Programmable Optical Differential Ising Machines for Phase Transition Simulation: supplemental document

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ABSTRACT

Drawing inspiration from the physical Ising model, optical Ising machines have emerged as a promising paradigm in optical computing. These machines typically utilize optical phase or amplitude modulators to represent the analog spin variables, which determine the final Hamiltonian by the combinatorial configuration of the complex optical field. However, current optical modulators exhibit relatively low modulation resolution and significant technical challenges arise when scaling up optical Ising systems to maintain consistent performance across integrated device clusters. Here we propose a programmable optical differential Ising machine that provides direct access to changes in the Hamiltonian. This system streamlines unnecessary matrix calculations, thereby reducing the requirements for the size and precision of the modulator matrix. Through numerical simulations on 60 to 100 spin MaxCut benchmark problems, we determine that the minimum required resolutions for the DAC and ADC are 6 bits and 5 bits, respectively, along with a significant tolerance of 0.3 rad for phase errors and over 10 multiplexing operations. Additionally, 100-spin experiments with a dual-channel system demonstrate its potential for phase transition simulation and solving complex optimization challenges.

1 Numerical simulation results

1.1 Influence of AD bit resolution

The probability of reaching the ground state at 90%, 99%, and 100% for 60 and 80 spin instances is visualized in Figure1(a)-(b). We can observe that the performance inflection point for reaching the ground state at 90% is at 3 bits, while the inflection point for reaching 99% and 100% is at 6 bits. A resolution of ≥ 7 bits can be considered a stable high-performance region, where increasing the number of bits does not yield a noticeable performance improvement. Furthermore, Figure 1(c) visually summarizes the minimum required and optimal bit resolutions for the "g05_N.x" problem set. This plot is color-coded by problem size (N=60,80,100), with the horizontal axis corresponding to the problem number and the vertical axis representing the AD bit resolution for each respective problem. The optimal bit and the minimum bit are displayed through a line and bar graph, respectively. The optimal bit resolution for the 30 instances demonstrate variability, likely due to the inherent differences among the problems. It is predominantly distributed between 8 and 12 bit, with a minimum of 6 bit. But there exists a stable value on the minimum required bit resolution of 6 bit. And this advantage does not seem to change as the problem size increases.

1.2 Influence of DA bit resolution

The impact of DA bits on the probability of reaching the maximum cut value with "ising3.0-N_x" instances is shown in Figure 2(a). Similarly, the probability of reaching 90% and 97% of the maximum cut value gradually increases with the number of DA quantization bits, and the performance inflection points occur at 3 bits and 5 bits, respectively. The probability of reaching 99% is nearly zero, and it only succeeds in 6-bit and 8-bit resolution. Figure 2(b) shows that, with the exception of "ising3.0-100_5555" (Instance 3), which requires 15 bits, the optimal DA bit resolution for the remaining instances is distributed between 6 and 10 bits. And a DA resolution range as low as 3 to 5 bits is sufficient to ensure the acquisition of the optimal solution, a result that significantly reduces the resolution requirements for the modulator.

1.3 Influence of random phase error

As shown in Figure 3 (a)-(b), under a random phase error of less than 0.3 rad, the success rate in identifying the maximum cut value or a cut value that exceeds 99% of the maximum cut value remains relatively stable. However, when the phase error exceeds 0.4 rad, the success rate starts to decline sharply and drops to nearly 0 once the noise reaches 0.5 rad. Figure 3(c) presents the statistics for optimal random phase errors in 30 instances. An interesting phenomenon is that the optimal performance point is not near zero error but is distributed between 0.1 and 0.3 rad. It is speculated that a certain level of noise helps the system escape from local optima, thereby finding the global optimal solution.

