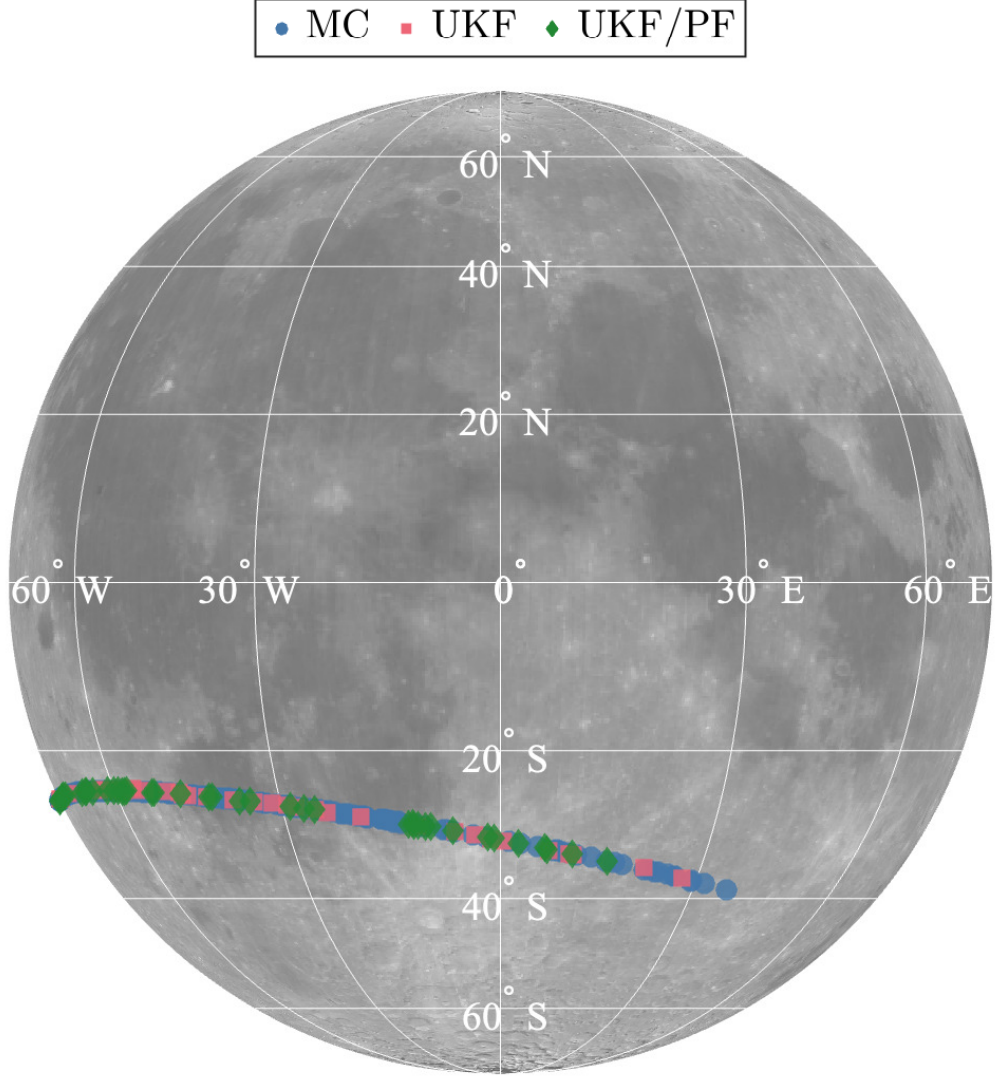


Fig. 9 The lunar impact corridor for 2024 YR₄ in the lunar ME frame predicted by the MC, UKF, and the UKF/PF (HZ* = 1.06), respectively.

angle. The impact probability is the fraction of ensemble members that collide with the Moon. The impact velocity is the magnitude of the velocity vector at impact relative to the Moon. The impact angle is the angle between this velocity vector and the local horizontal plane, calculated as

$$\text{impact angle} = \cos^{-1} \left(\frac{-\mathbf{r}_{\text{ME}} \cdot \mathbf{v}_{\text{ME}}}{|\mathbf{r}_{\text{ME}}| |\mathbf{v}_{\text{ME}}|} \right). \quad (20)$$

To determine the accuracy of these three estimated parameters, the UKF and UKF/PF are compared with the large MC.

