Supplementary information

A. Absolute and convective instabilities

When considering problems of propagation of excitations, two approaches can be used: either the development of the excitation in time at a given point in space is considered, or the development of the excitation in space at a given moment in time. In the first case, the value of complex frequencies $\omega = \omega' + i\omega''$ are found from the dispersion relations for real wave vectors $\mathbf{k}'$. In the second, the complex wave vectors $\mathbf{k} = \mathbf{k}' + i\,\mathbf{k}''$ are found from the dispersion relations for real frequencies. If some real value of the wave vector $\mathbf{k}'$ corresponds to a complex value of the frequency ω with $\omega''(\mathbf{k}') < 0$ then the disturbance, which has the form of a plane monochromatic wave $e^{i(\mathbf{kr}-\omega t)}$, will grow indefinitely with time, thus, the dynamic system will be unstable. In fact, small perturbations do not have the form of plane monochromatic waves, but are packets, which are a superposition of monochromatic waves. The asymptotic behavior of wave packets can differ significantly from the behavior of individual waves. We will show that at $t \to \infty$ a wave packet can remain confined at a fixed point even at $\omega''(\mathbf{k}') < 0$. Write the wave packet for the point $x = 0$ in the form

$$U(0,t) = \int_{-\infty}^{\infty} u_{0k} e^{i\omega'(\mathbf{k}')t} e^{-\omega''(\mathbf{k}')t} d\mathbf{k}' . \tag{A1}$$

If $\omega'' < 0$, then the term $\exp(-\omega'' t)$ approaches infinity when $t \to \infty$. But the term $\exp(i\omega' t)$ is the rapidly oscillating function, which after multiplying on continuous function u_{0k} and integration it over wave vector $\mathbf{k}'$, evaluates to zero in the limit $t \to \infty$. Therefore, expression (A1) represents uncertainty of $0 \cdot \infty$, thus it can achieve values as infinitesimal small as infinitely large. Thus, it is insufficient to merely ascertain the existence of complex frequencies among the roots of the dispersion equation investigating the nature of wave growth over time. Rather, it is necessary to examine the behavior of a wave packet at a fixed point as $t \to \infty$.

Suppose one has to solve a one-dimension problem. When the perturbation $U(x,t)$ infinitely grows in fixed point *x* at $t \to \infty$, the instability is named absolute instability

$$\lim_{\substack{t\to\infty \\ x=const}} U(x,t) = \infty \ . \tag{A2}$$

If the disturbance remains confined, the instability is called convective

$$\lim_{\substack{t\to\infty \\ x=const}} U(x,t) = A, \qquad 0 \le A < \infty . \tag{A3}$$

Absolute instability is spontaneous infinite growing fluctuations over the time in the fixed point inside the system. Such system can play a role of generator of oscillations. In convective instability, the limitation of the growth of a disturbance at a fixed point in space is associated with the removal of the excitation by its moving in a space. This type of instability corresponds to the amplification of waves.

Other words, the key feature of the convective instability phenomenon is that if a disturbance is observed at a fixed position, it will be seen to pass the observation point, increasing in amplitude, and then disappear, carried away by the flow. At this fixed point, the system returns to its initial state after the disturbance has passed. This implies that condition (A3) with A = 0 is satisfied. For this to occur, the propagation speed of the disturbance must be less than the flow speed of the medium; that is, the flow "carries away" the disturbance faster than it can propagate back. The result is an amplification of the signal or disturbance, but this usually does not cause chaos or a complete destruction of the stationary state at a particular location, since the growing disturbance is continuously advected downstream (or "carried away").

The concept of amplification in a system exhibiting convective instability can be explained as follows: An external perturbation, such as a surface plasmon wave with a frequency ω (or a defined frequency spectrum), is introduced into the flow (the system). The system is characterized by the kinetic energy associated with the flow shear (electric current in the case under consideration). The interaction between the wave and the flow enables the wave to extract energy from the flow, converting this kinetic energy into the energy of wave motion. This energy transfer process leads to an exponential increase in the amplitude of the wave as it propagates along the flow. This is typically modeled as $A(x) \propto A_0 \cdot \exp(k_i x)$, where $k_i > 0$ is the spatial amplification factor. As a result, the wave (surface plasmon) leaves the system with an amplitude significantly larger than its initial value, while maintaining its original frequency (a process known as linear amplification). Thus, convective instability is precisely the property that allows a system to amplify disturbances introduced from the outside without internal self-excitation (which is characteristic of absolute instability).

