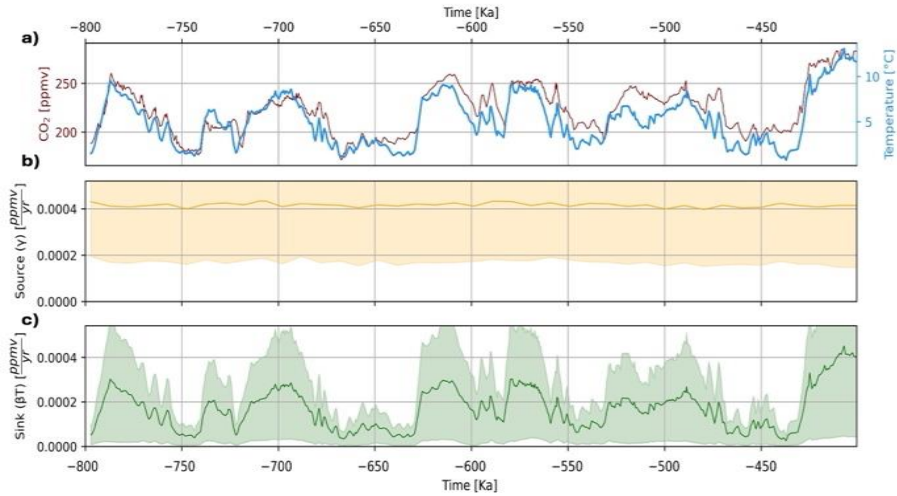


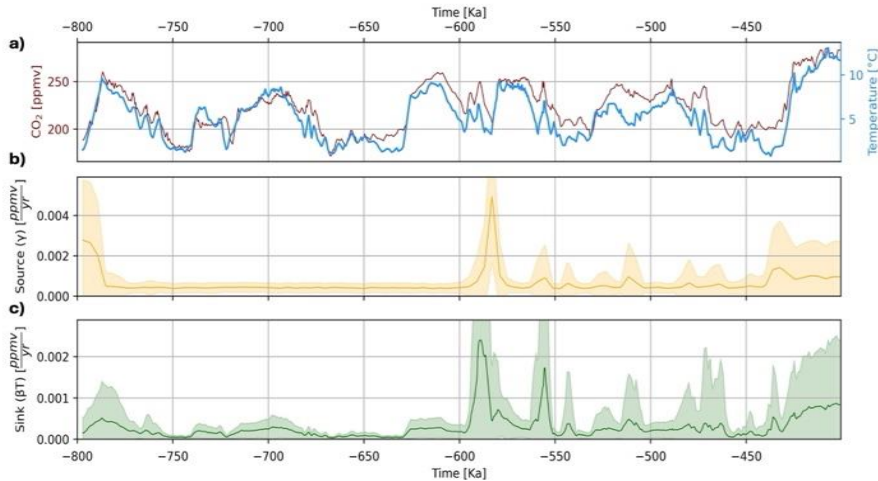
Figure 1. Central derivatives (red lines) used for the inversion and number of data-points every 5 kyrs from Ref.1 (a) and Ref.2 (b) (Methods).

Model parameter	Min	Max
$t$ [ka]	-797,099.0	0.0
$\gamma$ [ $\frac{ppmv}{yr}$ ]	0.0	0.08
$\beta$ [ $\frac{ppmv}{yr\ ^\circ C}$ ]	0.0	0.008
$k$	1	100
$\omega$	-1.0	3.0

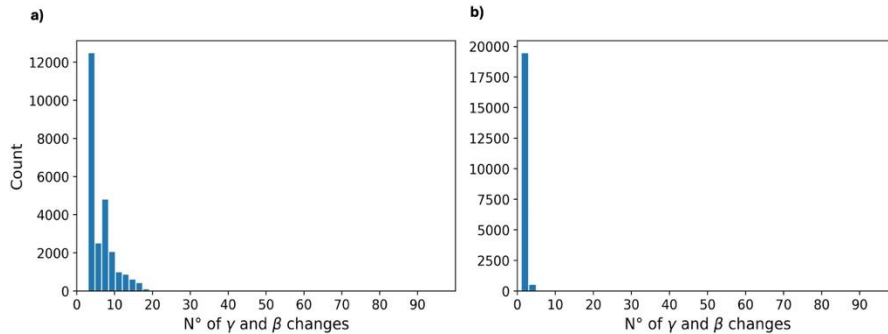
*Table I. Range values for uniform sampling of  $t$ ,  $\gamma$ ,  $\beta$ ,  $k$  and  $\omega$  used for the inversion.  $k$  and  $\omega$  are dimensionless quantities.*



*Figure 2. Results from the first test as a simple MCMC inversion. (a) 800-400 ka subset including the atmospheric  $\text{CO}_2$  and  $T$  reconstructions from Refs.1, 3 shown for reference with flux results. (b)  $\gamma$  source flux time history. (c)  $\beta T$  sink flux time history. The reduced variability and magnitude align with results from the main text over this period.*



