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Research Article

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Posted Date: August 10th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-7929688/v1>

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Additional Declarations: No competing interests reported.

Version of Record: A version of this preprint was published at Signal, Image and Video Processing on October 1st, 2026. See the published version at <https://doi.org/10.1007/s11760-026-05729-0>.

Modified Pseudo-Huber Loss Based Zero-Attraction LMS Algorithm for Sparse Systems

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Abstract. The zero-attraction based least mean square (LMS) algorithms have gained popularity for sparse system identification due to their low computational complexity and easy implementation. This paper introduces a new sparsity aware function based on Pseudo-Huber Loss (P-HL) namely modified P-HL (MP-HL). This loss function is designed to enhance the sparsity promotion while maintaining smoothness and differentiability for stable adaptation. This is then utilized to develop novel sparsity aware zero attraction-based MP-HL LMS (ZA-MP-HL-LMS) adaptive filter. The proposed adaptive filter integrates zero-attraction mechanisms with the conventional LMS to effectively identify sparse systems. The paper also presents bound on the learning rate, ensuring convergence in both the mean and mean square error sense. Additionally, mean and mean square error (MSE) theoretical analysis and state state MSE is also performed. Finally, guidance on the selection of zero attraction parameter to ensure steady state MSE lesser than LMS is also provided. Simulation results demonstrate that the proposed algorithms outperform the state-of-the-art approaches in the system identification examples.

Keywords: Pseudo-Huber loss, sparsity, convergence, steady state mean square error, complexity.

1 Introduction

Adaptive filters play a pivotal role in the field of advanced signal processing. The key advantage is that there is no requirement to know about the input and reference signal to adjust the filter coefficients. The cost function is one of the major criteria that determine the efficiency of adaptive filters. Very often, sparse systems are seen in many real time applications. A sparse system is the one whose impulse response contains more number of close to zero coefficients than non-zero coefficients. Examples of such systems are underwater acoustic channels [1], television transmission channels

etc.,. The well-known least mean square (LMS) algorithm uses mean square error cost function which does not make use of sparsity.

To address the limitation, proportionate and Zero-attraction mechanisms are the two major strategies used for sparse adaptive filtering. Proportionate methods enhance convergence by scaling adaptation gains based on coefficient magnitudes but often fail to fully suppress small taps. Zero-attraction mechanisms overcome this by introducing a new term called sparsity term into the existing algorithm that drives insignificant coefficients toward zero, yielding lower steady-state error and cleaner sparse solutions [3-5]. With lower complexity, better noise robustness, and possibilities for further refinements, the later are particularly effective for highly sparse scenarios such as echo cancellation and sparse channel estimation.

Based on the above, several sparsity-aware variants of LMS have been proposed for performance improvement. The well-known l_1 norm-based algorithm is the zero-attracting LMS (ZA-LMS) algorithm[6]. The problem with ZA-LMS algorithm is that it offers more penalties to the larger coefficients than to the smaller ones. To overcome this limitation, the re-weighted zero-attracting LMS algorithm was proposed where a function that grows less quickly for the large magnitude coefficients is used where the l_1 norm attract the smaller coefficients to zero more strongly than the large magnitude ones [6]. However, it still affects/shrinks the large magnitude coefficients, and it is not convex. Thus the l_0 norm based LMS algorithm, and their variants were proposed [7, 8]. Motivated by the information theory-based criteria in the context of sparsity exploitation [9-14] and to further enhance the convergence behavior of zero-attracting algorithms, a new sparsity-aware adaptive filtering approach is introduced in this brief. The formulation of this approach involves the utilization of the Pseudo-Huber loss function [15].

