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Tax treatment of commuter cost

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Abstract

The paper discusses the tax treatment of commuting where wages and housing cost vary across locations. An income tax distorts the locational choices of agents, who dislike commuting and have preferences for consumption and place of residence. Wages, housing cost and commuting cost determine how subsidising or taxing commuting affects behaviour and social efficiency. A subsidy encourages commuting and induces agents to choose a more favourable living place. The analysis clarifies the circumstances in which the behavioural responses alleviate or exacerbate the tax distortions, also where housing is tax favoured, as is often the case in practice. A central purpose is to develop a framework that lends itself to empirical application. An empirical illustration based on Norwegian data shows how one can infer efficiency effects of responses to subsidies on commuting.

Keywords: income tax; commuting, commuter cost; subsidies on commuting, place of residence

JEL Classification: H21, H24

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1. Introduction[1]

Many workers incur sizeable travel expenses commuting between their home and their workplace. In many countries commuting to work is subsidised, for instance by making travel expenses deductible against gross income when computing taxable income[2]. A central question is whether there is a social efficiency case for this arrangement. A conceivable argument is that it may mitigate the distortionary effect of income taxes by lowering the cost of working to stimulate labour supply. This is an argument in the spirit of a strand of literature on how commodity taxes or subsidies can alleviate labour supply distortions (see, for example, Corlett and Hague (1951), Christiansen (1984)). This is however a contentious issue in the commuting context. It has been argued that, rather than encouraging people to travel to well-paid jobs, a travel subsidy prompts people to choose attractive places to live even if it involves commuting to work. A further circumstance of relevance is that in many countries housing is subsidised due to various tax favours, which constitutes another pre-existing distortion.

The current paper discusses subsidies on commuting from a social efficiency perspective in a context where people choose place of residence and choose among workplaces offering different wages. The government taxes wages by an income tax, which discourages workers from pursuing higher earnings through choice of workplace, thus creating a tax distortion.

We analyse the tax treatment of commuting in a setting where the attainable wage and the housing cost are increasing along any line from a point on the periphery to a central point. Based on pure preferences, each agent has a preferred place of residence, and a living place is less valued the further away it is located from the favourite place of residence. The geographical wage dispersion reflects that labour is not equally productive in all places. Besides the spatial variation, people have different innate abilities and are not equally productive even in the same place, as is a standard assumption in optimal income tax models following Mirrlees (1971). How steeply the wage of an agent increases along a centre-bound line depends positively on his/her productive skill. Each agent chooses a place of residence and a workplace along the wage line. The commuting distance is the distance between the two points. Beyond the concern with living place, the individuals derive utility from consumption and shortness of commuting distance. The circumstances determining the choices of the agent are the after-tax wage, the commuting cost, the cost of housing, and the preferences.

Our analysis differs from previous studies in various ways both with regard to purpose and model assumptions, to be detailed below. Beyond modelling theoretically the trade-offs of commuters, a key purpose is to establish a framework for empirical assessment of introducing/changing subsidisation of commuting. Distinctive assumptions are the presence of a continuum of workplaces and the existence of preferences for places of residence.

By including residential preferences, we deviate from the common assumptions in previous literature (that we elaborate upon below) where people are indifferent between living places because saving on commuting costs or achieving a larger income are always offset by a larger housing cost. Our assumption opens for the possibility that agents may refrain from a move that

[1] This paper is part of the research at Norwegian Fiscal Studies at the Department of Economics, University of Oslo. We gratefully acknowledge comments by XX and other participants at the conference of the International Institute of Public Finance in 2024 and comments by Thor Olav Thoresen.

[2] Two methods are common: The deduction is based on declared actual cost or commuting distance.

would be financially beneficial because it would be a move to an otherwise less attractive place. These are observable empirical cases. We also deviate from the common assumption that agents only incur a pecuniary travel cost by adding a direct disutility due to commuting.

Where people have a preferred place of residence, a subsidy has two effects. It encourages commuting and it induces an agent to move closer to the preferred place of residence, which may be a move towards a higher-paying or a lower-paying workplace. In the former case the overall effect on earnings is unambiguously positive and a subsidy mitigates the income tax distortion of earnings. In the latter case the opposite effect is a conceivable outcome. One should note that when introducing a subsidy, it is the change in *earnings* that determines the impact on social efficiency. Several empirical studies have estimated the effect on the length of commutes. This is an interesting positive issue, but from a normative perspective a crucial question is whether a longer commute leads to a larger wage and not only to longer journeys to better living places.

Combining our theory of marginal trade-offs of commuters with data on the prices, wages and taxes determining behaviour, we show how one can infer empirically the efficiency effects of marginally changing a subsidy on commuting. Where a marginal relocation of place of residence or workplace is economically harmful (beneficial) the commuter must obtain a non-pecuniary benefit (incur a disbenefit) from the move for the initial position to be a private optimum. It follows whether the move is desirable or not according to the pure preferences of the commuter. We can then infer the geographical direction of the responses to a marginal subsidy. Also computing empirically, the tax/subsidy wedges, we can then assess whether the induced responses are socially beneficial or harmful under the prevailing tax distortions. We illustrate this approach by using data for the region around Oslo in Norway. To capture differences in productive skill, we distinguish between more and less educated agents. We conclude that a larger subsidy on commuting enhances efficiency for those with lower education while it is uncertain, but plausible, that it also enhances efficiency for those with higher education, and accordingly aggregate efficiency. One should note that this is a case-specific result in our illustration. The general contribution is the approach developed to enable empirical inference about efficiency effects based on a limited set of data.

Our approach should be interpreted as taking a long-term perspective. In the short term there are presumably considerable inertia, adjustment costs and constraints when it comes to change of residence and workplace. We shall briefly mention the polar cases where place of residence is fixed, but otherwise we consider the outcome of long-term optimisation that is unconstrained by short-term concerns.[3]

In practice, workers choose labour supply at many different margins (working hours, work intensity, occupation, type of job, retirement age, etc.). To focus on commuting, we abstract from other margins in our analysis. We should however bear in mind that in cases where commuting is an unavoidable cost of working a change in the commuter cost is likely to affect an agent's willingness to participate in the labour market.

The current paper has some features in common with Wrede (2009), while deviating in other respects. In both models, people reside along a straight line, earn wage income, pay a distortive income tax, and face a, possibly subsidised, commuting cost. Both papers have spatial wage

[3] However, there are findings in the literature indicating that our caveat may be of less relevance. Boehm (2013) claims that individuals react swiftly to changes in the tax treatment of commuting expenses.

dispersion which is modelled as discrete with only two workplaces in Wrede's model and continuous in this paper. (We shall come back to the continuity assumption below.) Also, in both models a worker earns an exogenous wage at a given workplace. In the former model everybody earns the same income in each of the two workplaces, while in our model workers earn different incomes at each of a continuum of workplaces. In Wrede's model individuals have identical utility functions defined over consumption and land (housing lot size). In contrast to our model, all individuals achieve the same utility level in equilibrium. In the current model, agents have utility functions defined over consumption, commuting distance, and, more importantly, heterogeneous preferences for place of residence. In Wrede's model the housing cost has a local maximum at each workplace and a global maximum at the one paying the higher wage. The housing cost declines as one moves away from a given workplace. The reason is that the commuting cost increases and, to have an equilibrium, the larger commuting cost must be offset by a lower housing cost. The decline will continue until reaching the point where agents become better off by switching to commute to a different workplace. As in the current model there is a continuum of workplaces, moving away from a particular workplace will immediately lead to a new workplace where the wage is larger or smaller

In the current model choosing a place of residence further away from a particular workplace will increase the commuting cost and change the housing cost as in Wrede's model but a difference is that also the attractiveness of the living place may change according to each worker's preferences. A decision to move may raise or lower the housing cost depending on each worker's preference for living place. We note that in both models the housing cost is higher at a higher-paying workplace.

