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Spectral CT reconstruction algorithm combining total variation and low-rank regularizations based on the Chambolle-Pock method

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Abstract: Spectral computed tomography (CT) utilizes photon-counting detectors to acquire multi-energy data for reconstruct images. Due to the limited number of photons in each energy bin, spectral CT imaging suffers from severe quantum noise, making clinical diagnosis more challenging. To address this limitation, we proposed a spectral CT reconstruction model combining total variation (TV) and low-rank regularizations. In the proposed model, the TV regularization was applied to suppress staircase artifacts in spatial domain, while the low-rank regularization was utilized to reduce noise by exploiting the correlation among the spectral CT images in the spectral dimension. To improve the efficiency of the spectral CT reconstruction algorithm, the Chambolle-Pock method was employed to solve our proposed model. Firstly, we constructed the spectral CT reconstruction model combining TV and low-rank regularizations. Secondly, within the Chambolle-Pock optimization framework, the dual variables were introduced to reformulate the reconstruction model as a convex-concave saddle-point problem, which was then transformed into a different equivalent form. Finally, we derived the closed-form solution for each sub-problem in this different equivalent form, thereby obtaining a single-loop iterative reconstruction algorithm. To validate the algorithm's efficacy, simulated head and thorax phantom data experiments were performed. Extensive experimental results demonstrate the proposed algorithm's outstanding performance in noise suppression, artifact reduction and detail preservation compared with state-of-the-art methods.

Keywords: spectral CT, Total Variation, Low-Rank, Chambolle-Pock method, single-loop

1. Related Work

Recently, spectral computed tomography (CT) has emerged as a significant research direction in medical imaging, because of the great potential for material decomposition[1,2] and disease detection[3]. To obtain multiple

sets of projection data with different spectral information in a single exposure, the photon-counting detector is usually equipped in the spectral CT system. However, spectral projections measured by the photon-counting detector usually suffer from strong noises[4], due to the limited number of received photons in each energy bin [5]. This factor could result in the artifacts in spectral CT images. To address this issue, many state-of-art spectral CT reconstruction algorithms[6-10] are proposed.

Ignoring the correlation between different energy bins, the conventional analytical reconstruction algorithms (e.g., filtered back-projection algorithm[11] and iterative reconstruction algorithms[12-15] can directly employed to reconstruct channel-independent images for each energy bin separately. Due to the strong noises in the spectral projections, these reconstruction algorithms[16] will inevitably lead to obvious artifacts and noise in the reconstruction results, negatively affecting the diagnostic accuracy in the subsequent procedure. Such reconstruction strategies ignore the data correlation among different energy bins, leading to sub-optimal reconstruction results. To overcome this drawback, many algorithms have been developed to make full use of the data correlation in the spectral domain.

To address the aforementioned issue, model-based iterative reconstruction algorithms are currently widely adopted in spectral CT imaging. On the one hand, many sparsity regularizations based reconstruction algorithms have been developed for spectral CT. Tensor dictionary learning is introduced to explore the correlation among the spectral CT images from different energy bins[4]. Wu et al.[17] integrated the gradient l_0 norm and tensor dictionary learning to eliminate artifacts effectively. Hu et al.[18] proposed a spectral image similarity-based tensor-enhanced reconstruction (SISTER) method that integrates l_0 norm minimization, tensor decomposition, and non-local similarity for sparse-view CT reconstruction. Cueva et al.[19] proposed a synergistic multi-spectral CT reconstruction with directional TV to preserve cross-channel structural information, enhancing image quality and computational efficiency.

On the other hand, numerous works has been proven that the low-rank regularization play an important role in spectral CT image reconstruction. The nuclear norm is an efficient constraint for low-rank regularization[20]. Semerci et al. [21] presented an iterative reconstruction algorithm for spectral CT that incorporated tensor nuclear norm regularization. The non-local low-rank cube-based tensor is applied for low-dose spectral CT reconstruction[22].

Meanwhile, combining both sparsity and low-rank prior information can further improve the reconstruction performance of spectral CT. Li et al.[23] introduced a low-rank tensor decomposition with spatial-temporal total variation regularization method to the reconstruction model. Niu et al. [24] presented a nonlocal low-rank and sparse

matrix decomposition for spectral CT reconstruction, which leveraged the similarity of patches. Xia et al.[25] utilized the similarity in image-spectral tensors to assist spectral image reconstruction and obtained promising results. Chen et al.[26] proposed a fourth-order nonlocal tensor decomposition model, where low-rank and sparsity constraints are enforced via tensor-based weighted nuclear norms and TV norms. These iterative reconstruction algorithms involving both sparsity and low-rank prior information are effective in improving the reconstruction results. However, due to the nested loop structures, they are usually computationally expensive[18].

In this work, making full use of prior information to suppress noise and eliminate artifacts in the reconstruction results, we proposed a TV and low-rank based spectral CT reconstruction algorithm (TV-LR-SCT). In the spectral CT reconstruction model, the TV regularization was utilized to suppress staircase artifacts in each tomographic image slice and the low-rank regularization was employed to reduce noise by exploiting the correlation among the spectral CT images. To improve the efficiency of the iterative reconstruction algorithms, the Chambolle-Pock (CP) method was applied to solve our proposed reconstruction model, obtaining a single-loop iterative reconstruction algorithm for spectral CT. Avoiding the nested loop structures, our proposed algorithm effectively reduces the computational complexity. Our contributions were mainly summarized as follows:

1. To sufficiently suppress staircase artifacts in each tomographic image slice and exploit the similarities among different energy bins in spectral dimension, we proposed a spectral CT reconstruction model combining TV and low-rank regularizations.
2. To improve computational efficiency, we innovatively employed the CP method to solve the spectral CT reconstruction model, obtaining a single-loop iterative spectral CT reconstruction algorithm.
3. Extensive experimental results demonstrate that our proposed algorithm outperforms several state-of-the-art methods in both subjective visual assessment and objective quantitative metrics.

