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Nanna Bergmann Winther

nwi@liftra.dk

Aalborg University: Aalborg Universitet <https://orcid.org/0009-0004-5306-2608>

Niklas Michael Bauer

Fraunhofer-Institut für Betriebsfestigkeit und Systemzuverlässigkeit LBF

Marcus Faß

TU Darmstadt: Technische Universität Darmstadt

Jörg Baumgartner

Fraunhofer-Institut für Betriebsfestigkeit und Systemzuverlässigkeit LBF

Jens Henrik Andreasen

Aalborg University: Aalborg Universitet

Jan Scjødt-Thomsen

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Non-Proportional Fatigue Stress Criteria in Welded Joints: A Design Code Perspective

N. B. Winther^{1,2,a}, N. M. Bauer³, M. Faß³, J. Baumgartner³, J. H. Andreasen¹, J. Schjødt-Thomsen²

¹ Aalborg University, Department of Materials and Production, Fibigerstræde 16, Aalborg East, 9220, Denmark

² Liftra, Stationsmestervej 81, Aalborg, 9000, Denmark

³ Fraunhofer LBF, Bartningstrasse 47, Darmstadt, 64289, Germany

^anwi@liftra.dk,

Abstract

Welded joints often exhibit a significant reduction in fatigue life when subjected to non-proportional (NP) multiaxial loading conditions. Therefore, it is essential to evaluate how current design codes account for NP fatigue and whether their assessment criteria remain adequate. This study examines five NP fatigue life assessment methods from major design codes: IIW [1], DNV [2], ASME [3], FKM [4] and BSI [5] using a database of 22 experimental studies involving NP fatigue in welded steel joints. The evaluation is based on two structural stress assessment approaches derived from finite element analysis: the fine hotspot stress method, as recommended by IIW, DNV, FKM, and BSI, and the equivalent(-equilibrium) structural stress method (E(E)SS) method as proposed by ASME. These are also compared to two benchmark nominal stress-based criteria: maximum principal stress and von Mises stress.

The results confirm that NP stress states lead to a notable reduction in fatigue life. While none of the evaluated design codes completely correct for this effect, the IIW and ASME method perform best overall in terms of prediction accuracy and consistency. Furthermore, the study proposes calibrated NP correction factors for each method based on the database. These factors yield near-perfect alignment between predicted and experimental fatigue life, offering a potential basis for improving current design practices.

Keywords

Non-Proportional Fatigue, Multiaxial Fatigue, Welded Joints, Fatigue Review

Introduction

Fatigue in steel components is a critical consideration in the design process, as failure can lead to significant structural issues. Welded joints, in particular, are more susceptible to fatigue-related failures. An often overlooked factor in fatigue assessment is the non-proportional (NP) multiaxial stress state, where the main stress direction changes over time [6, 7].

This study aims to contribute to a broader understanding of how multiaxial fatigue can be addressed in practice. Special attention is given to evaluating the performance of NP fatigue criteria in relation to existing engineering design codes and recommendations based on structural stress approaches.

An example of an NP stress state occurs when applied forces differ, such as when one force acts faster or is delayed relative to another. This phenomenon has been shown to reduce the fatigue life in both welded and non-welded components, often leading to deviations from fatigue predictions based on traditional S-N curve methodologies [8, 9, 10].

In this context, it should be noted that neutral or even lifetime-increasing effects under NP loading have been observed [11]. These effects are typically associated with brittle or semi-ductile materials, such as cast iron [12]. However, since welding is uncommonly performed on non-brittle materials. Hence, for welded steels, a reduction in fatigue life under NP loading is generally anticipated.

The limited attention to NP stress effects can be attributed to the inherently conservative nature of fatigue design methodologies, where unintended reductions in fatigue life are absorbed within the applied safety factors [13]. As a result, NP stress states are frequently overlooked and not explicitly considered in standard fatigue assessment approaches.

A reasonable question is whether further study of this phenomenon is necessary if it is already absorbed within the safety factor. However, it is important to recognize that safety factors are intended to account for uncertainties and variations in real-world conditions rather than systematically recurring effects. The influence of NP stress states on fatigue life has been documented since the 1940s for non-welded components [14] and the 1980s for welded components [15], suggesting that it should be directly accounted for in fatigue design rather than implicitly absorbed within safety margins.

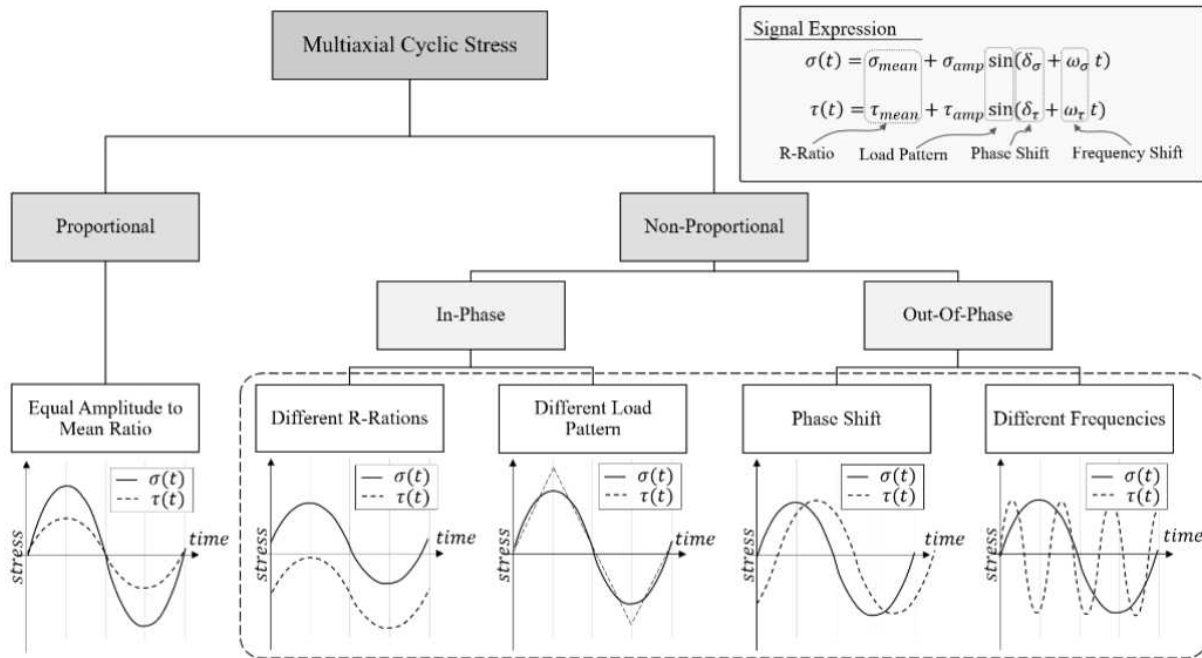


Fig. 1 Classification of multiaxial cyclic stress states, distinguishing between proportional and non-proportional loading conditions. The non-proportional stress states are further categorized into in-phase (with different R-ratios and load patterns) and out-of-phase (with phase shift and different frequencies). The accompanying signal expressions describe the variation in normal stress $\sigma(t)$ and shear stress $\tau(t)$ over time.

This paper provides insight into NP stress states induced by one or more of the four NP aspects: phase and frequency shifts between stress components, variations in R-ratios, and different load patterns [16]. These classifications, illustrated in Fig. 1, describe how multiaxial cyclic stress conditions deviate from proportional

loading. The focus is specifically on welded steel components, as this is a critical area that has received comparatively less attention than non-welded components. The aim of this study is to evaluate five fatigue life assessment approaches prescribed by major design codes: IIW [1], DNV [2], ASME [3], FKM [4], and BSI [5], along with two benchmark methods based on maximum principal stress and von Mises stress. The evaluation assesses the adequacy of each method in correcting for the effects of NP loading on fatigue life. In addition, the experimental database is used to calibrate new NP correction factors/quantifiers for each of the seven methods, aiming to achieve near-perfect correction for NP effect.

Published Non-Proportional Experimental Studies

A comprehensive collection of experimental data on non-proportional stress state in welded steel joints has been compiled. This database specifically focuses on welded steel joints subjected to constant amplitude loading. As a result, 22 publications containing experimental data on NP loading conditions have been identified [9, 15, 17, 22-24, 25-27, 28-29, 30-34, 36-37, 44-50]. An illustration of the specimen designs used in these studies, along with the applied loading conditions, is presented in Appendix A, Fig. 8. Additionally, the key parameters, including loading type, NP aspects investigated, welding type, weld condition, and failure location, are summarized in Appendix A, Table 4.

1.1. Database Structure and Content

- **Publication and Specimen Information:** The entry section includes the year of the study, author, and specimen type, providing context on the experimental setup.
- **Material Properties:** The base materials used in the experiments are identified, along with their mechanical properties, including yield strength, ultimate tensile strength, elongation and hardness.
- **Weld Characteristics:** The database captures details of the welding method (e.g., TIC, MIG), weld type (e.g., fillet, butt), penetration type, and post-weld treatment conditions (e.g., stress-relieved or as-welded)
- **Geometric and Dimensional Properties:** Specimen thickness and units are recorded.
- **Fatigue Life and Failure Analysis:** The number of cycles to failure and failure location (e.g., weld toe, weld root) are specified. Additionally, cases where runout or retesting occurred are likewise registered.
- **Loading Conditions:** The database provides comprehensive details on loading conditions, including maximum and range values for both forces.
- **Non-Proportionality Aspects:** The database includes information regarding the four NP aspects as follows:
 - **Load Ratio:** Information on load ratios for different loading cases, with 9% of the NP data covering this aspect.
 - **Load Pattern:** Specifies whether the loading deviates from sinusoidal wave functions. No pure cases with alternative load patterns have been identified in the database. As the data study by Siljander [17] is a hybrid formulation of different pattern and phase shift but are not a pure case of change in load pattern.
 - **Phase Shift:** This is the most extensively investigated NP aspect, with 80% of the NP data focusing on this parameter. However, it should be noted that only two studies have examined phase shifts other than 90 degrees, making this a relatively under explored area within itself.
 - **Frequency Shift:** Differences in loading frequencies account for 11% of the NP data, with only two studies investigating this aspect.
- **Stress Parameters:** The database includes calculated nominal stress values for both stress ranges and mean stresses. In addition to nominal stress, the two stress evaluation concepts hot-spot and E(E)SS have also been included.

This collection likely represents the entirety of available experimental data on NP fatigue in welded steel joints under constant amplitude loading. Despite extensive literature searches, no additional experimental studies on welded steel components under NP loading have been identified. While some studies on NP fatigue behaviour exist for aluminium [18] and magnesium alloys [19, 20], they have been excluded as this review focuses solely on welded steel connections.

1.2. Selection and Exclusions of Data

Exclusions have been made where duplicate of data series was identified. Specifically, the study by Yousefi et al. [21] has been omitted, as it presents data originally obtained from Witt [9]. A similar case applies to Amstutz [22], which, despite not explicitly citing Witt [9], presents an identical data series. This conclusion was reached through direct comparison of reported values, highlighting the lack of proper citation in Amstutz et al. [22].

Additionally, the data study from Bufalari [23, 24] represents an extension of the earlier work by Lieshout [25]. Four data points have been removed in the transition from Lieshout [25] to Bufalari [24] due to revised interpretations based on further experimental insights. This refinement was confirmed through direct communication with the authors involved in the study. Consequently, the data study by Bufalari [24] is considered the reference for these experiments.