As discussed in Sec. 4.3, the HZ-GVB can be adjusted to prioritize either accuracy or efficiency. In the first implementation of the UKF/PF, we use a conservative value of HZ* = 1.06, calculated using Eq. (7) with parameters $\{n, d, \alpha_0\} = \{1000, 6, 0.003\}$. Figure 10 provides a visualization of the UKF/PF

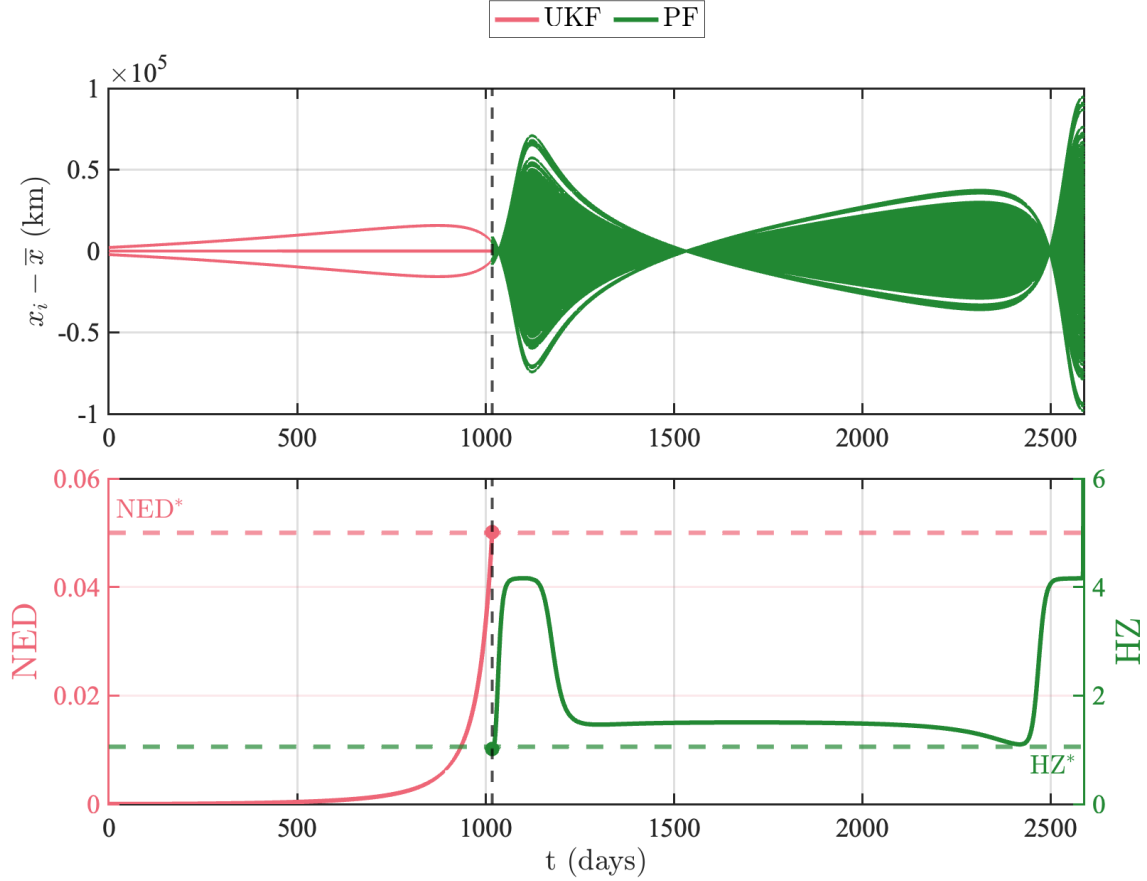


Fig. 10 UKF/PF applied to the YR₄ uncertainty propagation with a $NED^* = 0.05$ and a $HZ^* = 1.06$. (*top*) The UKF/PF members throughout the propagation, displayed as the difference in the member x_i from the estimated mean $\bar{x}$. (*bottom*) The NED/HZ values throughout the propagation, with the dotted black line showing where the transition from UKF to PF, or vice versa, occurs.

members throughout the propagation, as well as the NED and HZ values over time. In the visualization, the x -values of the sigma points/particles shifted by the estimated mean $\bar{x}$ of the sigma points/particles in the SSB-centered, equatorial-aligned frame is displayed over time. A single switch from the UKF to the PF occurs after 1,017.86 days of propagation on 2028-Sep-03 20:38:49 TDB. From this epoch through the close approach, the evolving ensemble never returns below the HZ-GVB, thus the UKF/PF stays in PF mode.