2. Methodology

2.1 Spectral CT Image Reconstruction Model

To improve image quality by making full use of the data correlation in the spectral domain, we proposed a spectral CT reconstruction model combining TV and low-rank regularizations. Taking into account the two-dimensional fan-beam scanning geometry, the proposed spectral CT reconstruction model is formulated as:

$$\arg \min_u \left\{ \sum_{i=1}^S \left[\frac{1}{2} \|Au_i - g_i\|_2^2 + \lambda_1 \|(\nabla u_i)\|_1 \right] + \lambda_2 \|u\|_* \right\}, \quad (1)$$

where A denotes the system matrix, g is the projection data, g_i is the projection data and the CT image of the i th ($i = 1, 2, \dots, S$) energy level, u is the spectral CT image, u_i is the CT image of the i th energy level.

$\|Au_i - g_i\|_2^2$ represents the data fidelity of i th energy level. $\|(\nabla u_i)\|_1$ represents the TV regularization, which was utilized to suppress staircase artifacts of each energy level in spatial domain. $\|u\|_*$ represents the nuclear norm, which was employed as the low-rank regularization to reduce noise by exploiting the correlation among the spectral CT images in the spectral dimension. λ_1 and λ_2 are the weighting parameters that balance the data fidelity term and regularization terms. In this work, the parameters λ_1 and λ_2 were set to 0.0001 and 0.05, respectively.

2.2 TV-LR-SCT

In this work, to yield better image quality and shorten reconstruction time, the CP method[27] was applied to solve the image reconstruction model in equation (1). We determined u as the primal variable and introduced two dual variables p and q . Then, the image reconstruction model in equation (1) was reformulated to the form of the saddle point problem with the following assignments:

$$\begin{aligned} F(p, q) &= F_1(p) + F_2(q), \quad F_1(p) = \sum_{i=1}^S \frac{1}{2} \|p_i - g_i\|_2^2, \quad F_2(q) = \sum_{i=1}^S \lambda_1 \|(|q_i|)\|_1, \\ G(u) &= \lambda_2 \|u\|_*, \\ K &= \begin{bmatrix} A \\ \nabla \end{bmatrix}, \end{aligned}$$

where F is the dual function and G is the primal function. The linear transform K from the primal variable to the dual variables is written as:

$$\begin{bmatrix} p \\ q \end{bmatrix} = Ku. \quad (2)$$

The convex conjugate functions of the dual functions $F_1(p)$ and $F_2(q)$ can be obtained by the Legendre transform:

$$F_1^*(p) = \sum_{i=1}^S \left(\frac{1}{2} \|p_i\|_2^2 + \langle p_i, g_i \rangle \right), \quad (3)$$

$$F_2^*(q) = \sum_{i=1}^S \delta_{Box(\lambda_1)}(|q_i|), \quad (4)$$

where $\delta_{Box(a)}(x)$ is an indicator function which is defined as:

$$\delta_{Box(a)}(x) = \begin{cases} 0 & \|x\|_\infty \leq a \\ \infty & \|x\|_\infty > a \end{cases}. \quad (5)$$

Therefore, according to expression adopted by Sidky et al[28], the generic saddle point problem is finally derived as:

$$\min_u \max_{p, q} \left\{ \sum_{i=1}^S \left[\langle p_i, Au_i - g_i \rangle - \frac{1}{2} \|p_i\|_2^2 + \langle q_i, \nabla u_i \rangle - \delta_{Box(\lambda_1)}(|q_i|) \right] + \lambda_2 \|u\|_* \right\}. \quad (6)$$

The CP method can be employed to solve this generic saddle point problem by concurrently solving the primal and dual problems. Thus, this generic saddle point problem associated can be transformed into a different equivalent form:

$$\begin{cases} u^{n+1} = \arg \min_u \left\{ \sum_{i=1}^S \left[\langle p_i^n, Au_i - g_i \rangle + \langle q_i^n, \nabla u_i \rangle + \frac{1}{2\tau} \|u_i - u_i^n\|_2^2 \right] + \lambda_2 \|u\|_* \right\} \\ \bar{u}^{n+1} = u^n + \theta(u^{n+1} - u^n) \\ p^{n+1} = \arg \max_p \sum_{i=1}^S \left[\langle p_i, A\bar{u}_i^{n+1} - g_i \rangle - \frac{1}{2} \|p_i\|_2^2 - \frac{1}{2\sigma} \|p_i - p_i^n\|_2^2 \right] \\ q^{n+1} = \arg \max_q \sum_{i=1}^S \left[\langle q_i, \nabla \bar{u}_i^{n+1} \rangle - \delta_{\text{Box}(\lambda_1)}(|q_i|) - \frac{1}{2\sigma} \|q_i - q_i^n\|_2^2 \right] \end{cases}, \quad (7)$$

where σ and τ are step sizes of the primal and dual variables, θ is a fixed parameter.

The sub-problem for the primal variable u in equation (7) can be rewritten as:

$$\begin{aligned} u^{n+1} &= \arg \min_u \left\{ \sum_{i=1}^S \left[\langle A^T p_i^n, u_i \rangle + \langle \nabla^T q_i^n, u_i \rangle + \frac{1}{2\tau} \|u_i - u_i^n\|_2^2 \right] + \lambda_2 \|u\|_* \right\} \\ &= \arg \min_u \left\{ \frac{1}{2\tau} \sum_{i=1}^S [\|u_i + \tau A^T p_i^n + \tau \nabla^T q_i^n - u_i^n\|_2^2] + \lambda_2 \|u\|_* \right\}. \end{aligned} \quad (8)$$

The element of the vector x is defined as $x_i = u_i^n - \tau A^T p_i^n - \tau \nabla^T q_i^n$. The minimization problem (8) is equivalent to:

$$u^{n+1} = \arg \min_u \left\{ \frac{1}{2\tau} \|u - x\|_2^2 + \lambda_2 \|u\|_* \right\}. \quad (9)$$

The singular value decomposition of a matrix x of rank r is defined as:

$$x = U \Sigma V^*, \quad (10)$$

where U and V are the orthogonal tensors, Σ is the diagonal tensor whose diagonal elements ρ_i ($1 \leq i \leq r$).

According to the singular value thresholding algorithm (Cai et al., 2008), the closed-form solution of the minimization problem (9) can be expressed as:

$$u^{n+1} = U \mathcal{D}_{\tau\lambda_2}(\Sigma) V^*, \quad (11)$$

where $\mathcal{D}_{\tau\lambda_2}(\Sigma) = \text{diag}(\{\max(\rho_i - \tau\lambda_2, 0)\})$.