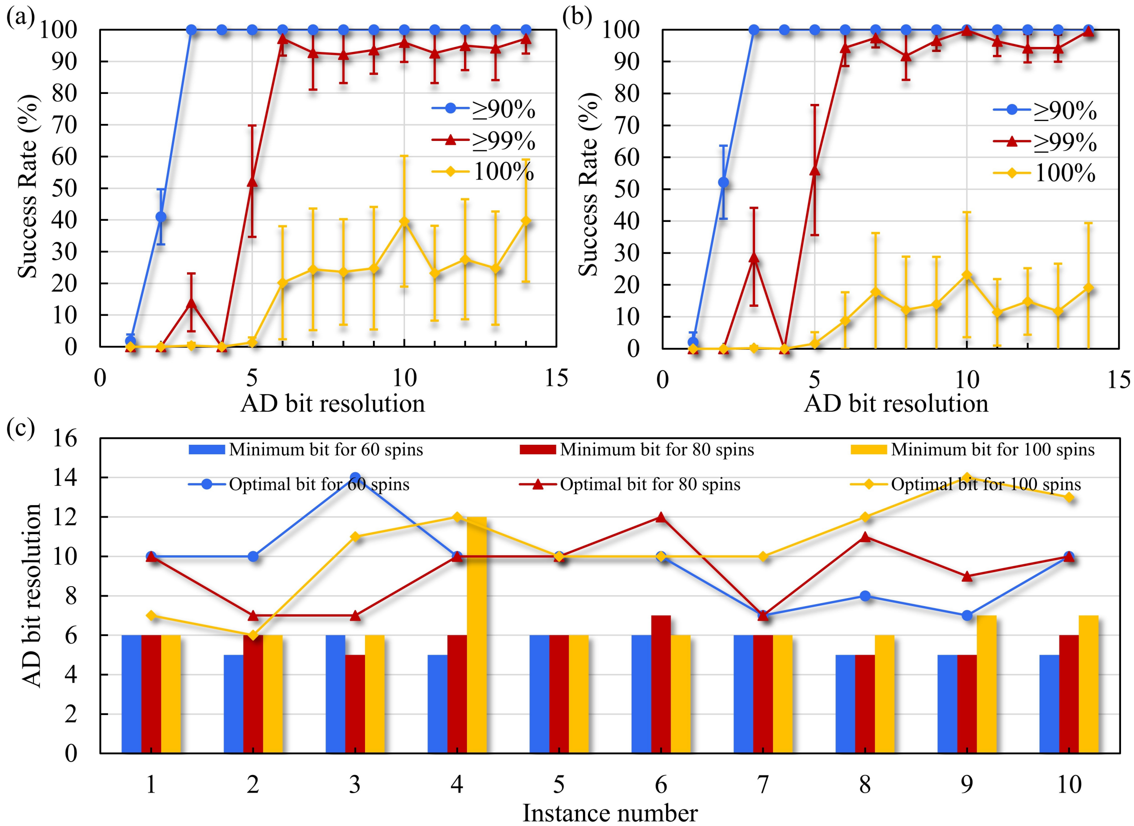

Figure 1. Influence of AD bit resolution on the ODIN. (a)-(b) The success rate of reaching the ground state over 90%, 99%, and 100% for three different problem sets (spin = 60 and 80) with different DA bits. (c) The required bit resolution and optimal bit resolution for three different problem sets.

1.4 Influence of time division multiplexing time

It can be observed from Figure 4 (a)-(b) that the success rate is high with small numbers of TDM and declines sharply after more than 10 multiplexing operations. Within 10 times, $\geq 99\%$ of the maximum cut can be reached and the limit to reach 90% of the maximum cut is 20. Based on the statistical analysis of the search results from 30 instances, Figure 4 (c) summarizes that the maximum number of multiplexing operations achievable is within the range of 10 to 12.