Wrede is explicit about the market for land, which is suppressed in our paper. We simplify by postulating directly geographical variation in housing cost. The housing price is assumed to vary monotonically with location between the periphery and the centre. Assumptions about the variation in housing cost may prompt concerns about general equilibrium effects on prices. We shall address this issue on Section 5. Before then, we may recognise that the issue is of no concern when considering individual behaviour of agents who are price takers. The current paper addresses more explicitly the locational choice of residence and the choice of commuting distance, and how the two decisions may reinforce or oppose each other. It describes more explicitly how responses to a commuting subsidy is conditional on prices, agent-specific wages and heterogeneous preferences. It also captures conceivable pre-existing distortions of housing prices due to favourable tax treatment, often observed in practice. This is important when applying our analysis to assess policy in many countries but absent in previous literature on the tax treatment of commuting expenses. A key contribution, facilitated by our set-up, is to present a framework for assessing empirically the efficiency effects of changing the subsidisation of commuting, based on a limited set of data on prices, wages, and tax rates.

In the next section we extend the presentation of related literature. Section 3 presents the model. In Section 4 we describe the agents' behaviour and responses to tax policy, while Section 5 addresses the social efficiency and welfare arguments. The subsequent section presents empirical illustrations based on Norwegian data. Section 7 concludes.

2. Other related literature.

Previous literature on tax treatment of commuting expenses takes different approaches and addresses various aspects both akin to and beyond the scope of our analysis.

In an early contribution, Wrede (2001) discusses tax deductibility of commuting expenses in the presence of wage taxes and some degree of attachment to home in a model with two regions. In each region land is scarce, and residents demand land for housing and firms use land and labour as inputs in production. Commuting is costly in terms of goods (commuting expenses) and time (leisure foregone). The government has a revenue requirement. The key question is whether (first best) efficient allocations can be supported as market equilibria where the government imposes a wage tax but allow commuting expenses to be deductible at a certain rate, which may optimally exceed 100%. A further contribution is a discussion of need for geographical differentiation of tax rates according to the residence principle.

Borck and Wrede (2005) establish a model where there is a single workplace located in the city centre, where some people are high-income earners (rich) with high commuting cost and others are low-income earners (poor) with low commuting cost. There are no residential preferences. Either the rich or the poor live in the central city and the other group in the suburbs. In each region agents are indifferent between places of residence because any extension of the commute is offset by lower housing cost.

Agrawal et al. (2024) present a positive and empirical analysis of how commuting subsidies affect commuting distances and the match quality of workers and firms. The basis is a model where workers are assumed to have a fixed residential location and choose work location trading off a higher wage against a longer commute. The empirical analysis exploits actual changes to the tax treatment of commuting expenses in German tax law. It documents that more generous commuting deductions increase commuting distance. A further result is that commuting subsidies allow for high-ability individuals to better match with higher-paying firms and thus disproportionally improving the earnings of already high-income workers. There is assortative matching.

Paetzold (2019) exploits Austrian data and tax rules to study the effect on commuting distances. of subsidising commuting expenses. The conclusion is that commuting subsidies do indeed enhance the length of the commute.

Boehm (2013) exploits data from Germany to estimate the effect of commuting costs on the decision to switch job and move houses. He concludes that individuals are more likely to move and switch jobs when tax breaks change, and they are more likely to do so for the resulting commuting distance to be shorter. A finding is that the average individual is more likely to be willing to switch jobs rather than to move houses.

Richter (2006) analyses tax treatment of commuting costs subject to the assumption that commuting does not generate additional income. Willingness to pay for commuting may rather be due to savings of housing cost. The analysis concludes that under these assumptions commuting should be taxed as such a tax would diminish the income tax distortion of labour supply defined as hours worked.

Borck and Wrede (2009) discuss the use of commuting subsidies to internalise agglomeration externalities. The paper also introduces a distinction between intracity and intercity commuting.

Wrede (2000) addresses a particular aspect of commuting, namely that commuters can cut their travel time by spending more on faster modes of travel. Time saving increases the time endowment available for work and leisure. Where the labour-leisure trade-off is distorted by an income tax, there is a case for letting commuting expenses be partially deductible when determining taxable income. The concern of that paper is neither the choice of locations of work and home nor commuting distance. The current paper, like most other papers in the field, leaves aside the choice of modes of travel.

We confine our attention to the case where there is no geographical differentiation of the tax rate across local communities within the commuting region. Wrede (2001) briefly discusses deductibility of commuting expenses where a wage tax is levied at different rates according to the residence principle. Agrawal and Hoyt (2018) address the spatial distortions created by inter-jurisdictional tax differentials.

Another aspect of transport subsidies, also beyond commuting subsidies, is concern with the spatial expansion of cities, which is addressed in Bruckner (2005). While most papers, like ours, take a normative approach focusing on social efficiency, Borck and Wrede (2005) present a political economy approach based on the commuting model outlined above in combination with assumptions about the market for land.

3. Model with preference for place of residence.

Suppose that, from some starting point, y is the distance to the workplace, and r is the distance to the place of residence, where the direction is from the periphery towards the centre. Obviously, the distance between the two places is the commuting distance. In practice there may be several routes to the centre corresponding to different starting points on the periphery. In any case, we assume that what matters is distance to living place and workplace. We denote by k the cost of travelling one unit of distance, say one kilometre or mile[4]. We assume that the output generated by an agent varies with location. We express the output as wy, where $w > 0$ is a constant parameter, which determines the wage of an agent at various locations assuming that the agent's wage is equal to the output he produces [5]. The assumption is that the wage increases with centrality. Consumption, c, is output (income) net of the commuting cost and cost of residence, p. We assume that the housing cost varies according to place of residence so that p is a function of r, $p(r)$. We shall assume that $p'(r) > 0$, i.e. $p(r)$ increases with proximity to the centre. We shall use a linear approximation so that $p'(r) = p'$ is a positive constant. We assume that an agent derives utility from consumption, c, but dislikes commuting. We note that $y \geq r$. If the converse were true ($y < r$) the agent could both diminish commuting (the distance between r and y) and obtain a larger income by increasing y for a fixed r. Denoting the commuting distance by z, we have $z = y - r$.

A variety of residential preferences are discussed in the literature. Both properties of locations (remoteness, tranquility, urban proximity, amenities, etc.) and characteristics of people influence preferences for place of residence. (A couple of examples are Schultheiss et al. (2024) and Sinniah et al. (2016)). Taking any location as point of departure, a move in either direction

[4] Travelling can cause environmental and other external costs. This is not our concern in this paper. We assume that any external costs are internalised and included in k.
[5] We may assume that nobody works at locations given by y-values close to zero.

will be accompanied by changes in locational properties that people may value differently. We shall assume that according to pure preferences, i.e., neglecting economic costs and resources, each agent has a preferred place to live, r^*, and what matters for utility is the departure of the actual place of residence, r, from r^*. The utility function is then

$u(c, z, r - r*)$ (1)

The partial derivatives of the utility function, denoted by subscripts, are

$u_c > 0, u_z < 0, u_{r-r*}$, where $u_{r-r*} > 0$ for $r < r^*$, $u_{r-r*} = 0$ for $r = r^*$ and $u_{r-r*} < 0$ for $r > r*$.