A further limitation arises in the study by Archer [15], which, while included in the database for reference, has been excluded from the final analysis. The primary reason for this exclusion is the use of strain gauge measurements at an unspecified location, making it impossible to replicate the stress data ideally in finite element modelling, without making significant assumptions.

The study by Garcia and Baptista [26, 27] is the only one in the database that reported a significant increase, by a factor of eleven, in fatigue life for NP cases, rather than a reduction. This unexpected result has started further investigation into this publication. As this study examines a biaxial loading case, where two perpendicular tensile forces are applied instead of a pure shear force in cases with e.g., torsion, it has only one comparable study in the database, conducted by Takahashi et al. [28, 29]. Their findings, however, showed an average fatigue life reduction of 28% under NP loading conditions as expected. It should be noted that NP stress states have, in some cases, been documented to increase fatigue life rather than reduce it. This phenomenon has been observed in brittle materials, such as cast iron [11]. However, since welding is rarely performed on brittle materials, NP loading in welded steel connections is generally expected to reduce fatigue life. Given that the study by Baptista and Garcia focuses on ductile materials, the significant increase in fatigue life observed in their results was unexpected. Due to the unusually high fatigue life reported in this study, it has been determined that this case should be considered an outlier. Consequently, this study has been excluded from the database until further investigations can be conducted.

Given the focus of this review on structural stress approaches, only fatigue failures occurring at the weld toe are considered. As a result, studies where failures occurred at the weld root or other locations have been excluded, further reducing the database to the final 9 publications including Siljander [17], Sonsino [30], Razmjoo [31], Witt & Zenner [32], Bäckström [33], Witt [9], Takahashi [29, 28], Lieshout & Bufalari [25, 24] and Winther [34]. This additional constraint further highlights the limited availability of experimental NP fatigue data for welded steel components.

1.3. Assessment of Dataset Quality

For a dataset to be considered fully complete, it must include the necessary baseline data, consisting of the investigated NP case, the corresponding proportional case, and pure uniaxial data. The uniaxial data typically

consists of two series to capture both pure normal stress and shear stress. Consequently, a complete dataset study should ideally consist of four experimental series to provide a robust basis for analysis.

Typically, individual data series are presented in the form of a log-log S-N curve, derived using Basquin's power-law approximation [35]. The S-N curve provides a critical link between applied stress and fatigue life and is essential for the evaluation of NP fatigue criteria discussed in 1.8. Therefore, it is imperative to assess whether all required information is available and whether each data series contains a sufficient number of repetitions to reliably define an S-N curve.

According to the DNV design codes [2], a minimum of 15 specimens and at least three load levels are required to establish a statistically valid S-N curve. Additionally, these specimens should be tested at unique load levels rather than being repetitions at the same load, ensuring a distributed dataset for curve fitting. In the current collected data, only 6 out of 59 individual datasets contain 15 or more repetitions, and when considering unique load levels within a ± 10 MPa range, no series exceeds 11 distinct load levels. The average repetition count is 9, while the average number of unique load levels is 6. Given the limited availability of data, values in the orange range have been deemed acceptable within these constraints.

Upon reviewing the available datasets, several limitations were identified, primarily due to missing data or an insufficient number of data points. A full summary of the datasets is presented in the Appendix A.

A notable limitation is observed in the dataset from Sonsino [30], where the original studies involving flange-to-tube and tube-to-tube welded specimens did not include all corresponding uniaxial S-N curves. In the case of the flange-to-tube specimens, it was possible to identify similar geometries in other studies [36, 37], which were used as substitutes. However, for the tube-to-tube specimens, no equivalent experimental data for pure torsion was found. As a result, only design code- or recommendation-based S-N curves could be used, preventing a fully experimental comparison.

A similar issue exists in the dataset by Bufalari [24]. This study includes a pure torsion S-N curve only for the case of $R = -1$, whereas the combined loading tests were performed at $R = 0$. Despite the mismatch in load ratio, the available torsion S-N curve was used as a reference due to the absence of alternative experimental data.

The dataset by Siljander contains only two repetitions and load levels for the pure torsion tests, which limits its statistical reliability. Bäckström's dataset includes a maximum of three repetitions per load case, making it statistically weak as well. Additionally, in the study by Winther et al., only a single load level was tested under combined loading, making it impossible to establish a full S-N curve for these cases. However, since the evaluation primarily relies on pure uniaxial data, this limitation does not significantly impact the overall applicability of the dataset.

While the absence of pure uniaxial data presents a significant limitation when evaluating directly against experimental results, this issue is mitigated when comparing results to design code S-N curves, such as the focus in this study, where the necessary reference curves are available through the design codes. These limitations do not render the dataset unusable, but they indicate that many series do not fully align with best practices for experimental validation. Consequently, while the dataset remains valuable for analysis, its usability depends on the intended evaluation method. When using design code S-N curves, missing data can be compensated for, but for strictly experimental analyses, certain data studies may lack the completeness required for reliable fatigue assessments.

Ultimately, despite the small collection of data studies, it represents the most comprehensive and reliable resource available for evaluating NP fatigue behaviour in welded joints for structural stress approaches. The findings and conclusions drawn from this study must, therefore, be understood within the constraints imposed by the current experimental evidence.

1.4. Residual Stress Handling

In the fatigue assessment of welded components, residual stresses introduced by the welding process play a critical role. However, in the conventional S-N approach, these residual stresses are not explicitly modelled. Instead, they are inherently accounted for in the S-N curves themselves, which are typically derived from full-scale welded specimens tested in the as-welded condition. These curves therefore include the influence of tensile residual stresses, making them suitable for design without further adjustment.

In the present study, the fatigue database includes both as-welded and stress-relieved specimens. Since residual stresses affect crack initiation and early crack growth, their absence in stress-relieved specimens can lead to artificially improved fatigue life. Consequently, a correction factor has been applied to the stress-relieved data [2], given by:

$$f_m = \frac{1 + 0.8|R|}{1 + |R|}$$

Stress Evaluation Concepts and Multiaxial Fatigue Design Basis

Before going into detail with the selected NP correction methods, the fundamental principles of stress evaluation, the S-N relationship and the multiaxial fatigue assumptions are first addressed in this section. It begins with an overview of three commonly used structural stress evaluation methods. The section then outlines the formulation of S-N curves used to relate stress range to fatigue life, which form the basis for design and comparison in fatigue analysis. Finally, the underlying assumptions involved in applying uniaxial S-N curves to multiaxial loading conditions are discussed.

1.5. Stress Evaluation Concepts

To ensure a practical and focused investigation, only established design codes have been considered that explicitly address NP loading in a structural stress manner. This includes: IIW [1], DNV [2], ASME [3], FKM [4] and BSI [5]. Although Eurocode 3 [38] is widely used and shares structural similarities with other recommendations, it does not specifically address NP stress state and has therefore not been included in this study.

Across the reviewed design codes and recommendations, three stress evaluation concepts are consistently referenced: the nominal stress approach, the hot-spot stress method, and the equivalent structural stress (ESS) method. The nominal stress approach is suitable only for simple geometries and will be used in this study as a baseline reference. For more complex geometries, the design codes generally recommend the hot-spot method except for ASME, which instead refers exclusively to the ESS method.

To ensure broad applicability and alignment with international practice, this study adopts all three methods: nominal stress as a benchmark, and both hot-spot and ESS as practical structural stress evaluation methods for complex welded joints. This selection ensures coverage of the full range of methods recommended by the major fatigue design standards which handles NP stress state.

1.1.1. Nominal Stress Method

The nominal stress method provides a simplified estimate of stress based on average values across the cross-section of a component. It assumes uniform stress distribution and is used for simple geometries under basic loading conditions. Nominal stress does not account for stress concentrations, such as welds or geometric discontinuities. However, it should be noted that such effects are addressed in design codes and recommendations through the selection of an appropriate corresponding S-N curve.

Due to its simplicity, this method is not compatible with finite element analysis and serves mainly as the benchmark in this study.

1.1.2. Fine Hot-Spot Method

The hot-spot stress method is widely used in fatigue assessments of welded structures to estimate the structural stress at the weld toe. There are several variations of the hot-spot stress method [1, 2], primarily differing in the choice of extrapolation technique, e.g., linear or quadratic, and the distances from the weld toe at which stresses are evaluated. A commonly used variant is the fine hot-spot method, which typically uses linear extrapolation based on stress values at $0.4t$ and $1.0t$ with one element in the thickness direction. In this study, the fine hot-spot method is selected, based on the assumption that modern computational tools and mesh refinement are sufficient to capture the stress gradient accurately near the weld toe.

The original fine hot-spot extrapolation formula, based on normal stress values, is given by:

$$\Delta\sigma_{hotspot} = 1.67 \Delta\sigma_{0.4t} - 0.67 \Delta\sigma_{1.0t}$$

An illustration of this concept is shown in Fig. 2, where stresses at distances of $0.4t$ and $1.0t$ from the weld toe are linearly extrapolated to estimate the structural stress at the critical location.

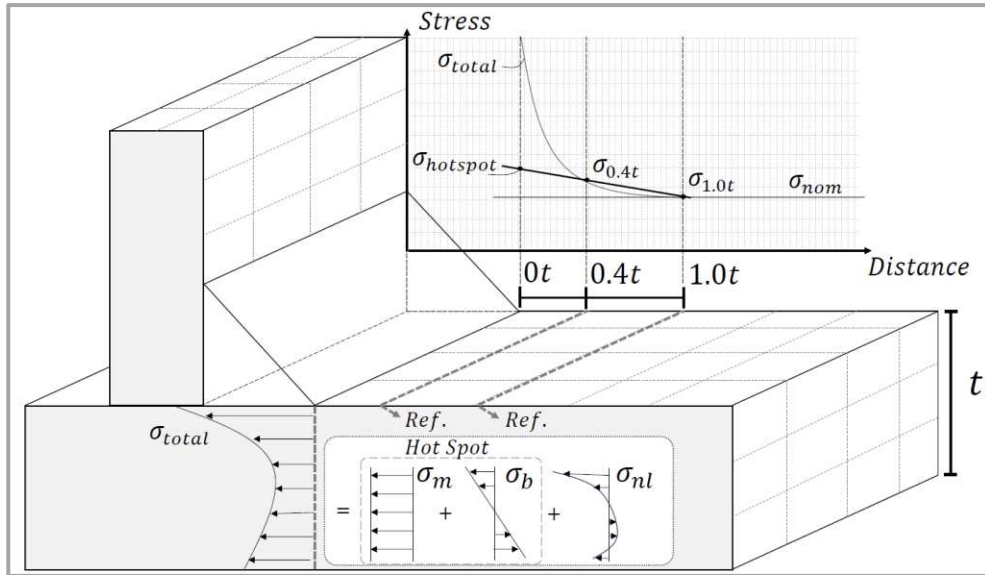


Fig. 2 Illustration of the hot-spot stress concept, showing stress evaluation at a welded joint. The stress is extrapolated from points at $0.4t$ and $1.0t$ to estimate the hot-spot stress at the weld toe, as visualized in the stress vs. distance plot.