The sub-problems for the dual variable p and q in equation (7) can be rewritten as:

$$p^{n+1} = \arg \min_p \left\{ \frac{\sigma+1}{2\sigma} \sum_{i=1}^S \left[\left\| p_i - \frac{1}{\sigma+1} p_i^n - \frac{\sigma}{\sigma+1} (A\bar{u}_i^{n+1} - g_i) \right\|_2^2 \right] \right\}, \quad (12)$$

$$q^{n+1} = \arg \min_q \left\{ \sum_{i=1}^S \left[\delta_{\text{Box}(\lambda_1)}(|q_i|) + \frac{1}{2\sigma} \|q_i - q_i^n - \sigma \nabla \bar{u}_i^{n+1}\|_2^2 \right] \right\}. \quad (13)$$

The closed-form solution of minimization problem (12) is:

$$p_i^{n+1} = \frac{1}{\sigma+1} p_i^n + \frac{\sigma}{\sigma+1} (A\bar{u}_i^{n+1} - g_i). \quad (14)$$

The optimal solution to problem (13) can be expressed in closed-form as:

$$q_i^{n+1} = \lambda_1 \frac{q_i^n + \sigma \nabla \bar{u}_i^{n+1}}{\max(\lambda_1 \mathbf{1}_I, |q_i^n + \sigma \nabla \bar{u}_i^{n+1}|)}, \quad (15)$$

where $\mathbf{1}_I$ is a vector with all components set to 1.

To summarize, the proposed TV-LR-SCT algorithm for spectral CT image reconstruction is shown in Algorithm

1. It can be observed in Algorithm 1 that our proposed TV-LR-SCT algorithm is a single-loop iterative algorithm,

which can effectively reduce the computational complexity.

Algorithm 1. Pseudocode for N steps of the TV-LR-SCT algorithm

1: $L \leftarrow \|(A; \nabla)\|_2$; $\tau \leftarrow 1/L$; $\sigma \leftarrow 1/L$; $\theta \leftarrow 1$; $n \leftarrow 0$

2: initialize u^0 , p^0 and q^0 to zero values

3: $\bar{u}^0 \leftarrow u^0$

4: **repeat**

5: $x_i \leftarrow u_i^n - \tau A^T p_i^n - \tau \nabla^T q_i^n$

$x \leftarrow \{x_1; x_2; \dots; x_S\}$

6: $x \leftarrow U \Sigma V^*$

$u^{n+1} \leftarrow U \mathcal{D}_{\tau \lambda_2}(\Sigma) V^*$

7: $\bar{u}^{n+1} \leftarrow u^{n+1} + \theta(u^{n+1} - u^n)$

8: $p_i^{n+1} \leftarrow \frac{1}{\sigma+1} p_i^n + \frac{\sigma}{\sigma+1} (A \bar{u}_i^{n+1} - g_i)$

$p^{n+1} \leftarrow \{p_1^{n+1}, p_2^{n+1}, \dots; p_S^{n+1}\}$

9: $q_i^{n+1} \leftarrow \lambda_1 \frac{q_i^n + \sigma \nabla \bar{u}_i^{n+1}}{\max(\lambda_1 1, |q_i^n + \sigma \nabla \bar{u}_i^{n+1}|)}$

$q^{n+1} \leftarrow \{q_1^{n+1}, q_2^{n+1}, \dots; q_S^{n+1}\}$

10: $n \leftarrow n + 1$

11 **until** $n \geq N$

3 Experiments and Results

To evaluate the performance of our proposed method, simulated data were used in the experiments. The X-ray-based Cancer Imaging Simulation Toolkit (XCIST)[29] is a new implementation of CatSim[30], which is developed to simulate X-ray imaging for the phantom. In this work, XCIST was used to simulate 2D scans of the Shepp-Logan head phantom and the clinical CT data from Low-Dose CT Image and Projection Data provided by the Mayo Clinic[31]. The images of these two kinds of phantoms were converted into water and bone density maps based on the value of pixels and their CT number, respectively. Then, these density maps were input to XCIST to yield the corresponding projection data. The voltage and current of the X-ray source were set to 120 keV and 175 mA. To obtain the spectral projection data, the energy spectrum of X-ray source was divided into 12 energy bins with a 10 keV increment. The size of each energy bin was set to 512×512 pixels. There were 720 views evenly distributed over

360°. As we known, the spectral projection data suffer from strong noise, due to the limited number of received photons in each energy bin. We set the current of the X-ray source to 1750 mA to suppress the noise in each energy bin and the corresponding reconstruction results were treated as the reference images.

In the experiment, our proposed TV-LR-SCT algorithm was compared with several representative state-of-the-art algorithms, including ASD-POCS[12], MDM-PCCT[32], FTVNNR[33], ASSIST[25], and SISTER[18]. In these algorithms, the ASD-POCS algorithm is a promising algorithm for CT image reconstruction based on TV regularization. The MDM-PCCT algorithm is a high-performance spectral CT reconstruction algorithm combining multiple dynamic modulations. The FTVNNR algorithm is a novel reconstruction algorithm of the nested loop structure based on both the TV regularization and the nuclear norm. The ASSIST and SISTER algorithms are the spectral CT reconstruction algorithms utilizing the non-local feature similarity of patches in both spatial and spectral domains. All experiments were performed on a workstation computer platform with an Intel® Core™ i7-9700K CPU, 32 GB RAM, and an NVIDIA RTX 3090 GPU. In this work, to quantitatively evaluate our proposed TV-LR-SCT algorithm, four metrics were chosen to perform image quality evaluation[34], including peak-signal-to-noise ratio (PSNR), structural similarity index (SSIM), root-mean-square error (RMSE), and visual information fidelity (VIF).

3. 1 Shepp-Logan head phantom

(1) Visual evaluation

To visually demonstrate the reconstruction performance of our proposed TV-LR-SCT, Fig. 1 presents one representative Shepp-Logan head phantom slice reconstructed using different algorithms. Four typical energy bins including (10, 20] keV, (40, 50] keV, (70, 80] keV and (100, 110] keV were chosen. The first column displays the reference images. The second to sixth columns show reconstruction results obtained by the ASD-POCS, MDM, FTVNNR, ASSIST and SISTER methods, respectively. The seventh column presents the reconstruction results of our proposed algorithms (TV-LR-SCT).

