Supplementary Information: How social contagions shape collective consensus in the presence of scale-free networks

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Influence of the fraction of activated nodes

In the main text (Figs. 1, 4, 7), the results are generated and shown when v is defined as the speed to reach a fraction of 30% activated nodes (cascade size). Figure S1-S8 shows the same results for nine different fractions of activated nodes—ranging from 10% to 90%, by increments of 10%, and including 30% for the sake of comparison.

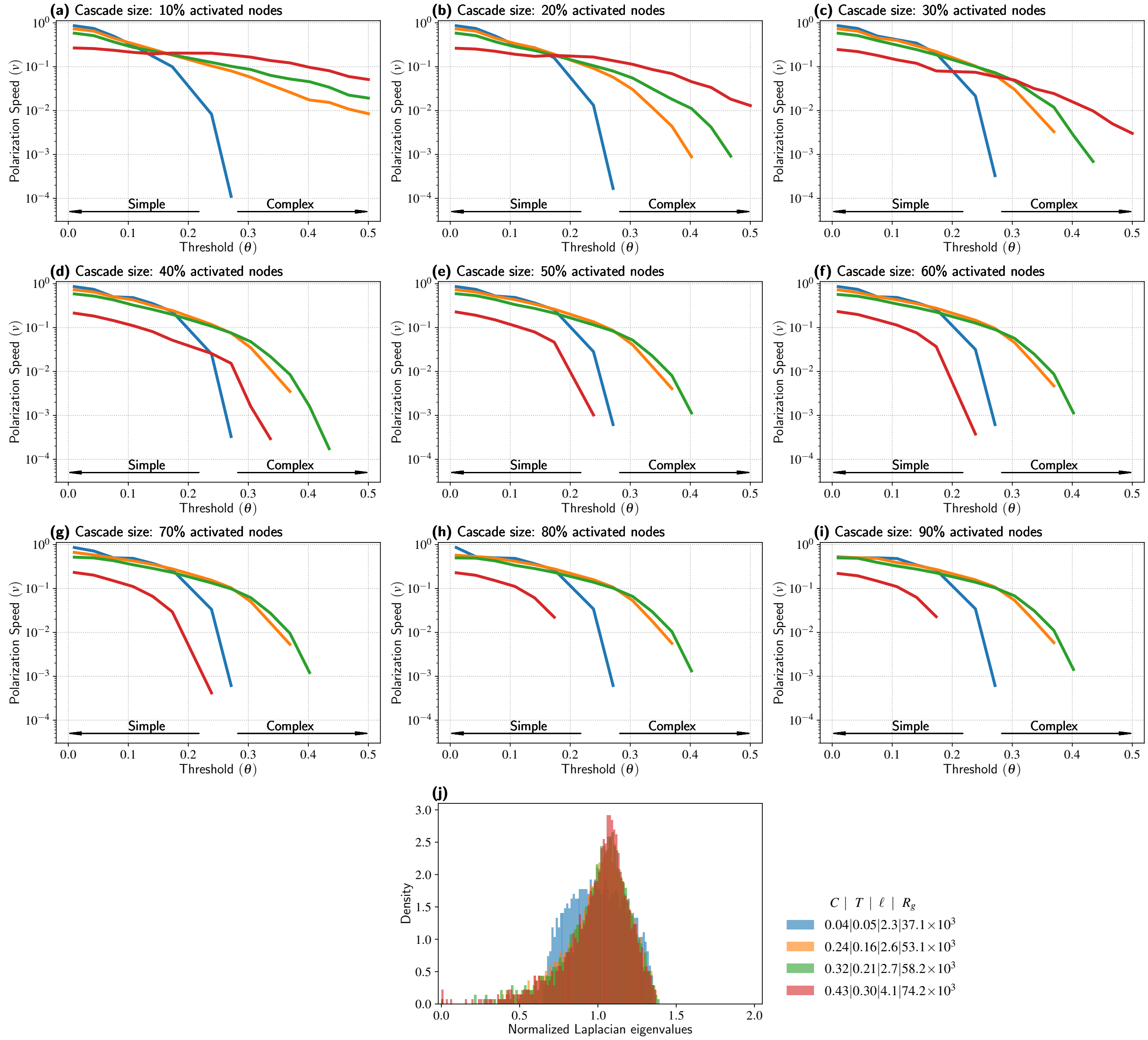

Figure S1. Linear threshold model for MHK networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **top 10% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks.