The traditional Pseudo-Huber loss function is widely considered do to its several attractive features. It is strongly convex resulting in an unique solution and stable convergence. Its positive second order derivative and symmetric, monotonic form, along with an adjustable scaling parameter enables a smooth transition between quadratic and linear behavior [16-19]. These features make it as a suitable candidate for sparsity-aware adaptive filtering [20]. However, for sparse system identification, its smoothness near zero causes penalization of small coefficients, reducing its ability to promote exact sparsity. Additionally, its gradient is proportional to the magnitude of the coefficients, leading to uniform penalization. Finally it lacks robustness under impulsive or heavy-tailed noise and in online, its smoothness leads to poor convergence and insufficient suppression of weights. While Pseudo-Huber offers smoother gradients and is more compatible with gradient descent than l_0 or ZA-LMS, its softness leads to weaker sparsity enforcement. To address these shortcomings, we propose a modified Pseudo-Huber loss by incorporating a sign-based sparsity-shaping term. This modification enhances sparsity promotion by sharpening the gradient so that the near to zero coefficients are encouraged to become exact zeros, selectively shrinking small coefficients while preserving larger ones. Additionally, it flattens the gradient for significant weights to avoid bias and improves robustness to impulsive noise. At the same time, it maintains smoothness and differentiability, ensuring stable and efficient optimization. With the proposed modified Pseudo-Huber loss function, a modified Pseudo-Huber loss function-based zero-attracting LMS (ZA-MP-HL-LMS) algorithm is developed. Using the energy conservation relation, analytic treatment on

step size boundary, convergence in mean and mean square error are derived. Then, simulations under system identification and echo cancellation applications proves that the integration has shown to enhance the performance, leading to faster convergence speed and improved modeling accuracy.

The major contributions of the paper are as follows: 1. Development of novel zero attraction-based MP-HL-LMS (ZA-MP-HL-LMS) adaptive filter based on proposed cost function called modified Huber loss function. 2. Study of the convergence condition and steady state mean square performance analysis and the step size boundary condition. Also the guidance on the selection of zero attraction parameter to ensure steady state MSE lesser than LMS is also provided. 3. Simulations are conducted to demonstrate the superiority of the proposed adaptive filter when compared to existing sparsity aware adaptive filters.

The rest of the paper is described as follows: In section 2, the proposed modified Huber loss is introduced and the zero attraction-based MP-HL-LMS (ZA-MP-HL-LMS) adaptive filter is derived. Section 3 presents analytical investigation into the convergence of the ZA-MP-HL-LMS algorithm, along with the analysis of mean and mean squared error (MSE). Section 4 provides simulation results that compare the performance of various sparsity-aware LMS algorithms. Finally, section 5 concludes the study.

2 Proposed MP-HL cost function and ZA-MP -HL - LMS adaptive filter

Let's consider an unknown system identification problem in which an adaptive filter is used to identify an unknown system. The unknown system is modeled as a finite impulse response system in which $\mathbf{g}_o$ where $\mathbf{g}_o \in R^{N \times 1}$ represent the unknown system coefficient vector that need to be identified. The input signal to the system of length N is $\mathbf{x}(n) = [x_1(n), x_2(n), \dots, x_N(n)]^T$. The desired signal is modeled as a linear regression model given by $d(n) = \mathbf{g}_o^T \mathbf{x}(n) + v(n)$. Here, $v(n)$ is the noise source.

The estimated output of the adaptive filter is given by

$$y(n) = \mathbf{g}^T(n) \mathbf{x}(n), \quad (1)$$

where $\mathbf{g}(n) = [g(n), g(n-1), \dots, g(n-N+1)]^T$, is the vector of the estimated filter weights. The error signal is obtained as

$$e(n) = d(n) - y(n). \quad (2)$$

The cost function $L(n)$ of standard LMS algorithm is given by $L(n) = \frac{1}{2} e^2(n)$. Based on this, the filter coefficient for LMS is updated as

$$\mathbf{g}(n+1) = \mathbf{g}(n) - \mu \frac{\partial L(n)}{\partial \mathbf{g}(n)} = \mathbf{g}(n) + \mu e(n) \mathbf{x}(n) \quad (3)$$

where μ is the step size parameter.