We let $m = \frac{-u_z}{u_c} = \frac{-u_y}{u_c}$ denote the marginal valuation of reduced commuting distance, which is equivalent to the marginal disbenefit incurred by travelling an extra unit of distance.

$n = \frac{u_{r-r*}}{u_c}$ denotes the marginal valuation of distance from the preferred place of residence, i.e. the positive marginal benefit of moving closer to the preferred place or the negative marginal benefit of moving further beyond it.

We shall make the following assumptions.

Assumption 1. $m_z > 0$ *and* $m_c > 0$

It seems natural that one is more willing to sacrifice general consumption to shorten the commuting distance when the initial distance is larger and when the initial consumption level is larger.

Assumption 2. $m_{r-r*} = n_z = 0$.

How the valuation of a shortening of commuting distance is affected by living closer to or further away from the preferred place of residence, i.e., the sign of m_{r-r*}, is an open question. Likewise, it is hard to tell how the valuation of place of residence is affected, if at all, by the commuting distance. Absent any clear notion of these signs, we shall at various stages set their values equal to zero.

Assumption 3. $n_{r-r*} < 0$. *Where* $n > 0$, $n_c > 0$. *Where n<0,* $n_c < 0$

We assume that the marginal benefit of moving closer to the preferred place of residence is diminishing with proximity to the favourite place, whereas the marginal willingness to pay for such a move is larger at a larger consumption level.

An example of a utility function satisfying these assumptions is $u = \alpha c^{\beta} - \sigma z^{\gamma} - \rho(r - r^*)^2$

where $0 < \beta < 1 < \gamma, \alpha > 0, \sigma > 0, \rho > 0$

Before we turn to the budget constraint, we also introduce taxes and subsidies. We assume there is an income tax with tax rate *t*. Moreover, commuting is subsidised at a rate *s* and the housing cost may be subsidised at a rate *x*. The reason for considering the latter possibility is that an important part of housing cost is mortgage interest which can be tax favoured for instance by being tax deductible. If the alternative to investing in housing is to deposit the wealth into a bank account, the interest foregone is interest net of tax where the interest is taxed, while the imputed housing rent is untaxed.

To focus on a conceivable case for subsidising, or possibly taxing, commuting, we treat the taxation of income as exogenous. This does not necessarily mean that the tax rate, *t*, is non-modifiable. To maintain a balanced budget, the income tax may change because of a subsidy (or tax) on commuting. However, where the initial *t* is optimal, there is no first order effect of a change at the margin. The crucial assumption is that there is a preexisting distortionary income tax to be allowed for when considering a subsidy on commuting. On welfare grounds, subsidies on housing may be hard to justify. However, in practice it is a fact that subsidies exist and have political support. Being pragmatic and realistic, taking housing subsidies as given may be the more interesting case when considering subsidization of commuting, but we also include the special case where housing subsidies are absent.

The budget constraint is then:

$$c = \big((1-t)w - (1-s)k\big)z + (1-t)wr - (1-x)p(r) + a \tag{2}$$

or, rearranging terms,

$$a = c + [(1-x)p(r) - (1-t)wr] - \big((1-t)w - (1-s)k\big)z \tag{3}$$

where we interpret *a* as a fixed endowment, a lump sum transfer or, if negative, a lump-sum tax. The exogenous income is available for acquiring general consumption, place of living and a desirable (preferably short) commuting distance. The net cost of moving house towards the centre, i.e., increasing *r* for a fixed commuting distance, is the additional housing cost minus the attained pay rise. The unit cost of shortening the commuting distance (lowering z) for a fixed place of residence is the wage foregone net of the saving of commuting cost. The effect of a tax on earnings is to lower the private cost of shortening the commuting distance by depressing the earnings foregone, and to lower the return to a move towards a workplace offering a higher wage. We shall assume that it is possible for all agents to have earnings exceeding the housing cost. Moreover, we confine attention to the case where the following assumption holds.

Assumption 4. $w - k > 0$ *and* $w(1-t) - k(1-s) > 0$.

A longer commute increases income net of commuter cost.

4. Agent behaviour

Based on our model, a positive analysis of agent behaviour is of interest in its own right and will prepare the ground for our subsequent analysis of social efficiency. The agent is assumed to choose place of residence and commuting distance (*r* and *z* and hence $y = r + z$) to maximise utility (1) subject to the constraint (2) acting as a price taker, i.e. treating *w* and *p'* as exogenous. We may note that if one changes *z* for a fixed *r* or changes *r* for a fixed z there is in either case a change of workplace. The first order conditions are given by equations (4) and (5) below.

$$(1-t)w - (1-s)k - m = 0 \tag{4}$$

The pay rise acquired by extending the commuting distance, $(1-t)w$, is equated to the marginal pecuniary cost, $(1-s)k$, plus the marginal disbenefit in monetary terms, *m,* of extending the commuting distance.

$$n + (1-t)w - (1-x)p' = 0 \tag{5}$$

The marginal benefit, *n*, of moving house closer to the centre (increasing *r*), plus the pay rise acquired by changing workplaces, $(1-t)w$, is equated to the increase in housing cost, $(1-x)p'$. According to pure preferences, the agent would like to move closer to the centre when $n>0$, but the benefit from such a move is just offset by a financial cost, which is the additional housing cost over and above the attained wage increase.

Agents will make different choices based on (4) and (5) because they have different wages and heterogenous preferences determined by r*.

We should note that condition (5) enables us to infer the agents' marginal valuation of changing location of residence. We shall make use of this revealed preference approach in various places in the subsequent analysis.

As is well known, substitution effects are crucial in analysis of social efficiency, to be carried out below. The substitution effects of subsidising commuting are derived in appendix A. Denoting substitution effects by subscript *s*, the following effects are found.

$$z_s = \frac{k}{D}(n_c n - n_{r-r*}) > 0 \qquad \text{(see a2)} \tag{6}$$

The effect is signed by invoking Assumption 2. Workers will respond to a (larger) subsidy by commuting a longer distance.

$$r_s = k\frac{m_c n}{D} \qquad \text{(see a3)} \tag{7}$$

$r_s > 0$ or $r_s < 0$ according as $n > 0$ $(r < r*)$ or $n < 0$ $(r > r*)$.

The upshot is that a compensated subsidy on commuting provides an incentive to move closer to the preferred place of residence. The crucial question is then in which direction workers prefer to move. Where $r < r^*$ a more favourable place of residence is reached by moving towards the centre and higher-paying workplaces. This happens where $n = (1-x)p' - (1-t)w > 0$. The housing cost increases more steeply than the wage. Where $r > r*$ a more favourable place of residence is reached by moving house towards lower-paying workplaces. This happens where $n = (1-x)p' - (1-t)w < 0$. The wage increases more steeply than the housing cost.

$$c_s = -\frac{k}{D}mn_{r-r*} > 0 \qquad \text{(see a4)} \tag{8}$$

A (larger) subsidy on commuting leads to higher general consumption.

To get some intuition for the results, we may note that a longer commute is compensated by substitution towards general consumption and a more attractive place of residence.

We can summarise effects in the following:

Proposition 1*. Where workers have preference for place of residence and choose place of residence and commuting distance, a compensated subsidy on commuting leads to longer commuting distance and induces workers to increase general consumption and to move closer to the preferred place of residence. The move will be towards the centre if the housing cost rises more steeply than does the disposable income* $(n = (1-x)p' - w(1-t) > 0)$*, and the move will be away from the centre if the disposable income rises more steeply than does the housing cost* $(n = (1-x)p' - w(1-t) < 0)$.