B. Sturrock's criteria

The interaction of waves under certain conditions can lead to instability in the system. Simple and intuitive criteria for studying the stability of a system when two waves interact are Sturrock's criterions. Their derivation is based on the above-mentioned principles in Appendix A.

The dispersion equation describing the interaction of two waves in some domain near the point $(\omega, \mathbf{k})$ often has a form

$$(\omega - \mathbf{k}\mathbf{v}_1)(\omega - \mathbf{k}\mathbf{v}_2) + m = 0, \qquad \text{(B1)}$$

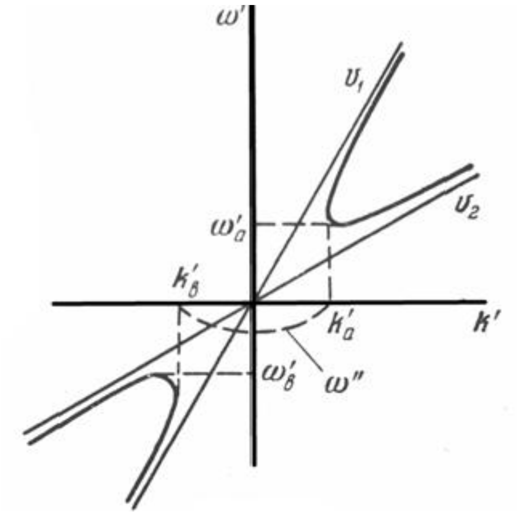

Fig.B1. The splitting of the curves of dispersion relations for two waves in the case $\mathbf{v}_1 \cdot \mathbf{v}_2 > 0$, $m > 0$, which corresponds to convective instability in the range (k_a', k_b').

where $\mathbf{v}_1, \mathbf{v}_2$ are the phase velocity of the waves, m is the parameter of interaction between the waves. If there is no interaction between the waves, then the parameter $m = 0$ and equation (B1) decomposes into two equations, each of which describes an independent wave. If the interaction between the waves is small, then it most significantly affects the dispersion curves of independent waves at the point of phase synchronism, that is, at the point of intersection of the dispersion curves of independent waves. At the point of phase synchronism, both waves have the same frequency and wave vector. In the theory of oscillations, such a state of the system is called degenerate. If there is interaction between the waves in the system, then the degeneracy is removed and the splitting arises at the point of synchronism. Sturrock's criteria allow to determine the type of instability in the system by the type of splitting [20]. For example, if the asymptotes (v_1 , v_2) of the curves are inclined to an one side ($\mathbf{v}_1 \cdot \mathbf{v}_2 > 0$), then convective instability occurs at $m > 0$. This case is shown in Fig.B1. Thus, to determine the convective instability in the system under consideration, one should study the dispersion relations near the point of phase synchronism

For more details, one should see, for example, Refs.[30-35]

C. Effective susceptibility of a layer of plasmonic nanoparticles

To determine the effective susceptibility (the linear response to an external field) for a nanosystem comprising of plasmon nanoparticles and units of active layer, we will solve the self-consistency equation known as the Lippmann-Schwinger equation

$$\begin{aligned} E_i(\mathbf{R},\omega) &= E_i^{(0)}(\mathbf{R},\omega) - \\ &- k_0^2 \sum_{l=1}^{N_l} \int_{V_{Pl}} d\mathbf{R}'_l G_{ij}^{(33)}(\mathbf{R},\mathbf{R}'_l,\omega)\chi^{Np}(\omega)E_j(\mathbf{R}'_l,\omega) - \\ &- k_0^2 \sum_{m=1}^{N_m} \int_{V_{Pl}} d\mathbf{R}'_m G_{ij}^{(33)}(\mathbf{R},\mathbf{R}'_m,\omega)\chi^{d}(\omega)E_j(\mathbf{R}'_m,\omega) \ , \end{aligned} \qquad \text{(C1)}$$