As per the standard LMS algorithm, it converges if μ satisfies the condition, $0 < \mu < \frac{1}{\lambda_{max}}$ where λ_{max} is the maximum eigen value of the covariance matrix $\mathbf{R}_x$ of the input vector $\mathbf{x}(n)$ [6]. Based on this, the Excess Mean Square Error under steady state ($EMSE_\infty$) of LMS is given as

$$EMSE_{\infty(LMS)} = \frac{\mu\sigma_v^2 Tr(\mathbf{R}_x(\mathbf{I} - \mu\mathbf{R}_x)^{-1})}{2 - \mu Tr(\mathbf{R}_x(\mathbf{I} - \mu\mathbf{R}_x)^{-1})}. \quad (4)$$

where $Tr(\cdot)$ is the Trace operator. In order to put forth the proposed algorithm, the pseudo-Huber loss function is given by

$$J(\mathbf{g}(n)) = a^2 \left[\sqrt{1 + \left(\frac{\|\mathbf{g}(n)\|^2}{a^2} \right)} - 1 \right]. \quad (5)$$

Taking the derivative of (5) w.r.t $\mathbf{g}(n)$, we obtain

$$J'(\mathbf{g}(n)) = \frac{a\mathbf{g}(n)}{\sqrt{\|\mathbf{g}(n)\|^2 + a^2}} \quad (6)$$

where $a > 0$ is called the scaling parameter.

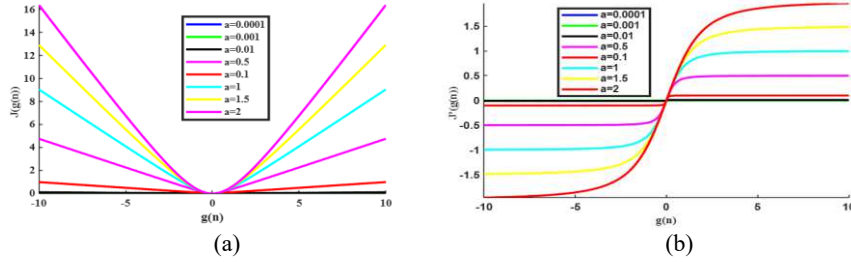


Fig. 1 Variation of **a.** pseudo-Huber loss function **b.** Gradient of pseudo-Huber loss function for different values of a .

The variation of pseudo-Huber loss function $J(\mathbf{g}(n))$ for various values of a is illustrated in Fig.1(a). From Fig.1(a) it can be found that it possesses monotonically non-decreasing and symmetrical function. The scaling parameter a in the Pseudo-Huber loss function plays an important role in the transition between l_2 and l_1 norms. Specifically, when $|\mathbf{g}(n)| < a$, equation (5) behaves as a quadratic function, which is suitable for Gaussian noise together with smooth and convex optimization. Conversely, when $|\mathbf{g}(n)| > a$, the function becomes similar to the l_1 norm, which enhances robustness against outliers and impulsive noise. This transition allows the Pseudo-Huber loss to combine the benefits of both norms. Fig. 1(b) indicates that the gradient function $J'(\mathbf{g}(n))$ of pseudo-Huber loss has non-increasing and monotonic nature. The parameter a also regulates the gradient of the loss function that is smaller values of a lead to sharper transitions, promoting sparsity by encouraging near-zero coefficients to become exact zeros, while also limiting the influence of large errors and improving robustness. In contrast, larger a values result in behavior closer to the traditional l_2 loss, emphasizing smooth error minimization but reducing robustness. Thus, tuning a appropriately is essential for achieving a balance between stability, sparsity promotion, and robustness in adaptive filtering, especially under impulsive or heavy-tailed noise environments.

Thus from Fig. 1 it is inferred that the pseudo-Huber loss and its derivative satisfies the requirement of sparsity function [9,22]. However, the gradient function is operated in a manner such that it provides uniform attraction to all filter coefficients irrespective of whether they are far away from zero or near to zero which is the same problem that prevails with ZA-LMS algorithm [4].

To leverage the inherent sparsity within the system and to eliminate the disadvantage of uniform penalty to all terms, a novel cost function is introduced, where the gradient function given in (6) is modified by replacing the $\mathbf{g}(n)$ by $\text{sgn}(\mathbf{g}(n))$. The

resultant proposed modified gradient of pseudo-Huber loss based function is expressed as follows.