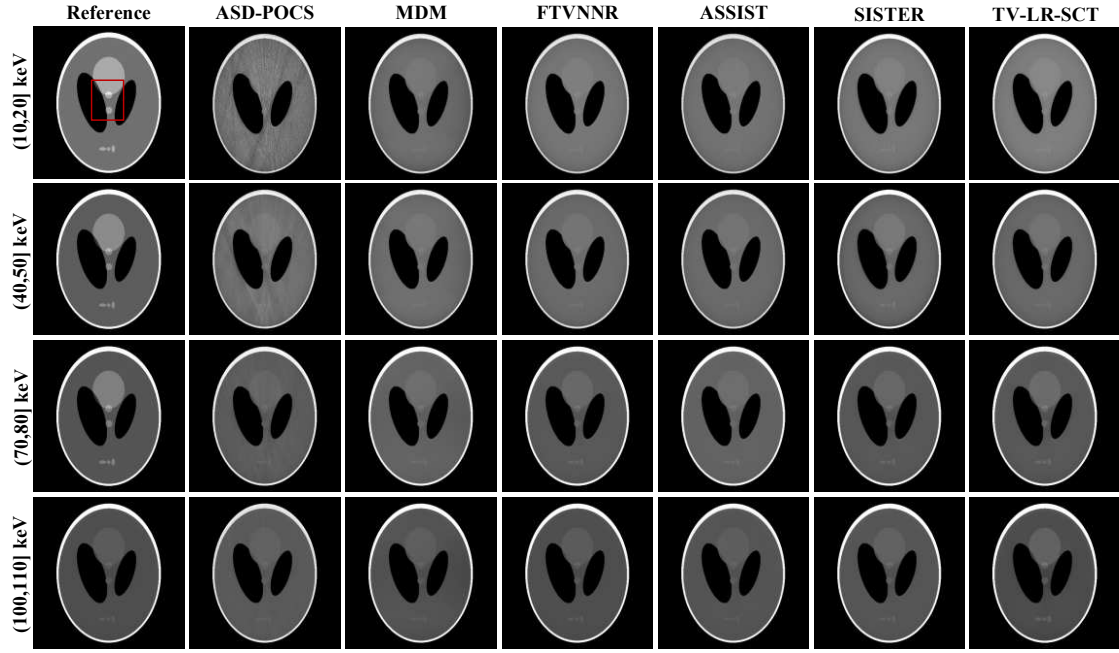


Fig. 1 Reconstruction results of different methods with the Shepp-Logan head phantom.

From visual assessment perspective, the reconstruction results demonstrate distinct performance characteristics across algorithms and energy bins in Fig. 1: the ASD-POCS algorithm performs poorly in the low-energy bin ((10, 20] keV), retaining a significant amount of noise and artifacts in the image. And in the high-energy bin ((100,110] keV), there may be loss of details. It can be seen that some details in the MDM result ((40, 50] keV) are clear, but there may be excessive smoothing in the high-energy bin ((100,110] keV), resulting in loss of details. The images reconstructed by the ASSIST and SISTER algorithms obtained better visual effect in (10, 20] keV and (40, 50] keV spectral bins. However, there may be more noise and artifacts in the (100,110] keV spectral bin, which can affect image quality. It is obvious that FTVNNR can effectively suppress noise and provide high contrast resolution in the (70, 80] keV spectral bin. But there may be detail loss in the (100,110] keV spectral bin. In the last row, the proposed TV-LR-SCT algorithm suppresses most of the noise and artifacts than other competing algorithms.

For a better evaluation of the visual quality of the reconstructed images in Fig. 1, the images over the region of interest (ROI) marked by the red rectangle are zoomed-in and depicted in Fig. 2. The red arrow indicates the location containing features revealed differently by the competing algorithms. There are some obvious artifacts in the results of ASD-POCS, as indicated by the red arrows. In Fig. 2, it can be seen that the reconstruction results of the MDM method contain obvious ring artifacts. The FTVNNR preserve more details than ASSIST and SISTER, but edge information indicated by red arrows are blurred in comparison with the reference images. Compared with other algorithms, we can observe that TV-LR-SCT not only suppresses the noise and artifacts effectively but also preserves edge features successfully, especially in the parts indicated by red arrows.

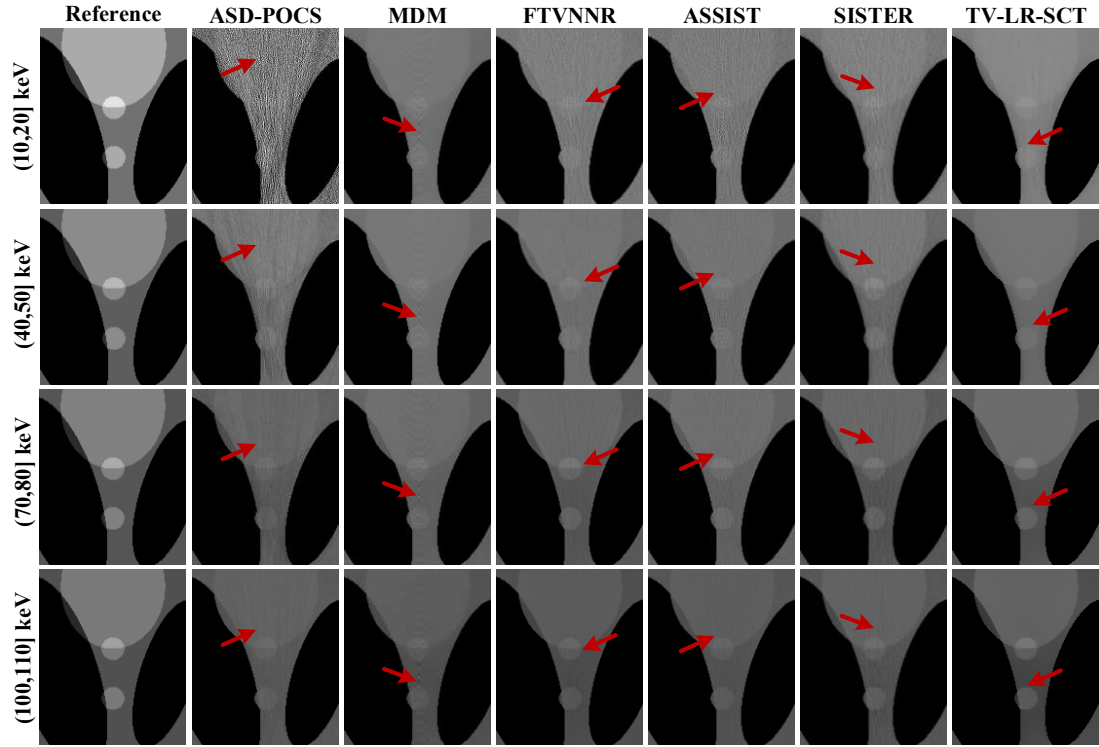


Fig. 2 Zoomed regions indicated by the red box in Fig1.