A further question is what happens to the choice of workplace and hence income. We have concluded that *z* increases. Then it follows that if the agent prefers to move house closer to the centre ($n = (1-x)p' - w(1-t) > 0$) the unambiguous conclusion is that y=z+r increases. A higher-paying workplace is chosen. Where the agent chooses a living place further away from the centre, ($n < 0$ and $r_s < 0$) there are opposing effects on y. To get further insight into the case where *r* declines, let us invoke the conclusion that consumption rises. The change in consumption is given by:

$$\begin{aligned} dc &= \left[(1-t)w-(1-s)k\right]dz+\left[(1-t)w-(1-x)p'\right]dr \\ &= \left[(1-t)w-(1-s)k\right]dy-\left[(1-x)p'-(1-s)k\right]dr \end{aligned} \tag{9}$$

where $dy = dz + dr$.

The change in c is decomposed into two terms. We note that according to assumption 4 $(1-t)w-(1-s)k>0$. $(1-x)p'-(1-s)k$ is the net gain (augmentation of c) achieved by lowering *r* and increasing *z* to keep *y* unchanged, i.e., moving further away from the given workplace and accepting a longer commute. The net gain is equal to a decline in housing cost minus the increase in commuter cost. Where the latter effect dominates the lowering of *r* and extension of *z* diminish consumption and a higher-wage workplace must be chosen to obtain larger consumption, i.e. $dy > 0$. Where lowering r and extending z for fixed workplace increases consumption, choosing a lucrative workplace is not necessary to achieve larger consumption but will of course contribute to a further increase. In mathematical terms, we have two cases where the subsidy unambiguously prompts a rise in *y*: $(1-x)p' > w(1-t)$ and $(1-x)p' < (1-s)k < w(1-t)$. To summarise our findings, we state

Proposition 2*. Where workers have preference for place of residence and choose commuting distance and place of residence, a compensated subsidy on commuting will induce workers to increase earnings in the case where the rise in housing cost exceeds the wage increase, and in the case where the rise in housing cost is below the commuting cost per unit of distance. Otherwise, the impact on earnings is indeterminate.*

The comparative statics results above express shifts in the behaviour of agents where they act as price takers. A caveat is that the market outcome may be influenced by general equilibrium effects. Where people tend to move, the shift in behaviour may change the geographical housing price gradient. The responses may drive up prices closer to the centre where the more favoured places of residence are centrally located. In the opposite case, the outcome may be a weaker price decline when moving away from the centre. The impact is to modify the quantitative effects of the initial shift in locational housing demand. We present a brief formalisation in Appendix B.

Revisiting the effects on earnings according to Proposition 2, we realise that earnings will increase when there is a positive shift in both *z* and *r*, and earnings will increase even if the rise in *r* is lessened by a general equilibrium effect. Where *r* diminishes but the decline shrinks due to general equilibrium effects our conclusion still holds that *y* must increase where *c* would otherwise diminish.

5. Social efficiency[6]

The effect on social efficiency of introducing a subsidy on commuting can be measured as a behavioural-induced change in tax revenue, which can be expressed as

$$d\tau = (tw - sk)z_s + (tw - xp')r_s \quad (10)$$

The social gain from increasing *z* at the margin (and hence increasing *y* for a fixed *r*) is equal to the extra output, given by *w*, minus the additional pecuniary and non-pecuniary commuting cost, *k+m,* which yields the net effect $w - k - m$. Substituting for *m* from the first order conditions of the agent's optimisation, (eq. 4), we get $(tw - sk)dz$. We can interpret $tw - sk$ as a net-of-subsidy tax wedge, i.e., the discrepancy between the social and the private gain. Where the tax wedge is positive *tw-sk>0*, as is obviously the case where *s=0*, there is a downward distortion of commuting. Commuters refrain from commuting extra kilometres to earn a larger income because part of the additional income is taxed away. A longer commute to a higher-paying workplace, $z_s > 0$, would then alleviate the pre-existing distortion and yield a beneficial social effect as captured by the former term in (10).

The social gain from increasing *r* at the margin (and hence increasing *y* for a fixed *z*) is equal to the extra output, given by *w*, plus the extra benefit from changing place of residence minus the additional housing cost, i.e. $w + n - p'$. Substituting for *n* from the first order condition of the agent's optimisation, (eq. 5), we get $(tw - xp')dr$. Also $tw - xp'$ can be interpreted as a net-of-subsidy tax wedge. Where *x=0*, there is a pure income tax wedge, which discourages people from taking up residence in higher-wage, but more expensive, locations. The latter effect is however diminished where housing is subsidised through tax favours curtailing the additional housing cost. Where the subsidy is large it may override the income tax ($xp' > tw$) such that there would in fact be a net subsidy encouraging moves towards the centre.

In general, we can consider the effect of slightly increasing a subsidy on commuting for any value of *x* and assuming that $tw > sk$. Then the former term in (10) is positive, whereas the latter term can have either sign. We can state

Proposition 3.

Assuming that $tw > sk$*, there is an unambiguous social efficiency case for increasing a subsidy on commuting*

- *i. where moving closer to the centre is desirable according to pure preferences,* $n = (1 - x)p' - w(1 - t) > 0$*, and there is a positive net tax wedge associated with r,* $tw - xp' > 0$*, which is always the case where* x=0.
- *ii. where moving further away from the centre is desirable according to pure preferences,* $n = (1 - x)p' - w(1 - t) < 0$*, and there is a negative net tax wedge associated with r,* $tw - xp' < 0$.

[6] Beyond efficiency effects, a subsidy on commuting will of course have distributional effects, which the current analysis leaves aside. It is efficiency effects which are at the fore in discussions of a subsidy. It is hardly a significant instrument for dealing with distributional concerns.

In other cases, the net effect is ambiguous because the beneficial effect of increasing the commuting distance is to a smaller or larger extent offset by a change of living place with a negative impact on social efficiency.

A central finding is that a subsidy on commuting will induce commuters to move closer to their preferred place of residence. If this move is towards the centre, it is socially beneficial if there is a net tax wedge associated with centre-bound moves. People initially live too far from the centre from a social efficiency point of view because their residential choices are distorted. If the preferred move is away from the centre, it exacerbates the distortion and is socially harmful. Where the move in either direction prompts a general equilibrium effect (cf. Appendix B), the extent of the move is modified and so is the efficiency effect without being reversed.

An initial situation without subsidies is a starting point for addressing the introduction of a subsidy. Returning to (10) and considering a departure from the situation without subsidies (x=*s=0*), we have

$$d\tau = tw(z_s + r_s)ds = twy_s ds \tag{11}$$

The following conclusion is straightforward.

Proposition 4.

Departing from a situation without subsidies there is a case for introducing a subsidy on commuting if and only if it stimulates earnings.

The circumstances in which a subsidy stimulates earnings and enhances utility are given in Proposition 2 above. The result in Proposition 4 is similar to that of Wrede (2009, Proposition 1). In his model there are only two workplaces (central business districts) and earnings will increase where workers move from the low-productivity to the high-productivity location. In the current model workers increase earnings by opting for workplaces with higher productivity along a line with a continuum of workplaces. A regime with many workplaces is more conveniently modelled as a continuum in a theoretical setting, even though our empirical data below will be discrete. As set out above, our model studies more in depth the interaction between residence choice and choice of commuting distance. A reason for this concern is that it has a bearing on the choice of workplace. It throws light on the question whether the effect of a commuting subsidy is simply to induce agents to choose more attractive places of residence, as sometimes claimed in public debate. A second reason is that the residence choice has efficiency ramifications where the housing cost is subsidised, which is often an important concern, especially when considering empirical effects.