$$J'_M(\mathbf{g}(n)) = \text{sgn}[\mathbf{g}(n)] \frac{|a|}{\sqrt{\|\mathbf{g}(n)\|^2 + a^2}} \quad (7)$$

where $\text{sgn}[\mathbf{g}(n)]$ is the component wise sign function given by $\text{sgn}[g(n)] = \begin{cases} \frac{g}{|g|} & \forall g \neq 0 \\ 0 & \text{otherwise} \end{cases}$ and J'_M is the modified cost function. When (7) is integrated with respect to $\mathbf{g}(n)$ with $|a| = 1$, the cost function becomes

$$J_M(\mathbf{g}(n)) = \sinh^{-1} \left(\frac{|\mathbf{g}(n)|}{|a|} \right) \quad (8)$$

Here $\sinh^{-1}$ refers to the inverse hyperbolic sine function

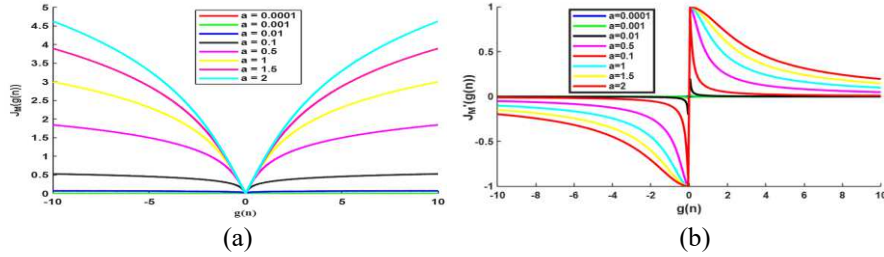


Fig. 2 a. Proposed sparsity function **b.** modified pseudo-Huber loss function

To validate the appropriateness of the proposed modified pseudo-Huber loss function as sparsity function, the plot of the variation of the proposed cost function and the modified pseudo -Huber loss function for various values of a is shown in Fig. 2. Thus it can be concluded that the proposed cost function exhibits non linearity and it is also evident from Fig. 2.a that for a given value of a , the cost function results in a narrower range where coefficients close to zero are attracted to zero. Consequently, the proposed method exerts less influence on coefficients that are significantly far away from the origin. Considering these insights, the proposed MP-HL cost function exhibits superior convergence behavior compared to other sparsity aware cost functions.

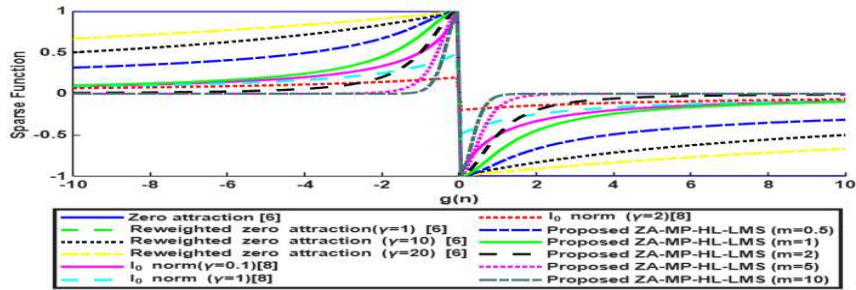


Fig. 3 Proposed sparsity function with other sparse functions for different values of m

The plot of (7) for different values of a is shown in Fig. 2.b. It shows that the proposed gradient of cost function is a monotonically increasing function and symmetrical for all $g(n) > 0$. Hence the modified cost function can be used as the sparse function. comparison of the proposed sparsity function with existing sparse functions for various values m is illustrated in Fig. 3. Notably, it can be seen that $m = 2$ corresponds to the VZA-LMS algorithm [9]. As the value of m is increased, the curve becomes sharper around the origin, promoting sparsity by attracting small coefficients

and suppressing larger ones. Unlike ZA-LMS [6], which applies uniform attraction, RZA-LMS [6], l_0 -LMS [8] and the proposed function exert stronger influence on smaller coefficients. This selective shrinkage addresses ZA-LMS limitation and supports the proposed norms as effective sparsity-promoting functions [9] with improved convergence behavior.

The generalized modified Huber loss function can be written as $\frac{\text{sgn}[\mathbf{g}(n)]}{\left(\sqrt{\|\mathbf{g}(n)\|^2 + a^2}\right)^m}$.