(2) Quantitative evaluation

To quantitatively measure the overall performance of each algorithm, we employ four metrics (PSNR, SSIM, RMSE, and VIF) to evaluate: Table 1 presents the quantitative assessment results of reconstructed images across different energy channel. For more intuitive comparison, the data in Table 1 are visualized with the following configurations: The x-axis represents different energy levels (Bin1: (10, 20] keV, Bin2: (40,50] keV, Bin3: (70,80] keV, Bin4: (100,110] keV), while the y-axis indicates the quantitative metric values (including PSNR, SSIM and VIF). Distinct color bars correspond to results from different reconstruction algorithms.

Table 1 The quantitative evaluation values of reconstruction results with different algorithms on Shepp-Logan head phantom

Metrics	keV	Methods					
		ASD-POCS	MDM	FTVNNR	ASSIST	SISTER	TV-LR-SCT
PSNR (dB)	(10,20]	18.5761	23.0767	22.9679	22.4000	22.9644	23.1445
	(40,50]	22.6086	24.6220	24.6394	24.4748	24.6116	24.6724
	(70,80]	25.6597	26.0857	27.1055	26.2807	27.0180	28.1117

	(100,110]	25.8475	26.5968	26.9048	27.2390	27.1320	27.8395
	(10,20]	0.6199	0.9403	0.8941	0.8472	0.8837	0.9504
SSIM	(40,50]	0.8465	0.9486	0.9524	0.9282	0.9177	0.9621
	(70,80]	0.9421	0.9584	0.9483	0.9464	0.9438	0.9791
	(100,110]	0.9624	0.9564	0.9641	0.9549	0.9568	0.9796
	(10,20]	0.1178	0.0701	0.0711	0.0759	0.0711	0.0696
RMSE	(40,50]	0.0740	0.0587	0.0586	0.0597	0.0588	0.0584
	(70,80]	0.0521	0.0496	0.0441	0.0485	0.0446	0.0393
	(100,110]	0.0510	0.0468	0.0451	0.0435	0.0439	0.0406
	(10,20]	0.1080	0.4140	0.3009	0.2819	0.2451	0.3866
VIF	(40,50]	0.2460	0.4357	0.4186	0.3873	0.2856	0.4548
	(70,80]	0.3766	0.4476	0.3726	0.4207	0.3527	0.5226
	(100,110]	0.4666	0.4333	0.4361	0.4352	0.3911	0.5363

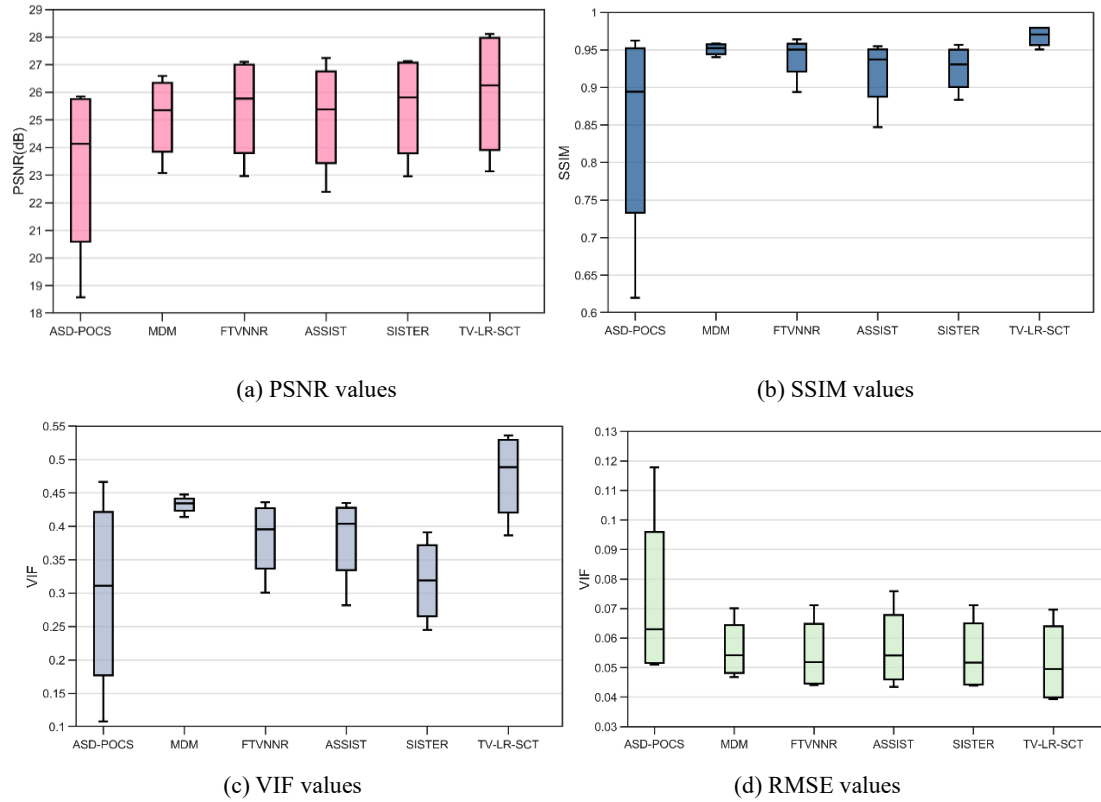


Fig. 3 The quantitative evaluation box plots of reconstruction results with different algorithms on Shepp-Logan head phantom

Combining Fig. 3 and Table 2, it is found that there are significant differences in the performance of different spectral CT reconstruction algorithms. Within the four energy bins, the PSNR, SSIM and VIF values of the ASD-POCS algorithm are relatively low, while the RMSE value is relatively high. We can see that the MDM algorithm indicates stable performance in PSNR and SSIM values, while exhibiting potential for improvement in RMSE and VIF values. The FTVNNR algorithm demonstrates favorable performance in terms of PSNR and SSIM, particularly within the high-energy bins (e.g., (70, 80] keV). However, it exhibits slightly elevated RMSE values at certain energy bins and suboptimal VIF performance. It is clear that the SISTER algorithm performs close to optimal in several values, but there is still a certain gap compared with our proposed algorithm. Consistent with the visual results, the proposed TV-LR-SCT achieved the highest PSNR, SSIM and VIF, and the lowest RMSE. It can be seen that TV-LR-SCT indisputably improved these four metrics.

