Supplementary Material S2

Analysis of TC and SO indices for Tree Drawing Test Classification .

Supplementary Material for:
"Phi and Alzheimer's Disease: Is the Tree Drawing Test for diagnosing cognitive impairment an inner view of the golden proportion?"

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1   Introduction

This supplementary material presents the performance analysis of traditional Tree Drawing Test (TDT) indices :

- **TC index**: Trunk-to-Crown ratio (trunk height/crown height $\times$ 10)

- **SO index**: Space Occupation ratio (tree area/paper area)

The analysis employs the four classification metrics described in the main paper and reported here for reader's convenience.

$$\text{Distance-to-Diameter Ratio, DDR} = \frac{\text{Inter-group distance}}{\text{Maximum intra-group spread}}$$

$$\text{Overlap Coefficient, OC} = 1 - \text{DDR}$$

$$\text{Fisher Ratio, FR} = \frac{\text{Between-group variance}}{\text{Within-group variance}}$$

$$\text{Coefficient of Variation, CV} = \frac{\text{Standard Deviation}}{\text{Mean}}$$

These indices are used to assess discriminative power across Alzheimer's Disease (AD), Mild Cognitive Impairment (MCI), and Control (CNTRL) groups.

2   Global results

Table 1 presents the global classification performance across all indices without demographic data stratification. Sample sizes n for each diagnostic group are as follows

$$\text{AD } n = 613, \qquad \text{MCI } n = 328, \qquad \text{CNTRL } n = 438.$$

Figure 1 provides a visual summary of these classification metrics.   Based on Table 1 we

	Mean ± SD			Classification Metrics			
	AD	**MCI**	**CNTRL**	**DDR**	**OC**	**FR**	**CV**
TC index	13.41±8.28 [12.77, 14.10]	10.26±4.72 [9.75, 10.77]	7.52±3.69 [7.17, 7.87]	0.354	0.646	0.247	0.523
SO index	0.063±0.08 [0.056, 0.069]	0.104±0.09 [0.094, 0.115]	0.312±0.23 [0.289, 0.333]	0.558	0.442	0.611	0.976

Table 1: Comparison of traditional Tree Drawing Test indices across diagnostic groups. TC = Trunk-to-Crown ratio; SO = Space Occupation ratio. DDR = Distance-to-Diameter Ratio; OC = Overlap Coefficient; FR = Fisher Ratio; CV = Coefficient of Variation. Values in brackets represent 95% confidence intervals for group means.

observe that the TC (Trunk-to-Crown) index demonstrates a clear ascending pattern from normal controls (7.52 ± 3.69) to MCI (10.26 ± 4.72) to Alzheimer's disease (13.41 ± 8.28), suggesting that individuals with greater cognitive impairment draw trees with proportionally larger trunks relative to their crowns.

Conversely, the SO (Space Occupation) index exhibits a descending pattern from normal controls (0.312 ± 0.23) to MCI (0.104 ± 0.09) to AD (0.063 ± 0.08), indicating that cognitively impaired individuals draw progressively smaller trees that occupy less space on the paper.

The AD group shows the highest variability in TC index ($\sigma = 8.28$), suggesting more heterogeneous drawing patterns in this population, while normal controls demonstrate the most consistent performance ($\sigma = 3.69$).
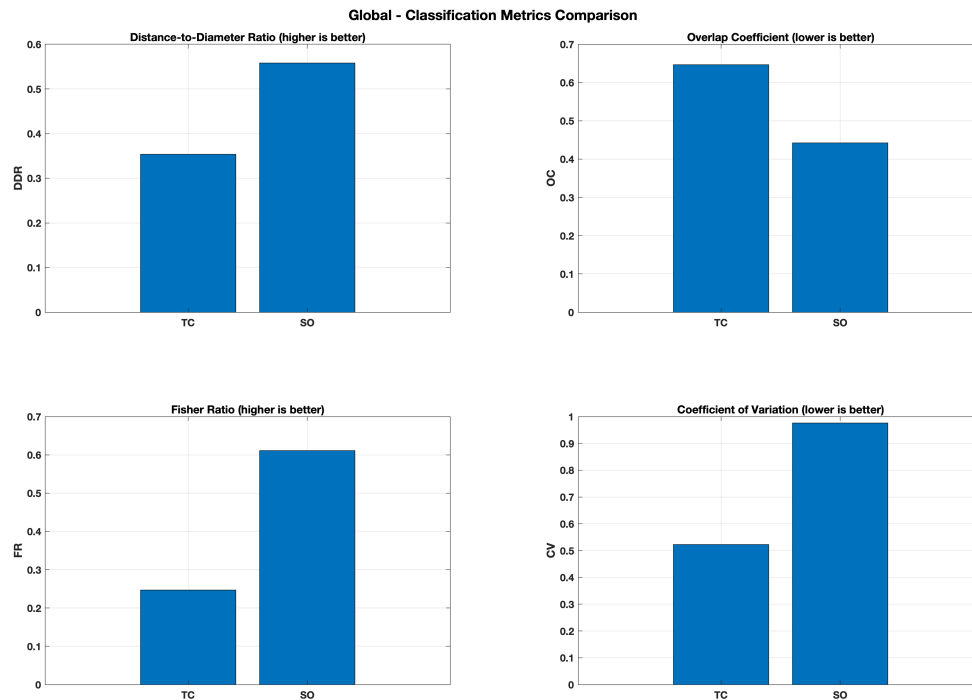
Figure 1: Comparison of classification metrics across different deviation measures. The four panels show: (top left) Distance-to-Diameter Ratio (DDR) where higher values indicate better group separation; (top right) Overlap Coefficient (OC) where lower values are preferable; (bottom left) Fisher Ratio (FR) where higher values indicate better discriminative power; (bottom right) Coefficient of Variation (CV) where lower values indicate better consistency.