Based on the new cost function, a Zero attraction feature is added into the conventional LMS algorithm cost function which is given as

$$L_1(n) = \frac{1}{2} e^2(n) + \beta J_M(\mathbf{g}(n)) \quad (9)$$

where β is the regularization parameter. Using Gradient descent approach, the filter update of ZA-MP-HL-LMS is defined as

$$\mathbf{g}(n+1) = \mathbf{g}(n) + \mu e(n)\mathbf{x}(n) - \rho J'_M(\mathbf{g}(n)). \quad (10)$$

Substituting $J'_M(\mathbf{g}(n))$ given in (6) into (10) we get

$$\mathbf{g}(n+1) = \mathbf{g}(n) + \mu e(n)\mathbf{x}(n) - \rho \frac{\text{sgn}[\mathbf{g}(n)]}{\sqrt{\|\mathbf{g}(n)\|^2 + a^2}}. \quad (11)$$

Here $\rho = \mu\beta|a|$ is the zero-attraction parameter which attracts the filter coefficient to zero. The update rule of the ZA-MP-HL-LMS adaptive filter is (11).

3 Performance analysis

In this section, the mean and mean square performance analysis of the proposed ZA-MP-HL-LMS are analysed using energy conservation relation.

3.1 Mean Performance analysis

The stability of the proposed ZA-MP-HL-LMS algorithm can be analyzed from the mean convergence condition derived in (S6). The algorithm remains stable in the mean sense when the step size (S8 in online supplement) is chosen as

$$0 < \mu < \frac{1}{\lambda_{max}} \quad (12)$$

which is same of standard LMS algorithm (derivation is provided as online supplement). The convergence in mean (derivation is provided as online supplement) is given as

$$E[\mathbf{g}(\infty)] = \mathbf{g}_o - \frac{\rho}{\mu} \mathbf{R}_x^{-1} f(\mathbf{g}(\infty)) \quad (13)$$

From (13), it is found that the proposed approach returns a biased estimate of the true unknown coefficients. However with proper selection of ρ , the proposed approach can able to achieve lower MSE than the LMS for a sparse system. Also when $\rho = 0$, the algorithm returns to standard LMS [4]

3.2 Mean Square Performance analysis

In the section the mean square performance analysis of the proposed ZA-MP-HL-LMS is analyzed using energy conservation relation. The mean square stability condition is given by (S23 in supplementary file)

$$0 < \mu < \frac{2}{(2+\eta)\lambda_{max}} \quad (14)$$

The steady-state EMSE of the proposed algorithm (derivation is provided as online supplement) is given by

$$EMSE_{\infty(ZA-MHL-LMS)} = \frac{\eta}{2-\eta} \sigma_v^2 + \frac{\rho q'}{\mu(2-\eta)} \left(\rho - \frac{p'}{q'} \right). \quad (15)$$

The upper bounded of q is given by (S36) and p is given by (S38). The value of q is non-negative and hence q' is also positive. Thus if p' is positive, then it is possible that the proposed approach produces lower EMSE than LMS if the value of $\rho < \frac{p'}{q'}$.

Additionally the computational complexity (presented in Table. 1 in supplementary file) reveals that the proposed ZA-MP-HL-LMS demonstrates a balanced computational complexity with an advantage of better performance in terms of faster convergence and lower steady-state MSE.

4 Simulation

In order to study the performance of the proposed ZA-MP-HL-LMS algorithm, an unknown system of length $N = 16$ is chosen. The non-zero coefficients are chosen as any value between -1 to 1. Throughout, the input and the background noise are white Gaussian with zero mean and unity variance, the signal-to-noise ratio (SNR) is 30 dB.

Firstly the variation of p' for different levels of sparsity is studied. The value of ρ is chosen as 0.0001 and a as 0.08. Fig. 4 illustrates the variation of p' for normalized sparsity levels (varying from nonsparse to highly sparse), where p' is averaged after 250 iterations. As seen in Fig. 4, for highly sparse system, the value of p' is positive and as the sparsity level decreases, the value becomes negative. Thus the simulation confirms that the algorithm performance is favorable in sparse system due to the effective zero-attraction mechanism. The increasing trend in p' with more sparsity highlights the suitability of the proposed sparsity-aware filter in improving the performance in terms of faster convergence and lower steady state MSE.