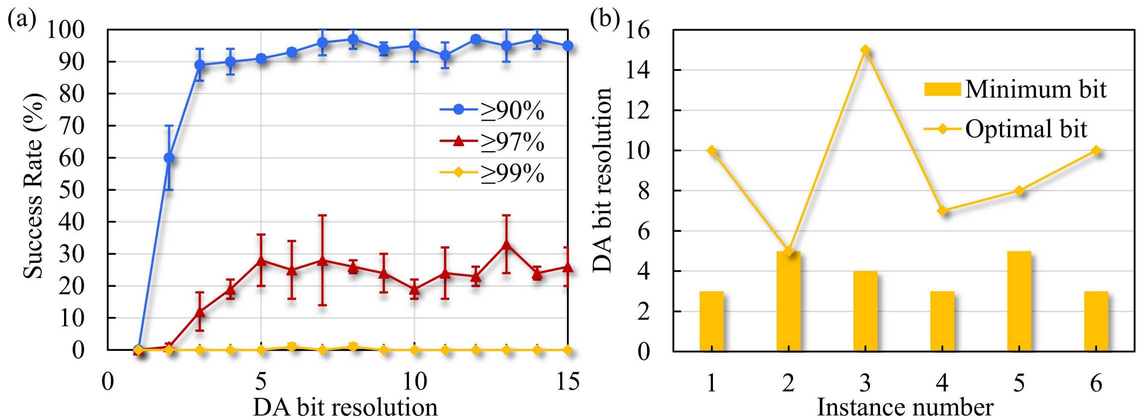

Figure 2. Influence of DA bit resolution on the ODIN. (a)-(b)The success rate of reaching the ground state over 90%, 97%, and 99% for 3 "ising3.0-N_x" instances with different DA bits. (b) The required bit resolution and optimal bit resolution for 3 "ising2.5-N_x" and 3 "ising3.0-N_x" instances.

