Non-line-of-sight imaging of moving objects obscured by a random corridor

Supplementary Information

Supplementary Note 1：

Imaging principle of RSESIC method

In the paper, the cross-correlation of speckle intensity images $C_{\mathrm{rd}}\left(\Delta\boldsymbol{\rho}\right)$ calculated by the RSESIC method is approximately equivalent to the transmitted field autocorrelation of a hidden object $C_{\mathrm{ho}}\left(\Delta\boldsymbol{\rho}\right)$ under the condition of multiple scattering. Hidden objects in random corridors can be imaged with the approximation. The approximate relationship can be expressed as,

$$C_{\mathrm{rd}}\left(\Delta\boldsymbol{\rho}\right) \approx C_{\mathrm{ho}}\left(\Delta\boldsymbol{\rho}\right) \tag{1}$$

This relationship can be derived from the definition of $C_{\mathrm{rd}}\left(\Delta\boldsymbol{\rho}\right)$ based on three assumptions. $C_{\mathrm{rd}}\left(\Delta\boldsymbol{\rho}\right)$ is defined as,

$$C_{\mathrm{rd}}\left(\Delta\mathbf{r}\right) = \left\langle \frac{\left[I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}\right) - \left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right]\left[I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right) - \left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right]}{\sigma_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{o}}\right)\sigma_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)} \right\rangle_{\mathbf{r}_{\mathrm{c}},\mathbf{r}_{\mathrm{o}}} \tag{2}$$

The ensemble average $\left\langle \ldots \right\rangle_{\mathbf{r}_{\mathrm{o}}}$ is used to increase the overall average times. Therefore, $C_{\mathrm{rd}}\left(\Delta\boldsymbol{\rho}\right)$ is expressed as,

$$C_{\mathrm{rd}}\left(\Delta\mathbf{r}\right) = \frac{\left\langle\left[I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}\right) - \left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right]\left[I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right) - \left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right]\right\rangle_{\mathbf{r}_{\mathrm{c}}}}{\sigma_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{o}}\right)\sigma_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)} \tag{3}$$

The numerator is expanded in formula (3) to obtain,

$$C_{\mathrm{rd}}\left(\Delta\mathbf{r}\right) = \frac{\left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}\right) I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}} - \left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}}\left\langle I_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\mathrm{c}}}}{\sigma_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{o}}\right)\sigma_{\mathrm{rd}}\left(\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho}\right)} \tag{4}$$

The reduced electric field of scattered light is denoted as E_{rd}, and then the reduced intensity of scattered light can be expanded as $I_{\mathrm{rd}} = E_{\mathrm{rd}}E_{\mathrm{rd}}^{*}$.

Assumption 1: E_{rd} follows a Gaussian distribution with zero mean. Apply a moment theorem for complex Gaussian [1], and the first term of the numerator in Eq(4) can be expanded as,

$$\begin{aligned}
&\left\langle I_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) I_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c} \\
=\ &\left\langle E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho}) E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c} \\
=\ &\left|\left\langle E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}\right|^2 + \left\langle \left|E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o)\right|^2\right\rangle_{\mathbf{r}_c} \left\langle \left|E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right|^2\right\rangle_{\mathbf{r}_c} \\
=\ &\left|\left\langle E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}\right|^2 + \left\langle I_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle_{\mathbf{r}_c} \left\langle I_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}
\end{aligned} \tag{5}$$

At the same time, the denominator in Eq. (4) reads[2],

$$\begin{aligned}
&\sigma_{\mathrm{rd}}(\mathbf{r}_o)\sigma_{\mathrm{rd}}(\mathbf{r}_o+\Delta\boldsymbol{\rho}) \\
=\ &\left\langle I_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle_{\mathbf{r}_c} \left\langle I_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c} \\
=\ &\left\langle \left|E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o)\right|^2\right\rangle_{\mathbf{r}_c} \left\langle \left|E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right|^2\right\rangle_{\mathbf{r}_c}
\end{aligned} \tag{6}$$

Substituting (5) and (6) into (4), we have,

$$C_{\mathrm{rd}}(\Delta\mathbf{r}) = \frac{\left\langle E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}}{\left\langle \left|E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o)\right|^2\right\rangle_{\mathbf{r}_c}} \cdot \frac{\left\langle E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}}{\left\langle \left|E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right|^2\right\rangle_{\mathbf{r}_c}} \tag{7}$$