3.2.2 Clinical CT Data

(1) Visual evaluation

Fig.4 shows the typical thoracic slice reconstructed results by different methods. It can be clearly seen from Fig.4 that the visual effect of the proposed method is superior to other methods. In the results of ASD-POCS, the artifacts are not well suppressed in high-energy bins. Moreover, in the low-energy bins, due to the influence of noise and artifacts, the details may not be clear enough. Although the ASSIST algorithm can effectively improve detail preservation and suppress artifacts, it fails to maintain structural integrity in complex structures and low-contrast regions, exhibiting residual artifacts and detail loss. The MDM algorithm effectively reduces noise and artifacts but suffers from consistent information loss and over-smoothed across all the energy bins. In Fig.4, the FTVNNR algorithm successfully suppresses noise and preserves structures through LR and TV regularizations, but artifacts are not well suppressed. It can be seen that the SISTER reconstructed results blurred with indistinct edges. Compared to the competing algorithms, TV-LR-SCT effectively removes artifacts and completely preserves the structures, yielding the most similar reconstructed results to the reference images.

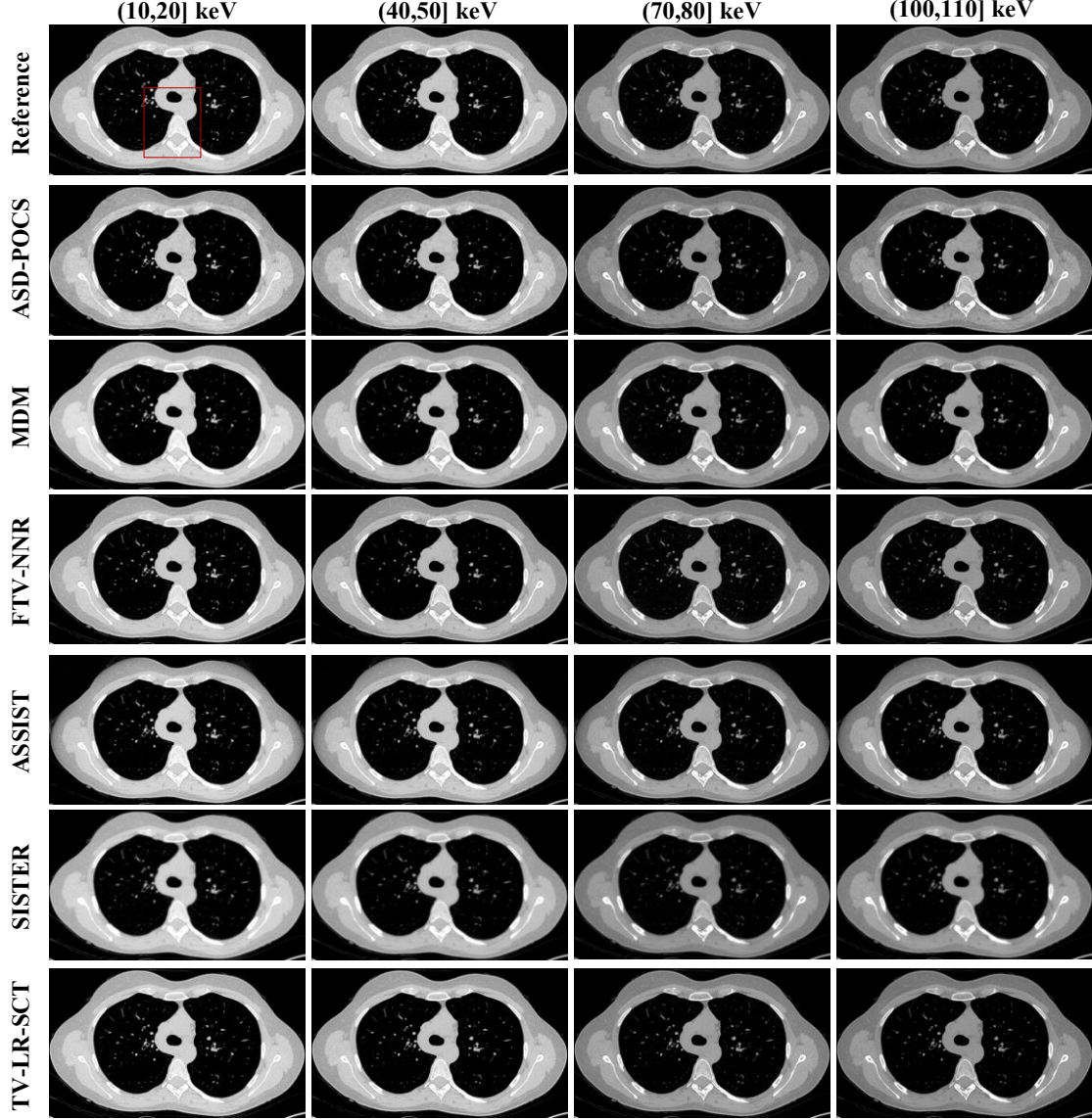


Fig. 4 Reconstruction results of different methods with the thoracic image.

To evaluate the detail preservation capability of different algorithms, the ROI indicated by the red box in Fig. 4 are enlarged in Fig. 5. The red arrows reflect the detailed information reconstructed by different methods. It can be found from Fig. 5 that the ASD-POCS algorithm still has more noise and artifacts compared with the reference images. As indicated by red arrows in Fig. 5, the MDM algorithm suffers from detail loss due to over-smoothing. Observing ROI images of the ASSIST method, it is relatively close to the reference images, but there are still obvious artifacts. The FTVNNR method effectively suppresses artifacts but introduces new noise that degrade image quality. Further observation, compared to the reference images, the SISTER method results in an overall blurry appearance with unclear edges, exhibiting relatively poor image quality. In contrast, our proposed method (TV-LR-SCT) demonstrates superior performance in terms of both detail preservation and artifact reduction, achieving the closest approximation to the reference images.