where $G_{ij}^{(33)}(\mathbf{R},\mathbf{R}'_l,\omega)$ is the electrodynamic Green's function, describing the electrodynamic properties of the medium containing the plasmon nanoparticles and the active layer particles. Furthermore, $\chi_{jl}^{Np}(\omega)$ represents the susceptibility of a single metal nanoparticle on the surface of the magnetic film, and $\tilde{\chi}_{jl}^{d}(\omega)$ is the susceptibility of a single unit within the absorber layer (e.g., a semiconductor quantum dot or an organic dye molecule). The effective susceptibility, in this case, is a global characteristic of a sub-monolayer coverage. Supposing that all nanoparticles of the layer of plasmonic nanoparticles (LPNPs) are located from the surface at the same distance z_l, let us introduce the effective susceptibility of the LPNPs as the non-local linear response of the system to an external field, defined by the following relation

$$J_i(\mathbf{r},z_l,\omega) = -i\omega\varepsilon_0 \int_{V_{LPNP}} d\mathbf{r}' \mathrm{X}_{ij}^{(S)}(\mathbf{r}-\mathbf{r}',z_l,\omega)E_j^{(0)}(\mathbf{r}',z_l,\omega) , \qquad \text{(C2)}$$

with $J_i(\mathbf{r},z_l,\omega)$ the effective electric current induced inside the film of LPNPs, $\mathrm{X}_{ij}^{(S)}(\mathbf{r}-\mathbf{r}',z_l,\omega)$ effective susceptibility of nanoplasmonic system. From the other side, the current defined in Equation (C2) can be connected with a local field through a microscopic relation, offering an alternative formulation for the effective susceptibility

$$J_i(\mathbf{r},z_l,\omega) = -i\omega\varepsilon_0\chi_{jl}^{Np}(\omega)E_l(\mathbf{r},z_l,\omega) \ . \qquad \text{(C3)}$$

Taking into account that the system under consideration is macroscopically homogeneous along the surface and supposing that nanoobjects are distributed along the surface homogeneously, one can write second and third terms in the right part of (C1)

$$\overline{-k_0^2\sum_{l=1}^{N_l}\int_{V_{Pl}} d\mathbf{R}'_l G_{ij}^{(33)}(\mathbf{R},\mathbf{R}'_l,\omega)\chi^{Np}(\omega)E_j(\mathbf{R}'_l,\omega)} -$$

$$-\overline{k_0^2\sum_{m=1}^{N_m}\int_{V_d} d\mathbf{R}'_m G_{ij}^{(33)}(\mathbf{R},\mathbf{R}'_m,\omega)\chi^{d}(\omega)E_j(\mathbf{R}'_m,\omega)} \approx \tag{C4}$$

$$\approx \overline{-k_0^2\sum_{l=1}^{N_l} G_{ij}^{(33)}(\mathbf{r}-\mathbf{r}',z_l,z_l,\omega)V_{Pl}\chi^{Np}(\omega)E_j(\mathbf{r}-\mathbf{r}',z_l,\omega)} -$$

$$-\overline{k_0^2\sum_{m=1}^{N_m} G_{ij}^{(33)}(\mathbf{r}-\mathbf{r}',z_l,z_d,\omega)V_d\chi^{d}(\omega)E_j(\mathbf{r}-\mathbf{r}',z_d,\omega)}\ ,$$

where

$$\overline{(...)} = \frac{1}{S^{N_\alpha}}\int_S d\mathbf{r}_1\int_S d\mathbf{r}_2...\int_S d\mathbf{r}_{N_\alpha}(...)\ ,\qquad \alpha = l,m \tag{C5}$$

means the mathematical procedure of averaging of the system over locations of the particles. Thus, each term in Eq.(C4) has a form