We denote the transmitted field of the hidden object as E_{ho}. Previous studies have shown that E_{rd} is a function of E_{ho} [3],

$$E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) = \int d\mathbf{r} K_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r}) \cdot E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_o) \tag{8}$$

$$E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho}) = \int d\mathbf{r} K_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r}) \cdot E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_o+\Delta\boldsymbol{\rho}) \tag{9}$$

Here $\mathbf{r}$ represents the position vector in three-dimensional space with random media, $K_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r})$ represents the subspace reduced generalized propagator (SRGP) from $\mathbf{r}$ to $\mathbf{r}_c$. Substituting (8) and (9) into (7),

$$\begin{aligned}
&\left\langle E_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o) E^*_{\mathrm{rd}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c} \\
=\ &\left\langle \int E_{\mathrm{ho}}(\mathbf{r}';\mathbf{r}_o) K_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r}') d\mathbf{r}' \int E^*_{\mathrm{ho}}(\mathbf{r}'';\mathbf{r}_o+\Delta\boldsymbol{\rho}) K^*_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r}'') d\mathbf{r}'' \right\rangle_{\mathbf{r}_c} \\
=\ &\left\langle \int\int E_{\mathrm{ho}}(\mathbf{r}';\mathbf{r}_o) E^*_{\mathrm{ho}}(\mathbf{r}'';\mathbf{r}_o+\Delta\boldsymbol{\rho}) K_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r}') K^*_{\mathrm{rd}}(\mathbf{r}_c,\mathbf{r}'') d\mathbf{r}' d\mathbf{r}'' \right\rangle_{\mathbf{r}_c}
\end{aligned} \tag{10}$$

Assumption 2: When $\mathbf{r}' \neq \mathbf{r}''$, $\int K_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}},\mathbf{r}')K_{\mathrm{rd}}^{*}(\mathbf{r}_{\mathrm{c}},\mathbf{r}'')d\mathbf{r}'=0$, namely, SRGP from different spatial locations to $\mathbf{r}_{\mathrm{c}}$ is independent of each other. Therefore, equation (10) can be reduced to,

$$\begin{aligned}&\left\langle E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E_{\mathrm{rd}}^{*}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}\\ =&\left\langle\int E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})E_{\mathrm{ho}}^{*}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\left|K_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}},\mathbf{r})\right|^{2}d\mathbf{r}\right\rangle_{\mathbf{r}_{\mathrm{c}}}\\ =&\int E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})E_{\mathrm{ho}}^{*}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\left\langle\left|K_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}},\mathbf{r})\right|^{2}\right\rangle_{\mathbf{r}_{\mathrm{c}}}d\mathbf{r}\end{aligned} \tag{11}$$

Assumption 3: $P_G=\left\langle\left|K_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}},\mathbf{r})\right|^{2}\right\rangle_{\mathbf{r}_{\mathrm{c}}}$ is a constant and independent of $\mathbf{r}$ and $\mathbf{r}_{\mathrm{c}}$, then,

$$\left\langle E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E_{\mathrm{rd}}^{*}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}=P_G\int E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})E_{\mathrm{ho}}^{*}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\mathrm{d}\mathbf{r} \tag{12}$$

Similarly,

$$\left\langle\left|E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right|^{2}\right\rangle_{\mathbf{r}_{\mathrm{c}}}=P_G\int\left|E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})\right|^{2}d\mathbf{r} \tag{13}$$

$$\left\langle\left|E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right|^{2}\right\rangle_{\mathbf{r}_{\mathrm{c}}}=P_G\int\left|E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right|^{2}d\mathbf{r} \tag{14}$$

Therefore, it can be approximated as,

$$\frac{\left\langle E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E_{\mathrm{rd}}^{*}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}}{\left\langle\left|E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right|^{2}\right\rangle_{\mathbf{r}_{\mathrm{c}}}}\approx\frac{\int E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})E_{\mathrm{ho}}^{*}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})d\mathbf{r}}{\int\left|E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})\right|^{2}d\mathbf{r}} \tag{15}$$