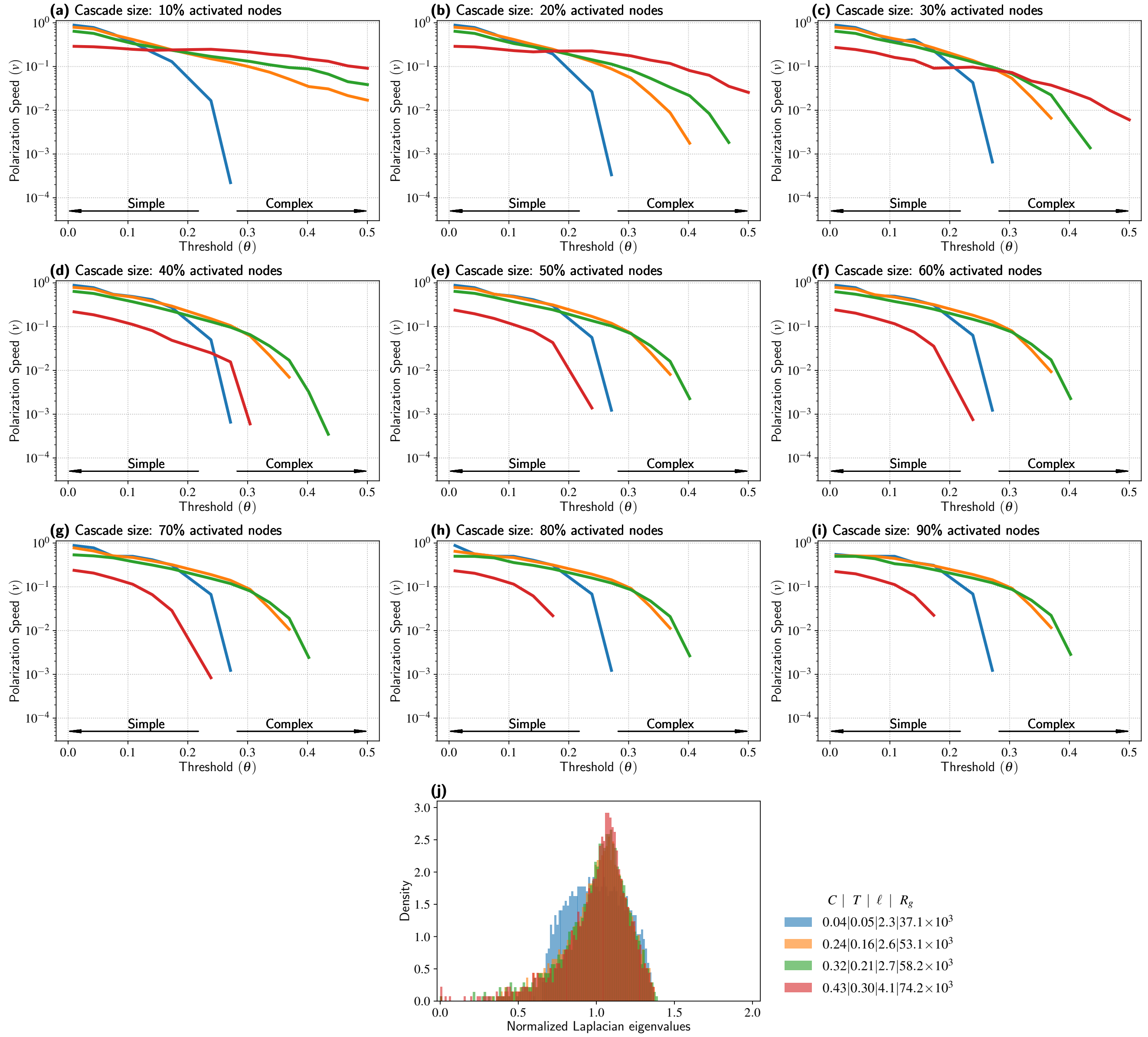

Figure S2. Linear threshold model for MHK networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **top 5% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks.