We note that the efficiency effect, dR, tends to zero where the change of earnings, y_s , vanishes. This confirms the finding of Richter (2006) that there is no case for a commuting subsidy if it has no effect on earnings ("does not earn taxable wage income", see *op cit*. p.689)[7].

Where there is a negative social efficiency effect of a (positive) subsidy, there would of course be an argument for a tax on commuting.

[7] Unlike the assumption of this paper, Richter assumes that there is an endogenous labour supply in terms of hours worked, being distorted by an income tax. Then one could enhance efficiency by taxing commuting to raise revenue enabling the government to lower the income tax and ease the distortion.

Considering also the special case where commuting is subsidised but there is no housing subsidy, we get

$$d\tau = twdy - skdz \tag{12}$$

There is a negative impact due to longer commutes, $-skz_s < 0$, beyond the earnings effect because the preexisting subsidy on commuting taken alone causes an upward distortion of commuting, which counteracts the negative income tax distortion. It will weaken the positive effect where earnings increase and exacerbate the distortion where earnings decrease.

The analysis above has adopted the assumption of fixed pretax wages and prices, which is common in much of the optimum tax literature, often invoking a linear technology assumption. Deviating from this assumption, it may be of interest to consider how the effects under survey may be modified by conceivable general equilibrium effects. In particular, changes of residence may affect local scarcity of land and housing prices, as assumed in several previous papers. Where, neglecting this effect, a subsidy induces agents to move house towards higher-wage locations, $r_s > 0$, these responses may drive up housing prices in more central areas dampening, without reversing, the inducement to move. Consequently, there would also be a weaker effect on income. Where, absent any general equilibrium effects, a subsidy induces people to move further away from the centre, $r_s < 0$, the responses may drive up housing prices in less central areas with a moderating influence on the move to less central areas. This effect would make an increase in income more likely. We see this from the equation $y_s = r_s + z_s$. As commuting is stimulated, $z_s > 0$, shrinking the negative impact, $r_s < 0$, (diminishing the absolute value), strengthens the case for a positive effect on income, $y_s > 0$. In our framework, the direction of subsidy-induced moves may vary between individuals with different characteristics. Where some are encouraged to move towards the centre and others to move towards the periphery there will be counteracting effects. The impact on housing prices becomes indeterminate and may even vanish.

It is important to note that, as agents are price takers, we can rely on the first order optimality conditions of the agents to infer their marginal valuation of changing location of residence even if general equilibrium effects were to occur.

We have considered how the change in individual behaviour affects social efficiency measured by the behavioural-induced contribution to tax revenue. With a heterogeneous population there may be conflicting individual effects, and aggregating the effects is required to get the total impact on social efficiency. Whereas a subsidy stimulates commuting and enhances efficiency where tw > sk for all *w*, the induced change of living place may go in either direction, depending on the sign of $n = (1-x)p' - w(1-t)$, which may vary between agents with different wage rates. We have assumed that there is a continuum of types, each characterised by a preference parameter, *r**, and a value of *w*. Denoting the simultaneous density function by $h(r^*, w)$, the aggregate change in tax revenue is $\int\int d\tau\, h(r^*, w)\; dr^* dw = \int\int \big((tw - sk)z_s + (tw - xp')r_s\big)\, h(r^*, w) dr^* dw$, where the size of the population is normalised to unity. With conflicting positive and negative effects empirical observations are required to determine which effects that dominate.

6. Empirical illustration

There are three key factors determining behaviour in our model: pecuniary travel expenses, spatial variation in earnings and spatial variation in housing cost. We want to show how empirical data on these factors can be exploited to assess the efficiency effects of changing actual subsidies on commuting. Combining theory and data, we can both compute the initial

tax wedges and infer the direction of responses to a marginal reform, which in conjunction determine the sign and magnitude of the resulting efficiency effect.

To provide an empirical illustration of this approach, we have considered the region around Oslo in Norway. That is, we take Oslo as the centre in our model.

Each year, the tax authority in Norway announces a travel cost per kilometre that can be deducted against gross income by those eligible for a deduction. We use this figure for the pecuniary travel cost corrected for tax deduction.

We use register data on earnings in 2021, reported by the employers, to establish average earnings in each municipality in the region.[8] We distinguish between highly educated workers (university or college education) and workers with lower education. The reason is that we expect centrality of a workplace to be more important for some categories of workers than for others. Close to 60 per cent of the employees in the region have higher education.

For housing costs in 2021 $\left(p'(1-x)\right)$, we use data commissioned by The National Federation of House Owners in Norway.[9] For each municipality several types of housing costs are assessed for an average 120 square metres sized dwelling. The items we include in the housing costs are interest payments (net of tax benefits due to deductions), maintenance, insurance, energy consumption, and municipal charges for water, wastewater treatment, garbage collection, etc. Moreover, we adjust the housing costs according to high or low education. Highly educated individuals will, due to higher income, enjoy higher housing standards (defined by size and quality). This is taken care of by estimating the difference in housing values between individuals with high and low education based on register data from the Income and Wealth Statistics.[10][11]

In addition to variation in housing standards across income groups, housing consumption in terms of standards will normally vary as part of behavioural responses to commuting subsidies. Larger consumption of most goods is obtained where a higher wage is attained at a more productive workplace. However, we do not single out housing standards as a separate good when considering the commuters' responses. Where the consumption bundle is optimally composed there is indifference between marginal consumption of various goods, and we can consider a change in housing standards as part of the change in general consumption, c.

By using Google maps, we measure the distance between the Oslo City Hall and the administrative centre of each of the surrounding municipalities. In Figures 1 and 2 we have plotted the average private housing costs, $(1-x)p'$, and the average earnings for the two educational groups in each municipality, depending on the distance to Oslo. It appears that this

[8] For more information regarding the data on earnings, see https://www.ssb.no/en/arbeid-og-lonn/lonn-og-arbeidskraftkostnader/statistikk/lonn.

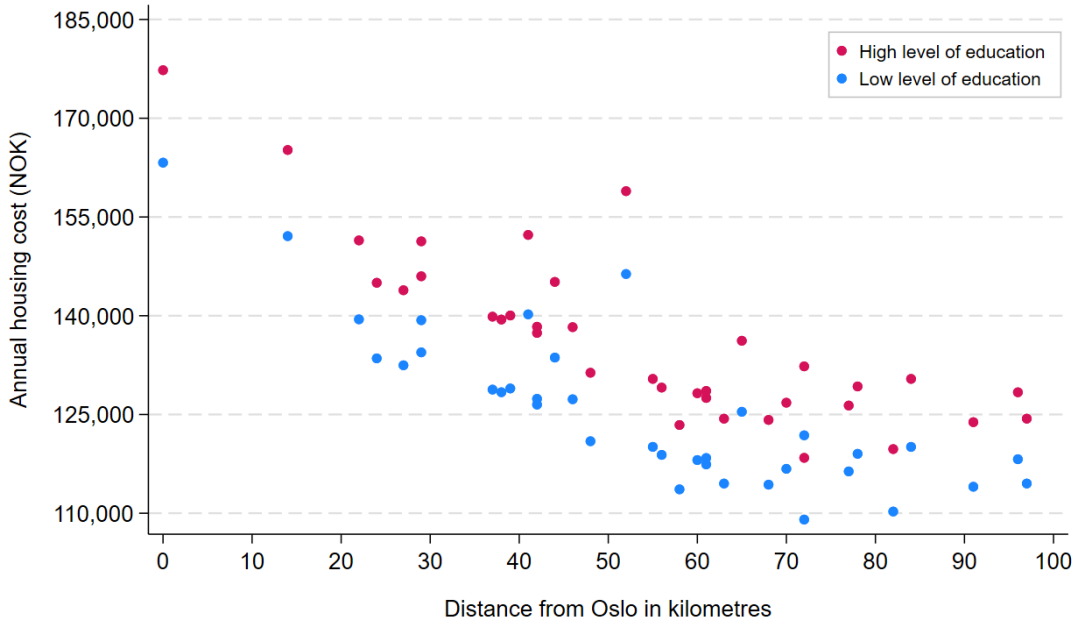
[9] The data are retrieved from several sources and compiled by the consulting company Economics Norway (see https://www.huseierne.no/huseiernes-bokostnadsindeks/).