*Figure 3. Results from the second test as a rj-McMC inversion. (a) 800-400 ka subset including the atmospheric  $\text{CO}_2$  and  $T$  reconstructions from Refs. 1, 3 shown for reference with flux results. (b)  $\gamma$  source flux time history. (c)  $\beta T$  sink flux time history. The higher variability compared to the first test arises from the use of a rj-McMC, where the small  $\gamma$  and  $\beta T$  magnitudes align with reduced pre-400 ka flux variability in the main text.*



*Figure 4. Distribution for the number of  $\gamma$  and  $\beta$ T changes from the main analysis (a) and the inversion from the second test (b). Although results from the second test express pre-400 ka variability, the number of  $\gamma$  and  $\beta$ T changes is lower (i.e., ~1), confirming the reduce flux variability pre-400 ka.*

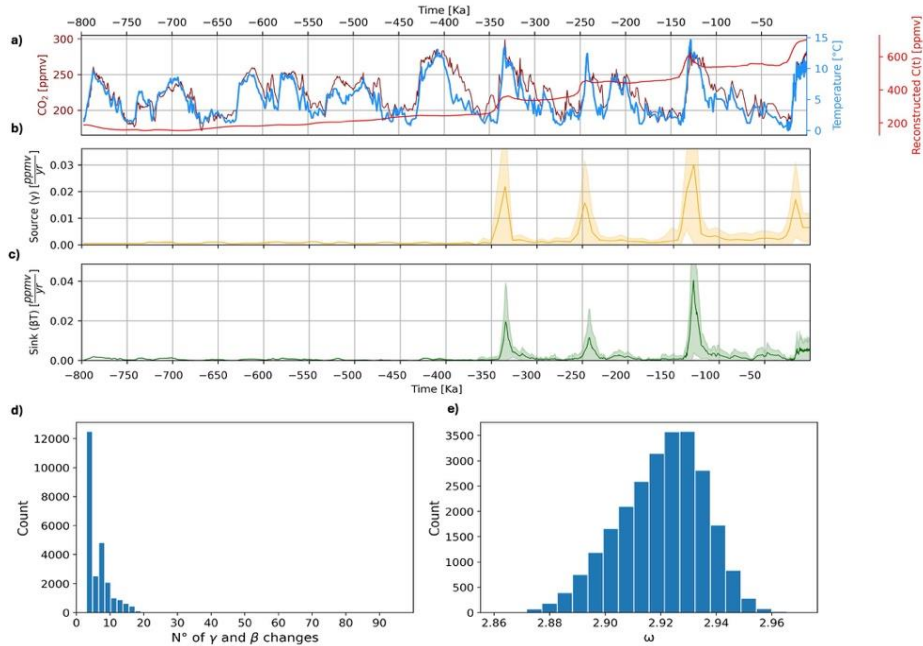


Figure 5. Results from the main text inversion as a comparable reference with results from test in Fig.5. (a) CO<sub>2</sub>, T, and reconstructed C(t) curve using the retrieved  $\gamma$  and  $\beta T$  fluxes. (b-c) Retrieved  $\gamma$  and  $\beta T$  flux time series. (d) Distribution for the number of  $\gamma$  and  $\beta T$  changes. (e) The distribution of the uncertainty scaling parameter  $\omega$ . The high uncertainty on inverted data leads to high value of  $\omega$  and reduced variability of fluxes.