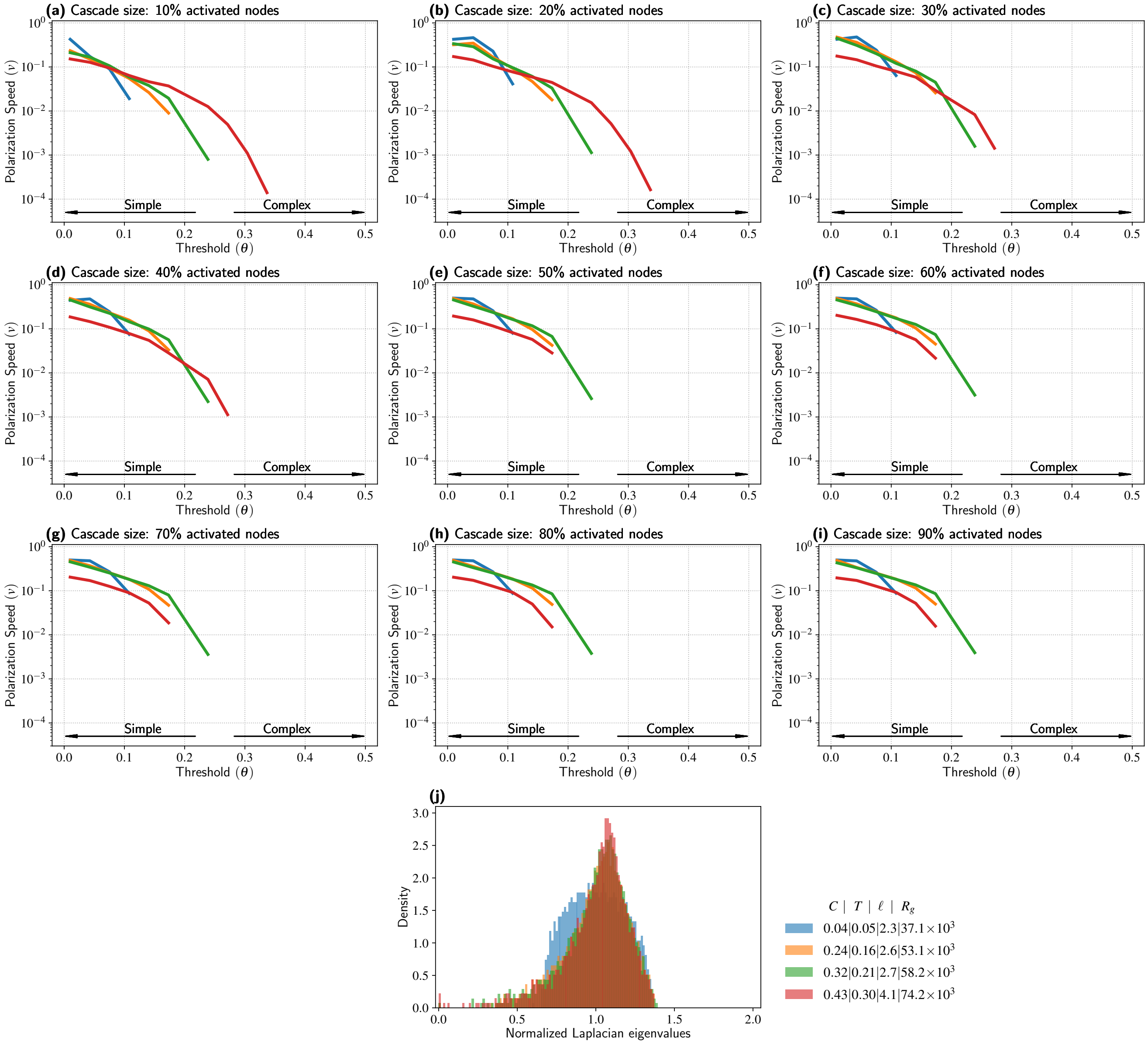

Figure S3. Linear threshold model for MHK networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **bottom 5% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks.