To extend this approach to multi-axial loading conditions, it is assumed in this study that the same extrapolation logic can be applied to all components of the stress tensor. This leads to the generalized form:

$$\Delta\sigma_{hotspot} = 1.67 \Delta\sigma_{0.4t} - 0.67 \Delta\sigma_{1.0t} , \quad \Delta\sigma = \begin{bmatrix} \Delta\sigma_x & \Delta\tau_{xy} & \Delta\tau_{xz} \\ \Delta\tau_{xy} & \Delta\sigma_y & \Delta\tau_{yz} \\ \Delta\tau_{xz} & \Delta\tau_{yz} & \Delta\sigma_z \end{bmatrix}$$

1.1.3. Equivalent(-Equilibrium) Structural Stress Approach

The ASME design code introduces an Equivalent Structural Stress (ESS) method for fatigue assessment of welded joints, which is stated to be mesh-insensitive and in internal equilibrium [3]. The structural stress is defined as the sum of membrane and bending stress components and is evaluated along a Structural Classification Line

(SCL), corresponding to the A-A section illustrated in Fig. 3. The equilibrium structural stress is formulated as follows:

$$\sigma_{ESS} = \sigma_m + \sigma_b$$

where the membrane component is σ_m and a bending component is noted as σ_b .

Although the ASME design code does not explicitly reference Dong's original work, there is strong indication that the equivalent-equilibrium structural stress (EES) method by Dong [39] is the funding work for the ESS included in the ASME. This is evident not only from the methodological similarities but also from the near-identical figures presented in both sources.

The ASME design code states that the structural stress method inherently satisfies internal equilibrium. However, this assumption is only valid if the underlying finite element model is sufficiently refined and appropriate element types is applied to ensure that equilibrium conditions are met. As such, the claim of mesh insensitivity can be questioned. Although it is not incorrect, since mesh-independent results can indeed be achieved after adequate refinement, the design code does not specify the level of refinement required. Consequently, the reliability of the method under coarse or inconsistent meshing, particularly near weld toes or geometric discontinuities, remains uncertain and may necessitate additional convergence studies to confirm mesh-independent behaviour.

This is in contrast, to Dong's original method which enforces equilibrium explicitly between two parallel sections, such as A-A and B-B, spaced at a known distance d illustrated in Fig. 3. This formulation guarantees mesh insensitivity also for coarse meshing by computing the structural stress from force and moment equilibrium.

$$\sigma_m = \frac{1}{t} \int_0^t \sigma_x(y) dy \quad \sigma_m \frac{t^2}{2} + \sigma_b \frac{t^2}{6} = \int_0^t \sigma_x(y) y dy + d \int_0^t \tau_{xy}(y) dy$$

Here, t is the plate thickness, d is the distance between Sections A-A and B-B, and $\sigma_x(y)$ and $\tau_{xy}(y)$ are the through-thickness distributions of normal and shear stresses, respectively. These expressions are presented as per unit length.

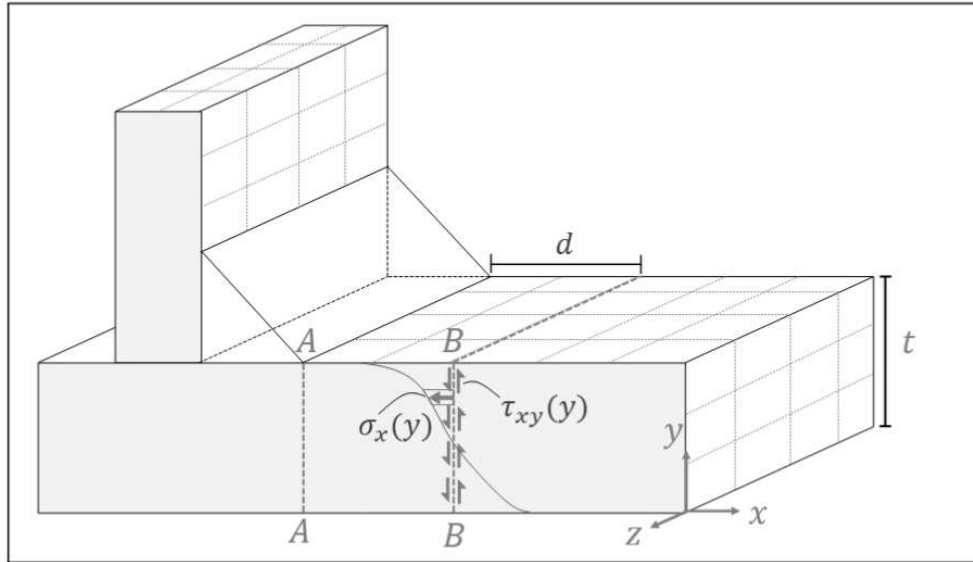


Fig. 3 Illustration of the EES concept, showing stress evaluation across planes A-A and B-B to satisfy equilibrium. Both the normal and shear stress distributions ($\sigma_x(y)$, $\tau_{xy}(y)$) are used to compute this structural stress concept.

This distinction highlights the trade-off between practical applicability (ASME) and theoretical robustness (Dong). While ASME provides a user-friendly framework aligned with engineering practice, Dong's method remains the more rigorous formulation for achieving guaranteed mesh-independent results.

Since it is desirable to maintain a consistent mesh for both the hot-spot and ESS method, it has been decided to apply Dong's original approach in this study to ensure true mesh insensitivity. Henceforth, the abbreviation for Dong's method used in connection with the ASME design code will be E(E)SS. Details on the interpretation and implementation of Dong's equilibrium equations are provided in Appendix B.

As with the hot-spot method, an assumption must be made when extending the E(E)SS approach to multiaxial loading. For the normal stress components, the structural stress is computed by satisfying the force and moment equilibrium conditions. For the shear stress, Dong does not provide an explicit equilibrium-based formulation. However, he proposes a simplified representation of the structural shear stress as the sum of its membrane and bending components:

$$\tau_{ESS} = \tau_m + \tau_b$$

here, τ_m is the membrane shear stress, and τ_b is the corresponding bending component. In this study, this relation is applied along the A-A section to estimate the structural shear stress.

One of the key advantages of this method lies in its use of a Master S-N curve. Unlike traditional approaches, there is no need to identify and assign a corresponding FAT class or detail category for each geometry. The concept behind the Master S-N curve is that all S-N curves for welded joints are essentially parallel, differing only by a scalar factor. This context the scalar is attributed to the stress concentration factor (SCF) at critical locations such as the weld toe. By incorporating the SCF into the structural stress calculation, as the E(E)SS method is designed to do, the fatigue assessment can be performed using a single Master S-N curve. Although different curves may still be needed for variations in material properties or operating temperature, multiple curves for different geometries are no longer required.

1.6. S-N Relationship

To estimate fatigue life from stress for any fatigue method, the logarithmic relationship by Basquin [35] is used in this study. This expression relates the stress range to the number of cycles to failure through the slope (m) of the S-N curve and the logarithmic fatigue resistance coefficient ($\log C$):

$$\log(\Delta\sigma_{eq,f}) = \log(C) - m \log(N_f)$$

Here, $\Delta\sigma_{eq,f}$ represents the equivalent stress range corresponding to a specific number of cycles to failure. It can also be interpreted as the fatigue resistance at that particular number of cycles, and N_f denotes the number of cycles to failure. Please note that the term '*equivalent stress*' is used to describe stress states that combine both normal and shear components. Therefore, it is not limited to von Mises or maximum principal stress but serves as a general term for combined stress expression.

In design codes and fatigue classifications e.g., FAT or detail categories, this relation is often expressed in the following form:

$$\Delta\sigma_{eq,f}^m N_f = \Delta\sigma_{Ref}^m N_{Ref}$$

From this formulation, either the number of cycles to failure or the equivalent stress range can be isolated:

$$N_f (\Delta\sigma_{eq,f}) = N_{Ref} \left(\frac{\Delta\sigma_{Ref}}{\Delta\sigma_{eq}} \right)^m \quad \Delta\sigma_{eq,f}(N_f) = \Delta\sigma_{Ref} \left(\frac{N_{Ref}}{N_f} \right)^{1/m}$$

Typically, the reference number of cycles is taken as $N_{Ref} = 2 \cdot 10^6$ [1], depending on the chosen design code. This formulation will be used to evaluate NP correction methods that yield an equivalent stress range as their output.

In cases where the NP correction method does not directly yield an equivalent stress range, such as interaction equation-based formulations, an iterative approach is required to determine the number of cycles to failure. These methods describe fatigue life in terms of the cyclic degree of utilization, defined as the ratio between the

applied stress range and the fatigue resistance, i.e., the reference fatigue resistance range $\Delta\sigma_{Ref}(N)$ obtained from the uniaxial S-N curve for a given cycle count N . Examples include methods from IIW and FKM, which will be presented in the following sections. A general interaction formulation is shown below:

$$\left(\frac{\Delta\sigma}{\Delta\sigma_{Ref}(N)}\right)^{n_\sigma} + \left(\frac{\Delta\tau}{\Delta\tau_{Ref}(N)}\right)^{n_\tau} = A$$

To determine the number of cycles to failure, the reference fatigue resistance $\Delta\sigma_{Ref}(N)$ can be substituted with the expression with the equivalent fatigue resistance at a specific number of cycles to failure $\Delta\sigma_{eq,f}(N_f)$. The same methodology applies for the shear fatigue resistance. The interaction equation then becomes a non-linear expression in terms of the number of cycles to failure N_f :

$$\left(\frac{\Delta\sigma}{\Delta\sigma_{Ref}\left(\frac{N_{Ref}}{N_f}\right)^{1/m_\sigma}}\right)^{n_\sigma} + \left(\frac{\Delta\tau}{\Delta\tau_{Ref}\left(\frac{N_{Ref}}{N_f}\right)^{1/m_\tau}}\right)^{n_\tau} = A$$

Here, m_σ and m_τ are the slopes of the normal and shear uniaxial S-N curve, n_σ and n_τ are the exponents of the normal shear utilization term and A is usually equal 1 but it depends on the methods, which will be presented for the individual methods respectively. As no closed-form solution exists for this type of equation, an iterative numerical method is employed to solve for the fatigue life. The iterative solution process is based on the methodology outlined by Fass et al. [40].

1.7. Multiaxial in relation to S-N Curves

It is important to recognise that the S-N curve relationships used to relate stress states to fatigue life are typically based on uniaxial fatigue data. Specifically, data derived from pure normal stress loading. Such data are widely available in design codes and recommendations. However, pure shear fatigue data are considerably less frequently available, with most design codes providing only two reference cases [1, 5]. This applies to the previously mentioned standards, with exception of ASME [3], which instead refers to a master S-N curve that does not differentiate between shear and normal stresses. In cases involving pure shear, it is assumed that the same Master S-N relationship can be directly applied by shear stress as that of normal stress.

Applying the S-N curve approach in a multiaxial setting is challenging, as the S-N curve is based on uniaxial data. To reflect common engineering practice, the S-N curves in this study are interpreted in a way that allows direct fatigue life prediction for combined loading scenarios by converting them into an equivalent stress range. Hence, this does not apply to interaction equations, as the number of cycles is determined by an iterative approach, yielding no equivalent stress range. A common way to achieve this equivalent stress range is through models such as the maximum principal Stress or the von Mises approach, both of which reduce the multiaxial stress states to a scalar equivalent value. To provide a broader context and establish reference points for comparison, these two widely used methods have been included in this study as benchmarks as the NP loading effects are not explicitly considered.