For the SO index, normal controls exhibit the greatest variability ($sigma = 0.23$), which may reflect the broader range of drawing styles in healthy individuals.

The SO index demonstrates superior classification performance with a coefficient of variation (CV) of 0.976 compared to the TC index's CV of 0.523, suggesting that space occupation is a more reliable discriminator between groups.

The confidence intervals show clear separation between groups for both indices, particularly between normal controls and AD patients, supporting the diagnostic utility of these measures.

The visualization plots shown in Figures 2 and 3 highlight the distinct patterns emerging across the cognitive spectrum.

Figure 2 demonstrates a monotonic increase in TC index values with cognitive impairment severity: normal controls ($\mu \approx 7.5$), MCI ($\mu \approx 10.0$), and AD ($\mu \approx 13.5$). This represents an approximate 36% increase from CNTRL to MCI and an additional 31% increase from MCI to AD, yielding an overall 78% elevation in AD relative to controls.

Figure 3 exhibits an inverse relationship, with values decreasing by approximately 68% from normal controls ($\mu \approx 0.31$) to MCI ($\mu \approx 0.10$), and an additional 40% reduction from MCI to AD ($\mu \approx 0.06$). The total reduction from CNTRL to AD represents an 81% decrease in space utilization.

The minimal overlap between CNTRL and AD groups in both indices supports strong effect sizes, while the intermediate positioning of MCI values confirms the progressive nature of the underlying cognitive decline. The SO index demonstrates superior group separation, particularly between CNTRL and the cognitively impaired cohorts, consistent with its higher classification coefficient ($CV = 0.976$ vs. 0.523).
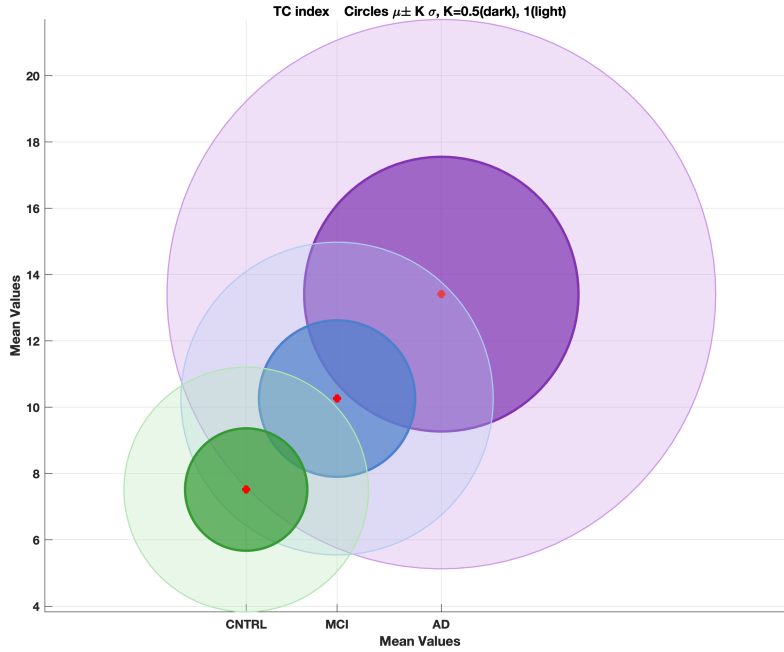
Figure 2: Dual-scale visualization of diagnostic group distributions using the TC index. Each diagnostic group is represented by concentric circles centered at the group mean μ (red dots): inner circles (darker shading, radius $= \sigma/2$) represent the core distribution containing approximately $\approx 38\%$ of observations, while outer circles (lighter shading, $= \sigma$) show the extended distribution including approximately $\approx 68\%$ of observations. Here σ represents the Standard deviation SD.