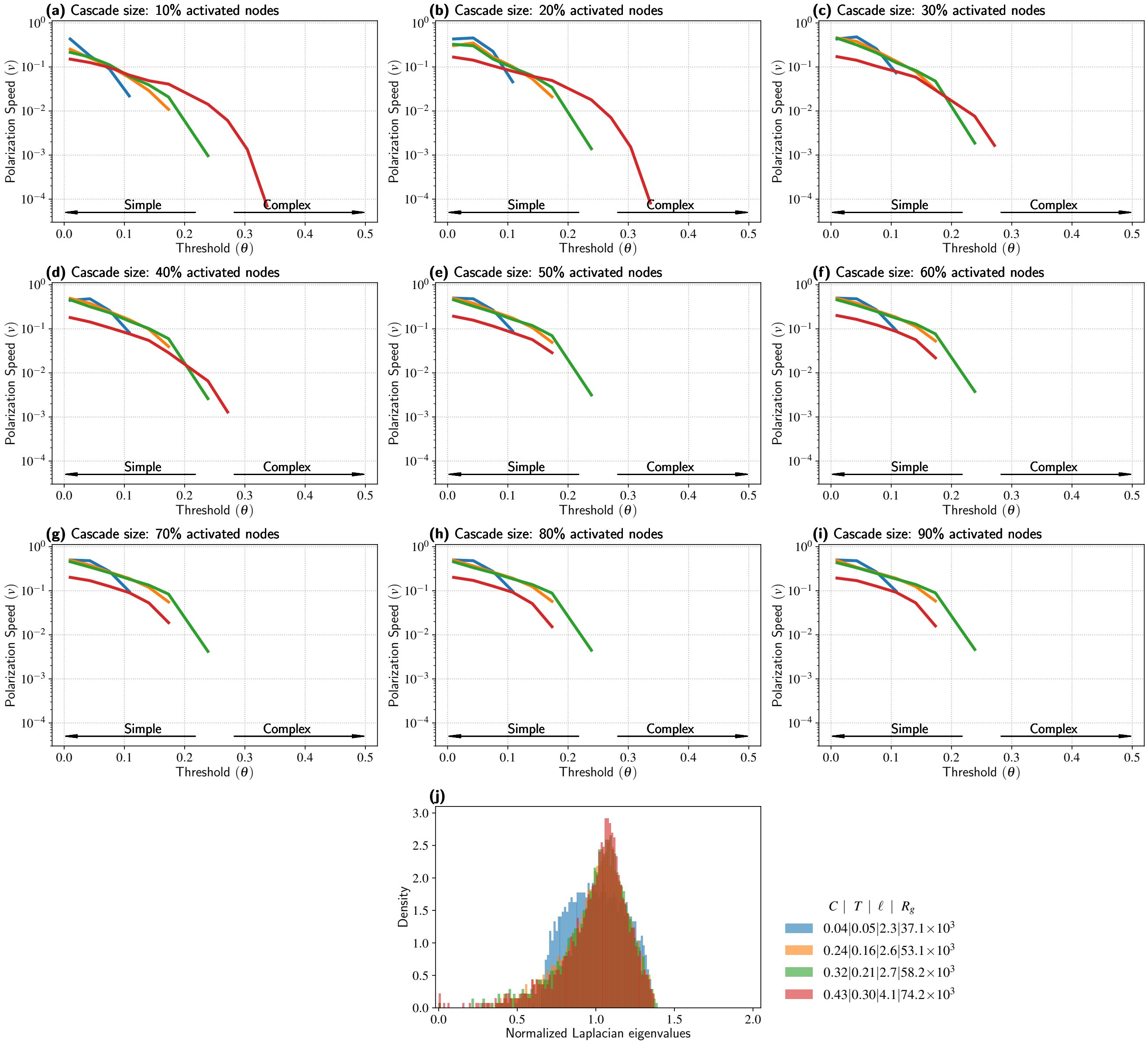

Figure S4. Linear threshold model for MHK networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **bottom 10% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks.