1.8. Selection of NP Fatigue Life Methods for Evaluation

As previously mentioned, only a selected group of design codes were identified to include methods for handling NP loading conditions. These are IIW [1], DNV [2], ASME [3], FKM [4], and BSI [5]. The aim of this section is to introduce each of these assessment approaches and highlighting their advantages and limitations. Additionally, the two commonly used multiaxial methods, maximum principal stress and von Mises, are included for benchmark purposes. A short overview of the selected methods is presented in Table 1.

Table 1 Overview of NP fatigue treatment in common design codes, including assessment types and key correction methods.

Method	Assessment Type	Description
Max. Prin.	Equivalent Stress Range	Benchmark – Based on the maximum of the principal stress components. No account for NP effects.
Von Mises	Equivalent Stress Range	Benchmark – Uses distortion energy theory. Suitable for proportional loading. No account for NP effects.
IIW	Interaction Equation	Based on Gough-Pollard with added parallel stress terms. A reduced from equal 1.0 to 0.5 in NP cases.
DNV	Equivalent Stress Range	Uses a modified Gough-Pollard relation. A correction factor of 1.1 is applied to account for NP effects.
ASME	Equivalent Stress Range	Von Mises-based expression with a phase-shift correction factor to account for NP loading.
FKM	Interaction Equation	Uses max. principal stress for proportional cases and a Gough-Pollard variant with exponents for NP.
BSI	S-N Curve Classification	Uses shear S-N curves for proportional loading. For NP loading, the endurance limit is reduced by 50%.

1.1.4. Maximum Principal Stress

The maximum principal stress is often used as a fatigue damage parameter in multiaxial loading conditions. In two-dimensional stress states, such as it is present on the surface of a component, it is calculated directly from the in-plane normal and shear stresses. The maximum principal stress σ_1 is given by:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

In this study, the maximum principal stress is used as one of the reference methods to compare multiaxial fatigue methods under proportional loading conditions.

1.1.5. Von Mises Stress

The von Mises equivalent stress is a widely used yield criterion for ductile materials, formulated based on the second deviatoric stress invariant J_2 defined as:

$$\sigma_{vM} = \sqrt{J_2}, \quad J_2 = \frac{1}{2} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \right] + 3 (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$$

It should be noted that von Mises stress was originally developed as a yield criterion, it is often employed in multiaxial analysis for its ability to represent the stress states with a single scalar value. However, its use in fatigue, which is an inherently crack initiation and propagation driven phenomenon, may be questioned from a physical standpoint, as fatigue does not necessarily involve yielding.

1.1.6. IIW-Recommendation

The third edition of the IIW-Recommendations [1] accounts for the stress component parallel to the weld [41, 42]. In the second edition [43] the method was based on the original Gough-Pollard interaction equation [44], which only considers the normal and shear stress components. The extended formulation introduces an ellipsoidal relationship between the cyclic degrees of utilization of the normal stress to the weld $\sigma_{\perp}$, the parallel stress $\sigma_{\parallel}$, and the shear stress τ :

$$\left(\frac{\Delta\sigma_{\perp}}{\Delta\sigma_{\perp,Ref}(N)} \right)^2 + \left(\frac{\Delta\sigma_{\parallel}}{\Delta\sigma_{\parallel,Ref}(N)} \right)^2 + \left(\frac{\Delta\tau}{\Delta\tau_{Ref}(N)} \right)^2 \leq CV$$

The terms $\Delta\sigma_{\perp,Ref}(N)$, $\Delta\sigma_{\parallel,Ref}(N)$, and $\Delta\tau_{Ref}(N)$ correspond to the reference fatigue resistances under uniaxial loading for a given number of cycles N .

The comparison value CV is defined as CV = 1 for proportional loading and CV = 0.5 for NP loading. However, loading can still be classified as proportional if the principal stress direction varies by less than 20°, or if one stress component is dominant, i.e., the other components are less than 15% of the dominant stress.

1.1.7. DNV-Recommended Practice

The NP correction method from DNV-Recommended Practice introduces an equivalent stress to account for NP loading. In cases with NP loading the γ takes the value of 1.1 corresponding to an increase in the stresses of 10%. The proposed formulation is given as:

$$\Delta\sigma_{eq, DNV} = \gamma \sqrt{\Delta\sigma_{\perp}^2 + 0.81 \Delta\tau_{\parallel}^2}$$

Although DNV does not explicitly reference its theoretical origin, this method is believed to stem from the Gough-Pollard interaction equation [44]. In this context, the effective stress formulation may be interpreted as a geometrically simplified equivalent of the original interaction relation, assuming fixed ratios between stress components. A detailed derivation of this equivalence and the reduction to the form used in DNV is provided in Appendix C.

1.1.8. ASME-International Code

The ASME-International Code [3] introduces a NP correction method based on the von Mises equivalent stress. The approach accounts for out-of-phase loading by scaling the equivalent stress using a correction factor $F(\delta)$, which depends on the phase angle δ between the normal and shear stress components. The equivalent stress range is given by:

$$\Delta\sigma_{eq, ASME} = \frac{1}{F(\delta)} (\Delta\sigma^2 + 3\Delta\tau^2)^{0.5}$$

Two options are provided depending on whether the out-of-phase angle δ is known or not:

$$F(\delta) = \frac{1}{\sqrt{2}} \left(1 + \left(1 - \frac{12 \Delta\sigma^2 \Delta\tau^2 \sin^2(\delta)}{(\Delta\sigma^2 + 3\Delta\tau^2)^2} \right)^{0.5} \right)^{0.5}, \quad F(\delta) = \frac{1}{\sqrt{2}}$$

This correction is designed to capture the additional fatigue damage observed under out-of-phase loading.

1.1.9. FKM-Guideline

The FKM-guideline [4] considers NP loading by distinguishing between a method for proportional loading:

$$\frac{1}{2} \left(\frac{\Delta\sigma_{\perp}}{\Delta\sigma_{\perp, Ref}(N)} + \frac{\Delta\sigma_{\parallel}}{\Delta\sigma_{\parallel, Ref}(N)} \right) + \sqrt{\left(\frac{\Delta\sigma_{\perp}}{\Delta\sigma_{\perp, Ref}(N)} - \frac{\Delta\sigma_{\parallel}}{\Delta\sigma_{\parallel, Ref}(N)} \right)^2 + \left(\frac{\Delta\tau}{\Delta\tau_{Ref}(N)} \right)^2} \leq 1$$

and another method for NP loading:

$$\frac{\Delta\sigma_{\perp}}{\Delta\sigma_{\perp, Ref}(N)} + \frac{\Delta\sigma_{\parallel}}{\Delta\sigma_{\parallel, Ref}(N)} + \frac{\Delta\tau}{\Delta\tau_{Ref}(N)} \leq 1$$

1.1.10. BSI-Standard

The BSI design code [5] applies a simplified classification-based approach for assessing fatigue under multiaxial loading. In this method, the equivalent stress is defined as the maximum of either the normal or shear stress range:

$$\Delta\sigma_{eq} = \max(\Delta\sigma, \Delta\tau)$$

- *In-Phase Combined Loading*: For loading conditions where the normal and shear stresses are in phase, the code recommends evaluating fatigue life using normal stress-based S-N curves.

- **Out-of-Phase Combined Loading:** For cases involving out-of-phase stress components, the design code recommends evaluating fatigue life using the shear-based S-N curve parameters, with an additional reduction of the endurance limit by a factor of two.

1.9. Observation: General Formulation

It is noted that a majority of NP correction methods, including those in the IIW [1], DNV [2], and FKM guidelines [4], are based on this interaction equations likely inspired from the Gough-Pollard formulation [44]. This is particularly interesting considering that the original Gough-Pollard model is empirical and based solely on proportional loading data and deducted from fatigue tests of ductile non-welded materials.

As a sub-investigation, the interaction equation was explored to assess the influence of different exponent combinations for the normal and shear stress components, as originally proposed by Archer [15]. The primary objective was to evaluate whether specific exponent values could provide a better fit to experimental fatigue data. The results, shown in Fig. 4, include several interaction curves plotted using varying exponent values. The analysis was based on nominal stress data combined with experimentally derived S-N parameters, thereby inherently accounting for the notch effect at the weld toe, in order to be true to experimental results. The resulting data exhibit considerable scatter, which makes it difficult to identify a clearly superior exponent combination. This outcome is not unexpected given the natural variability in fatigue test data.

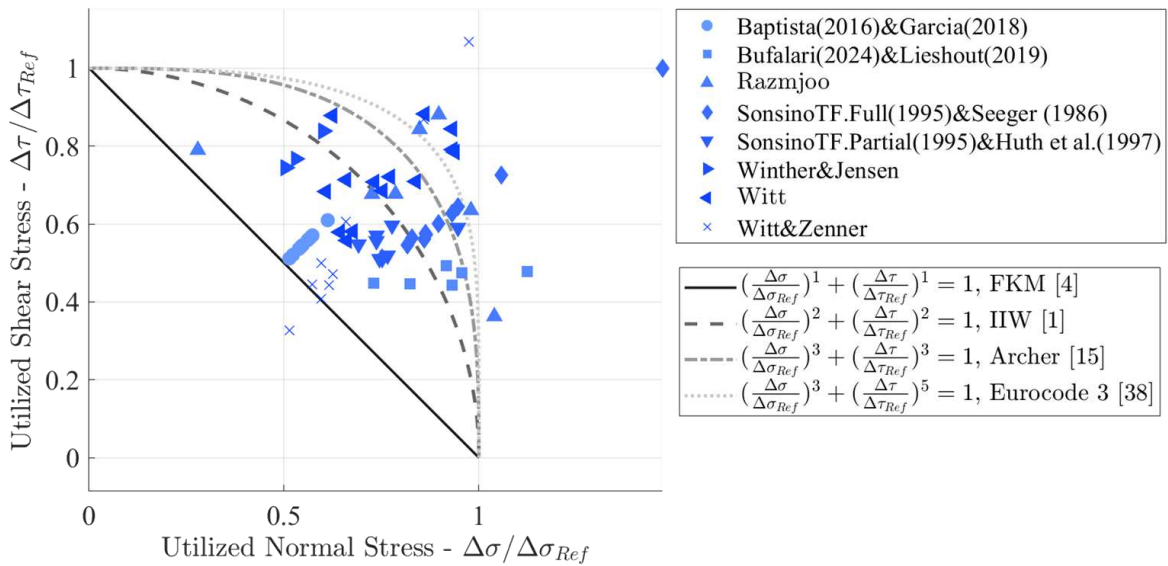


Fig. 4 Comparison of experimental fatigue data with common interaction equations for proportional multiaxial stress states. The data points represent studies from the database, while the plotted curves correspond to interaction models with different exponents. For comparison the Eurocode 3 [38] has been added.

In conclusion, while the choice of exponent influences the shape of the interaction curve, the question of optimal exponent selection remains unresolved. Further research into this topic has been carried out by Bauer et al. [41] and proposed possible exponents. Overall, when considering welded fatigue data, the results tend to follow an interaction-type relationship, supporting its applicability for fatigue life estimation. However, the choice of a perfectly fitted interaction relationship cannot be concluded from this sub-investigation, highlighting the uncertainties that remain in this field.