To demonstrate the flexibility of the UKF/PF, we implement a second simulation with a more relaxed HZ-GVB of $HZ^* = 1.5$. This value, chosen arbitrarily to ensure a switch from PF mode back to UKF mode occurs, intends to demonstrate that the UKF/PF can maintain high accuracy even when the Gaussianity threshold is loosened. Because the hybrid filter remains in UKF mode for a greater portion of the propagation, this implementation is expected to be more computationally efficient. Figure 11 provides a visualization of the UKF/PF members throughout the propagation, as well as the NED and HZ values over time. In this implementation, a switch from the UKF to the PF occurs after 1,017.86 days of propagation on 2028-Sep-03 20:38:49 TDB, the same epoch as in the previous implementation. Then, due to the more relaxed HZ-GVB, a switch from the PF back to the UKF occurs after 1,228.36 days of propagation on 2029-Apr-02 08:38:49 TDB. After 1,744.36 days of propagation on 2030-Aug-31 08:38:49 TDB, the filter switches back to PF mode and remains there through the close approach.

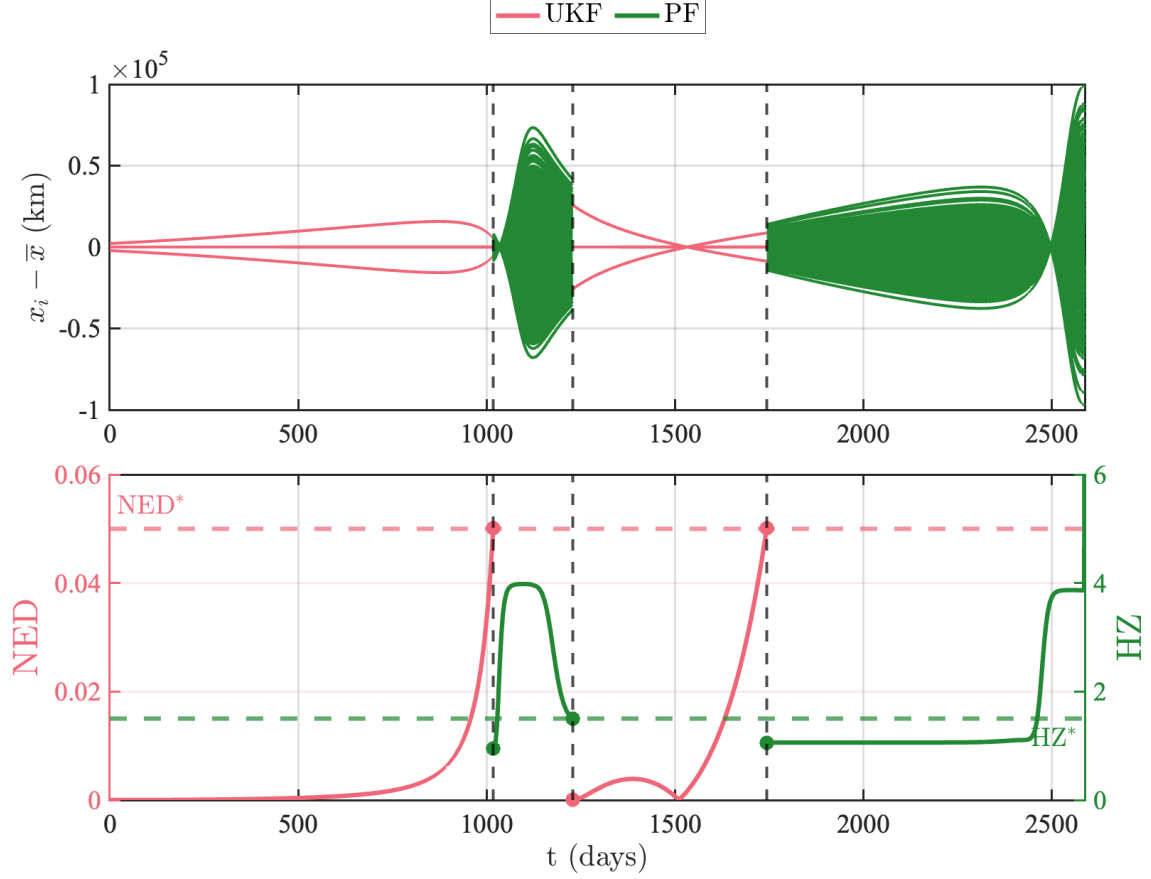


Fig. 11 UKF/PF applied to the YR₄ uncertainty propagation with a $NED^* = 0.05$ and a $HZ^* = 1.5$. (*top*) The UKF/PF members throughout the propagation, displayed as the difference in the member x_i from the estimated mean $\bar{x}$. (*bottom*) The NED/HZ values throughout the propagation, with the dotted black line showing where the transition from UKF to PF, or vice versa, occurs.