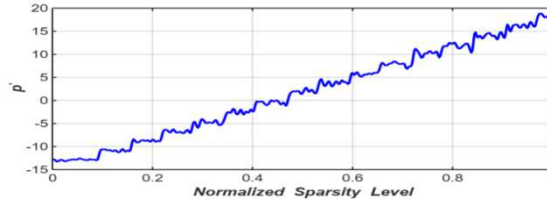


Fig. 4 Variation of p' for different sparsity levels

Next the tracking ability of the proposed algorithm is evaluated. For the first 500 iterations, the system is extremely sparse with only the 8th coefficient is set to 1 while the rest are all set to 0 and for the next 500 iterations, the system suddenly changes to very sparse where 2nd, 5th, 7th and 12th coefficients being non-zero. Followed by this the system becomes moderately sparse with 8 non-zero coefficient. In the 1500 -2000 iteration, the system is said to be sparse with 12 non-zero coefficients. Finally a non-sparse system is taken with 16 non-zero filter coefficients. Figure 5 illustrates the performance of the proposed algorithms along with the ZA-LMS[6], RZA-LMS[6], l_0 based LMS[8], VZA-LMS[22] algorithms for different levels of sparsity.

The step size is $\mu=0.05$, $\rho=0.00074$ and other parameters are chosen in such a way that all the considered algorithms have the same initial convergence speed. From Fig.

5 it is evident that the proposed algorithms outperform the existing sparsity aware algorithms in terms of convergence and steady state error even when the system changes. Also the proposed approach is said to exhibit good tracking capability when compared to other existing algorithms. Additionally, it is evident that the behaviour of the proposed algorithm is strongly dependent on a . Smaller values of a ($a \leq 0.1$) pushes small coefficients toward zero more strongly. Therefore the convergence is faster as it enforces more sparsity, but the steady state MSE is more especially when the system is extremely sparse. On the other hand larger values of a ($a \geq 2$), makes the algorithm to pulls coefficients toward zero more gently. This results in slower convergence and lower steady state MSE. From these observations, it is suggested to carefully select a so as it balances convergence rate and steady-state performance effectively.

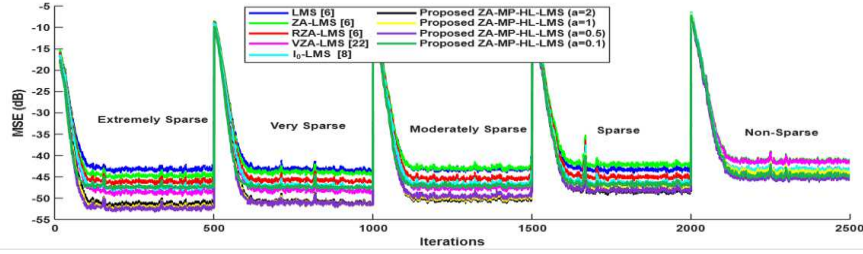


Fig. 5 Performance comparison of the proposed sparsity aware adaptive filter at SNR=30 dB with $\mu = 0.05$ $\rho = 0.00074$ and $a = 0.8$.

The effect of μ on the performance of the proposed algorithms is studied in Fig. 6 (a). For this, the same system as the previous experiment is selected where $\rho = 1 \times 10^{-4}$ and $a = 1$. It can be observed that, as μ increases, the steady-state error decreases with improved convergence speed up to certain value of μ . When μ is increased further, steady state error decreases, but convergence speed is reduced. Therefore, careful selection of μ is essential. To study the variation of ρ on the performance of the proposed adaptive filter, ρ is varied from 0.00001 to 0.1 and the steady state MSE values are plotted as shown in Fig 6 (b). The steady state MSE are obtained by averaging over last 200 iterations. As expected, initially, as ρ increases, the MSE decreases, indicating improved sparsity enforcement and convergence. However, beyond a certain threshold, further increases in ρ lead to an increase in MSE, suggesting over-penalization of filter coefficients and degraded performance.