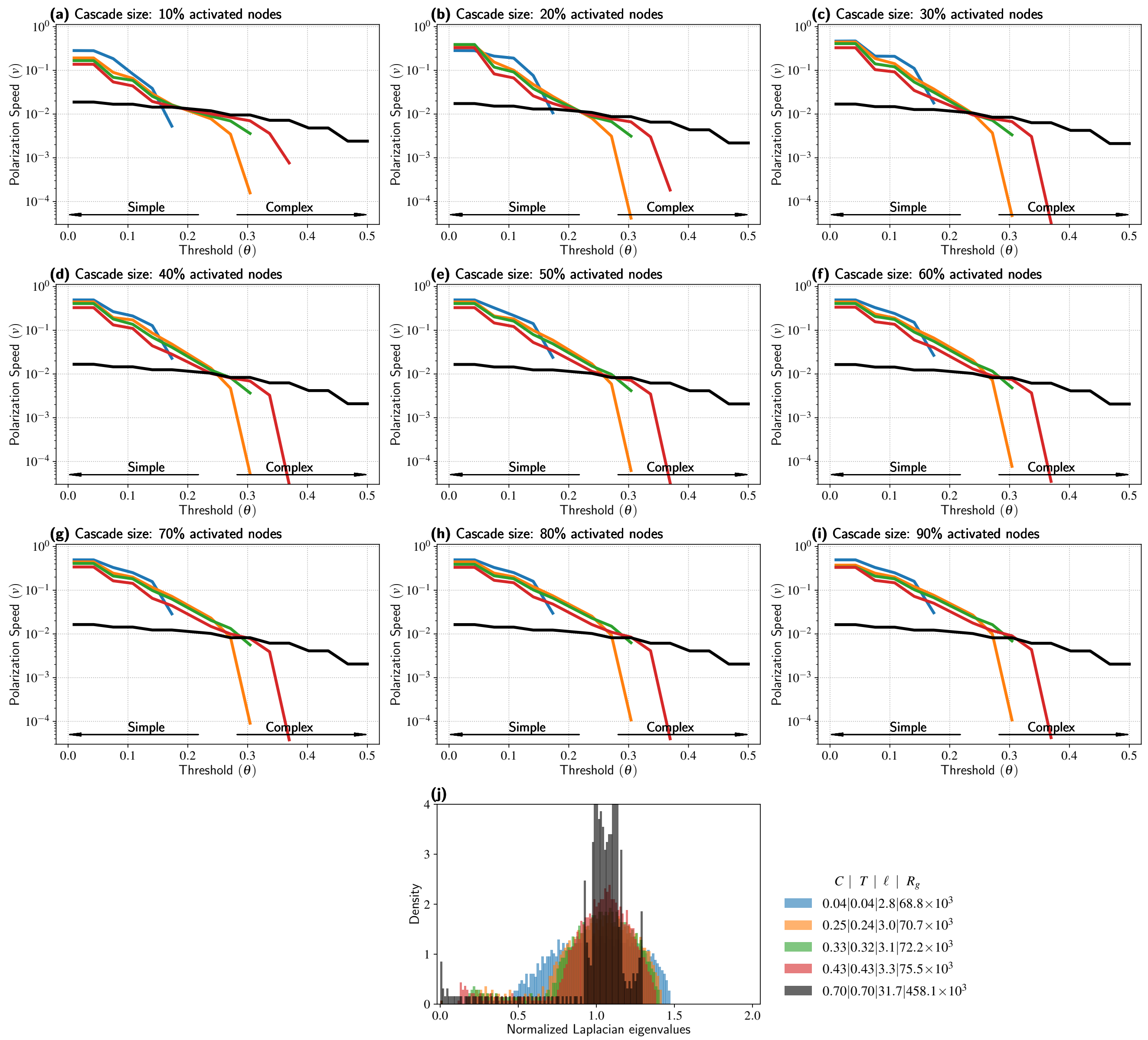

Figure S5. Linear threshold model for WS networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **top 10% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks (some of the histograms have been clipped in favor of the readability of the figures).