We define the autocorrelation of the transmitted field of a hidden object as,

$$C_{\mathrm{ho}}(\Delta\boldsymbol{\rho})=\left|\frac{\int E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})E_{\mathrm{ho}}^{*}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})d\mathbf{r}}{\int\left|E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})\right|^{2}d\mathbf{r}}\right|^{2} \tag{16}$$

By substituting Eq. (15), (16) into Eq. (7), eventually we obtain the imaging principle of the RSESIC method,

$$C_{\mathrm{rd}}(\Delta\mathbf{r})\approx C_{\mathrm{ho}}(\Delta\boldsymbol{\rho})=\left|\frac{\int E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})E_{\mathrm{ho}}^{*}(\mathbf{r};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})d\mathbf{r}}{\int\left|E_{\mathrm{ho}}(\mathbf{r};\mathbf{r}_{\mathrm{o}})\right|^{2}d\mathbf{r}}\right|^{2} \tag{17}$$

Assumption 2 is satisfied by strong scattering of random media. The subspace reduction we proposed guarantees that Assumption 3 is satisfied regardless of the shape of random medium. Moreover, subspace reduction also ensures the validity of Assumption 1 for any observation position $\mathbf{r}_\mathrm{c}$ and any size of spatial statistical region $\langle \ldots \rangle_{\mathbf{r}_\mathrm{c}}$.

In expression (16), $\int d\mathbf{r}$ represents the integral of three-dimensional space including random medium, $E_\mathrm{ho}(\mathbf{r};\mathbf{r}_\mathrm{o})$ and $E_\mathrm{ho}(\mathbf{r};\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})$ represent the three-dimensional transmitted field illuminating on a hidden object. Therefore,

$$\begin{aligned}&\int E_\mathrm{ho}(\mathbf{r};\mathbf{r}_\mathrm{o})E^*_\mathrm{ho}(\mathbf{r};\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})d\mathbf{r}\\ =\ &\iiint E_\mathrm{ho}(x,y,z;\mathbf{r}_\mathrm{o})E^*_\mathrm{ho}(x,y,z;\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})dxdydz\end{aligned} \tag{18}$$

When the hidden object is a transmission plate and the integral interval of $\int dz$ is relatively small, $E_\mathrm{ho}(x,y,z) \approx E_\mathrm{ho}(x,y)$, thus,

$$\begin{aligned}&\int E_\mathrm{ho}(\mathbf{r};\mathbf{r}_\mathrm{o})E^*_\mathrm{ho}(\mathbf{r};\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})d\mathbf{r}\\ =\ &b_z\cdot\iint E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o})E^*_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})dxdy\end{aligned} \tag{19}$$

Here $b_z = \int dz$ is a constant. Applying the Eq. (19) to the Eq. (16), we have,

$$C_\mathrm{ho}(\Delta\boldsymbol{\rho}) = \left|\frac{\iint E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o})E^*_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})dxdy}{\iint \left|E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o})\right|^2 dxdy}\right|^2 \tag{20}$$

We define the reduced dimensionless transmitted field as,

$$\hat{E}_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o}) = \frac{E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o})}{\sqrt{\iint \left|E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o})\right|^2 dxdy}} \tag{21}$$

$$\hat{E}_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho}) = \frac{E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o}+\Delta\boldsymbol{\rho})}{\sqrt{\iint \left|E_\mathrm{ho}(x,y;\mathbf{r}_\mathrm{o})\right|^2 dxdy}} \tag{22}$$

By inserting Eq. (21), (22) to Eq. (20) and applying the autocorrelation theorem [4], we have,

$$
\begin{aligned}
C_{\mathrm{ho}}(\Delta\boldsymbol{\rho}) &= \left|\iint \hat{E}_{\mathrm{ho}}(x,y;\mathbf{r}_{\mathrm{o}})\,\hat{E}^{*}_{\mathrm{ho}}(x,y;\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\,dxdy\right|^{2} \\
&= \left|\mathcal{F}^{-1}\left\{\left|\mathcal{F}\left\{\hat{E}_{\mathrm{ho}}(x,y)\right\}\right|^{2}\right\}\right|^{2}
\end{aligned}
\tag{23}
$$