1.10. Observation: Practical Implementation

Another important aspect is the practical implementation of each method. When ranking the criteria by ease of use, the BSI approach is clearly the most user-friendly. It requires only a classification of the load case and

selection of an appropriate S-N curve. Next in terms of practicality are the equivalent stress range methods, such as those in DNV and ASME, which rely on clearly defined stress combinations and simple correction factors. At the more complex end of the spectrum are interaction equation-based methods, such as those from IIW and FKM, which require iterative numerical solutions to estimate fatigue life. This added complexity is only necessary when a direct fatigue life calculation is required, as it involves additional post-processing with the iterative solution. In contrast, if the goal is merely to verify the stress state, the method involves simply comparing the stress against a comparison value, which keeps the approach relatively straightforward.

These observations provide useful context for the comparative evaluation of NP correction methods in the following chapters, particularly regarding their balance between accuracy, theoretical consistency, and usability in engineering practice.

Evaluation of Design Code NP Correction Methods

The focus is on assessing the conservatism of the selected NP correction methods when applied strictly in accordance with their respective design code formulations. This follows the combinations explicitly recommended in each code, where both the stress evaluation concept and S-N parameters are predefined.

The IIW, DNV, FKM and BSI methods are all formulated based on the hot-spot stress method and have therefore been evaluated using hot-spot stress ranges in conjunction with the corresponding S-N parameters provided in each respective design code. The ASME method, in contrast, is based on the equivalent structural stress (EESS) concept and has been evaluated accordingly using the recommended procedure and associated fatigue parameters from the Master S-N curve. As for the two selected benchmark methods, maximum principal Stress and von Mises, they are assessed using the nominal stress concept in combination with the corresponding design S-N curves. All the S-N parameter has been set to the 97.7% confidence level, to assess the conservative from the design code perspective.

Illustrations of the design code based, and benchmark methods are presented using N_{calc} - N_{exp} plots, which compare the predicted number of cycles to failure (N_{calc}) with the experimentally observed number of cycles to failure (N_{exp}). These plots are provided in Fig. 5 and Fig. 6.

Each case has been evaluated based on the following statistical metrics: prediction ratio (Pre. Ratio), mean absolute log error (Scatter) and standard deviation of log error (StDv). These metrics quantify the systematic offset, spread, and variability of the prediction accuracy, respectively, and are calculated as follows. A summary of the computed values for each method is presented in Table 2.

$$\begin{aligned}
 Pre.Ratio &= 10^{-\bar{b}} & \bar{b} &= \frac{1}{n} \sum_{i=1}^n \log_{10} \left(\frac{N_{calc,i}}{N_{exp,i}} \right) \\
 Scatter &= \frac{1}{n} \sum_{i=1}^n \left| \log_{10}(N_{calc,i}) - \log_{10}(N_{exp,i}) \right| \\
 StDv &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\log_{10}(N_{calc,i}) - \log_{10}(N_{exp,i}) - \bar{b})^2}
 \end{aligned}$$

Here, $\bar{b}$ represents the mean offset between predicted and experimental fatigue lives in logarithmic space. The corresponding prediction ratio, defined as $10^{-\bar{b}}$, expresses this offset in linear space and can be interpreted as a multiplicative factor deviation from the ideal prediction line $N_{calc} = N_{exp}$. In these expressions, n denotes the total number of data points, $N_{calc,i}$ is the predicted fatigue life for sample i , and $N_{exp,i}$ is the corresponding experimentally measured fatigue life.

1.11. Benchmark Methods

Fig. 5 presents the $N_{calc} - N_{exp}$ plots for the benchmark methods: maximum principal stress and von Mises. It is evident that the data under proportional loading conditions lie on the conservative side, suggesting that the use of a 97.7% confidence interval contributes to a conservative fatigue life prediction.

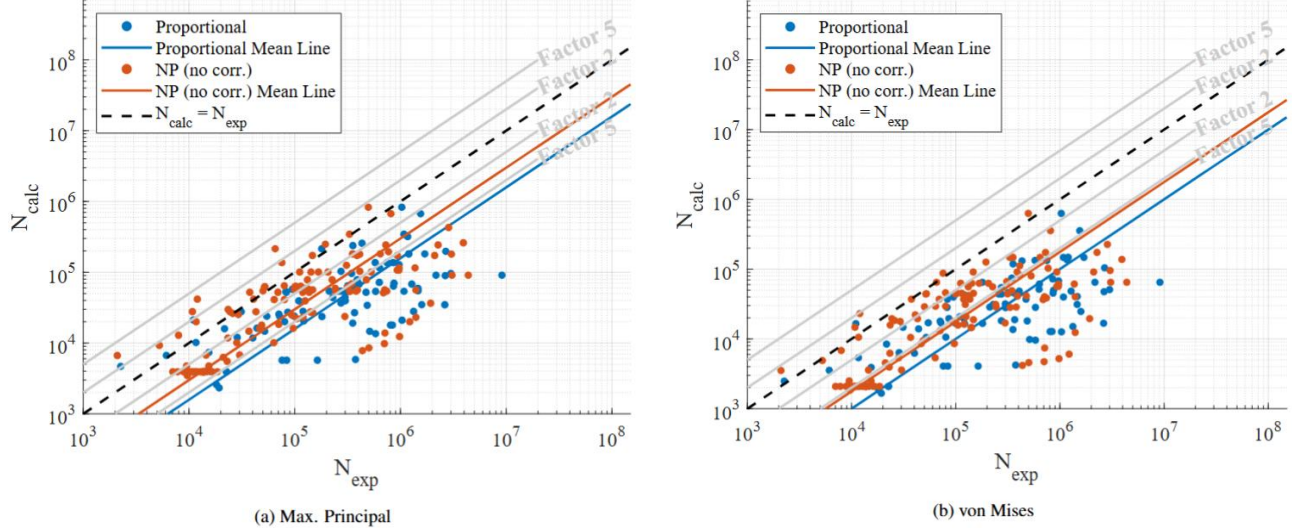


Fig. 5 $N_{calc} - N_{exp}$ graphs for the two benchmark methods fatigue life prediction methods. Each includes data points for proportional and NP loading with and without NP correction.

In this context, conservativeness is represented by a prediction ratio greater than 1, while a ratio below 1 would indicate non-conservativeness. The prediction ratios for proportional loading are 6.33 and 10.00 for the maximum principal stress and von Mises methods, respectively. These values reflect the expected conservative behaviour inherent in design methods and serve as a baseline for evaluating other approaches. While the von Mises method is generally more conservative than the maximum principal stress method, both remain within the bounds of safe design practice.

Under non-proportional loading, a reduction in predicted fatigue life is observed. Specifically, the prediction ratio decreases by 47% for the maximum principal stress method (from 6.33 to 3.34) and by 43% for the von Mises method (from 10.00 to 5.56). This reduction supports the widely accepted understanding that non-proportional stress states in welded steel structures have a reducing effect on fatigue life. Highlight the need for a correction method to achieve the expected inherent conservatism.

1.12. Design Code Methods

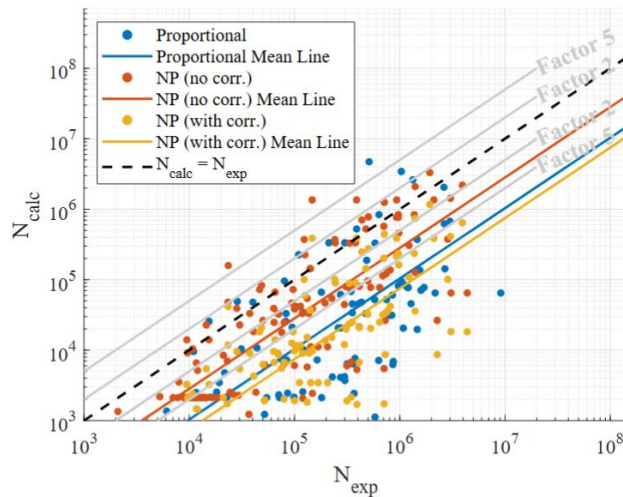
After establishing the standard inherent conservativeness expressed by the prediction ratios, the same approach can be applied to the five design codes. The results are visually illustrated in the Fig. 6 and the bar plot in Fig. 7, further summarized in Table 2, and discussed in the following.

Fig. 7 Prediction ratio across methods and loading types. A ratio of 1.0 indicates agreement between experimental and calculated fatigue lives; however, since S–N curves include a 97.7–99% confidence interval (depending on the design code), conservative results with ratios above 1.0 are expected. The blue bars show proportional loading (inherent conservatism), the orange bars show NP loading without correction, and the yellow bars show NP loading with correction. Ideally, yellow and blue bars should align, indicating full compensation of NP effects. Percentage labels give the relative change compared to proportional loading.

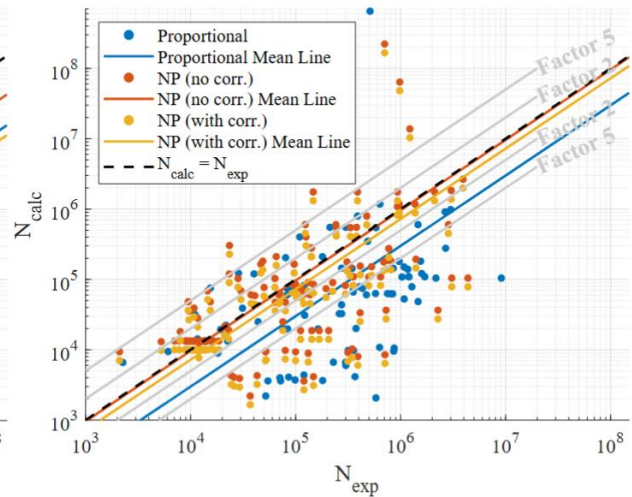
For reference, an ideally well-calibrated NP correction would bring the prediction ratio for NP loading (yellow bar) to the same level as the proportional case (blue bar), restoring the same level of inherent conservatism while avoiding excessive overprediction.

Table 2 Summary of statistical metrics for fatigue life prediction methods under proportional and NP loading. Metrics include Mean Log. Bias Offset (conservativeness), standard deviation (StDv), and scatter (average log error).

Method	Loading Type	Pre. Ratio	StDv	Scatter	Comment
Max. Prin.	Proportional	6.33	0.49	1.00	Conservative predictions with moderate scatter and low variability.
	NP (no NP correction)	3.34	0.52	0.76	Reduced conservativeness but improved accuracy under NP loading.
Von Mises	Proportional	10.00	0.51	0.82	Moderate conservativeness and low scatter.
	NP (no NP correction)	5.65	0.51	0.58	Less conservative but more accurate under NP conditions.
IIW	Proportional	9.66	0.39	0.64	Conservative but with the lowest variability for proportional loading.
	NP (no NP correction)	3.55	0.58	0.64	Less conservative but slightly higher variability.
	NP (with NP correction)	13.36	0.57	1.14	Highly conservative but with increased scatter and variability.
DNV	Proportional	3.34	1.12	0.95	Moderate conservativeness with high variability and scatter.
	NP (no NP correction)	1.04	0.99	0.63	Near non-conservative but better accuracy with high variability.
	NP (with NP correction)	1.38	0.99	0.65	Minimal conservativeness but good accuracy, though variability is high.
ASME	Proportional	11.36	0.64	1.09	Strongly conservative with moderate variability and high scatter.
	NP (no NP correction)	4.85	0.48	0.72	Reduced conservativeness but improved accuracy.
	NP (with NP correction)	7.72	0.49	0.91	Moderately conservative with balanced scatter and variability.
FKM	Proportional	7.06	0.66	0.90	Conservative and moderately accurate.
	NP (no NP correction)	2.65	0.55	0.54	Low conservativeness but high accuracy.
	NP (with NP correction)	30.98	0.73	1.50	Very high conservativeness with high variability.
BSI	Proportional	3.29	0.81	0.78	Low conservativeness but elevated variability.
	NP (no NP correction)	1.03	0.70	0.56	Near non-conservative results with balanced accuracy.
	NP (with NP correction)	7.52	1.05	1.06	Very conservative with high variability and scatter.



