Supplementary Information

Dispersion-Engineered Broadband Transparent Meta-Cloak

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A. The underlying physics of the Huygens' unit structure

For the transparent cloak, it is essential to maintain high transmittance, and thereby achieving low scatting to be rarely detected. While the wavefront manipulation is also necessary through electromagnetic (EM) waves phases. Based on the above conditions, the Huygens' metasurface, which is constructed based on Huygens' principle, characteristics high transmittance, is the appropriate options for constructing a transparent cloak. In this section, we illustrate the Huygens' meta-atom and underlying physics via analyzing the equivalent circuit model characteristics of the unit-cell.

The Fig. S1(a) shows the free view of the meta-atom, which contains three cascaded metallic layers (copper, σ=5.96×10^7 S/m), which are separated by two 2-millimeter-thick dielectric medium (Rogers RO4003C, ε=3.55, tanδ=0.0027). The quadruple rotational symmetric pattern distribution ensures polarization insensitivity for x-and y- polarized waves with period *p*=10.35 mm. The outer layers (top and bottom layers) contain four corner strips which vary in length. For miniaturization, in some cases the strips become folded[1]. The middle layer contains four straight strips along the four sides of the unit cell. The special size parameters of the unit cell are shown in Fig. S2 (b)-(c), which is the top layer and middled layer respectively.

The equivalent circuit model is shown in Fig. S2(d), which consists three cascaded sheet admittances. In general, each sheet admittance is anisotropic[2,4] as:

$$\overline{\overline{Y}}_{\mathrm{S}} = \begin{bmatrix} Y_{\mathrm{s}}^{\mathrm{xx}} & 0 \\ 0 & Y_{\mathrm{s}}^{\mathrm{yy}} \end{bmatrix} \tag{1}$$

in order to simplify, the unit cell in this work is isotropic, thus the tensor form of admittance is reduced to the scalar form, as $Y_{\mathrm{s}}^{\mathrm{yy}} = Y_{\mathrm{s}}^{\mathrm{xx}} = Y_{\mathrm{S}}$. Using transmission matrix approach, the ABCD matrix is found:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \left[\begin{pmatrix} 1 & 0 \\ Y_{\mathrm{s1}} & 0 \end{pmatrix} \begin{pmatrix} \cos(\beta d) & j\eta_d \sin(\beta d) \\ \dfrac{j\sin(\beta d)}{\eta_d} & \cos(\beta d) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ Y_{\mathrm{s2}} & 0 \end{pmatrix} \begin{pmatrix} \cos(\beta d) & j\eta_d \sin(\beta d) \\ \dfrac{j\sin(\beta d)}{\eta_d} & \cos(\beta d) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ Y_{\mathrm{s1}} & 0 \end{pmatrix} \right] \tag{2}$$

When the ABCD matrix is related to the S-parameters of the structure:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{B/\eta_0 - C\eta_0}{2A + B/\eta_0 + C\eta_0} & \dfrac{2}{2A + B/\eta_0 + C\eta_0} \\ \dfrac{2}{2A + B/\eta_0 + C\eta_0} & \dfrac{B/\eta_0 - C\eta_0}{2A + B/\eta_0 + C\eta_0} \end{bmatrix} \tag{3}$$

The (3) is simplified for the isotropy of the meta-atom[3], thus A=D. Based on the above derivations, the transmittance and phase of the transmission coefficient are calculated as the function of Y_{s1} and Y_{s2}, assuming that the sheet is lossless, as shown in Fig. S2(e)-(f). The perfect transmission is highlighted points by hollow circle superimposed in Fig. S2(e), and the corresponding phase in Fig. S2(f). Results shown the meta-atom could keep high transmission while phase asymptotically approaches 360° coverage. The excellent characteristics provide a encouraging prerequisite supports for flexible wavefront manipulation.