Here $\mathcal{F}$ and $\mathcal{F}^{-1}$ represent Fourier transform and inverse Fourier transform, respectively. Then combining Eq. (23) with (1),

$$
\hat{E}_{\mathrm{ho}}(x,y) = \mathcal{F}^{-1}\left\{\sqrt{\mathcal{F}\left\{\sqrt{C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})}\cdot e^{i\varphi_{1}(\Delta\boldsymbol{\rho})}\right\}}\cdot e^{i\varphi_{2}(\mathbf{k})}\right\}
\tag{24}
$$

In this equation, $\varphi_1(\Delta\boldsymbol{\rho})$ is the phase of $\iint \hat{E}_{\mathrm{ho}}(x,y;\mathbf{r}_{\mathrm{o}})\,\hat{E}^{*}_{\mathrm{ho}}(x,y;\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\,dxdy$. When the hidden object is a transmission plate, we find that the imaginary part of $\iint \hat{E}_{\mathrm{ho}}(x,y;\mathbf{r}_{\mathrm{o}})\,\hat{E}^{*}_{\mathrm{ho}}(x,y;\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\,dxdy$ is much smaller than its real part, so $\varphi_1(\Delta\boldsymbol{\rho})\approx 0$. $\varphi_2(\mathbf{k})$ can be recovered by an iterative phase recovery algorithm[5, 6]. Substituting $\varphi_1(\Delta\boldsymbol{\rho})$, $\varphi_2(\mathbf{k})$ and $C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$ into (24), $\hat{E}_{\mathrm{ho}}(x,y)$ can be recovered. $\hat{E}_{\mathrm{ho}}(x,y)$ is the reduced dimensionless transmitted field on an XY-plane near the hidden object, and its amplitude $\left|\hat{E}_{\mathrm{ho}}(x,y)\right|$ is the image of the hidden object. Finally, the image of the hidden object is successfully reconstructed.

Supplementary Note 2:

Calculation of the autocorrelation of hidden objects

In our experiment, the hidden object to be imaged is a transmission plate. The transmittance of the aperture of the transmission plate to the laser is 1, and the transmittance of other parts of the plate is close to 0. Because the size of the aperture on the transmission plate is in the order of mm, which is 10^3 times of the laser wavelength, the diffraction effect can be ignored. Based on the analysis above, the transmitted field of different hidden objects can be simulated.

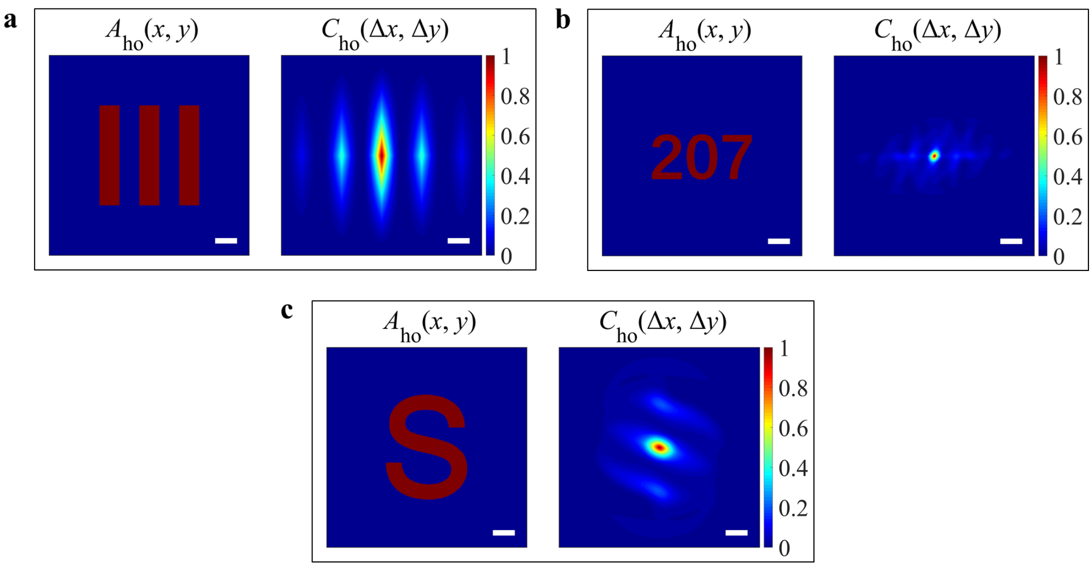
The steps to simulate the transmitted field of a hidden object are as follows. First, an amplitude function $A_{\mathrm{ho}}(x,y)$ is generated. Then the center of the transmission plate is taken as the coordinate origin, and $A_{\mathrm{ho}}(x,y)$ is binarized according to the geometric characteristics of the transmission plate. The rule of binarization is,