[10] We regress the log of housing value on a dummy for education, controlling for municipality and age. By knowing the average housing costs and the share of highly and less educated individuals, respectively, we can derive housing costs for each educational group. We estimate the costs to be 9 percentage higher for individuals with high education.

[11] For more information regarding the Income and Wealth Statistics, see https://www.ssb.no/en/inntekt-og-forbruk/inntekt-og-formue.

substantiates our assumption that both variables decline as we move away from Oslo.[12] To estimate the effects on earnings and housing costs of distance to Oslo, we run OLS regressions for each educational group. We report the results in Table 1 and, as expected, we get a significant negative effect on earnings and housing costs of moving away from Oslo. For instance, moving 10 kilometres closer to Oslo will generate more than NOK 11,400 in gross earnings for a highly educated individual. The housing cost rises by more than NOK 4,600. Furthermore, we need to estimate the marginal net subsidies on housing $(p'x)$. Hence, we run regressions with the tax benefits from interest deductions. The results reported in Table 1 show that housing subsidies rise with proximity to Oslo.

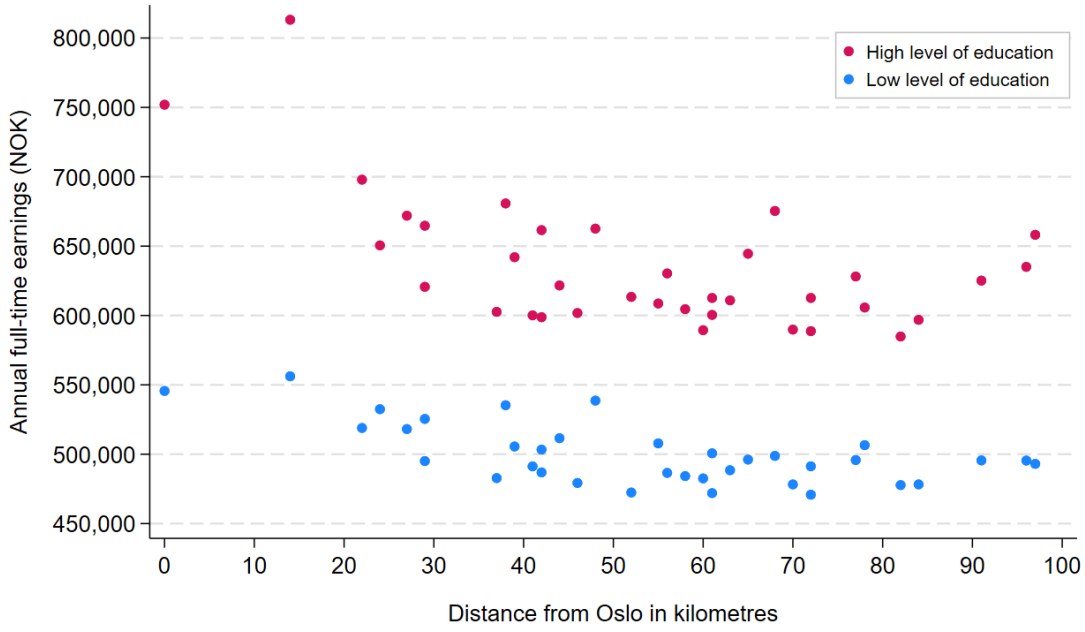
Figure 1. Average private housing costs by distance to Oslo for individuals with high and low level of education, respectively.



Note. Average private housing costs in each municipality in 2021. Distance by road from the administrative centre of the municipality to Oslo City Hall, estimated by using google maps.

[12] We have confined our attention to the geographical areas located less than 100 km from the centre, since the decline tends to vanish beyond this distance.

Figure 2. Average earnings by distance to Oslo for individuals with high and low level of education, respectively.



Note. Average full-time gross earnings in each municipality in 2021, as reported by the employers. Distance by road from the administrative centre of each municipality to Oslo City Hall, estimated by using google maps.

Table 1. Estimates from OLS regressions based on data from 2021

	Low education			High education		
	Private housing costs	Tax benefit from interest deductions	Earnings	Private housing costs	Tax benefit from interest deductions	Earnings
Distance (10km)	- 4,282.1*** (507.8)	-1,260.4*** (160.6)	-5,758.3*** (1255.1)	-4,650.4*** (551.4)	-1,368.8*** (174.38)	-11,455.9*** (2,882.2)
Const.	148,739.1*** (2975.6)	17,368.0*** (940.9)	531,005.0*** (7,355.96)	161,530.7*** (3,231.4)	18,861.6*** (1,021.9)	696,685.9*** (16,889.42)
R-sq.	0.68	0.64	0.38	0.68	0.64	0.32
Obs.	36	36	36	36	36	36

Note: Earnings and housing costs in Norwegian kroner (NOK) and distance from Oslo in 10 kilometres. We use register data on earnings, reported by employers, to establish average earnings in municipalities. Data on housing costs are taken from The National Federation of House Owners in Norway, and data on distance to Oslo computed by using Google maps. Standard errors in parenthesis and *** $p < 0.001$.

Since we are interested in the marginal increase in disposable income due to higher earnings, we tax the marginal increase in earnings by using an average marginal tax rate based on the number of individuals in each tax bracket.[13] Based on this, the travel costs, and our estimation results, we compute values corresponding to different components and report them in Table 2.

The sign of $n = (1-x)p' - w(1-t)$ obviously depends on *w*. We note that if p' is independent of *w* there are three possibilities. It is possible that $n = (1-x)p' - w(1-t)$ is positive or negative for all *w* in the population, or it may be that it is positive for small values of *w* and negative for large values of *w*. It may, however, be that p', which measures how housing cost varies with distance, is not the same for all *w*. Due to income effects, it is plausible that that richer people have larger housing expenses so that p' increases with *w*. It is still possible that $(1-x)p'$ may remain above or below all values of *w* or above only lower values of *w*. However, a conceivable novel case is that $(1-x)p'$ is below *w* for smaller values of *w* and above for larger values of *w*. Which case that prevails is obviously an empirical question.