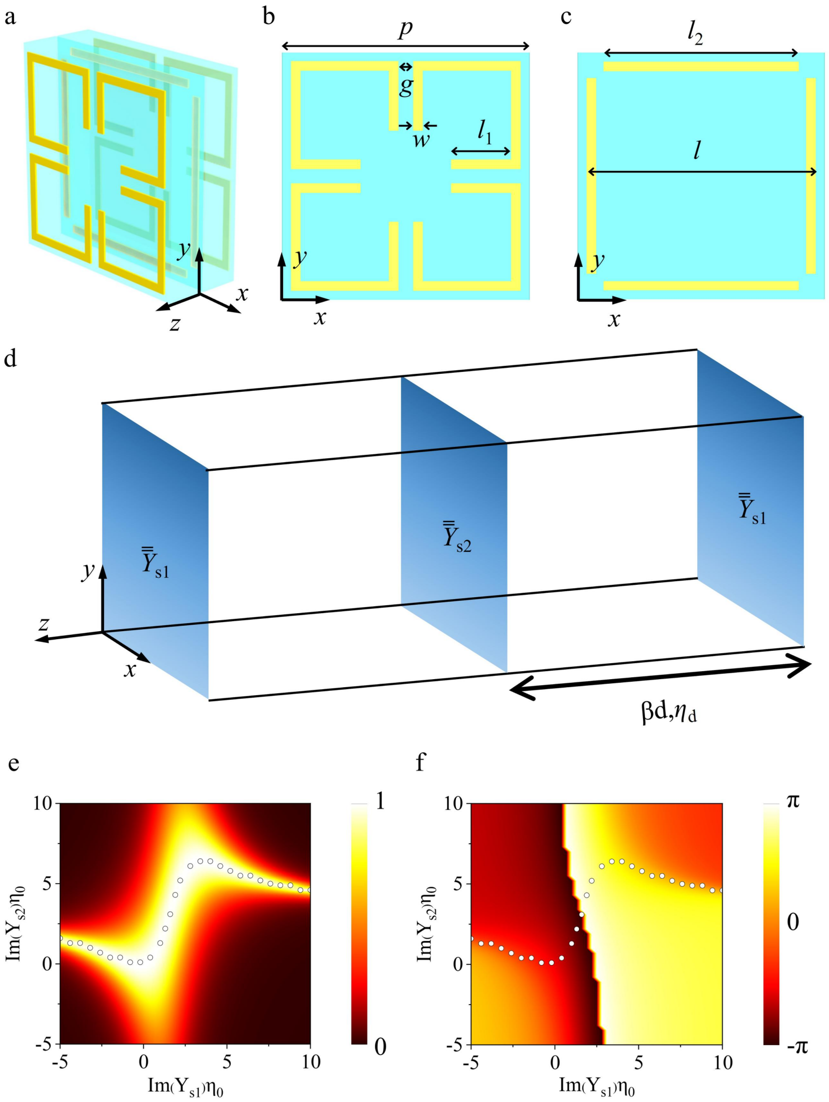

Figure S1. Schematic of Huygens' unit cell and the equivalent circuit model. a The free view of the Huygens' meta-atom. **b** The top view of the top metal layer and bottom metal layer of the meta-atom. **c** The top view of middle metal layer of the meta-atom. **d** The equivalent circuit modelling of the metasurface meta-atom. In this work, each sheet admittance is isotropic. **e** The transmission amplitude as a function of Y_{s1} and Y_{s2}. **f** The transmission phase as a function of Y_{s1} and Y_{s2}.

B. Principle of dispersion-engineered of the unit cell

For the broadband transparent cloak, the wavefront control is achieved by phase compensation, which the dispersion engineering is necessary, as analysis in the main text. When perfect transmittance is achieved, the dispersion-tunable capability is also important to be explored. In this section, three representative unit cells in exit surface are chosen to elucidates the principle of dispersion engineered, the corresponding parameters are shown in Table S1.