The lunar impact statistics for the MC, UKF, and UKF/PF with $HZ^* = 1.06$ and $HZ^* = 1.5$ are displayed in Fig. 12 and are provided in Tab. 1 in the following form (for impact velocity and angle): Median ($Q_1 - Q_3$), where Q_1 and Q_3 are the first and third quartiles, respectively. We use median and quartile values for comparison due to the skew of the distributions.

Table 1 Accuracy of filter-estimated lunar impact parameters compared with the MC simulation.

Filter	# of collisions	Impact velocity (km/s)	Impact angle ($^\circ$)
MC	205 of 5,000	14.103 (14.101-14.104)	45.082 (39.339-57.930)
UKF	45 of 1,000	14.090 (14.082-14.098)	50.096 (38.844-63.416)
UKF/PF ($HZ^* = 1.06$)	41 of 1,000	14.102 (14.101-14.104)	49.107 (39.322-54.548)
UKF/PF ($HZ^* = 1.5$)	40 of 1,000	14.103 (14.101-14.104)	49.493 (37.202-63.689)

Along with accuracy, the efficiency of each filter was evaluated. The criteria for efficiency was computational runtime compared with the large MC used as a benchmark. Each simulation was run in C on a

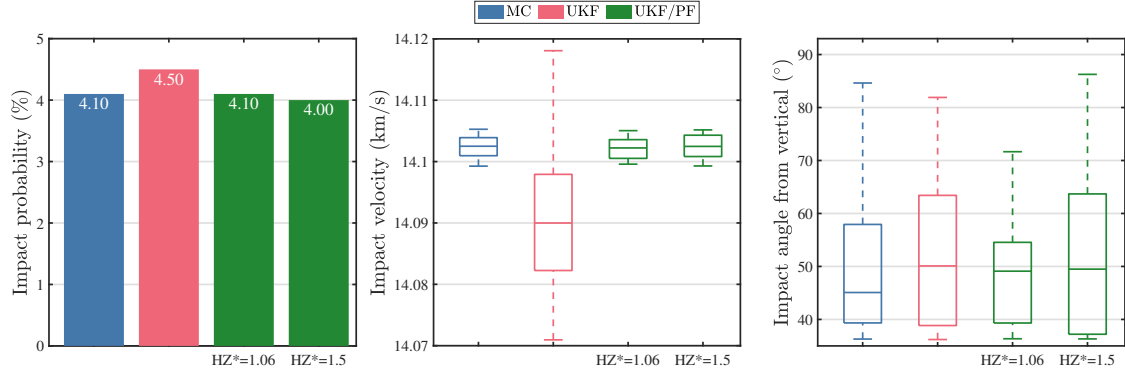


Fig. 12 YR₄ lunar impact statistics predicted by the MC, UKF, and UKF/PF, respectively. (*left*) The lunar impact probability, (*center*) the impact velocity, and (*right*) the impact angle for each of the filters.

single core of a 3.49 GHz Apple M2 Max chip. The size of the ensemble, total runtime, and corresponding speed-up relative to the MC of each filter is provided in Tab. 2.

Table 2 Efficiency of filters compared with the MC simulation.

Filter	Ensemble Size	Runtime (s)	Speed-up
MC	5000	4515.16	-
UKF	13	239.40	18.86
UKF/PF (HZ* = 1.06)	13 / 1000	1057.25	4.27
UKF/PF (HZ* = 1.5)	13 / 1000	912.07	4.95

6.2 Cratering and Ejecta Dynamics of Oblique Impacts

Having determined the expected impact location, probability, velocity and angle of YR₄ on the lunar surface using three different estimation methods, we now aim to characterize the size and shape of the resulting crater, along with the associated ejecta, as functions of the target material and impact angle. Specifically, we aim to quantify the mass expected to be ejected at velocities exceeding the lunar escape velocity (~ 2.4 km/s) and to estimate the size of the largest fragment within this population. To do so, we leverage the Π scaling framework cratering mechanisms developed by Housen and Holsapple across both strength and gravity regimes [35]. The lunar surface exhibits a wide variety of structures and compositions [36]; however, it can be broadly categorized into two geological categories: the highlands, dominated by coarse lunar regolith, and the mare regions, composed of basaltic hard rocks. Each of these surface categories can be represented using the following physical variables: average density ρ , surface cohesion Y , and porosity and granularity fitting coefficients $\{\mu, \nu, k\}$. The values of these variables for the two lunar-surface categories are provided in Tab. 3.

Table 3 Constant values for the Highlands and Mare target type [35].