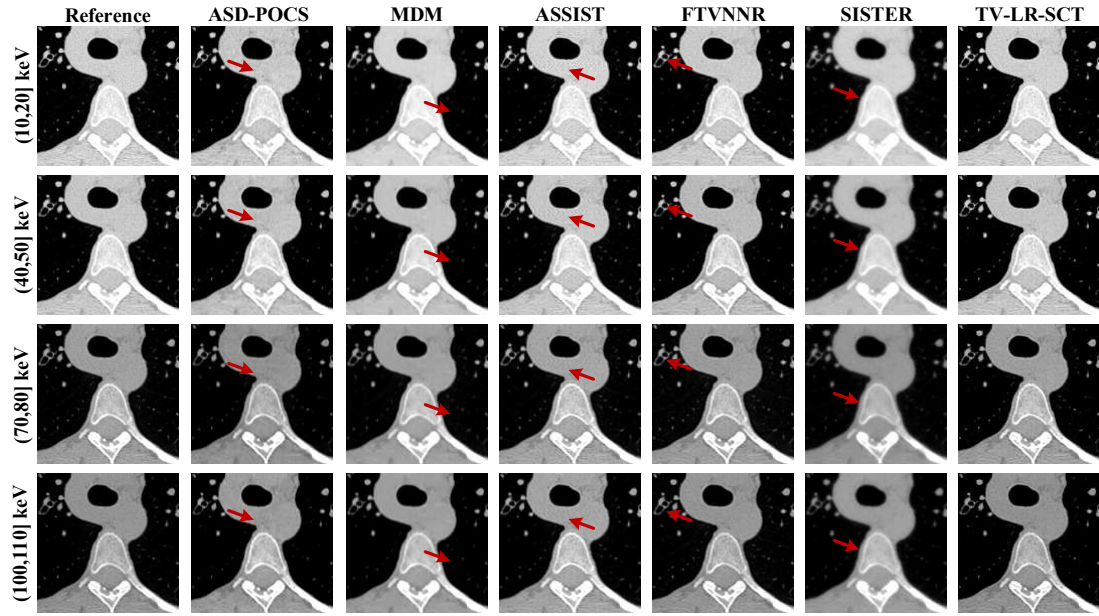


Fig. 5 Zoomed regions indicated by the red box in Fig4.

(2) Quantitative evaluation

Table 2 shows the PSNR, SSIM, RMSE and VIF values in different bins for quantitative assessment, with optimal values highlighted in red text. To better demonstrate the values of different algorithms, Fig. 6 shows the box plots of PSNR, SSIM and VIF values.

Table 2 The quantitative evaluation values of reconstruction results with different algorithms on the thoracic image

Metrics	keV	Methods					
		ASD-POCS	MDM	FTVNNR	ASSIST	SISTER	TV-LR-SCT
PSNR (dB)	(10,20]	24.7781	25.4926	25.5440	27.6240	28.3476	28.6168
	(40,50]	29.5379	28.7236	28.1514	28.4257	26.6913	33.6048
	(70,80]	30.8000	29.4650	30.0020	31.9292	32.8938	37.0698
	(100,110]	29.6616	30.4778	32.3157	32.2757	30.3967	36.6779
SSIM	(10,20]	0.8618	0.8902	0.9194	0.8492	0.8841	0.9235
	(40,50]	0.9190	0.9227	0.9220	0.9028	0.9129	0.9478
	(70,80]	0.9137	0.9396	0.9232	0.9317	0.9179	0.9475
	(100,110]	0.9249	0.9365	0.9553	0.9522	0.9364	0.9683
	(10,20]	0.0577	0.0531	0.0528	0.0416	0.0382	0.0370

RMSE	(40,50]	0.0334	0.0366	0.0391	0.0379	0.0462	0.0208
	(70,80]	0.0288	0.0336	0.0316	0.0253	0.0227	0.0140
	(100,110]	0.0329	0.0299	0.0242	0.0243	0.0302	0.0147
VIF	(10,20]	0.6459	0.7123	0.7425	0.5999	0.6356	0.7733
	(40,50]	0.7257	0.7413	0.6977	0.7376	0.6816	0.8621
	(70,80]	0.7536	0.7625	0.8564	0.8147	0.7297	0.8916
	(100,110]	0.6455	0.7812	0.8883	0.8585	0.7545	0.8965

Combined with Fig. 6 and Table 3, we evaluated the reconstruction performance in different algorithms. The ASD-POCS algorithm shows general performance in the all energy bins, with poor PSNR, SSIM and VIF values coupled with relatively high RMSE. As shown in Table3, the MDM, ASSIST and FTVNMR algorithms achieve good results in PSNR, SSIM, and VIF values, particularly excelling in high-energy bins. However, its RMSE values are relatively high, indicating that there still certain gaps between the reconstructed images and the references images. Compared SISTER algorithm, it is obvious that TV-LR-SCT obtained the best results in terms of both PSNR, SSIM, VIF and RMSE in different energy bins. From Fig.6, it is more intuitively presented that the distribution trends of PSNR, SSIM and VIF values. It also validated the advantage of TV-LR-SCT with providing better results.