At first, according to the boundary condition and the equivalence principle[4,5], the normalized admittance Y and impedance Z can be extracted from the complex coefficients of reflection (r) and transmission (t):

$$Y = Y_s \eta_0 = \frac{2(1-t-r)}{1+t+r} \tag{4}$$

$$Z = Z_s / \eta_0 = \frac{2(1-t+r)}{1+t-r} \tag{5}$$

Then, we plot the imaginary part of Y and Z as the function of frequency. The three unit cells as the Huygens' resonant frequency for the complete unit cell (which is the combination of Y and Z) occurs at which $\mathrm{Im}\{Y\} = \mathrm{Im}\{Z\}$[1,4], as shown in Fig. S2 (a)-(c).

The point of Huygens' resonances occur is marked with red hollow circles. A clear contrast can be observed in the number of resonant points, thus one to three from unit cell 2 to unit cell 11 in Metasurface-II. Accordingly, the transmission coefficient and reflection coefficient amplitude of the three units is shown in Fig. S2(d) and (f), the transmission peak occurs at resonance frequencies. The transmission phase also varies differently at the corresponding frequency band. Unit cell 2 shows a slow phase variation over 9-14 GHz, (i.e. a small group delay). There is only one resonance inside the band. Unit cell 11 achieves the fastest phase variation by having three resonances inside the band. The phase variation of unit cell 7 is between the above two, which occurs two resonances. At the interesting working band (the grey dotted lines marked in Fig. S2(d)), 10-12 GHz, the slop of phase is approximately linear and the group delay gradually increase (the gray shaded area). Thus, the dispersion of the unit cell could be engineered through manipulating the number of resonances and the strength of them among interesting band by tailoring the parameters of the metal layers.

Table S1: The geometrical parameters of three unit cells. (unit: mm)

	g	w	l	l_1	l_2
U2	0.8	0.5	9.6	2.4	5.2
U7	0.6	0.4	9.6	2.5	8.2
U11	2.1	1.1	10.35	0	7.9