Target Type	ρ (kg/m ³)	Y (MPa)	μ	ν	k
Highlands	1500	0.01	0.41	0.4	0.3
Mare	3200	10	0.55	0.4	0.3

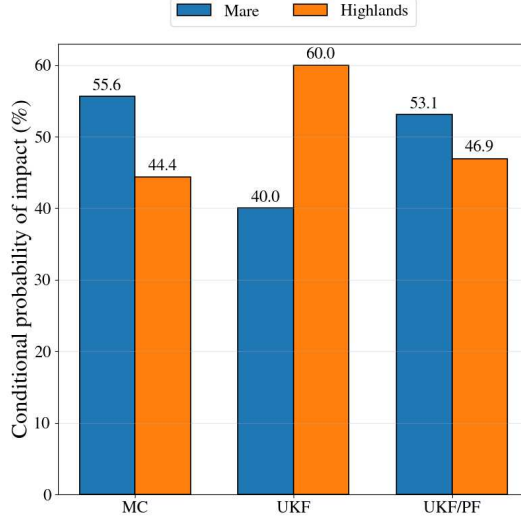


Fig. 13 Conditional probability of YR₄ impact across lunar geological regions for the three filters.

To determine in which regime craters will form, we use the following equation from [37]:

$$R = \frac{\rho g D}{Y}, \quad (21)$$

where g is the gravitational parameter of the Moon at its surface and D is the diameter of the crater. When $R < 1$, the crater is formed following strength processes and when $R > 1$ the gravity effects take over and drive the crater's formation. In the case of YR₄, if the impact occurs in the lunar highlands, where unconsolidated regolith dominates, the resulting crater formation will be governed by the gravity regime. If the impact occurs in a mare region dominated by basaltic rocks, the resulting cratering and ejecta dynamics are expected to follow strength-dominated scaling laws. Figure 13 shows the conditional probability of YR₄ impact as a function of the lunar geological region predicted by each of the three filters, with the predicted impacts from the two different UKF/PF implementations combined.

Using the equations from Housen and Holsapple, we find that a normal impact yields crater diameters of approximately 957 m and 1,086 m in the highlands and mare regions, respectively. These results are in accordance with Wiegert et al. However, as shown in Fig. 12, none of the predicted impacts occur with impact angle normal to the local surface of the Moon. For some cases, the impact angle is highly oblique, grazing the lunar surface. Because the Housen and Holsapple relationships assume near-normal impact angles, their employment for predicting the crater shape and size will likely be unreliable for impacts occurring at shallow angles. To model these potential impact conditions, we leverage the cratering dynamics equations from Raducan et al. [38], which bridges numerical simulations and analytical functions. When the impact angle becomes too oblique (above 45° from the vertical) the crater changes its formation symmetry from being point-wise (the crater has a constant radius with a bowl shape) to axial-symmetric along the impact trajectory. This symmetry change introduces an azimuthal dependency necessary to accurately model the shape and size of the crater. Rather than a bowl shape, the crater will become elongated in the downrange direction of the impact and shrink in the uprange direction. This phenomenon has been extensively characterized by numerous laboratory tests [39–41], simulations [38, 42], and observations [43]. Following this framework, the resulting crater shape and size at different azimuthal angles from the whole span of expected impact angles is described in Fig. 14.

In addition to modeling the crater morphology, we also simulate the velocity and angle of ejecta produced by the predicted YR₄ impact. As shown by Luo et al. [42], ejecta in the downrange direction are typically faster and more tangential to the surface, while those in the uprange direction are slower and

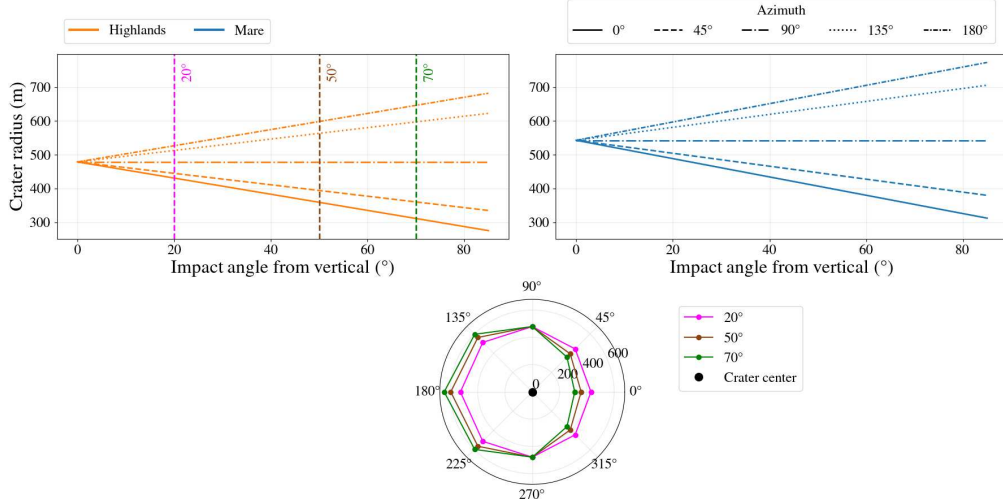


Fig. 14 Radial size of the crater for five azimuthal angles as a function of the impact angle from the vertical for the (*left*) highlands and (*right*) mare regions. (*bottom*) The planar projection of the crater size and shape at different impact angles.