$$A_{\mathrm{ho}}(x,y)=\begin{cases}1, & (x,y)\in \text{Hole area}\\ 0, & (x,y)\notin \text{Hole area}\end{cases} \tag{25}$$

Next, the phase of the transmitted field of the hidden object is simulated as a constant φ_{ho} that is independent of x or y. Finally, the simulated transmitted field of the hidden object is obtained,

$$E_{\mathrm{ho}}(x,y)=A_{\mathrm{ho}}(x,y)\cdot e^{i\varphi_{\mathrm{ho}}} \tag{26}$$

Substituting the simulated $E_{\mathrm{ho}}(x,y)$ into Eq. (21), (23), $C_{\mathrm{ho}}(\Delta x,\Delta y)$ can be obtained. The simulated $A_{\mathrm{ho}}(x,y)$ and $C_{\mathrm{ho}}(\Delta x,\Delta y)$ of three different hidden objects are shown in the supplementary Figure 1.



Supplementary Figure 1: Simulated $A_{\mathrm{ho}}(x,y)$ and $C_{\mathrm{ho}}(\Delta x,\Delta y)$ of three different hidden objects. Scale bar: 1mm.

Supplementary Note 3:

Elimination of the adverse effect of ambient noise

Since the Eq.1 of the imaging relation is obtained without concerns for the noise, it is no longer valid when the speckle intensity images contain noise. In the condition of taking the noise into consideration, the reduced electric field E_{rd} of scattered light is expressed as,

$$E_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})=E_{\mathrm{rd\text{-}ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})+E_{\mathrm{rd\text{-}b}}(\mathbf{r}_{\mathrm{c}};t_0+\Delta t) \tag{27}$$

Here $E_{\text{rd-ho}}$ represents the scattering field induced by the transmitted field E_{ho} interacting with the random medium. $E_{\text{rd-b}}$ represents the time-varying background noise field. The position vector of the hidden object at time t_0 is $\mathbf{r}_{\text{o}}$, and the velocity vector of the hidden object is $\mathbf{v}$, then $\Delta\boldsymbol{\rho}=\mathbf{v}\Delta t$. Based on equation (27), the cross-correlation of E_{rd} can be expanded into four terms,

$$\begin{aligned}&\left\langle E_{\text{rd}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\text{c}}}\\ =\;&\Big\langle E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)+E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)+\\ &E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)+E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\Big\rangle_{\mathbf{r}_{\text{c}}}\end{aligned}\tag{28}$$

Since $E_{\text{rd-ho}}$ and $E_{\text{rd-b}}$ are independent of each other,

$$\left\langle E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)+E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\text{c}}}=0\tag{29}$$

Substituting (29) into (28), then

$$\begin{aligned}&\left\langle E_{\text{rd}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle\\ =\;&\left\langle E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\text{c}}}+\left\langle E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)\right\rangle_{\mathbf{r}_{\text{c}}}\end{aligned}\tag{30}$$

Then substituting (30) into (7), we have

$$\begin{aligned}C_{\text{rd}}\left(\Delta\boldsymbol{\rho}\right)=\;&\frac{\left|\left\langle E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\text{c}}}\right|^2+\left|\left\langle E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)\right\rangle_{\mathbf{r}_{\text{c}}}\right|^2}{\left\langle\left|E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)\right|^2+\left|E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)\right|^2\right\rangle_{\mathbf{r}_{\text{c}}}^2}+\\ &\frac{\left\langle E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\text{c}}}\left\langle E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)\right\rangle_{\mathbf{r}_{\text{c}}}}{\left\langle\left|E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)\right|^2+\left|E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)\right|^2\right\rangle_{\mathbf{r}_{\text{c}}}^2}+\\ &\frac{\left\langle E_{\text{rd-ho}}^{*}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}+\Delta\boldsymbol{\rho}\right)\right\rangle_{\mathbf{r}_{\text{c}}}\left\langle E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)\right\rangle_{\mathbf{r}_{\text{c}}}}{\left\langle\left|E_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)\right|^2+\left|E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)\right|^2\right\rangle_{\mathbf{r}_{\text{c}}}^2}\end{aligned}\tag{31}$$