(a) IIW



(b) DNV

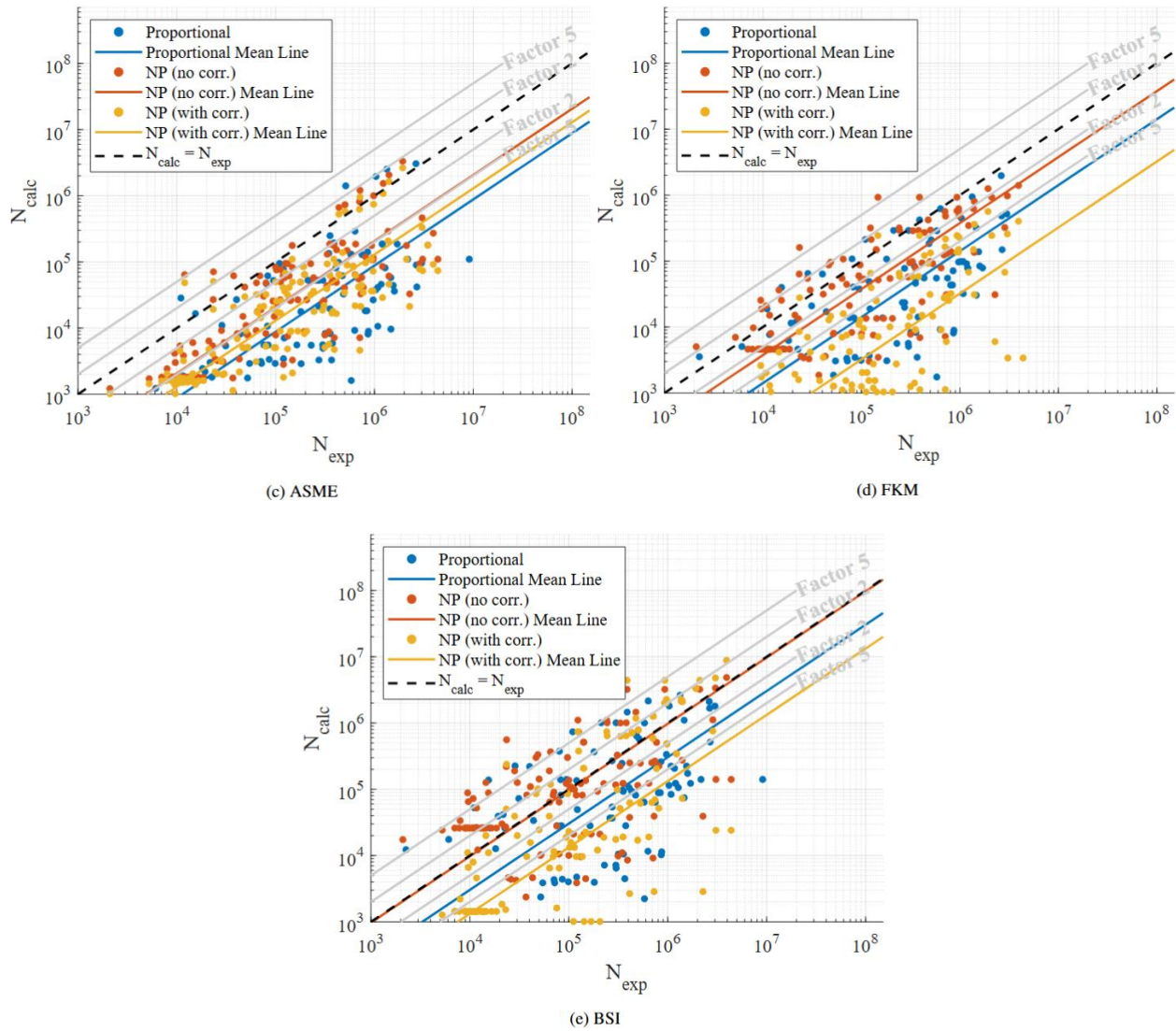


Fig. 6 $N_{calc} - N_{exp}$ graphs for the five design code fatigue life prediction methods. Each includes data points for proportional and NP loading with and without NP correction.

- IIW:** The $N_{calc} - N_{exp}$ plot and the corresponding prediction ratio bar chart for the IIW method, shown in Fig. 6, indicate that the inherent conservatism under proportional loading is consistent with the benchmark values. In contrast, the uncorrected NP data show a pronounced reduction in predicted fatigue life, exceeding the expected benchmark reduction of 63%. This confirms the detrimental effect of NP loading on fatigue performance. However, the application of the IIW non-proportionality correction significantly improves the prediction. The corrected NP prediction ratio exceeds that of the proportional case, resulting in an overcorrection of approximately 38%. While the correction restores conservativeness, it overshoots the desired target.
- DNV:** For the DNV approach, the prediction ratio under proportional loading falls outside the estimated standard range of inherent conservativeness. With a value of 3.34, it remains on the conservative side but is notably lower than those observed for other benchmark methods. A substantial reduction in predicted fatigue life is again observed under NP loading, with a decrease of approximately 69%, reinforcing the typical reducing effect of NP stress states.

However, the NP correction method provided by DNV yields only a marginal improvement. The correction increases the prediction ratio by just 10%, which is relatively limited in comparison to other methods. This makes DNV's correction approach one of the least effective in compensating for the impact of non-proportional loading.

- ASME:** The ASME method exhibits the highest inherent conservatism under proportional loading, with a prediction ratio of 11.36. This value slightly exceeds the estimated range observed across the benchmark methods, indicating a particularly more conservative design basis. The effect of NP loading is again evident, with a 57% reduction in predicted fatigue life compared to the proportional case. Among the five design code methods evaluated, the NP correction method from ASME provides the closest correction, reducing the prediction ratio deviation by approximately 32%. While this brings the corrected NP data closest to the proportional benchmark, the correction slightly underestimates fatigue life, in contrast to the IIW method, which tends to overestimate.
- FKM:** The proportional loading case yields an inherent conservatism of 7.06, which falls within the estimated standard range defined by the benchmark methods. As observed with other methods, the effect of NP loading is evident, with a 62% reduction in predicted fatigue life. However, this method yields an extremely conservative outcome, with a prediction ratio of 30.98, corresponding to an overestimation of conservatism by 338%. This overly conservative behaviour can be attributed to the use of a power of 1 in the interaction equation. As shown in Fig. 4, the corresponding interaction line lies almost entirely below all the experimental data points, indicating that no mean-based calibration of this power value has been applied.
- BSI:** The proportional prediction ratio of 3.29 is the lowest among all methods and falls below the estimated standard range of inherent conservatism. It closely aligns with the conservatism observed in the DNV approach. Once again, the detrimental impact of NP loading is evident, with a 69% reduction in predicted fatigue life. The NP correction method prescribed by the BSI design code results in a highly conservative outcome, increasing the prediction ratio by 129%. This makes it with FKM one of the very conservative correction approaches among the methods evaluated, even before applying the additional reduction of the endurance limit required for out-of-phase combined loading. If the halve endurance limit should be applied then the prediction ratio would increase with 6629%, making this correction method the most conservative. While this approach leads to significant overestimation of fatigue life, it is also the simplest to implement.

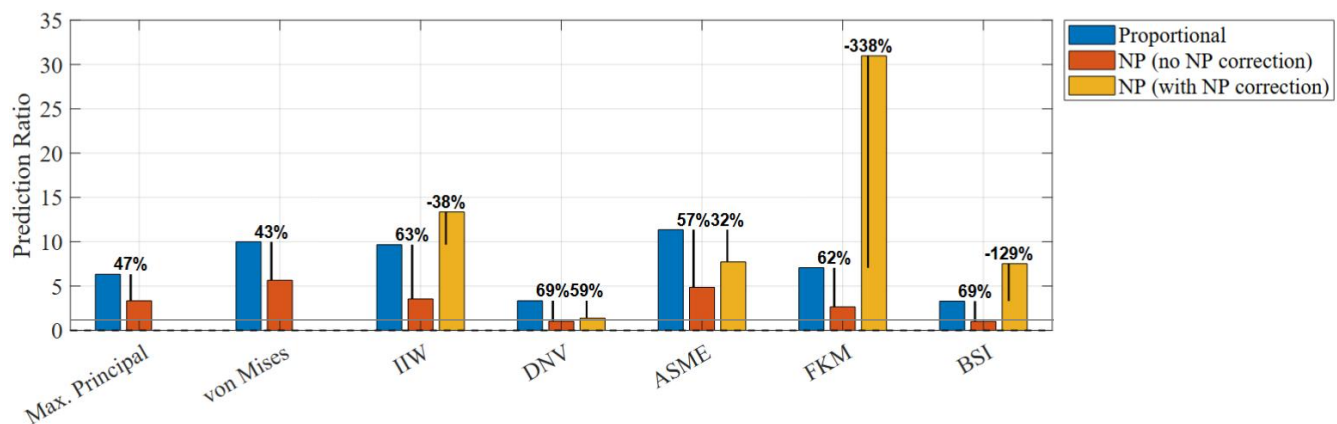


Fig. 7 Prediction ratio across methods and loading types. A ratio of 1.0 indicates agreement between experimental and calculated fatigue lives; however, since S–N curves include a 97.7–99% confidence interval (depending on the design code), conservative results with ratios above 1.0 are expected. The blue bars show proportional loading (inherent conservatism), the orange bars show NP loading without correction, and the yellow bars show NP loading with correction. Ideally, yellow and blue bars should align, indicating full compensation of NP effects. Percentage labels give the relative change compared to proportional loading.

The estimated range of inherent conservatism, based on the benchmark methods, lies between prediction ratios of 6.33 and 10.00. Within this context, the DNV and BSI methods fall below the range, ASME slightly exceeds it, while IIW and FKM lie within the expected bounds. Methods falling below this range are considered less conservative, but still demonstrate prediction ratios around 3, suggesting they remain on the safe side while providing estimates closer to experimental results. The IIW and FKM methods align well with the expected level of conservatism, making them more suitable for practical design purposes. ASME, although slightly more conservative than the range, still presents an acceptable safety margin for design purposes.

Regarding the NP stress state, all methods confirm a reduction in predicted fatigue life, consistent with the known effects of NP loading. However, the extent of this reduction varies across methods. When evaluating the NP correction approaches provided by the five design codes, none fully restores the original level of design conservatism observed under proportional loading.

Best Practice Recommendations

Overall, it can be stated that all correction methods shift the NP data toward the inherent conservatism observed under proportional loading. However, both the FKM and BSI methods tend to overcorrect, resulting in excessively conservative predictions. An opposite concern applies to the DNV method, which exhibits the lowest correction resulting in limited effectiveness in restoring the expected conservatism.