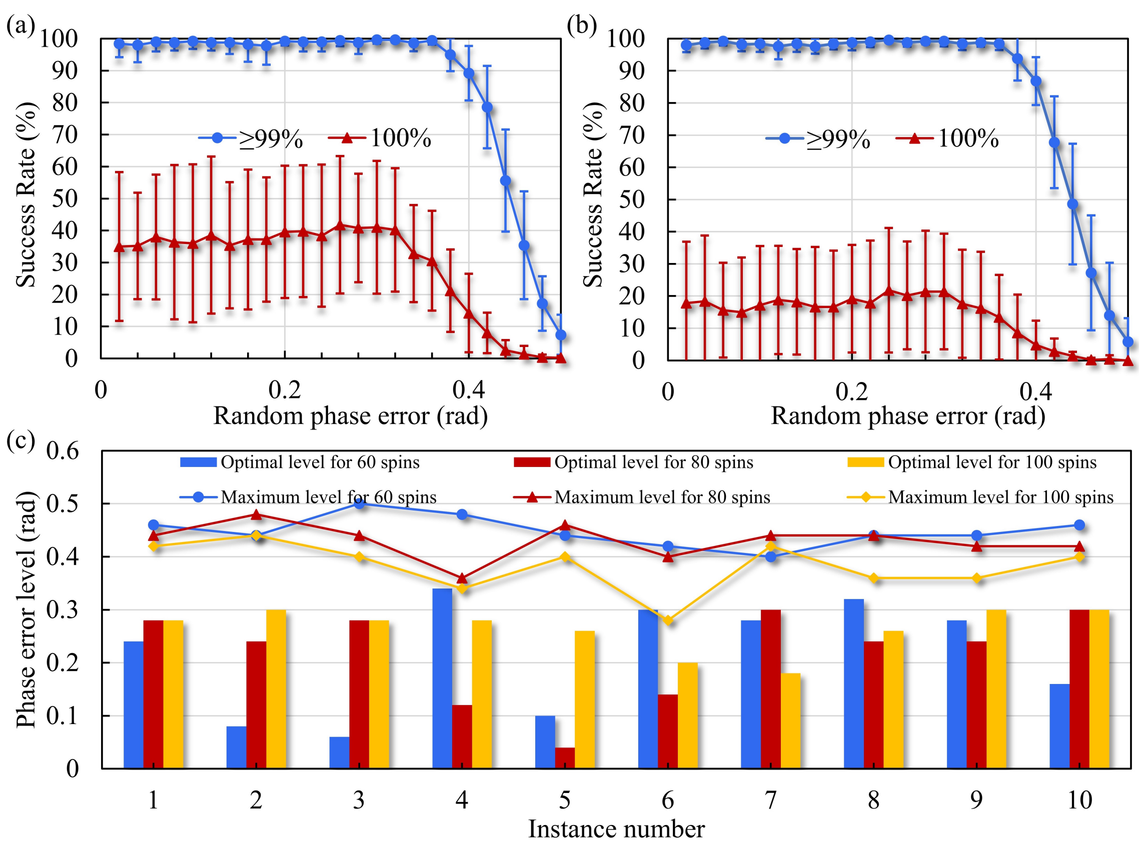

Figure 3. Influence of random phase error on the ODIN. (a)-(b) The success rate of reaching the ground state over 99%, and 100% for other "g05_N.x" MaxCut problem sets (spin = 60 and 80) with different level random phase error. (b) The maximum allowable error level and the optimal error level for three different problem sets.

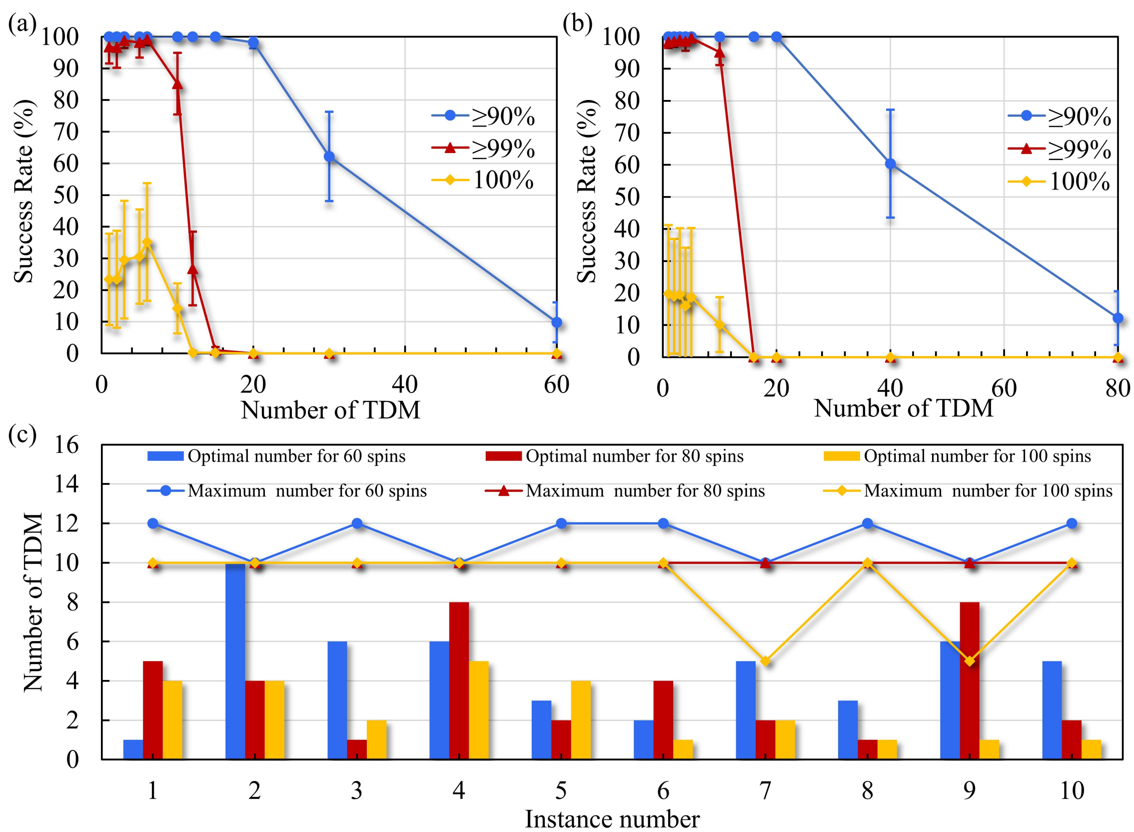

Figure 4. Influence of TDM number on the ODIN. (a)-(b) The success rate of reaching the ground state over 90%, 99%, and 100% for other "g05_N.x" MaxCut problem sets (spin = 60 and 80) with different TDM numbers.(b) The maximum and optimal TDM number for three different problem sets.