3 Sex and Education Stratifications

Tables 2 and 3 present the performance of SO and TC indices across demographic stratifications. The sample size n and percentage in each diagnostic group is reported below:

Sex(f), AD $n = 452(73.7\%)$, MCI $n = 184(56.1\%)$, CNTRL $n = 261(59.6\%)$

Sex(m), AD $n = 161(26.3\%)$, MCI $n = 144(43.9\%)$, CNTRL $n = 177(40.4\%)$

Education ≤ 5 years, AD $n = 370(60.4\%)$, MCI $n = 175(53.4\%)$, CNTRL $n = 68(15.5\%)$

Education > 5 years, AD $n = 243(39.6\%)$, MCI $n = 153(46.6\%)$, CNTRL $n = 370(84.5\%)$

Tables 2 and 3 present the performance of SO and TC indices across demographic stratifications. The stratified analyses demonstrate remarkable consistency in the diagnostic patterns across all demographic subgroups, with both indices maintaining their characteristic trends regardless of sex or educational level. For the SO index, all stratifications preserve the descending pattern from normal controls to AD patients, with classification coefficients (CV) ranging from 0.839 to 1.01, indicating robust discriminatory power across demographic categories. Notably, the SO index shows slightly superior performance in female participants (CV = 1.01) compared to males (CV = 0.839), and maintains stable performance across educational levels.

The TC index demonstrates equally consistent patterns across stratifications, with CV values ranging from 0.514 to 0.519, suggesting minimal influence of demographic factors on its diagnostic utility.

Interestingly, both indices show minimal variation in their mean values across sex and education subgroups within each diagnostic category, with confidence intervals demonstrating substantial overlap between demographic strata. This consistency indicates that both indices work
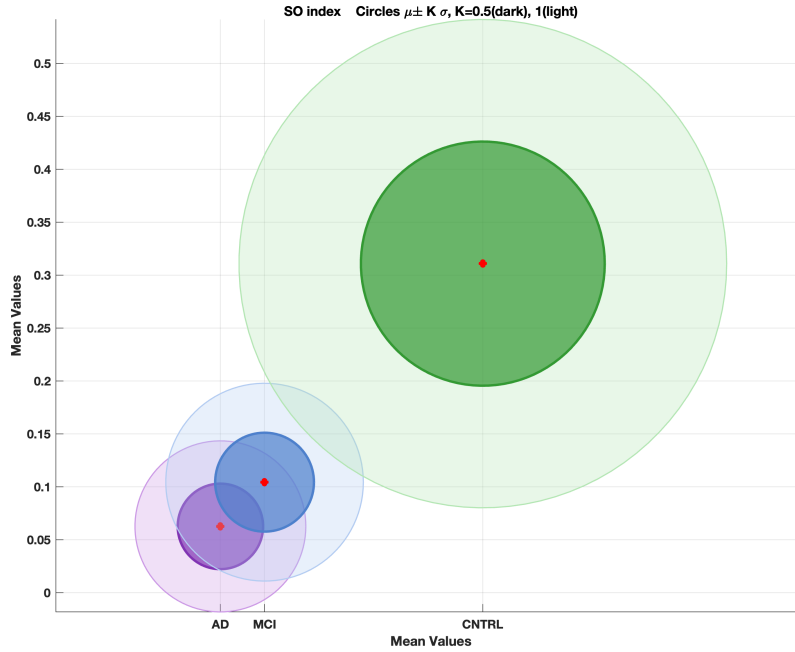
Figure 3: Dual-scale visualization of diagnostic group distributions using the SO index. Each diagnostic group is represented by concentric circles centered at the group mean μ (red dots): inner circles (darker shading, radius $= \sigma/2$) represent the core distribution containing approximately $\approx 38\%$ of observations, while outer circles (lighter shading, $= \sigma$) show the extended distribution including approximately $\approx 68\%$ of observations. Here σ represents the Standard deviation SD.

equally well across different demographic groups and that sex and education level do not interfere with the tests' ability to detect cognitive impairment.

4 Statistical Validation and Effect Size Analysis

Table 4 reports statistical analysis results, including F-statistics, significance levels, and effect sizes for all indices across diagnostic groups and demographic stratifications. It demonstrates highly significant differences across diagnostic groups for both indices, with all analyses yielding p-values < 0.001 even after False Discovery Rate correction. The SO index consistently outperforms the TC index in terms of statistical power, exhibiting substantially higher F-statistics across all stratifications (global analysis: $F = 556.764$ vs. $F = 251.840$).

Effect size analysis reveals that the SO index explains a larger proportion of variance in most stratifications, with particularly strong effects in males ($\eta^2 = 0.410$) and females ($\eta^2 = 0.385$) compared to the TC index ($\eta^2 = 0.182$ for both sexes). Notably, educational stratification reveals differential patterns, where individuals with lower education (≤ 5 years) show reduced effect sizes for the TC index ($\eta^2 = 0.099$) compared to those with higher education ($\eta^2 = 0.211$), while the SO index maintains robust effect sizes across both educational levels ($\eta^2 = 0.278$ and 0.337 respectively).

