Supplementary Information

S1 Coupled Mode Theory for Asymmetric Directional Couplers

Coupled mode theory (CMT) was used to mathematically model and optimize the behavior of the tunable PCM-based DC. According to CMT, the asymmetric coupling region can be modeled as [1]:

$$\kappa^2 = A\sin^2(\beta_c L + \phi), \tag{1}$$

where A denotes the maximum coupling between the two asymmetric waveguides and $\beta_c = \frac{\beta_o - \beta_e}{2}$, in which β_o and β_e are the propagation constants of, respectively, the odd and even supermodes. Also, ϕ takes into account the offset in coupling coefficient related to the input and output S-bend in the DC. As a result, the coupling coefficient of the DC when the PCM is in the amorphous state (κ_a^2) or is in the crystalline state (κ_c^2) can be modeled when the PCM on top of the waveguide is in either the amorphous or crystalline state as:

$$\kappa_a^2 = A\sin^2(\beta_c^a L + \phi), \tag{2}$$

$$\kappa_c^2 = A\sin^2(\beta_c^c L + \phi). \tag{3}$$

The coupling length values that result in $\Delta\kappa^2 \geq 0.98$ where $\Delta\kappa = \kappa_a^2 - \kappa_c^2$ can be used as the design point for our tunable PCM-based DC as changing the phase state of the PCM will toggle the β_c between β_c^a and β_c^c, and hence, the coupling coefficient can be tuned from 0 to 1. Moreover, the maximum coupling between the two waveguides (A) due to the asymmetric coupling region can be modeled using CMT. Thus, we can write:

$$A(W_1, W_2, W_{PCM}, \lambda) \propto \Delta\beta_{1,2}, \tag{4}$$

where $\Delta\beta_{1,2} = \beta_1 - \beta_2^{a,c}$. Here, β_1 denotes the propagation constant of the standalone passive waveguide and $\beta_2^{a,c}$ denotes the propagation constant of the standalone PCM-loaded waveguide [2]. The maximimum coupling between the waveguide for the design point selected in this paper is shown in Fig. 1.

S2 Heat Simulation of the Thermo-Optic Phase Shifter

Steady-state heat simulations were performed using Ansys Lumerical HEAT to simulate the thermo-optic phase shifters and capture the temperature distribution in a silicon-on-insulator (SOI) waveguide with a thickness of 220 nm and a width of 450 nm. The heater, made of TiW alloy, was placed 2 μm above the waveguide with a length of 200 μm and a thickness of 200 nm, following the Applied Nanotools (ANT) PDK specifications. Note that this specific design is used for both MZI-based network and LightPro network for a fair comparison, as it was fabricated before and experimentally tested according to [3]. The temperature profiles corresponding to each electrical power level were imported into Ansys Lumerical MODE to compute the effective refractive

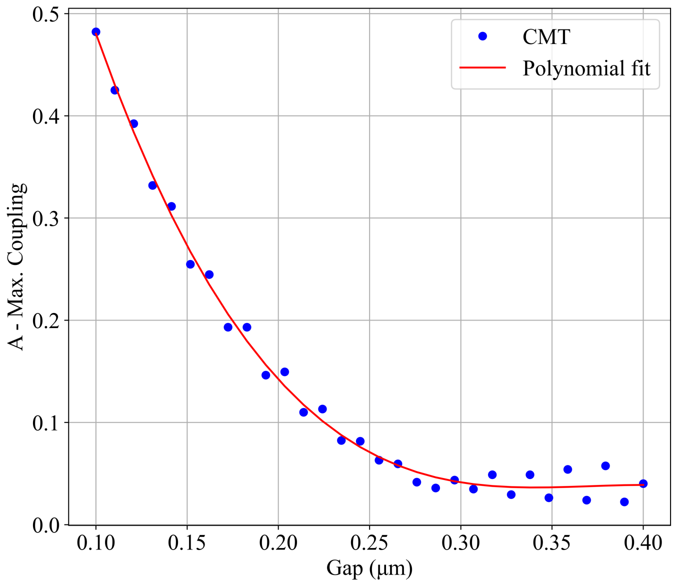

Fig. S 1 Maximum coupling between the coupled waveguides in the DC when the PCM is in the crystalline state for the selected design shown in Fig. **??**

index variation due to the thermo-optic effect by changing the heater's power. The simulated power–phase shift relationship is presented in Fig.2, and these results were used to evaluate the total power consumption of the phase shifter in LightPro, pruned LightPro, and MZI-based Clements network described in Section 2.