In contrast, the IIW and ASME approaches demonstrate better overall consistency. They exhibit more accurately recover the inherent conservatism characteristic of proportional loading. These attributes make IIW and ASME the most reliable correction methods among the five design codes evaluated in this study.

In choosing the most suitable correction method, practical implementation aspects must also be considered. Each method presents challenges that may affect its applicability in engineering practice. The following outlines the key considerations associated with the IIW and ASME methods:

- **The IIW method** requires the selection of an appropriate S-N curve based on the specific design detail, which demands a solid understanding of how to choose a detail correctly. This can be challenging for users unfamiliar with the process. Additionally, if the number of cycles to failure is to be determined, an iterative approach is necessary. However, if the goal is simply to assess the validity of a design, the formulation can be applied and compared to the CV parameter, keeping the process relatively simple. The method also relies on the hot-spot stress approach, which is well-documented and fairly easy to apply.
- **The ASME method** simplifies the process by using a single master S-N curve, eliminating the need to select a specific design detail. This also allows for a more direct calculation of the fatigue life, as the number of cycles to failure can be determined based on the von Mises equivalent stress using the master curve parameters. However, instead of the hot-spot stress approach, the ASME method relies on determining the ESS. This structural stress is more complex and is not thoroughly defined in the ASME design code, making it difficult to interpret without additional guidance. For this reason, a supplementary interpretation is necessary to clarify the specific approach adopted in this work, which is provided in 26Appendix B.

Given these considerations, the recommended approach is to apply the IIW correction method as the primary choice, as it yields a slightly conservative outcome under NP correction. The ASME method is recommended as a secondary option. While it performs slightly less conservatively for NP correction, it offers the advantage of using a single master S-N curve, which simplifies the process by removing the need for detail-specific curve selection, provided that the $E(E)SS$ can be accurately determined and applied.

Suggest NP Correction Factors/Quantifiers

Alternatively, based on these analyses, it is possible to evaluate the correction required to restore the inherent level of conservatism back to the level observed under proportional loading. For each of the seven methods examined in this study, an NP correction factor (f_{NP}) or quantifier (q_{NP}) has been applied to their fatigue life expression.

The factor/quantifier was determined by identifying the value needed to align the NP prediction ratio with the proportional baseline. The formulation and corresponding values of f_{NP} and q_{NP} are presented in Table 3.

Table 3 Summary of Evaluated NP Correction Factors (f_{NP}) for Design Codes and Associated Interpretations

Design Code	Suggested NP Correction		Comment
Max. Prin.	$\sigma_{1,1} = f_{NP} \left(\frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \right)$	$f_{NP} = 1.247$	No direct comparison is available, but the correction implies a 23.7% increase in equivalent stress.
Von Mises	$\sigma_{vM} = f_{NP}(\sqrt{J_2})$	$f_{NP} = 1.211$	Like the Max. Prin. case, no benchmark correction exists, but this yields a 21.1% stress increase.
IIW	$\left(\frac{\Delta\sigma_{\perp}}{\Delta\sigma_{\perp,Ref(N)}} \right)^2 + \left(\frac{\Delta\sigma_{\parallel}}{\Delta\sigma_{\parallel,Ref(N)}} \right)^2 + \left(\frac{\Delta\tau_{\parallel}}{\Delta\tau_{\parallel,Ref(N)}} \right)^2 = q_{NP}$	$q_{NP} = 0.591$	The IIW method is generally considered the most suitable. The derived f_{NP} is likewise close to the original CV of 0.5.
DNV	$\Delta\sigma_{eq,DNV} = f_{NP} \sqrt{\Delta\sigma_{\perp}^2 + 0.81\Delta\tau_{\parallel}^2}$	$f_{NP} = 1.475$	Compared to the original factor of 1.1 corresponding to 10% increase in stresses, this adjustment greatly increases the stress by 47.5%.
ASME	$\Delta\sigma_{eq,ASME} = f_{NP}(\Delta\sigma^2 + 3\Delta\tau^2)^{0.5}$	$f_{NP} = 1.310$	The original correction factor $1/\sqrt{0.5} \approx 1.41$ can be slightly reduced to 1.310 based on the data.
FKM	$\left(\frac{\Delta\sigma_{\perp}}{\Delta\sigma_{\perp,Ref(N)}} \right)^{q_{NP}} + \left(\frac{\Delta\sigma_{\parallel}}{\Delta\sigma_{\parallel,Ref(N)}} \right)^{q_{NP}} + \left(\frac{\Delta\tau_{\parallel}}{\Delta\tau_{\parallel,Ref(N)}} \right)^{q_{NP}} = 1$	$q_{NP} = 1.675$	A power adjustment method yielded a best-fit exponent of 1.675.
BSI	$\Delta\sigma_{eq,BSI} = f_{NP}(\max(\Delta\sigma, \Delta\tau))$	$f_{NP} = 1.475$	Based on the normal stress S-N curve, this correction implies a 47.5% increase in stress.

Benchmark correction methods for the maximum principal stress and von Mises approaches, both evaluated with the nominal stress method, yielded factors of 1.237 and 1.211, respectively. As these are factors, they can be directly correlated with a stress increase of 23.7% and 21.1%, required to compensate for the NP effect. The same approach can be done for the DNV and BSI methods, based on hot-spot stress evaluations, each exhibited a 47.5% increase in stress, while the ASME method, which uses the E(E)SS approach, showed a 31% increase.

For the IIW and FKM methods that utilize interaction equations, these can no longer be directly compared to a stress increase and instead are expressed as a quantifier. For this interaction equation formulation two different approaches were applied: adjusting the CV value for IIW and modifying the power exponent for FKM. The FKM formulation yielded a best-fit power value of 1.675. Although further refinement could be achieved by allowing the exponents on each term to vary independently, a uniform exponent was used for simplicity. For IIW, the NP-

corrected CV was found to be 0.591, which is reasonably close to the original CV of 0.50, suggesting minor correction was needed.

These suggested NP factors are not only useful for comparison purposes but may also serve as a reasonable basis for improving current design guidelines, especially given the limited availability of supporting data in the existing literature.

Conclusions

This paper has investigated the governing design codes, IIW, DNV, ASME, FKM, and BSI, which prescribe correction methods for addressing NP stress states in fatigue analysis. The study is based on a comprehensive literature-derived database comprising 801 data points, of which 200 represent NP loading conditions. The fatigue life predictions were evaluated using the nominal, hot-spot, and E(E)SS approaches, in accordance with the respective recommendations of each design code.

From this investigation, the following conclusions can be drawn:

- The degree of conservatism under proportional loading conditions has been analysed and further quantified through the prediction ratio for each design code.
- NP stress states result in a measurable reduction in predicted fatigue life. However, the degree of reduction is strongly dependent on the correction method employed.
- All correction methods recommended by the design codes contribute to recover lost conservatism from NP load cases, though to varying extents.
- Among the evaluated methods, the IIW correction approach provides the best overall performance in terms of accuracy and ease of application. The ASME method is a strong secondary option, offering reasonable accuracy within a simpler Master S-N curve framework, but it requires estimating the stress state using the E(E)SS approach.
- Suggested NP correction factors/quantifiers have been derived for each design code. These factors/quantifiers are valid for each of the five respected design codes and offer a practical and representative enhancement of the original guidelines.

These findings contribute to a more informed application of existing design codes and support the development of more robust and consistent method for addressing NP fatigue in welded steel constructions.

Further Work

While this study has focused on the correction methods prescribed by established design codes, further work will include a comprehensive evaluation of NP correction approaches proposed in the literature. The aim will be to assess whether more effective or accurate methods exist beyond the current design code-based recommendations. Such a comparison could provide valuable insight into the potential for future improvements to design codes and a better understanding of NP fatigue behaviour.

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Appendix A. Database overview on Non-Proportional Fatigue Studies

This appendix presents an overview of the experimental database on NP fatigue in welded steel joints, including both tabulated data and visual representations of specimen designs.

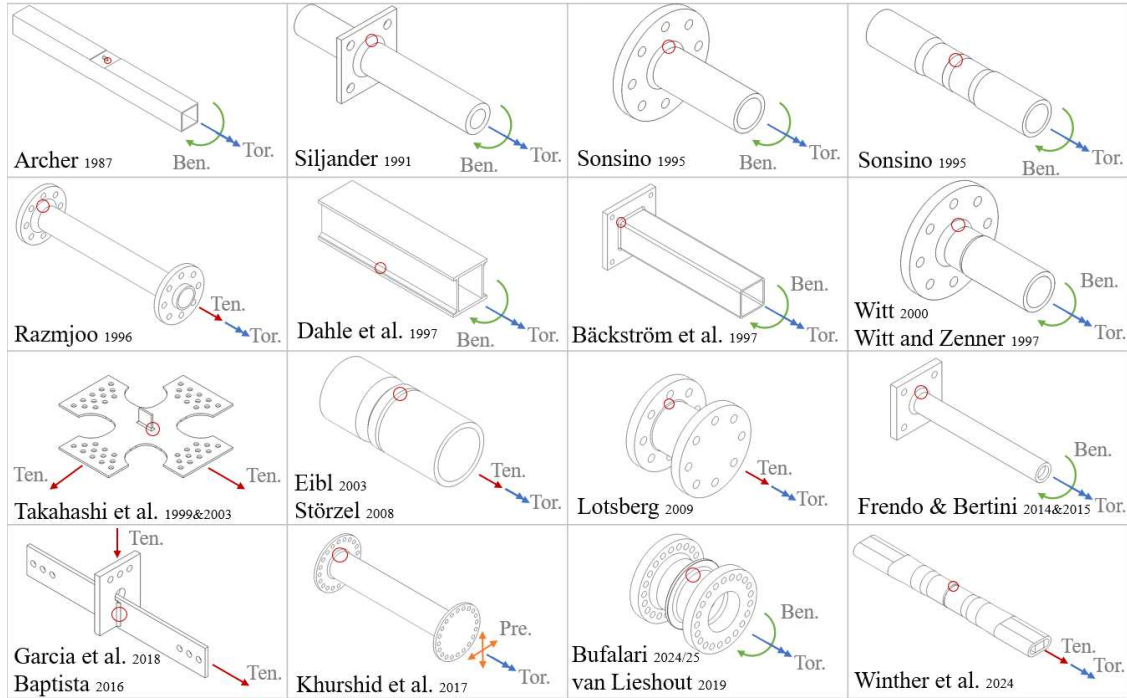


Fig. 8 CAD illustrations of specimen designs from the literature, showing various loading conditions applied in NP fatigue experiments. The applied loads include tension (ten.), torsion (tor.), bending (ben.), and pressure (pre.), with their respective directions indicated by arrows. The red circles mark the critical locations where fatigue failure was observed or analyzed in each study.

Table 4 Summary of publications on non-proportional fatigue experiments in welded steel specimens. The table details the specific NP aspects varied to induce an NP stress state, including differences in R-ratios, load patterns, phase shifts, and frequency variations. Additionally, it provides information on the weld type, weld condition, and the most frequently observed fatigue failure location.