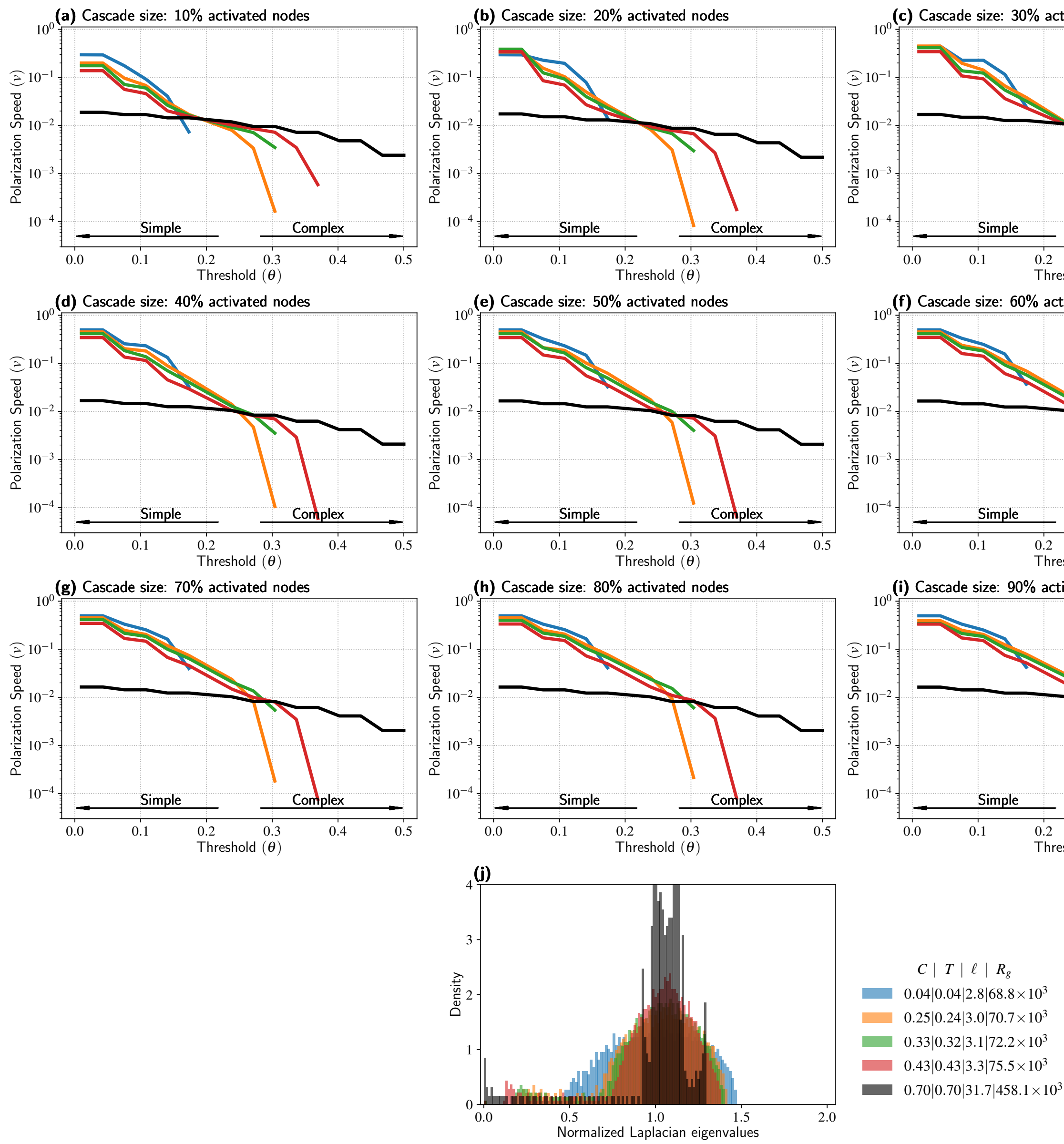

Figure S6. Linear threshold model for WS networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **top 5% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks (some of the histograms have been clipped in favor of the readability of the figures).