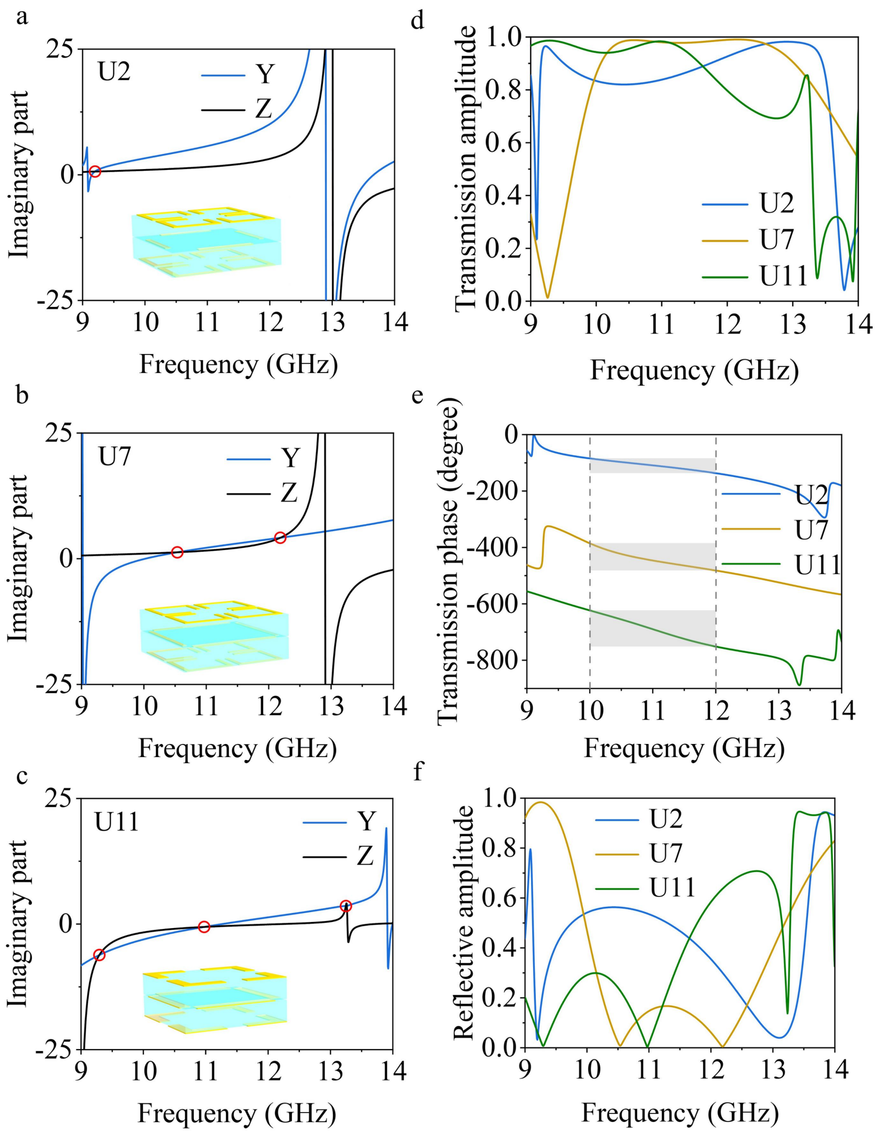

Figure S2. Schematic of the three units. Imaginary parts of normalized admittances Y and impedances Z of **a** unit cell 2, **b** unit cell 7 and **c** unit cell 11 of Metasurface-II. **d** Transmission coefficients amplitude of the three units. **e** Transmission coefficients phase of the three units. **f** Reflection coefficients amplitude of the three units.

C. Special unit structure

As described in the main text, to increase the phase coverage and enhance the flexibility of phase dispersion, additionally adjusted the structure of the Huygens' metasurface units with specific unit structure is applied. For the first special structure that there only dielectric substrate without metallic wire, as shown in Fig. S3(a), is applied as Unit Cell 1 of Metasurface-I. For the second special structure that there only top and bottom metallic wire without milled layer metallic wire, as the free view shown in Fig. 3(b) and top view shown in Fig. S3(c), is applied as Unit Cell 2 of Metasurface-I. Also, there is the third special structure that there only milled layer metallic wire without top and bottom metallic wire, as the free view shown in Fig. 3(d) and top view shown in Fig. S3(e), is applied as Unit Cell 1 of Metasurface-II. The free view of last type special structure is shown in Fig. S3(f). For this type of structure, compared with the structure in Fig. S1(a), the metal wires oriented perpendicular to the polarization direction of the incident wave have been removed, as shown in Fig. S3(g). Also, the top and bottom view of this type structure is shown in Fig. S3(h). This type special structure is applied in Unit Cell 3-4 and Unit Cell 10 of Metasurface-I, also in Unit Cell 5 of Metasurface-II.