For those with higher education, the finding is that one would lose financially by moving further away from the centre since the loss of income exceeds the decline of housing cost. The financial loss must then be balanced by a more attractive place of residence (r>r*). If it were not for the financial loss one would like to live further away from the centre. There is a non-pecuniary loss from moving closer to the centre (*n<0*) and a gain from moving further away from the centre. The commuting cost is NOK 2,799 and NOK 3,588 in the respective cases where the commuter is eligible or not for a deduction. In either case *n+m>0*. [14]

The efficiency effect of increasing *s* at the margin is $(tw-sk)z_s+(tw-xp')r_s=(tw-sk)z_s+(xp'-tw)(-r_s)$, where $-r_s$ is the decline of *r*. For those with higher education, we have *tw-sk*=3,450, $xp'-sk=580$, and $xp'-tw=-2{,}870$. With the figures in our empirical illustration, the marginal efficiency effect is $3{,}450z_s - 2{,}870(-r_s)$. We know that *z* will increase, and r will decrease because *n<0*. Then we have opposing effects in the expression for the efficiency effect. The incentive to increase the commuting distance is beneficial because there is a pre-existing downward distortion of commuting when $tw-sk>0$. However, there is an efficiency loss from lowering *r* ($-r_s>0$) because there is a pre-existing downward distortion since a lowering of *r* diminishes the income tax revenue by more than it diminishes the loss of revenue due to the subsidisation of housing. There will be a positive impact on social efficiency if $3{,}450z_s - 2{,}870(-r_s)>0$, which is equivalent to $z_s > -r_s 2{,}870/3{,}450 = -0.83r_s$ or $-r_s<1{,}20z_s$. The commuting distance must increase by more than 83 per cent of the decline of the distance to the place of residence, and the latter must not exceed the former by more than 20 per cent. Otherwise, there is an efficiency loss at the margin. It does not seem plausible that a subsidy on commuting should generate a change of distance to the living place that significantly exceeds the increase in commuting distance. One might even argue that the commuting subsidy has a more direct and hence a larger impact on the commuting distance than the more indirect effect on the distance to the place of residence. If this argument is accepted the subsidy obviously enhances efficiency. It is also interesting to note that for the well-educated *m*= 4,418, which is a fairly large disbenefit incurred by commuters.

[13] The marginal tax rate for individuals with high level of education is 37 percent, while for those with low level of education we use 36 percent.

[14] Commuting costs exceeding the amount of NOK 23,900 in 2021 were eligible for deductions. This implied that commuters had to live more than 33.5 kilometres away from the workplace to be eligible.

For those with lower education, there is a moderate private financial loss from moving closer to the centre since the housing cost rises by more than the increase in disposable income. The implication is that to be at an equilibrium there must a non-pecuniary gain from moving closer to the centre ($r<r^*$ and $n>0$). It follows that the agents will respond to a larger commuting subsidy by extending the commuting distance ($z_s > 0$) and moving towards the centre ($r_s > 0$), and earnings will increase.

The efficiency effect is $(tw - sk)z_s + (tw - xp')r_s$.

Using the numerical values from our empirical illustration, we get

$1{,}284z_s + 813r_s$

According to our analysis, the behavioural responses to a larger subsidy alleviate pre-existing distortions and enhances efficiency for those with lower education. For the less educated, we also find that m=886, which is a moderate disbenefit incurred by commuting.

Table 2. Derived values based on Norwegian data, 2021

	Low education	High education
$w(1-t)$	3,685	7,217
w	5,758	11,456
tw	2,073	4,239
$(1-x)p'$	4,282	4,650
p'	5,542	6,019
xp'	1,260	1,369
$(1-s)k$	2,799	2,799
k	3,588	3,588
sk	789	789
$xp' - tw$	-813	-2,870
$n = (1-x)p' - (1-t)w$	597	-2,567
$n + m = (1-x)p' - k$ *(s=0)*	694	1,062
m *(s=0)*	97	3,629
$n + m = (1-x)p' - (1-s)k$	1,483	1,851
m	886	4,418
$tw - sk$	1,284	3,450
$xp' - sk$	471	580

Note: The variables are defined in the main text. We use a marginal tax rate of 36 percent for less educated and a rate of 37 percent for highly educated. Commuting costs derived by using a cost of NOK 1.56 per kilometre and assuming 230 days of commuting.

Our empirical illustration shows the significance of variation in wage rates. On aggregate, 60 per cent of wage earners have higher education. We can conclude that a larger subsidy on commuting enhances efficiency for those with lower education while it is uncertain but plausible that it also enhances efficiency for those with higher education, and accordingly aggregate efficiency.

Another empirical finding is that there is a weak but positive relationship between commuting expenses and income. The correlation coefficient is approximately 0.05. This means that from a distributional perspective there is an argument against further subsidisation of commuting, but the case is indeed a very weak one. If one accepts that the more plausible outcome is that a

larger subsidy on commuting enhances efficiency, we have opposing efficiency and distributional effects, which would be offsetting at a social optimum. However, as the distributional effect is indeed minor, it seems plausible that, based on our data, a further increase of the subsidy on commuting would be welfare enhancing. Our main contribution is however to illustrate how one can analyse the effects of a subsidy on commuting both theoretically and empirically.

7. Conclusion

Our paper makes several contributions. First, it analyses the circumstances in which subsidising commuting can alleviate the income tax distortions of locational choices of residence and workplace in a setting where agents have a favourite place of residence according to pure preferences. It elaborates on the interaction between the two decisions. Then we extend the concern with efficiency to distortionary tax favours to housing often encountered in practice. We establish a framework for assessing the efficiency effects of changing a subsidy on commuting based on a limited set of empirical data on prices, wages, and tax rates. A key element of the approach is to infer information about residential preferences from the market and tax data. Finally, the approach is illustrated by considering empirically commuting in the region around Oslo in Norway.

Our starting point is the recognition that an income tax causes a downward distortion of earnings. Earnings depend on workplace, which is determined by commuting distance and the place of residence from which the commuter departs. Therefore, the tax distortion of earnings is made up of the distortion of commuting and the distortion of choice of living place. A subsidy on commuting stimulates commuting to higher-wage locations, which mitigates the pre-existing downward distortion of earnings. The impact via the residential choice is less transparent. A subsidy on commuting induces workers to move closer to their preferred location of residence. The reason is that the subsidy makes it more costly to shorten the commuting distance since the cost saving diminishes. When shortening the commuting distance becomes a more expensive good it is to some extent substituted by a more attractive living place. Whether the favourite place of residence is closer to or further away from the centre depends on the respective spatial variations in earnings and housing cost. Where the agents incur a financial loss due to unequal changes in housing cost and earnings when moving towards or away from the centre, the loss must be offset by a more attractive place of residence for the agent to be indifferent at the optimum. Where there is a preference for living closer to the centre, the subsidy will induce a move to better-paying locations and a further mitigation of the distortion of earnings. Where there is a preference for living further away from the centre the effect is reversed. Then the effects via the commuting decision and the residential choice are opposing each other and the net effect determines whether the earnings distortion is weakened or not.

However, there is a further complication. Housing is often subsidised through tax favours, the partial effect of which is to shrink, and conceivably reverse the sign of, the tax wedge created by the income tax. Then there are four possible cases when allowing for both the income tax and the subsidy on housing: i. There is a *net* downward distortion of the incentive to move to higher-wage locations and a subsidy on commuting encourages such a move. ii. There is a *net*

upward distortion of the incentive to move to higher-wage locations and a subsidy on commuting encourages a move in the opposite direction. iii. There is a *net* downward distortion of the incentive to move to higher-wage locations and a subsidy on commuting encourages a move in the opposite direction. iv. There is a *net* upward distortion of the incentive to move to higher-wage locations and a subsidy on commuting encourages such a move. In cases i. and ii. the distortionary effect is mitigated and there is a beneficial social effect through the impact on the residential choice, which adds to the positive effect through the impact on commuting. In cases iii. and iv. there is a harmful social effect through the impact on the residential choice, which opposes the positive effect through the impact on commuting. The overriding effect determines whether there is an efficiency enhancement or deterioration.