Publication	Load	R	Pattern	Phase	Freq.	Weld Type	Weld Condition	Failure
Archer [15]	Ben.-Tor.	-	-	✓	-	Partial, Fillet	As welded	Toe
Siljander [17]	Ben.-Tor.	-	Mix	Mix	-	Partial, Fillet	Stress relieved	Toe
Sonsino TF [30, 36]	Ben.-Tor.	-	-	✓	-	Partial, Fillet	Stress relieved	Toe
Sonsino TF [30, 37]	Ben.-Tor.	-	-	✓	-	Full, Fillet	Stress relieved	Toe
Sonsino TT [30, 36]	Ben.-Tor.	-	-	✓	-	Full, Fillet	Stress relieved	Toe
Sonsino TT [30]	Ben.-Tor.	-	-	✓	-	Flushed, Butt	Stress relieved	Toe
Razmjoo [31]	Ten.-Tor.	-	-	✓	-	Partial, Fillet	As welded	Toe
Bäckström et al. [33]	Ben.-Tor.	-	-	✓	-	Full, Fillet	As welded	Toe
Dahle et al. [45]	Ben.-Tor.	-	-	✓	-	Partial, Fillet	As welded	Root
Witt & Zenner [32]	Ben.-Tor.	-	-	✓	-	Partial, Fillet	Stress relieved	Toe
Witt [9]	Ben.-Tor.	-	-	✓	✓	Full, Fillet	Stress relieved	Toe
Amstutz et al. [22]	Ben.-Tor.	-	-	✓	-	Full*, Fillet	Stress relieved	Toe*
Takahashi et al. [28, 29]	Ten.-Ten.	-	-	✓	-	Partial*, Fillet	As welded	Toe*
Eibl [46]**	Ten.-Tor.	-	-	✓	-	Overlap, Full	As welded	Root
Störzel [47]**	Ten.-Tor.	-	-	✓	-	Overlap, Full	As welded	Root
Lotsberg [48]	Ten.-Tor.	✓	-	-	-	Partial, Fillet	As welded	Root
Bertini & Frendo [49, 50]	Ben.-Tor.	-	-	✓	-	Partial, Fillet	As welded	Root
Khurshid [51]	Pre.-Tor.	-	-	✓	-	Partial*, Fillet	As welded	Root
Garcia & Baptista [26, 27]	Ten.-Ten.	-	-	✓	-	Full, Fillet	As welded	Toe
Lieshout & Bufalari [25, 23, 24]	Ben.-Tor.	-	-	✓	✓	Full, Fillet	As welded	Toe*
Winther et al. [34]	Ten.-Tor.	-	-	✓	-	Full, Butt	As welded	Toe

* Assumption inferred from the information in the publication without being explicitly mentioned.

** Laser welded.

Appendix B. Interpretation of Dong's Method

In 2001, P. Dong proposed a structural stress method [52, 39] for evaluating the fatigue resistance of welded joints. The approach decomposes the total structural stress σ_s into two components: the membrane stress σ_m and the bending stress σ_b , such that:

$$\sigma_s = \sigma_m + \sigma_b$$

Rather than evaluating the stress directly at the weld toe (section A-A), Dong's method considers the stress distribution at two cross-sections, A-A and B-B, separated by a small distance d , as illustrated in Fig. 9. This enables mesh-insensitive stress extraction.

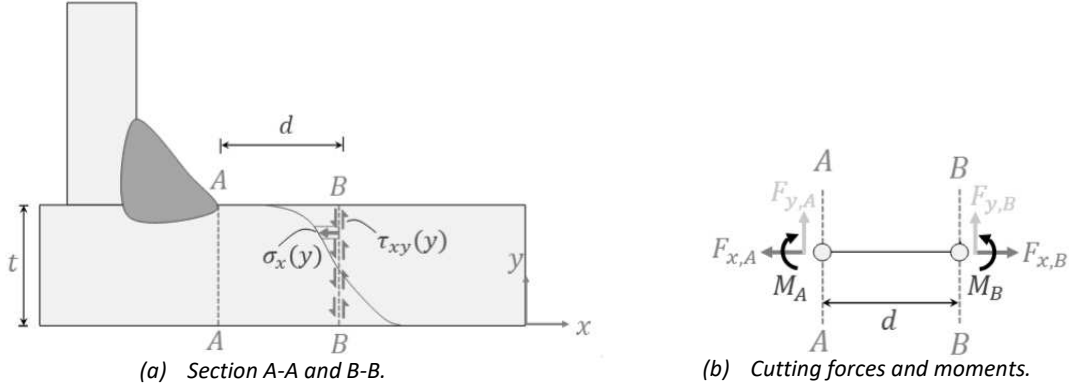


Fig. 9 Structural stress concept according to P. Dong [39]

The membrane stress is defined as the stress normal to the weld surface averaged through the plate thickness t :

$$\sigma_m = \frac{1}{t} \int_0^t \sigma_x \, dy$$

Although it is recommended to evaluate this at section B-B, the membrane stress should theoretically remain constant along the x -direction:

$$F_{x,A} = F_{x,B}$$

$$\sigma_{m,A} = \sigma_{m,B}$$

For bending around the z -axis, the moment and shear force at B-B must satisfy equilibrium with those at A-A:

$$M_A = M_B + d \cdot F_y$$

Due to the small distance d , classical beam theory (Euler–Bernoulli) may not apply. Thus, the moment is computed directly from the through-thickness stress distribution:

$$M = \int_0^t \sigma_x \cdot y \, dy$$

Assuming the coordinate origin is located at the bottom of the plate (as in Approach B), the moment at A-A becomes:

$$M_A = \int_0^t \sigma_{x,B}(y') \cdot y' \, dy' + d \cdot \int_0^t \tau_{xy,B} \, dy'$$

The same moment can also be expressed at A-A as the sum of membrane and bending contributions:

$$M_A = \int_0^t \sigma_{m,A} \cdot y' dy' + \int_0^t \sigma_{b,A}(y') \cdot y' dy$$

Assuming a linear bending stress distribution across the thickness, with $\sigma_{b,A}(y') = \frac{2\sigma_{b,A}}{t} y' - \sigma_{b,A}$, this becomes:

$$M_A = \sigma_{m,A} \cdot \frac{t^2}{2} + \sigma_{b,A} \cdot \frac{t^2}{6}$$

Combining the two expressions for M_A gives:

$$\frac{\sigma_{m,A} t^2}{2} + \frac{\sigma_{b,A} t^2}{6} = \int_0^t \sigma_{x,B}(y') \cdot y' dy' + d \cdot \int_0^t \tau_{xy,B} dy'$$

To compute the structural stress, the membrane component is first determined by integrating the normal stress at section B–B:

$$\sigma_{m,B} = \sigma_{m,A} = \frac{1}{t} \int_0^t \sigma_{x,B}(y') dy'$$

Then, the bending component can be isolated from the total moment:

$$\sigma_{b,A} = \frac{6}{t^2} \left(\int_0^t \sigma_{x,B}(y') \cdot y' dy' + d \cdot \int_0^t \tau_{xy,B} dy' - \frac{\sigma_{m,A} t^2}{2} \right)$$

Appendix C. Derivation of the Equivalent Stress Range of the Gough-Pollard Expression

The operation to obtain the equivalent stress range from the interaction equation is based on the formulation by Pederson [53]. The original formulation of the Gough-Pollard ellipse quadrant [44] is as follows:

$$1 = \sqrt{\left(\frac{\Delta\sigma}{\Delta\sigma_R}\right)^2 + \left(\frac{\Delta\tau}{\Delta\tau_R}\right)^2}$$

Due to the varying notations for penalty factors across different design codes (e.g., CV, γ), and the fact that these penalty factors represent different expressions on the left side, it has simplified the notation by using the symbol A. This symbol A can then be interpreted as the individual penalty factors depending on the design code being referenced.

The first step is to isolate the expression on one side, specifically the right side. The expression is written out for better understanding of the subsequent operations.

$$\begin{aligned} A &= \sqrt{\left(\frac{\Delta\sigma}{\Delta\sigma_R}\right)^2 + \left(\frac{\Delta\tau}{\Delta\tau_R}\right)^2} \\ &= \frac{1}{\sqrt{A}} \sqrt{\left(\frac{\Delta\sigma}{\Delta\sigma_R}\right)^2 + \left(\frac{\Delta\tau}{\Delta\tau_R}\right)^2} \\ &= \frac{1}{\sqrt{A}} \sqrt{\Delta\sigma^2 \left(\frac{1}{\Delta\sigma_R}\right)^2 + \Delta\tau^2 \left(\frac{1}{\Delta\tau_R}\right)^2} \end{aligned}$$

To extract an expression for the equivalent stress range, a stress term must be isolated. In this context, the normal fatigue resistance expression, noted in the box below, is added to the shear stress term underlined above.

$$1 = \left(\frac{1}{\Delta\sigma_R}\right)^2 \left(\frac{\Delta\sigma_R}{1}\right)^2$$

By adding this term, the term $\Delta\sigma_R$ can then be isolated on the left side of the expression by using the underlined term $(1/\Delta\sigma_R)^2$ in the expression below.

$$= \frac{1}{\sqrt{A}} \sqrt{\Delta\sigma^2 \left(\frac{1}{\Delta\sigma_R}\right)^2 + \Delta\tau^2 \left(\frac{1}{\Delta\tau_R}\right)^2 \left(\frac{1}{\Delta\sigma_R}\right)^2 \left(\frac{\Delta\sigma_R}{1}\right)^2}$$

The last step is to assume that $\Delta\sigma_R$ and the equivalent stress range $\Delta\sigma_{eq}$ are interchangeable. This assumption is questionable but will not be further elaborated in this work. However, by making this assumption, a usable expression for use with the S-N curve can be obtained as shown:

$$\begin{aligned} \Delta\sigma_{eq} &= \Delta\sigma_R = \frac{1}{\sqrt{A}} \sqrt{\Delta\sigma^2 + \Delta\tau^2 \left(\frac{1}{\Delta\tau_R}\right)^2 \left(\frac{\Delta\sigma_R}{1}\right)^2} \\ \Delta\sigma_{eq} &= \frac{1}{\sqrt{A}} \sqrt{\Delta\sigma^2 + \Delta\tau^2 \left(\frac{\Delta\sigma_R}{\Delta\tau_R}\right)^2} \end{aligned}$$

Equivalent Stress Range for DNV design code

For the DNV design code, the penalty factor γ can be expressed as $\gamma = 1/\sqrt{A}$. With the addition of the expression for the ratio of normal and shear stress terms, $0.81 = (\Delta\sigma_R/\Delta\tau_R)^2$, this changes the expression to the following:

$$\Delta\sigma_{eq, DNV} = \gamma \sqrt{\Delta\sigma^2 + 0.81 \Delta\tau^2}$$

Data Availability Statement

The experimental data analyzed in this study are available in the cited publications (see Appendix A, Table 4). The compiled Excel database created for this work is available from the corresponding author on reasonable request.

Authors Contributions

N. B. Winther: Conceptualization, Methodology, Data Curation, Analysis, Visualization, Writing – Original Draft, Writing – Review & Editing. **N. Bauer:** Data Collection, Methodology, Writing – Review & Editing. **M. Faß:** Methodology, Writing – Review & Editing. **J. Baumgartner:** Data Collection, Methodology, Writing – Review & Editing. **J. H. Andreasen:** Supervision, Writing – Review & Editing. **J. Schjødt-Thomsen:** Supervision, Writing – Review & Editing.

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Competing Interests

The authors declare no competing interests.