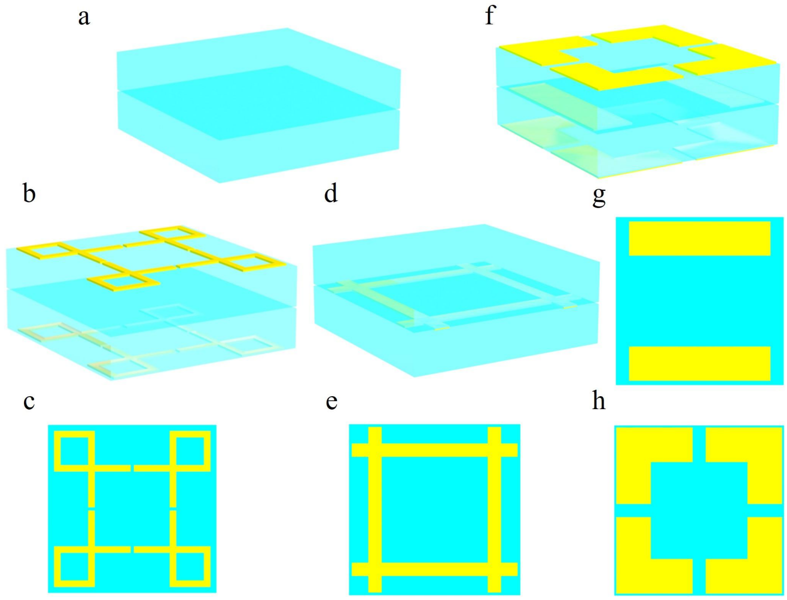

Figure S3. Schematic of the special units. a Free view of the first special unit structure which only contains dielectric substrate. **b** Free view of the second special structure which only contains top and bottom metallic wire. **c** Top and bottom view of the unit structure in **b**. **d** Free view of the third special structure that there only milled layer metallic wire without top and bottom metallic wire; **e** Middle layer of the unit structure in **d**; **f** The free view of last type special structure which in middle layer, the metal wires oriented perpendicular to the polarization direction of the incident wave have been removed; **g** Middle layer of the structure in **f**; **h** Top and middle layer of the structure in **f**.

D. Detailed design of the cloaked metasurface

According to main text, the broadband cloak should exhibit a linear phase spectrum for each meta-atom while the slope of phase profile is different. Section B provides guidance to design meta-atom with such phase distribution. According to the theoretical phase distribution in main text, each unit cell is optimized carefully. The EM spectrum of each unit cell applied in cloak is shown in Fig. S4. The detailed structure sizes are provided in Table S2-S3.

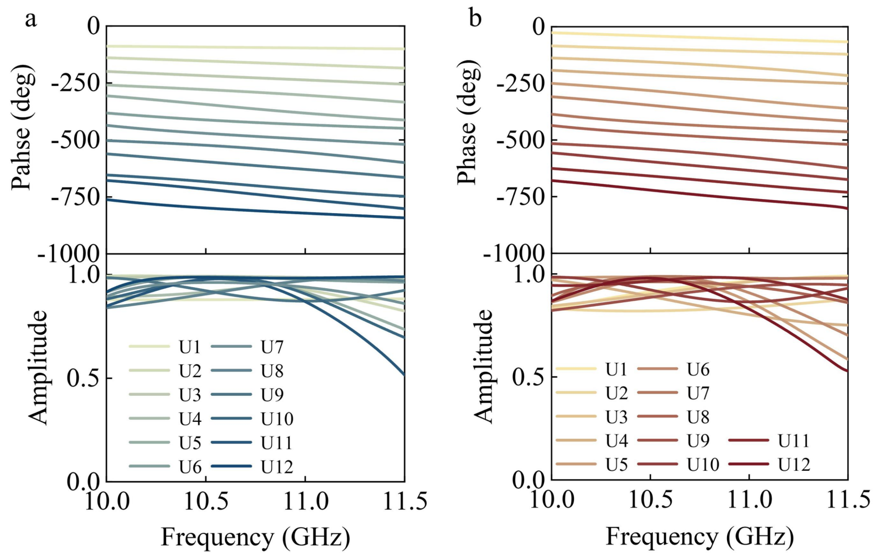

Figure S4. The EM response of each unit cell. a Phase (the top row) and amplitude (the bottom row) of transmission coefficients of each unit cell in metasurface-I. **b** Phase (the top row) and amplitude (the bottom row) of transmission coefficients of each unit cell in metasurface-II.

Table S2: The geometrical parameters of unit cells in metausrface-I. (unit: mm)

	g	w	l	l_1	l_2
U1	/	/	/	/	/
U2	4.6	0.4	9.6	4.3	/
U3*	1.9	3.8	10.1	0	8.7
U4*	0.9	2.3	10.1	0	9.2
U5	1.3	0.8	10.35	0	8.2
U6	1	0.4	9.6	2.2	8.5
U7	1	0.6	9.6	3	8.2
U8	2.6	0.4	9.6	1.1	6.2
U9	0.9	0.7	9.6	0	7
U10*	1.7	2.2	10.35	0	9.6
U11	1.3	0.6	10.35	0.9	8.2
U12	0.3	0.4	9.6	3.2	8.1