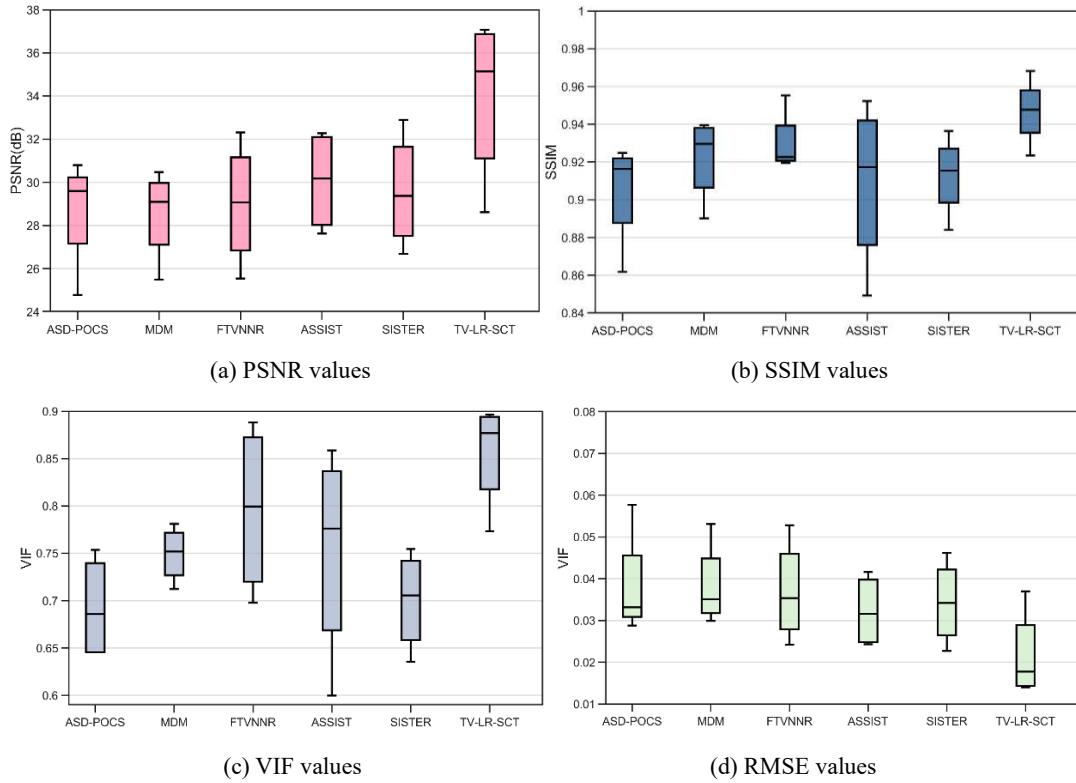


Fig. 6 The quantitative evaluation box plots of reconstruction results with different algorithms on the thoracic image

3.3 Parameter Selection

In this sub-section, we discuss the impacts of parameter selection in the proposed TV-LR-SCT, including regularization parameters λ_1 and λ_2 . Using the thoracic slice from the Mayo dataset, we systematically evaluated parameter combinations with λ_1 ranging from 0.0001 to 0.005 and λ_2 from 0.02 to 1. The Fig.7 visualizes the results of parameter selection, where the x- and y-axes represent λ_1 and λ_2 respectively. Among them, color intensity maps to PSNR values: brighter regions indicate higher PSNR values (better performance) and darker areas correspond to lower PSNR values. The results demonstrate significant performance variations across different parameter combinations, with distinct peaks (bright regions) and valleys (dark regions) in the PSNR landscape. Through comprehensive evaluation of both quantitative metrics and visual quality, we identified $\lambda_1=0.0001$ and $\lambda_2=0.05$ as the optimal parameter set, achieving an optimal balance between noise/artifact suppression and detail preservation for thoracic spectral CT reconstruction. This configuration delivered peak reconstruction accuracy while maintaining robust performance.

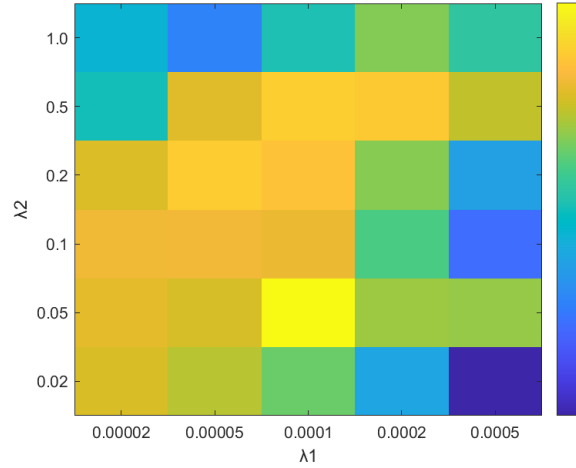


Fig. 7 Impact of regularization parameter selection on algorithm performance

4. Discussion

To better suppress staircase artifacts in each tomographic image slice in spatial dimension and exploit the similarities among different energy bins in spectral dimension, the TV and low-rank regularization terms were introduced for spectral CT image reconstruction. However, solving this spectral CT reconstruction model always results in an iterative reconstruction algorithm of the nested loop structure, which probably increases computational complexity. This study utilized the CP method to solve this TV-LR based spectral CT reconstruction problem, and finally developed a single-loop iterative TV-LR-SCT algorithm to improve computational efficiency. To evaluate the reconstruction quality of the proposed method, comparative experiments were conducted on the Shepp-Logan phantom and thoracic CT images in Mayo Clinic dataset. The results (Figs. 1–6) demonstrate that TV-LR-SCT

outperforms existing methods in noise suppression, artifact reduction, and detail preservation. Moreover, the quantitative evaluation values PSNR, SSIM, VIF and RMSE (Table 1-2) indicate that the TV-LR-SCT method achieves significant improvement.

As illustrated in Fig. 7, due to the choice of parameter directly affects the imaging quality, there are two parameters ($\lambda_1=0.0001$ and $\lambda_2=0.05$) need to be chosen through experimental analysis during the reconstructions. Through comprehensive analysis of the relationship between parameters and metrics (e.g., PSNR), we identified the optimal parameter values. Thus, just like with traditional spectral CT reconstruction algorithms, parameter selection remains a major challenge in spectral CT reconstruction. To further, to optimize parameter selection, we will explore effectively adaptive methods. In addition, we plan to further validate and assess the performance of our proposed TV-LR-SCT method on clinical data.

5. Conclusion

In this work, to improve the spectral CT images quality and make full use of prior information of spectral CT images, we have developed a novel spectral CT image reconstruction model combining total variation and low-rank regularizations (TV-LR-SCT). The application of the CP optimization framework to our model obtained a single-loop iteration scheme for spectral CT reconstruction, achieving significant improvements in computational efficiency. Extensive experimental results demonstrated that the proposed algorithm outperforms several state-of-the-art methods in both qualitative and quantitative aspects.

CRediT authorship contribution statement

Jiaqi Kang: Conceptualization, Data curation, Formal analysis, Methodology, Writing – original draft, Visualization. Pengcheng Zhang: Supervision, Writing – review & editing, Validation. Niu Guo: Investigation, Validation. Lei Wang: Data curation. Liyuan Zhang: Validation. Yi Liu: Investigation, Validation. Zhiguo Gui: Supervision, Project administration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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