ejected at higher angles. The ejecta angle, in particular, strongly influences their range, maximum height, and time of return to the surface. These quantities are computed using the range equation accounting for the sphericity of the Moon, as initially derived by Gault et al. [44], and the time of flight (TOF) is determined using Keplerian dynamics. Figure 15 shows that ejecta with velocities approaching the Moon’s escape velocity can remain aloft for more than 10 days and traverse the entire lunar surface, potentially thickening the tenuous exosphere of the Moon. The ejecta angle at each azimuth follows simulation results of Luo et al. [42].

The amount of material ejected by the predicted YR₄ impact is computed by combining the Housen and Holsapple formalism with the modification of physical characteristics of the target described by Raducan. For clarity, Fig. 15 displays the case of a normal impact. Both estimations of total ejected material, $\sim 10^{11}$ kg, and ejected material above the lunar escape velocity, $\sim 10^8$ kg, match Wiegert et al. In addition to this global perspective of ejected mass, we compute the maximum fragment size at each ejecta velocity using the methodology described in [45]. For a normal impact at the lunar escape velocity, the largest fragment is estimated to be approximately 2.7 m in diameter in the highlands and 3.0 m in a mare region.

7 Conclusions

Cartesian state uncertainty in the perturbed two-body problem has been shown to oscillate between near-Gaussian and highly non-Gaussian in the absence of measurement corrections. Moment-based filters, such as the UKF, efficiently estimate the state uncertainty when it is near-Gaussian, but struggle to accurately capture highly non-Gaussian distributions. Conversely, ensemble-based filters, such as the PF, can accurately represent highly non-Gaussian distributions, but are computationally expensive. To capitalize on the complementary strengths of these methods, hybrid filters have been developed that combine moment-based and ensemble-based approaches, achieving both computational efficiency and accurate representation of non-Gaussian uncertainties in perturbed two-body state estimation. To date, these approaches have relied on measurement updates to switch between the constituent moment and ensemble filters. However, in the absence of measurements, such as during long-term, measurement-free uncertainty propagation, these methods are not applicable. In this paper, we propose a novel UKF/PF hybrid filter that uses Gaussianity metrics to autonomously switch between the UKF and PF during measurement-free propagation. The NED, a measure of skew derived from the UT sigma points, is used to transition from the UKF to the PF when the state uncertainty becomes non-Gaussian, and the HZ,

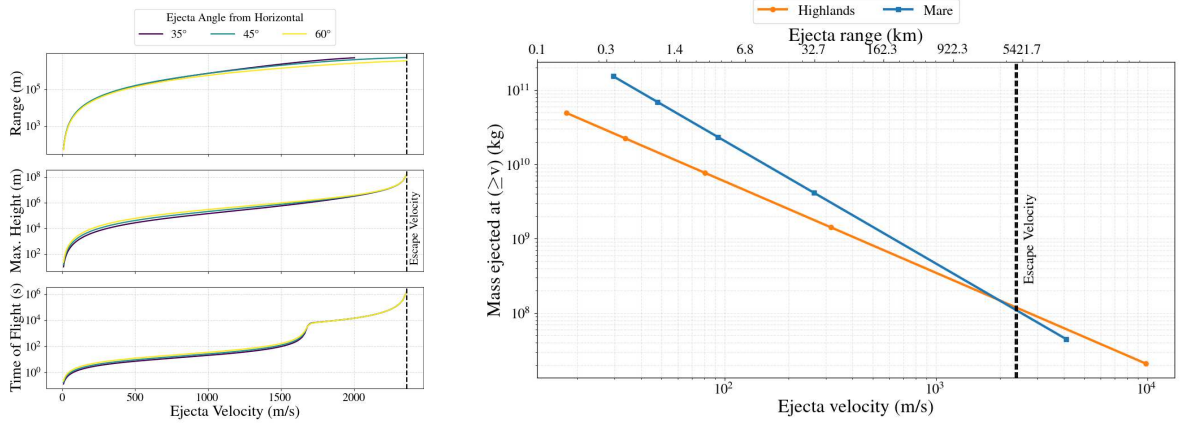


Fig. 15 Distribution of the range of the ejecta (*left, top*), maximum height (*left, middle*) and time of flight (*left, bottom*) for different ejecta angle and for ejecta velocities up to the lunar escape velocity. (*right*) Mass of material ejected with at least a velocity v for each lunar geological region. The associated range at each velocity is recalled on the top x-axis.

a MC-based measure of Gaussianity, is used to transition back to the UKF when the state uncertainty returns to approximately Gaussian.