Table S3: The geometrical parameters of unit cells in metausrface-II. (unit: mm)

	g	w	l	l_1	l_2
U1	/	0.8	8	/	10
U2	0.8	0.5	9.6	2.4	5.2
U3	2.2	0.4	9.6	0.3	6
U4	0.8	1.5	9.6	0	6.3
U5*	0.8	2.1	9.8	0	8.7
U6	1.2	0.64	10.35	0	8.2
U7	0.6	0.4	9.6	2.5	8.2
U8	1	0.6	9.6	3	8.2
U9	2.2	0.6	10.1	0	6.6
U10	2.5	0.6	10.35	0	6.6
U11	2.1	1.1	10.35	0	7.9
U12	1.0	0.5	10.35	0	8.2

*: The middle layer contains two straight strips along the x-axis of the unit cell and the strips along the y-axis is removed to meet the required.

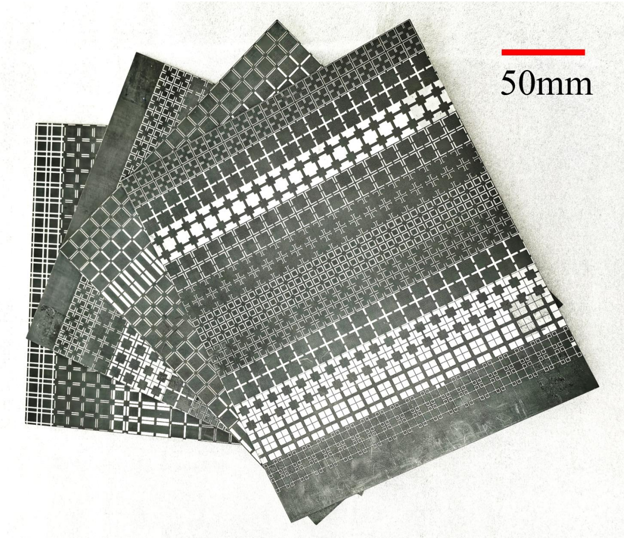

Figure S5. Photograph of the cloak metasurface prototype.

The sample of cloak metasurface fabricated by PCB process is shown in Fig. S5. The substrate is Rogers RO4003C (ε=3.55, tanδ=0.0027), while the metal is copper (σ=5.96×10^7 S/m). The metal surface has been treated with tin plating to prevent oxidation. From top to bottom is top layer of metasurace-I, middle layer of metasurace-I, top layer of metasurace-II, middle layer of metasurace-II.

E. Measurement setup of near field for the cloak

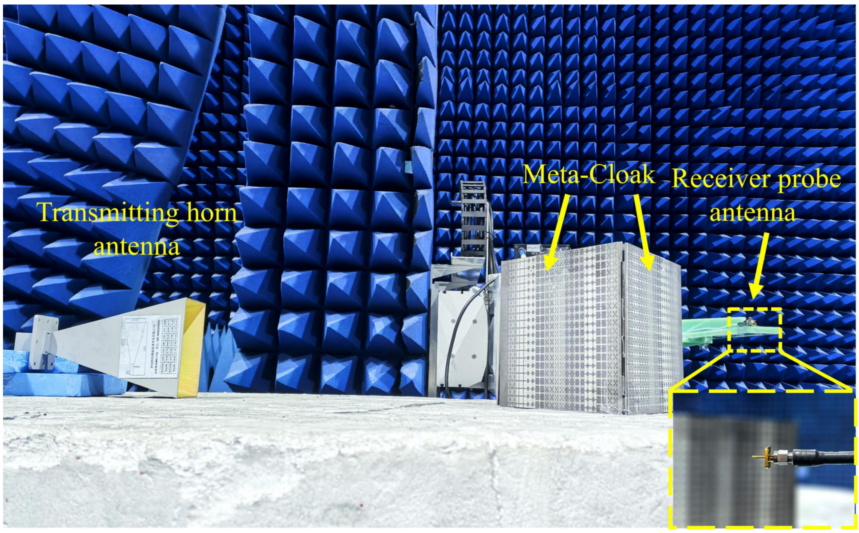

Figure S6. Schematics of the experimental setup to measure *E* distributions at the frequency band of 10-11.5 GHz as the cloak is illuminated by normally incident TM EM waves.

References

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