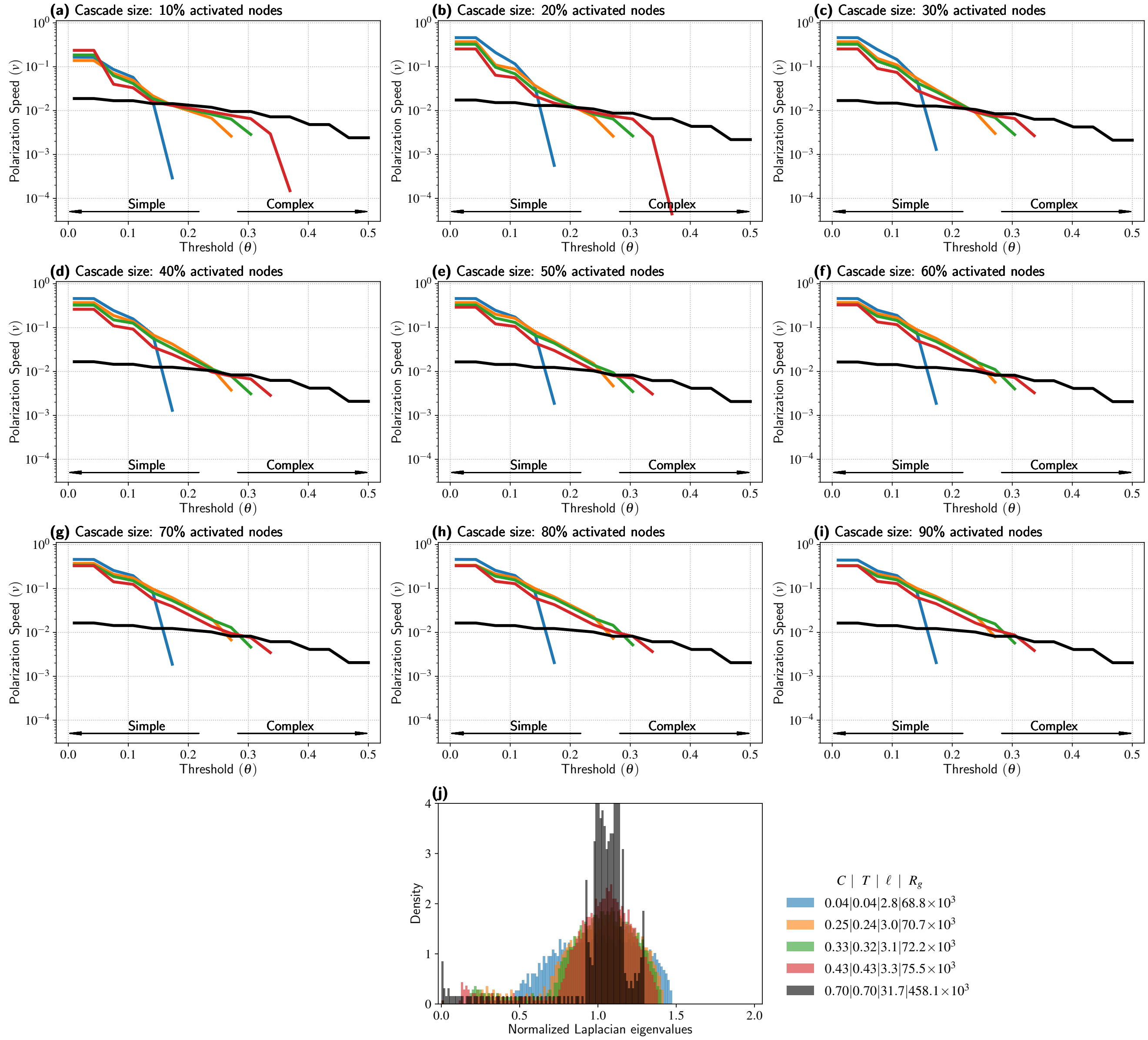

Figure S7. Linear threshold model for WS networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **bottom 5% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks (some of the histograms have been clipped in favor of the readability of the figures).