It can be found from Eq.(31) that $C_{\text{rd}}\left(\Delta\boldsymbol{\rho}\right)$ contains both the correlation of noise and of transmitted field of the hidden object. When the noise evolves rapidly with time, its time correlation can be seen as a δ function, therefore,

$$\left\langle E_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)E_{\text{rd-b}}^{*}\left(\mathbf{r}_{\text{c}};t_0+\Delta t\right)\right\rangle_{\mathbf{r}_{\text{c}}}=\begin{cases}\left\langle I_{\text{rd-b}}\left(\mathbf{r}_{\text{c}};t_0\right)\right\rangle_{\mathbf{r}_{\text{c}}}, & \Delta t=0\\ 0, & \Delta t>0\end{cases}\tag{32}$$

Substituting (32) into (31),

$$C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})=\begin{cases}1 & , \quad |\Delta\boldsymbol{\rho}|=0 \\ \dfrac{\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E^{*}_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right|^{2}}{\left\langle \left|E_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right|^{2}+\left|E_{\text{rd-b}}(\mathbf{r}_{\mathrm{c}};t_{0})\right|^{2}\right\rangle^{2}_{\mathbf{r}_{\mathrm{c}}}} & , \quad |\Delta\boldsymbol{\rho}|>0\end{cases} \tag{33}$$

It can be found from Eq.(33) that $C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$ is a piecewise function. When $|\Delta\boldsymbol{\rho}|$ increases gradually from 0, the noise causes $C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$ to drop sharply from 1 to $\left|\left\langle I_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right\rangle_{\mathbf{r}_{\mathrm{c}}} / \left\langle I_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right|^{2}$, it deviates significantly from $C_{\mathrm{ho}}(\Delta\boldsymbol{\rho})$. In the Supplementary Figure 2, C_{rd} calculated from the experimental data is compared with C_{ho}, which confirms the sudden drop behavior and shows the significant deviation between the two correlations.

In order to image hidden objects, the adverse effect of noise on $C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$ must be eliminated. We denote the cross-correlation of speckle intensity images with the noise elimination as $\widetilde{C}_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$,

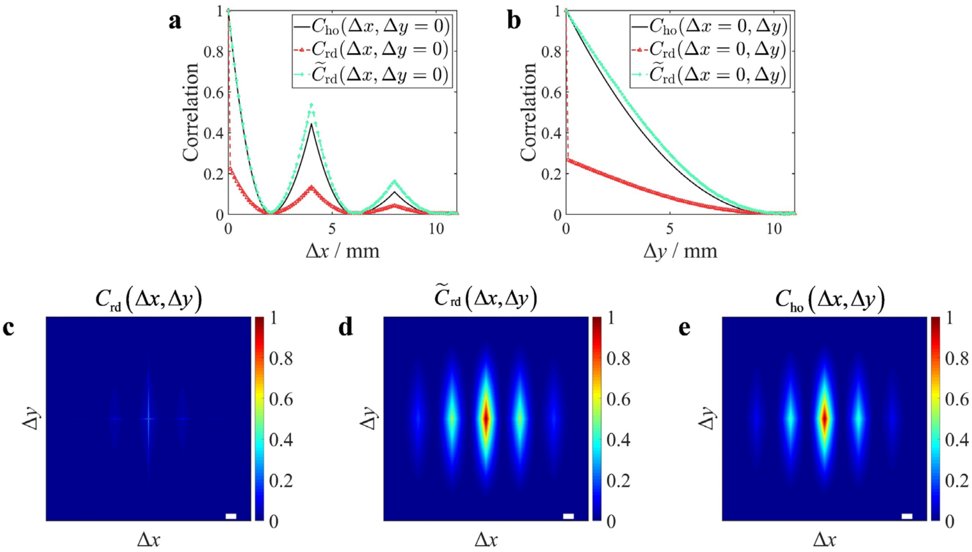
$$\widetilde{C}_{\mathrm{rd}}(\Delta\boldsymbol{\rho})=\frac{\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E^{*}_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right|^{2}}{\left\langle I_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right\rangle^{2}_{\mathbf{r}_{\mathrm{c}}}} \tag{34}$$