These findings confirm that both indices provide statistically robust discrimination between diagnostic groups, with the SO index demonstrating superior overall performance across demographic stratifications.

	AD	MCI	CNTRL	Classification Metrics			
		Mean ± SD		DDR	OC	FR	CV
Sex(f)	0.065 ± 0.089 [0.054, 0.076]	0.096 ± 0.087 [0.082, 0.110]	0.331 ± 0.250 [0.305, 0.356]	0.551	0.449	0.617	1.01
Sex(m)	0.055 ± 0.053 [0.049, 0.062]	0.116 ± 0.100 [0.100, 0.132]	0.282 ± 0.197 [0.262, 0.302]	0.621	0.379	0.697	0.839
Education ≤ 5 years	0.045 ± 0.049 [0.039, 0.051]	0.085 ± 0.069 [0.074, 0.096]	0.246 ± 0.194 [0.226, 0.266]	0.595	0.405	0.618	0.893
Education > 5 years	0.090 ± 0.108 [0.076, 0.103]	0.127 ± 0.112 [0.109, 0.145]	0.323 ± 0.235 [0.299, 0.347]	0.471	0.529	0.478	0.938

Table 2: SO index performance across sex and education stratifications. DDR = Distance-to-Diameter Ratio; OC = Overlap Coefficient; FR = Fisher Ratio; CV = Coefficient of Variation. Values in brackets represent 95% confidence intervals for group means.

	AD	MCI	CNTRL	Classification Metrics			
		Mean ± SD		DDR	OC	FR	CV
Sex(f)	13.26±7.80 [12.27, 14.24]	10.04±4.76 [9.28, 10.80]	7.54±3.65 [7.17, 7.92]	0.351	0.649	0.246	0.516
Sex(m)	13.85±9.52 [12.64, 15.05]	10.55±4.66 [9.81, 11.30]	7.48±3.77 [7.10, 7.87]	0.359	0.641	0.249	0.544
Education ≤ 5 years	13.36±8.06 [12.34, 14.38]	10.57±5.25 [9.74, 11.41]	7.30±3.35 [6.96, 7.65]	0.374	0.626	0.280	0.519
Education > 5 years	13.48±8.63 [12.39, 14.57]	9.91±4.01 [9.27, 10.55]	7.56±3.75 [7.17, 7.94]	0.355	0.645	0.240	0.514

Table 3: TC index performance across sex and education stratifications. DDR = Distance-to-Diameter Ratio; OC = Overlap Coefficient; FR = Fisher Ratio; CV = Coefficient of Variation. Values in brackets represent 95% confidence intervals for group means.

5 Conclusion

This analysis demonstrates that both traditional Tree Drawing Test indices provide robust diagnostic discrimination across the cognitive spectrum. The SO index consistently outperforms the TC index across all evaluation metrics, achieving superior effect sizes ($\eta^2 = 0.403$ vs. 0.182) and classification coefficients (CV = 0.976 vs. 0.523). Notably, both indices maintain their discriminative patterns across demographic stratifications, indicating their broad applicability regardless of sex or educational background.

Index/Stratification	F-statistic	p-value	p-adjusted	η^2
Global Analysis				
TC index	251.840	<0.001	<0.001	0.182*
SO index	556.764	<0.001	<0.001	0.403*
Sex Stratification				
Sex(f)				
TC index	164.196	<0.001	<0.001	0.181*
SO index	346.325	<0.001	<0.001	0.385*
Sex(m)				
TC index	89.248	<0.001	<0.001	0.182*
SO index	198.281	<0.001	<0.001	0.410*
Education Stratification				
Education ≤ 5 years				
TC index	62.331	<0.001	<0.001	0.099*
SO index	171.379	<0.001	<0.001	0.278*
Education > 5 years				
TC index	168.745	<0.001	<0.001	0.211*
SO index	258.753	<0.001	<0.001	0.337*

Table 4: Statistical Analysis Results. Test statistics, significance levels, and effect sizes across diagnostic groups (AD, MCI, CNTRL). All measures underwent normality testing using the Kolmogorov-Smirnov test; F-statistics are reported as all distributions satisfied parametric assumptions ($p > 0.05$ for normality). p-adjusted values incorporate False Discovery Rate correction using the Benjamini-Hochberg method ($\alpha = 0.05$). Effect sizes (η^2) quantify the proportion of total variance explained by diagnostic group membership. (η^2) values 0.01=small, 0.06=medium, 0.14=large effects (Cohen's conventions). $*p < 0.001$ after FDR correction for multiple comparisons.