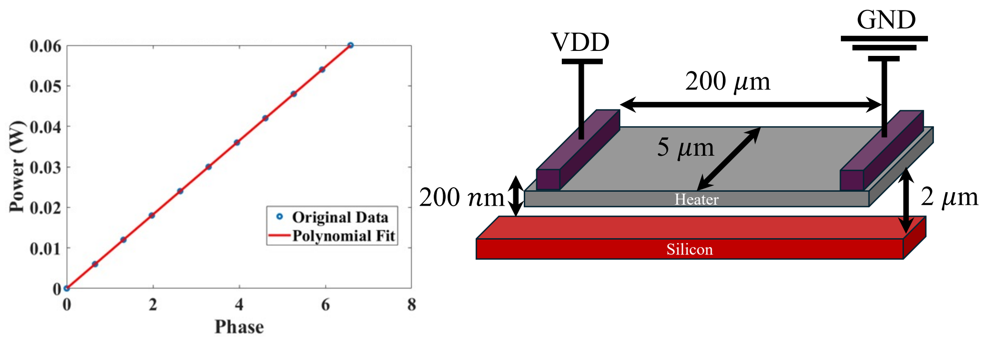

Fig. S 2 Power versus corresponding phase change for the 200-μm long TiN phase shifter using ANT PDK heater design.

S3 All-PCM LightPro and Clements Performance Comparison

To compare the performance of general-purpose LightPro and Clements network of different sizes when all phase shifters are based on Sb_2Se_3 in addition to tunable

DCs, we trained the Clements network of different sizes on the Gaussian dataset and then recorded the trained complex weights and the phase values related to MZIs in Clements network. The trained weights were used as a target matrix in LightPro. The design of the PCM-based phase shifter with the length of 10 μm to cover the full 2π phase shift in the phase shifters and the MZIs with a length of 71 μm is according to our prior work in [4]. The heat simulation was performed to capture the case where the PCM is 50% amorphized, 10% amorphized and 100% amorphized. Then an exponential curve were fitted to the data as the re-crysallization of PCM follows the Johnson-Mehl-Avrami as it was illustrated in [5, 6]. The trained phase values in Clements and optimized phases and coupling coefficient values in LightPro were then used to estimate the total programming energy of the PCMs in both networks. The results for footprint and programming energy are demonstrated in Fig. 3(a) and (b). Note that the design of the heater is similar to the design used for programming the tunable DCs illustrated in Section 2.

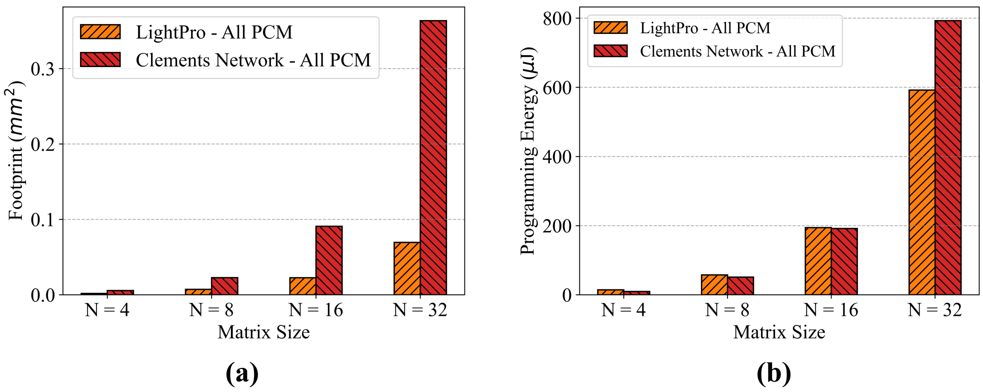

Fig. S 3 (a) Footprint area of the LightPro and Clements networks when all Pphase shifters are based on PCMs, (b) The programming energy for programming the PCMs in phase shifters and tunable DCs according to the trained and optimized values.

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