Diversity of geographical wage changes across agents implies that there may be individual responses with conflicting effects on social efficiency, and the overall impact depends on which response that dominates in the population.

To provide an empirical illustration of our theoretical analysis we have considered commuting in the region around Oslo in Norway, with Oslo as the centre in our model. Applying data on travel cost, spatial wage dispersion, and variation in housing cost, we can infer the responses to a subsidy on commuting for different wage groups, illustrated by highly educated and less educated people, respectively. In our case-specific illustration we find that increasing the subsidy on commuting enhances efficiency for those with lower education, while there are conflicting effects on efficiency for those with higher education, with a positive overall effect deemed to be the more plausible outcome.

We have adopted the assumption of fixed pretax wages and prices, which is common in much of the optimum tax literature. Some caveats may however be in order. As we have discussed already, changes of residence may affect local scarcity of land and housing prices, as assumed in Wrede (2001 and 2009). Where policy changes stimulate the employment density in certain locations, one may argue that there is a labour supply shift that tends to diminish local wages, and the converse is true in locations losing labour. On the other hand, one may argue that there can be agglomeration effects, especially in urban areas, implying that increasing employment raises the productivity of existing workers. Such effects are not taken into account by individual workers and can be perceived as agglomeration externalities.[15] On the other hand, bunching of activity may cause negative externalities due to congestion. Allowing for various deviations from our assumptions, a variety of cases may arise, and the analysis is likely to become much less tractable. Delving further into this matter is beyond the ambition of the current paper.

The recent Corona pandemic has provided an impetus for working from home, a phenomenon that seems to endure, at least to some extent, beyond the pandemic. Where a fixed part of the working week, say $1-\theta$, can be spent working at home, less commuting is required to earn a certain income while living away from the workplace. We can then replace z by θz in the utility function, and a certain commuting distance becomes less costly. On the other hand, working from home may cause inconvenience that must be traded off against the cost of

[15] Venables (2007) argues for inclusion of such effects in evaluation of urban transport projects.

commuting where working conditions are flexible. Subsidising commuting will distort this trade-off, being another source of excessive commuting. Elaborating on these aspects of commuting may be a topic for future research.

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Appendix A

We take as departure the fixed utility requirement:

$$u(c, z, r - r*) = u_0, \tag{a1}$$

which implicitly defines c as a function of z and r: $c\left(z, r-r^*\right) = c\left(y-r, r-r^*\right)$.

Differentiating (a1), we get

$$c_z = -\frac{u_z}{u_c} = m(c, z, r - r*)$$

$$c_{r-r^*} = -\frac{u_{r-r^*}}{u_c} = -n(c, z, r - r*)$$

Minimising the expenditure

$$e = c + [(1-t)w - k + sk](-z) + (1-x)p(r) - (1-t)wr$$

with respect to z and r, allowing for (a1), we get the first order conditions

$$e_z = c_z - \left[(1-t)w - k + sk\right] = 0$$

$$e_{r-r*} = c_{r-r*} + \left[(1-x)p' - (1-t)w\right] = 0$$

We find the following second order derivatives

$$c_{z,r-r*} = m_c c_{r-r*} + m_{r-r*} = -m_c n + m_{r-r*}$$

$$c_{r-r*,z} = -n_c c_z - n_z = -n_c m - n_z$$

$$c_{zz} = m_c c_z + m_z = m_c m + m_z$$

$$c_{r-r*,r-r*} = -n_c c_{r-r*} - n_{r-r*} = n_c n - n_{r-r*}$$

The second order conditions are

$$e_{zz} = c_{zz} > 0$$

$$e_{r-r*,r-r*} = c_{r-r*,r-r*} > 0$$

$$D = e_{zz}e_{r-r*,r-r*} - e_{z,r-r_*}^2 = c_{zz}c_{r-r*,r-r*} - c_{z,r-r_*}^2 > 0$$

To find the compensated effects of a subsidy by means of comparative statics, we differentiate the first order conditions and get

$$e_{zz}z_s + e_{z,r-r*}r_s - k = c_{zz}z_s + c_{z,r-r*}r_s - k = (m_c m + m_z)z_s + (-m_c n + m_{r-r*})r_s - k = 0$$

$$e_{r-r*,z}z_s + e_{r-r*,r-r*}r_s = c_{r-r*,z}z_s + c_{r-r*,r-r*}r_s = (-m_c n + m_{r-r*})z_s + (n_c n - n_{r-r*})r_s = 0$$

where (invoking Young's theorem), we have used $c_{r-r*,z} = c_{z,r-r*} = -m_c n + m_{r-r*} = -n_c m - n_z$.

Solving the equation system:

$$(m_c m + m_z)z_s + (-m_c n + m_{r-r*})r_s = k$$
$$(-m_c n + m_{r-r*})z_s + (n_c n - n_{r-r*})r_s = 0$$

for $z_s$and r_s, we obtain

$$z_s = \frac{k}{D}(n_c n - n_{r-r*}) > 0 \quad \text{(a2)}$$

where the sign follows from Assumption 2 and Assumption 3.

$$r_s = \frac{k}{D}(m_c n - m_{r-r*}) = \frac{k}{D}(n_c m + n_z) \quad \text{(a3)}$$

Subject to Assumption

$$r_s = \frac{k}{D}m_c n.$$

The fixed utility (a1) implies that

$$u_c\, c_s + u_z z_s + u_{r-r*}r_s = 0$$

$$c_s - m\frac{k}{D}(n_c n - n_{r-r*}) + n\frac{k}{D}(n_c m + n_z) = 0$$

Where $n_z = 0$ (Assumption 2),

$$c_s = -\frac{k}{D}mn_{r-r*} > 0 \quad \text{(a4)}$$

$$y_s = \frac{k}{D}(n_c n - n_{r-r*} + m_c n - m_{r-r*}) = \frac{k}{D}(n(m_c + n_c) - n_{r-r*} - m_{r-r*})$$
$$= \frac{k}{D}(n_c n - n_{r-r*} + n_c m + n_z) = \frac{k}{D}(n_c(n + m) - n_{r-r*} + n_z)$$

Where $m_{r-r*} = n_z = 0$ (Assumption 2),

$$y_s = \frac{k}{D}(n(m_c + n_c) - n_{r-r*}) = \frac{k}{D}(n_c(n + m) - n_{r-r*}) \quad \text{(a5)}$$

Where $n < 0$ (and $n_c < 0$) and $n + m > 0$, the sign is undetermined. Otherwise $y_s > 0$.

Appendix B

Assume that how steeply the housing price rises with proximity to the centre depends on the increase in housing demand (the number of people who want to live at a location). How demand increases with *r*

depends in turn on the price increase and the subsidy on commuting. We express the relationship by the function

$$p' = f(n'(r, p', s))$$

where n' is the increase in demand (number of people) as a function of *r*. A prime denotes the derivative $f' > 0$. Using subscripts to denote partial derivatives, we can write

$$p'_s = f'\left(n'_s + n'_{p'} p'_s\right)$$

$$p'_s = \frac{f' n'_s}{1 - f' n'_{p'}}$$

A larger housing price gradient (p') will lead to a lower increase in housing demand when approaching the centre, and $n'_{p'} < 0$. Where the dominant shift in locational housing demand is a move towards (away from) the centre $n'_s > 0$ $(n'_s < 0)$ and $p'_s > 0$ $(p'_s < 0)$.