To demonstrate the efficacy of the proposed UKF/PF with NED/HZ switching, we apply it to the uncertainty propagation of the near-Earth asteroid 2024 YR₄, which is predicted to have a close approach with the Moon on 2032-Dec-22 15:10:00 TDB. Using ASSIST, an ephemeris-quality N -body integrator, we propagate the initial state and covariance of YR₄ from 2025-Nov-21 00:00:00 TDB, the most updated orbit solution provided by the SBDB, through 2032-Dec-23 00:00:00 TDB with three different filters: the MC, the UKF, and our novel UKF/PF. The MC serves as a benchmark for both accuracy and efficiency, consisting of an ensemble of 5,000 members randomly sampled from the initial state and covariance and propagated through the final epoch while tracking lunar impact occurrences. The UKF consists of 13 UT weighted sigma points propagated up to the close approach epoch, at which point an ensemble of 1,000 members is randomly sampled from the reconstructed mean and covariance and propagated through the final epoch. The UKF/PF begins by propagating 13 UT weighted sigma points in UKF mode, tracking the NED at each time step. When the NED exceeds the value determined by Flegel and Bennett to correspond to the GVB, $NED^* = 0.05$, the UKF/PF transitions to PF mode by sampling an ensemble of 1,000 equally-weighted particles. The particles are then propagated and the HZ statistic is tracked. If the HZ value returns below the HZ-GVB, which may be determined by the user depending on whether accuracy or efficiency is prioritized, the UKF/PF transitions back to UKF mode by reconstructing the mean and covariance from the particle ensemble. The process repeats until the final epoch, with the UKF/PF remaining in PF mode after passing the close approach epoch to accurately capture the lunar impact. We compare two configurations of the UKF/PF distinguished by their HZ-GVB: one emphasizing accuracy, with a conservative value of $HZ^* = 1.06$, calculated using Eq. (7) with parameters $\{n, d, \alpha_0\} = \{1000, 6, 0.003\}$, and another emphasizing efficiency, with a relaxed, arbitrarily chosen value of $HZ^* = 1.5$.

The accuracies of the UKF and UKF/PF are assessed by comparing their predicted impact probability, impact velocity, and impact angle with those of the MC. Given the length of prediction, the UKF demonstrates surprising accuracy for the impact probability and impact angle, but poorly represents the impact velocity distribution. Compared with the MC, the UKF-predicted impact probability differs by 0.40%, while both implementations of the UKF/PF predict impact probabilities within 0.1%. For the impact velocities, the UKF-predicted median differs from the MC by 0.013 km/s, while both implementations of the UKF/PF predict median impact velocities within 0.001 km/s. For the impact angles, the

UKF-predicted median differs from the MC by 5.01° , while the UKF/PF with $HZ^* = 1.06$ differs by 4.03° and the UKF/PF with $HZ^* = 1.5$ differs by 4.41° . Overall, both implementations of the UKF/PF exceed the UKF across the impact statistics compared. In terms of efficiency, the UKF achieves a speed-up of $18.86\times$ compared to the MC, while the UKF/PF with $HZ^* = 1.06$ achieves a speed-up of $4.27\times$, and the UKF/PF with $HZ^* = 1.5$ achieves a speed-up of $4.95\times$. The flexibility of the UKF/PF is demonstrated by the fact that the more relaxed HZ-GVB implementation achieves higher efficiency while maintaining comparable accuracy in the lunar impact statistics.

Along with the uncertainty propagation results presented in this work, we estimate the crater dimensions and ejecta dynamics resulting from a potential YR₄ impact with the Moon. As these characteristics are dependent on the lunar surface composition at the impact location, we first determine the conditional probability of impact across the two main lunar geological regions: the highlands and mare regions. The UKF poorly estimates the conditional probability of the impact across lunar geological regions when compared to the MC, differing by 15.6%. On the other hand, the UKF/PF is within 3%. Furthermore, our predictions indicate that a potential impact is unlikely to occur at normal incidence; thus, we model the crater shape and size as a function of azimuthal angle and impact angle, where highly oblique impacts produce elongated craters in the downrange direction and contracted craters in the uprange direction. In regards to the ejecta created, we expect dramatic consequences should the potential impact occur, with more than 100,000 tons of material excavated and spread across the entire surface over more than a week period. An impact of this magnitude has the potential to generate co-orbital objects [46] ranging from independent particles to meter-sized fragments of the lunar surface, which may later re-impact the Moon or re-enter Earth’s atmosphere after temporary capture on orbits within the cislunar region [47]. Given the severity of these effects, accurate prediction of the impact location, velocity, and angle is crucial for reliable estimation of the crater dimensions and ejecta dynamics.