$$\overline{\sum_{\alpha=1}^{N_0} G_{ij}(\mathbf{r}-\mathbf{r}_\alpha,z,z_\alpha,\omega)\tilde{\chi}_{jl}(\omega)E_l(\mathbf{r}_\alpha,z_\alpha,\omega)} =$$

$$\frac{1}{S^{N-1}}\int_S d\mathbf{r}_1 d\mathbf{r}_2...d\mathbf{r}_{N-1}\sum_{\alpha=1}^{N_0}\int\frac{d\mathbf{k}}{(2\pi)^2}e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}_\alpha)}G_{ij}(\mathbf{k},z,z_\alpha,\omega)\times \tag{C6}$$

$$\times\tilde{\chi}_{jl}(\omega)\int\frac{\mathrm{d}\mathbf{k}'}{(2\pi)^2}e^{-i\mathbf{k}\mathbf{r}_\alpha}E_l(\mathbf{k}',z_\alpha,\omega)$$

Each of the N_0 terms on the right-hand side has the form

$$\frac{1}{S^{N_0}}\overbrace{\int d\vec{r}_1...d\vec{r}_{N_0}}^{=S^{N_0-1}}\cdot\int \mathrm{d}\mathbf{k}\int\frac{d\mathbf{k}}{(2\pi)^2}e^{-i\mathbf{k}\mathbf{r}}G_{ij}(\mathbf{k},z,z_\alpha,\omega)\tilde{\chi}_{jl}(\omega)$$

$$\times E_l(\mathbf{k}',z_\alpha,\omega)\overbrace{\int\frac{d\mathbf{r}_\alpha}{(2\pi)^2}e^{i\mathbf{r}_\alpha(\mathbf{k}-\mathbf{k}'')}}^{=\delta(\mathbf{k}-\mathbf{k}')} = \tag{C7}$$

$$= \frac{S^{N_0-1}}{S^{N_0}}\int\frac{d\mathbf{k}}{(2\pi)^2}e^{-i\mathbf{k}\mathbf{r}}G_{ij}(\mathbf{k},z,z_\alpha,\omega)\tilde{\chi}_{jl}(\omega)E_l(\mathbf{k},z_\alpha,\omega)$$

Thus,

$$\sum_{\alpha=1}^{N_0} G_{ij}(\mathbf{r}-\mathbf{r}_\alpha, z, z_\alpha, \omega)\tilde{\chi}_{jl}(\omega)E_l(\mathbf{r}_\alpha, z_\alpha, \omega) = \tag{C8}$$

$$= \frac{N_0}{S}\int \frac{d\mathbf{k}}{(2\pi)^2} e^{-i\mathbf{kr}} G_{ij}(\mathbf{k}, z, z_\alpha, \omega)\tilde{\chi}_{jl}(\omega)E_l(\mathbf{k}, z_\alpha, \omega) \ .$$

We can introduce the surface concentration of the particles as $n = N_0/S$. Equation (C1) holds true for any point inside the system under consideration, particularly at the specific locations of each plasmon nanoparticle. This allows us to calculate the field on the β-th object of the LPNP array, which is located at point

$$\begin{aligned} E_i(\mathbf{r}_\beta, z_l, \omega) &= E_i^{(0)}(\mathbf{r}_\beta, z_l, \omega) - \\ &- k_0^2 N_{Np}\int \frac{d\mathbf{k}}{(2\pi)^2} e^{-i\mathbf{kr}_\beta} G_{ij}(\mathbf{k}, z_l, z_l, \omega)\tilde{\chi}_{jl}^{Np}(\omega)E_l(\mathbf{k}, z_l, \omega) - \\ &- k_0^2 N\int \frac{d\mathbf{k}}{(2\pi)^2} e^{-i\mathbf{kr}_\beta} G_{ij}(\mathbf{k}, z_l, z_d, \omega)\tilde{\chi}_{jl}^{d}(\omega)E_l(\mathbf{k}, z_d, \omega) \ , \end{aligned} \tag{C9}$$

where N_{Np} and N are the surface density of plasmon nanoparticles and units of active layer, respectively. Taking into account that spatial averaging allows us to assume that the nano-objects are effectively 'spread' over the surface to form a homogeneous film, one can compare Eqs. (C2) and (C3) and transform to the Fourier representation (**k**-space) to obtain the connection between the local and external field via the effective susceptibility