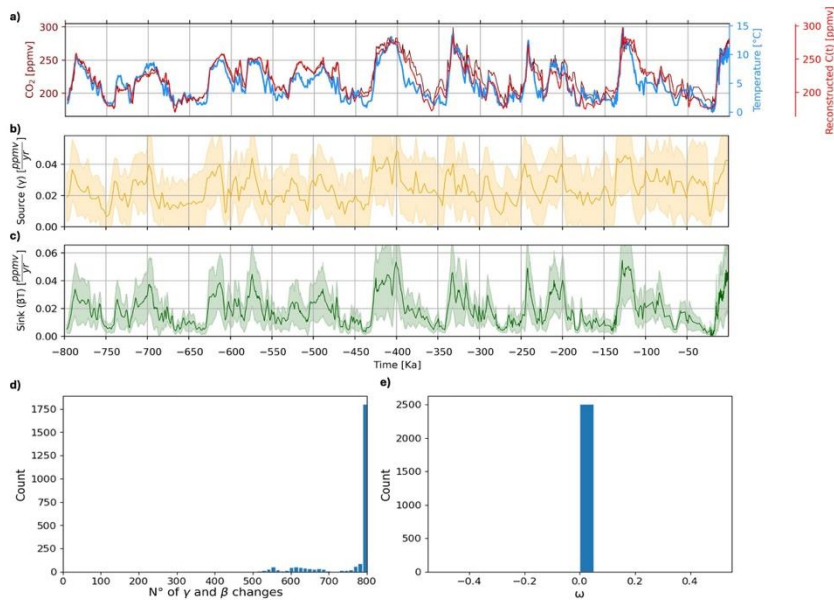
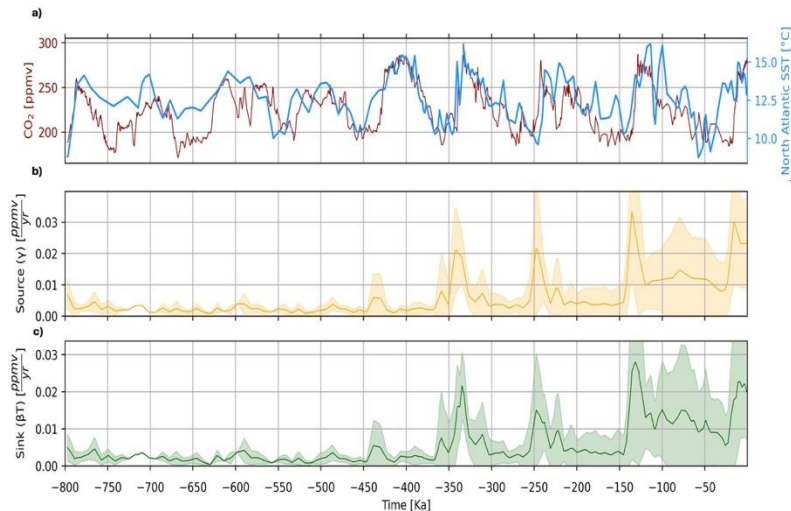
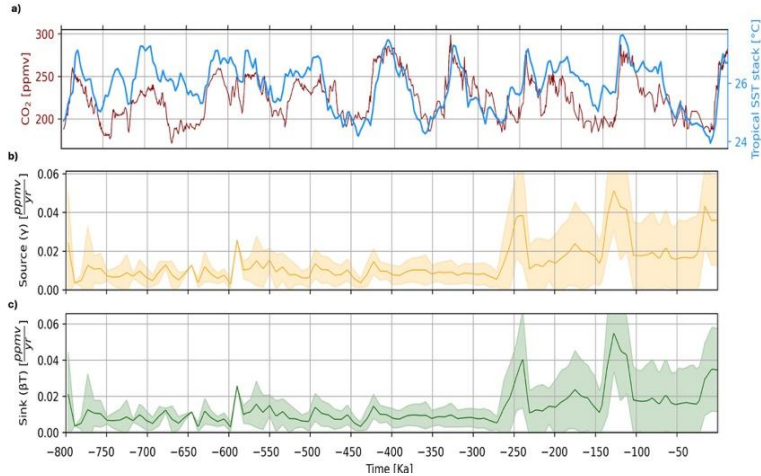


Figure 6. Results from the inversion when setting  $\omega=0$ . (a)  $\text{CO}_2$ ,  $T$ , and reconstructed  $\text{C(t)}$  curve using the retrieved  $\gamma$  and  $\beta T$  fluxes. (b-c) Retrieved  $\gamma$  and  $\beta T$  fluxes time histories. (d) Distribution for the number of  $\gamma$  and  $\beta T$  changes. (e) The distribution of the uncertainty scaling parameter  $\omega$ . The use of no scaling factor  $\omega$  leads to a large increment in the number of  $\gamma$  and  $\beta T$  changes, preventing to assess significant carbon cycle flux changes.

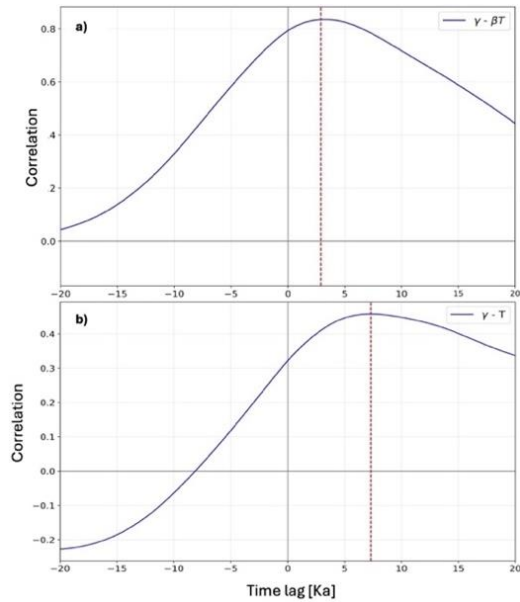


*Figure 7. Flux histories reconstruction using the global  $\text{CO}_2$  record and the alkenone SST reconstruction in the North Atlantic from main text Refs. 1, and 10, respectively. (a) Global  $\text{CO}_2$  and local North Atlantic SST record. (b) Source flux time history reconstruction. (c) Sink flux time history.*





*Figure 8. Flux histories reconstruction using the global  $\text{CO}_2$  record and the stacked alkenone SST reconstruction at the tropics from main text Refs.1, and 11, respectively. (a) Global  $\text{CO}_2$  and tropical stack SST record. (b) Source flux time history reconstruction. (c) Sink flux time history.*



*Figure 9. Cross-correlation analysis of  $\gamma$  against  $\beta T$  (a) and  $T$  (b). Maximum lag correlation is found with a lag of ~3 ka and ~7 ka.*