According to **Supplementary Note 1**, $\widetilde{C}_{\mathrm{rd}}(\Delta\boldsymbol{\rho})\approx C_{\mathrm{ho}}(\Delta\boldsymbol{\rho})$, $\widetilde{C}_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$, therefore, can be used to reconstruct the image of hidden objects. In order to recover $\widetilde{C}_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$, $\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E^{*}_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right|^{2}$ is extracted from $C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})$. By inverting Eq. (33), we have,

$$\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})E^{*}_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}}+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_{\mathrm{c}}}\right|^{2}=\begin{cases}\left\langle I_{\text{rd-ho}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right\rangle^{2}_{\mathbf{r}_{\mathrm{c}}} & , \quad |\Delta\boldsymbol{\rho}|=0 \\ C_{\mathrm{rd}}(\Delta\boldsymbol{\rho})\cdot\left\langle I_{\mathrm{rd}}(\mathbf{r}_{\mathrm{c}};\mathbf{r}_{\mathrm{o}})\right\rangle^{2}_{\mathbf{r}_{\mathrm{c}}} & , \quad |\Delta\boldsymbol{\rho}|>0\end{cases} \tag{35}$$

From equation (35), it can be found that all values of

$\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)E^*_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}\right|^2$ are known except for $\Delta\boldsymbol{\rho}=0$. Therefore, $\left\langle I_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle^2_{\mathbf{r}_c}$ can be extrapolated from the known values of $\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)E^*_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}\right|^2$. Then $\widetilde{C}_{\text{rd}}(\Delta\boldsymbol{\rho})$ can be recovered with equation (34).



Supplementary Figure 2: A comparison of C_{ho}, C_{rd} and $\widetilde{C}_{\text{rd}}$. Scale bar: 1mm.

We use quadratic polynomial to extrapolate $\left\langle I_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle^2_{\mathbf{r}_c}$. First, data points of $\left|\left\langle E_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)E^*_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o+\Delta\boldsymbol{\rho})\right\rangle_{\mathbf{r}_c}\right|^2$ in a monotonic decreasing region $(0<\Delta x\le l_x, 0<\Delta y\le l_y)$ near $|\Delta\boldsymbol{\rho}|=0$ are chosen. Then these data points are substituted into the following quadratic polynomials,

$$C_{\text{rd}}(\Delta x,\Delta y)\cdot\left\langle I_{\text{rd}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle^2_{\mathbf{r}_c}=\left\langle I_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle^2_{\mathbf{r}_c}+k_1(\Delta x+\Delta y)+k_2\left(\Delta x^2+\Delta y^2\right) \quad (36)$$

Since the expression contains three unknown coefficients $\left\langle I_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle^2_{\mathbf{r}_c}$, k_1 and k_2, at least three data points are required to solve these coefficients. We select three data points on x-axis, y-axis and 45 ° direction respectively, and substitute them into Eq. (36). Three groups of coefficients $\left\langle I_{\text{rd-ho}}(\mathbf{r}_c;\mathbf{r}_o)\right\rangle^2_{\mathbf{r}_c}$, k_1 and k_2 are solved. Then three values of $\left\langle I_{\text{rd-ho}}\left(\mathbf{r}_{\text{c}};\mathbf{r}_{\text{o}}\right)\right\rangle_{\mathbf{r}_{\text{c}}}^{2}$ are statistically averaged and substituted into Eq. (34) to obtain $\widetilde{C}_{\text{rd}}\left(\Delta\boldsymbol{\rho}\right)$.

The recovered $\widetilde{C}_{\text{rd}}$ is also drawn in Supplementary Figure 2. The good consistency between $\widetilde{C}_{\text{rd}}$ and C_{ho} indicates that the noise is successfully eliminated, and confirms the adaptability of the RSESIC method in an environment with strong noise.

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