$$\mathrm{X}_{ij}^{(S)}(\mathbf{k}, z_l, \omega)E_j^{(0)}(\mathbf{k}, z_l, \omega) = \chi_{jl}^{Np}(\omega)E_l(\mathbf{k}, z_l, \omega) \ , \tag{C10}$$

or

$$E_l(\mathbf{k}, z_l, \omega) = \left[\chi_{jl}^{Np}(\omega)\right]^{-1} \mathrm{X}_{ij}^{(S)}(\mathbf{k}, z_l, \omega)E_j^{(0)}(\mathbf{k}, z_l, \omega) \ . \tag{C11}$$

Performing 2D Fourier transformation and using Eq.(C11), one obtains from (C9)

$$\begin{aligned} \left[\chi_{jl}^{Np}(\omega)\right]^{-1} \mathrm{X}_{ij}^{(S)}(\mathbf{k}, z_l, \omega)E_j^{(0)}(\mathbf{k}, z_l, \omega) &= E_i^{(0)}(\mathbf{r}_\beta, z_l, \omega) - \\ &- k_0^2 N_{Np} G_{ij}(\mathbf{k}, z_l, z_l, \omega)\tilde{\chi}_{jl}^{Np}(\omega)\left[\chi_{jl}^{Np}(\omega)\right]^{-1} \mathrm{X}_{ij}^{(S)}(\mathbf{k}, z_l, \omega)E_j^{(0)}(\mathbf{k}, z_l, \omega) - \\ &- k_0^2 N G_{ij}(\mathbf{k}, z_l, z_d, \omega)\tilde{\chi}_{jl}^{d}(\omega)\left[\chi_{jl}^{Np}(\omega)\right]^{-1} \mathrm{X}_{ij}^{(S)}(\mathbf{k}, z_l, \omega)E_j^{(0)}(\mathbf{k}, z_l, \omega) \ . \end{aligned} \tag{C12}$$

Here, one assumes that the local electric field does not differ significantly at the planes of z_l and z_d. Under this simplification, one obtains an equation that defines the effective susceptibility of the LPNPs.

$$\Big\{\Big(\delta_{im} + k_0^2 N_{Np} G_{ip}(\mathbf{k}, z_l, z_l, \omega)\tilde{\chi}_{pm}^{Np}(\omega) + k_0^2 N G_{il}(\mathbf{k}, z_l, z_d, \omega)\tilde{\chi}_{lm}^{d}(\omega)\Big)\times \\ \times\Big[\chi_{lm}^{Np}(\omega)\Big]^{-1} \mathrm{X}_{lj}^{(S)}(\mathbf{k}, z_l, \omega) - \delta_{ij}\Big\} E_j^{(0)}(\mathbf{k}, z_l, \omega) = 0 \tag{C13}$$

Because the external field does not equal to zero, one obtains

$$\mathrm{X}_{lj}^{(S)}(\mathbf{k}, z_l, \omega) = \chi_{lm}^{Np}(\omega)\times \\ \times\Big[\delta_{jm} + k_0^2 N_{Np} G_{jp}(\mathbf{k}, z_l, z_l, \omega)\tilde{\chi}_{pm}^{Np}(\omega) + k_0^2 N G_{jl}(\mathbf{k}, z_l, z_d, \omega)\tilde{\chi}_{lm}^{d}(\omega)\Big]^{-1}. \tag{C14}$$

Denominator of this expression defines the pole part of the effective susceptibility and is the determinant of the matrix

$$\Omega_{jm}(\mathbf{k}, \omega) = \delta_{jm} + k_0^2 N_{Np} G_{jp}(\mathbf{k}, z_l, z_l, \omega)\tilde{\chi}_{pm}^{Np}(\omega) + k_0^2 N G_{jl}(\mathbf{k}, z_l, z_d, \omega)\tilde{\chi}_{lm}^{d}(\omega)\,. \tag{C15}$$

For more details, one should see, for example, Ref.[39]