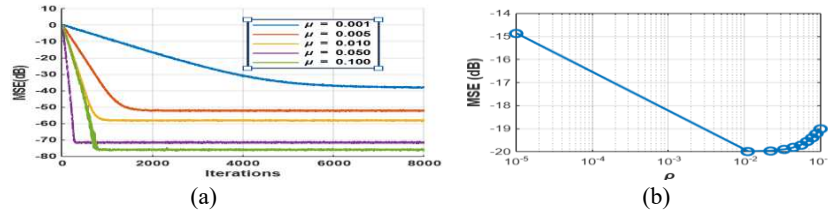


Fig. 6 Proposed sparsity aware adaptive filter for different values of **a. μ** **b. ρ** .

Table 1 illustrates the effect of filter length on the performance of the proposed algorithm. Simulations were conducted for tap lengths $N = 16, 128, 256, 512$ and 1024 respectively with the sparsity level maintained at 90% throughout. The corresponding steady-state MSE values are tabulated in Table.1. For the simulations, the step-size chosen is $\mu = 0.05$ for a system length of 16, $\mu = 0.01$ for 128, $\mu = 0.05$

for 256, $\mu = 0.004$ for 512, and $\mu = 0.0005$ for 1024 length system. The sparsity penalty is $\rho = 0.00074$ for 16 taps, $\rho = 0.00005$ for 128, $\rho = 0.00002$ for 256, $\rho = 0.0001$ for 512, and $\rho = 0.0005$ for 1024 taps for ZA-LMS, RZA-LMS and the proposed algorithms. The value of $\varepsilon = 10$ for RZA-LMS [6]. For the VZA-LMS algorithm, the parameter a is set as 10 for 16 taps, 20 for 128, 25 for 256, 30 for 512, and 40 for 1024 taps, and the penalty ρ is chosen as 0.001 for 16, 0.002 for 128, and 0.001 for 256, 512, and 1024 taps. For the l_0 -LMS algorithm, the penalty ρ is 0.0007 for 16 taps, 0.0005 for 128, 0.00002 for 256, 0.001 for 512, and 0.005 for 1024 taps, while the shrinkage parameter γ is set as 5 for 16 taps, 7 for 128 and 256 taps, 20 for 512 taps, and 1 for 1024 taps. For the proposed ZA-MP-HL-LMS algorithm, $a = 0.5$ is chosen throughout. These parameter settings were chosen to ensure optimal performance for each algorithm across all system lengths considered. The results in Table.1 proves that the proposed ZA-MP-HL-LMS algorithm consistently achieves lower steady-state MSE compared to other sparsity-aware algorithms, confirming its robustness and effectiveness even for longer tap-length systems.

Table.1. Performance comparison of the proposed sparsity aware adaptive filter for different value of tap length at SNR=30 dB

Tap length	LMS	ZA-LMS	RZA-LMS	VZA-LMS	l_0 -LMS	Proposed algorithm
N=16	-43.16	-47.93	-48.73	-50.40	-49.02	-52.14
N=128	-35.01	-41.32	-44.26	-48.26	-50.96	-51.03
N=256	-30.89	-40.70	-41.88	-42.40	-43.52	-46.42
N=512	-24.86	-33.47	-35.14	-36.89	-36.01	-42.25
N=1024	-20.98	-26.44	-28.50	-28.87	-28.67	-30.14

5 Conclusion

In this paper, a novel sparsity-aware function, termed the Modified Pseudo-Huber Loss (MP-HL), is proposed. Furthermore, a sparsity-aware adaptive filter, namely ZA-MP-HL-LMS is presented, and their convergence in terms of mean, mean square error and steady state MSE is studied. Further the importance of the zero attraction parameter in the final steady state error values is also exploited. Notably, through comprehensive simulation results, it is shown that the proposed algorithms surpass the performance of existing state-of-the-art approaches. These findings highlight the potential of MP-HL-based algorithms for improving the performance of sparse adaptive filters. In future, adaptive strategies for dynamically tuning the zero-attraction parameter will be explored.

Acknowledgements Not applicable.

Author Contributions S.R. and A.C. conceptualized the core idea and designed the algorithm and derived the theoretical analysis. , F.A implemented the MATLAB simulations and conducted performance evaluations. All authors reviewed and approved the final manuscript.

Funding No funding was received for conducting this study.

Data Availability No datasets were generated or analysed during the current study.

Competing interests The authors declare no competing interests.

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