Although the novel hybrid framework developed in this paper was applied to the uncertainty propagation of a natural satellite, the authors also believe the technique demonstrates considerable potential for use in spacecraft navigation. To date, ensemble filters have been underutilized for spacecraft state estimation due to their computational expense, especially when the state of the spacecraft is high-dimensional. Moment filters have sufficed thus far due to frequent measurement corrections from ground stations, but there exist trajectories where state uncertainty may become highly non-Gaussian between measurement updates, such as in highly perturbed environments in deep space. In these cases, a hybrid filter such as the UKF/PF proposed allows for a gentle transition from moment-based to ensemble-based filtering only when absolutely necessary, saving computational resources while still accurately capturing non-Gaussian uncertainties. Future work will focus on applying the UKF/PF with NED/HZ switching to these scenarios, as well as exploring how the NED/HZ relationship changes when subject to different force models.

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Appendix A Initial States and Covariances

The initial states and covariances used in this paper can be found in Tab. A1.

Table A1 Initial states and covariances at 2025-Nov-21 00:00:00 TDB.

YR ₄ - Orbital elements	q (AU)	e ()	i (°)	Ω (°)	ω (°)	t_p (JD)
State	8.51680517E-01	6.61472533E-01	3.40816040E+00	2.71363850E+02	1.34366017E+02	2.46063689E+06
Covariance	1.46470600E-12	-2.08156290E-13	1.72804592E-11	8.38386332E-14	-5.34004563E-12	5.26814631E-12
	-2.08156290E-13	3.03001146E-14	-2.45757819E-12	-1.18717410E-13	9.29613961E-13	-7.55331893E-13
	1.72804592E-11	-2.45757819E-12	2.12683928E-10	1.96981291E-11	-7.41970824E-11	6.31655622E-11
	8.38386332E-14	-1.18717410E-13	1.96981291E-11	6.84200965E-11	-6.23995872E-11	5.93684650E-12
	-5.34004563E-12	9.29613961E-13	-7.41970824E-11	-6.23995872E-11	8.76550996E-11	-2.45934864E-11
	5.26814631E-12	-7.55331893E-13	6.31655622E-11	5.93684650E-12	-2.45934864E-11	1.99502724E-11
Sun - ASSIST	x_A (AU)	y_A (AU)	z_A (AU)	$\dot{x}_A$ (AU/day)	$\dot{y}_A$ (AU/day)	$\dot{z}_A$ (AU/day)
State	-3.36382392E-03	-5.13068424E-03	-2.07481411E-03	7.28782466E-06	-1.50149499E-07	-2.14655923E-07
YR ₄ - ASSIST ¹	x_A (AU)	y_A (AU)	z_A (AU)	$\dot{x}_A$ (AU/day)	$\dot{y}_A$ (AU/day)	$\dot{z}_A$ (AU/day)
State	-3.23511640E+00	-8.70916766E-01	-5.88740818E-01	-2.46168396E-03	-6.45707651E-03	-2.97082877E-03
Covariance	5.58516118E-11	-3.28564255E-11	-8.65100566E-12	2.83175929E-13	-4.96473697E-14	-1.58137743E-15
	-3.28564255E-11	1.93383753E-11	5.09283203E-12	-1.66585566E-13	2.92200283E-14	9.36753690E-16
	-8.65100566E-12	5.09283203E-12	1.34617093E-12	-4.38594936E-14	7.69284886E-15	2.53628844E-16
	2.83175929E-13	-1.66585566E-13	-4.38594936E-14	1.43574491E-15	-2.51719110E-16	-8.01439951E-18
	-4.96473697E-14	2.92200283E-14	7.69284886E-15	-2.51719110E-16	4.41570995E-17	1.41173381E-18
	-1.58137743E-15	9.36753690E-16	2.53628844E-16	-8.01439951E-18	1.41173381E-18	5.74696454E-20

¹For conversion from the ecliptic to the equatorial frame, the IAU76 obliquity w.r.t. to the J2000 frame is used ($\varepsilon = 23.43929111^\circ$)

Supplementary Files

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