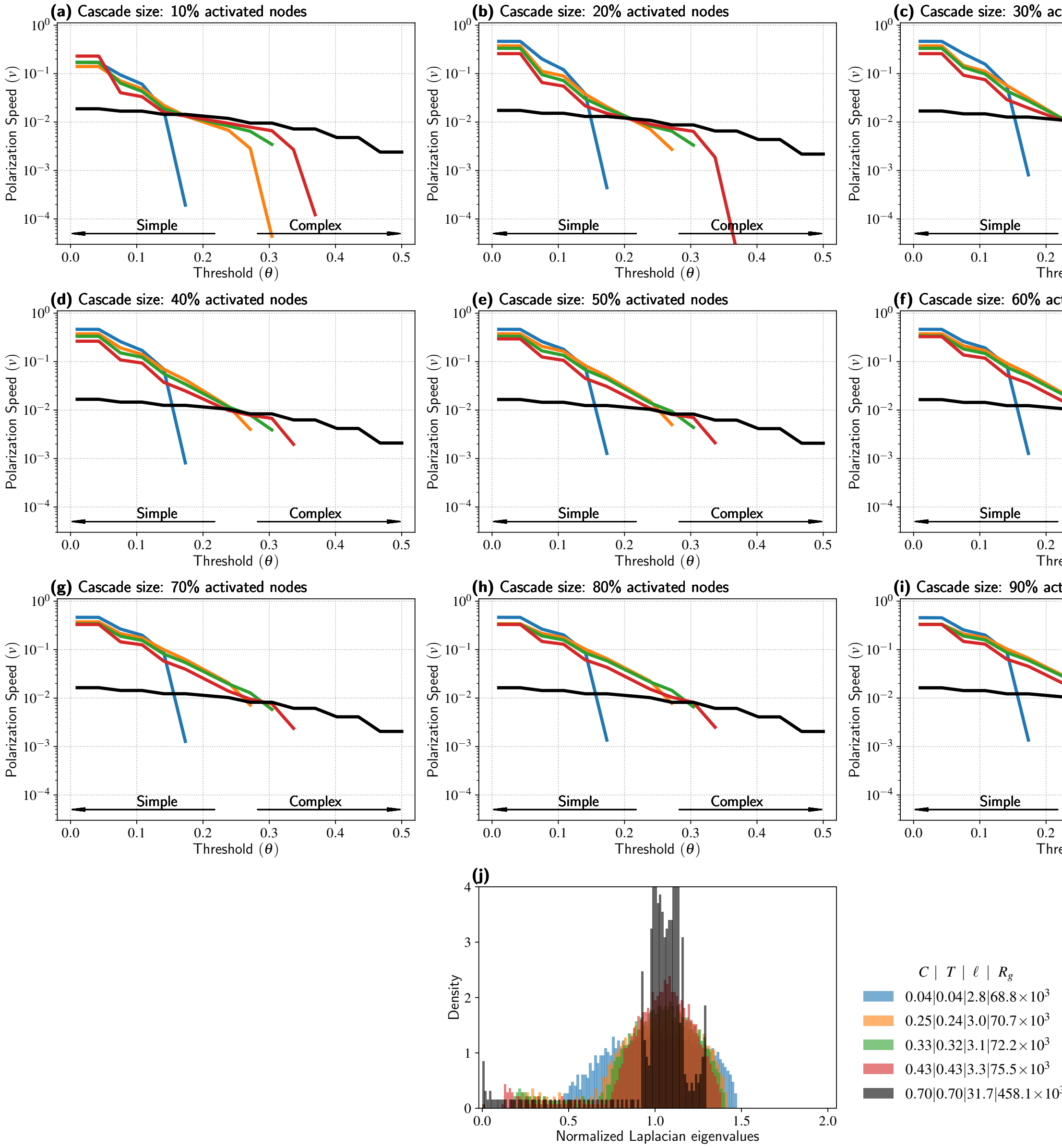

Figure S8. Linear threshold model for WS networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with different cascade sizes (**a-i**). Results are averaged over all seed nodes in the **bottom 10% by degree** that are initially and individually activated along with their neighbors. (**j**) Corresponding normalized Laplacian eigenspectrum density histogram for these networks (some of the histograms have been clipped in favor of the readability of the figures).

Seeding effect

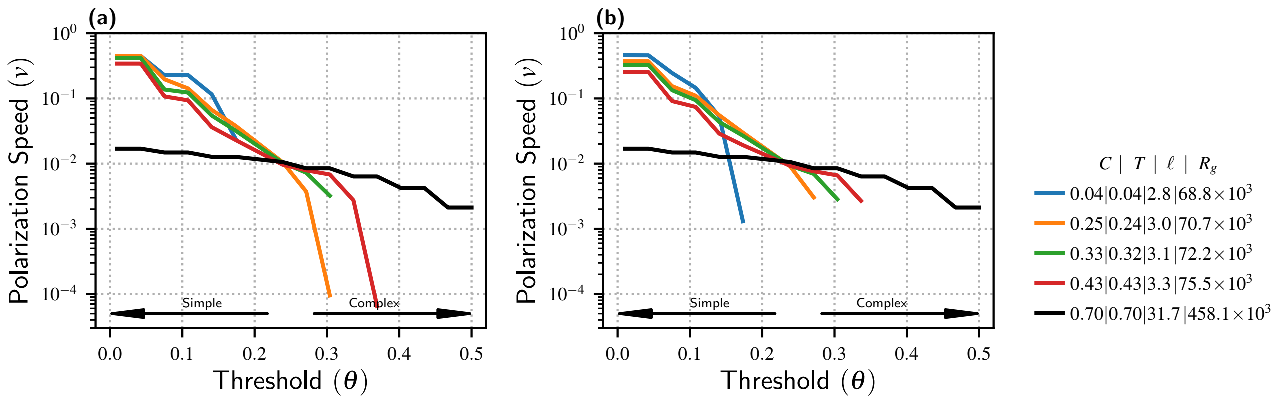

Figure S9. Linear threshold model for WS networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, and cascade size 0.3. Results are averaged over all seed nodes in (**a**) the top 5% by degree and (**b**) the bottom 5% by degree, that are initially and individually activated along with their neighbors. Networks are sampled to be the same.

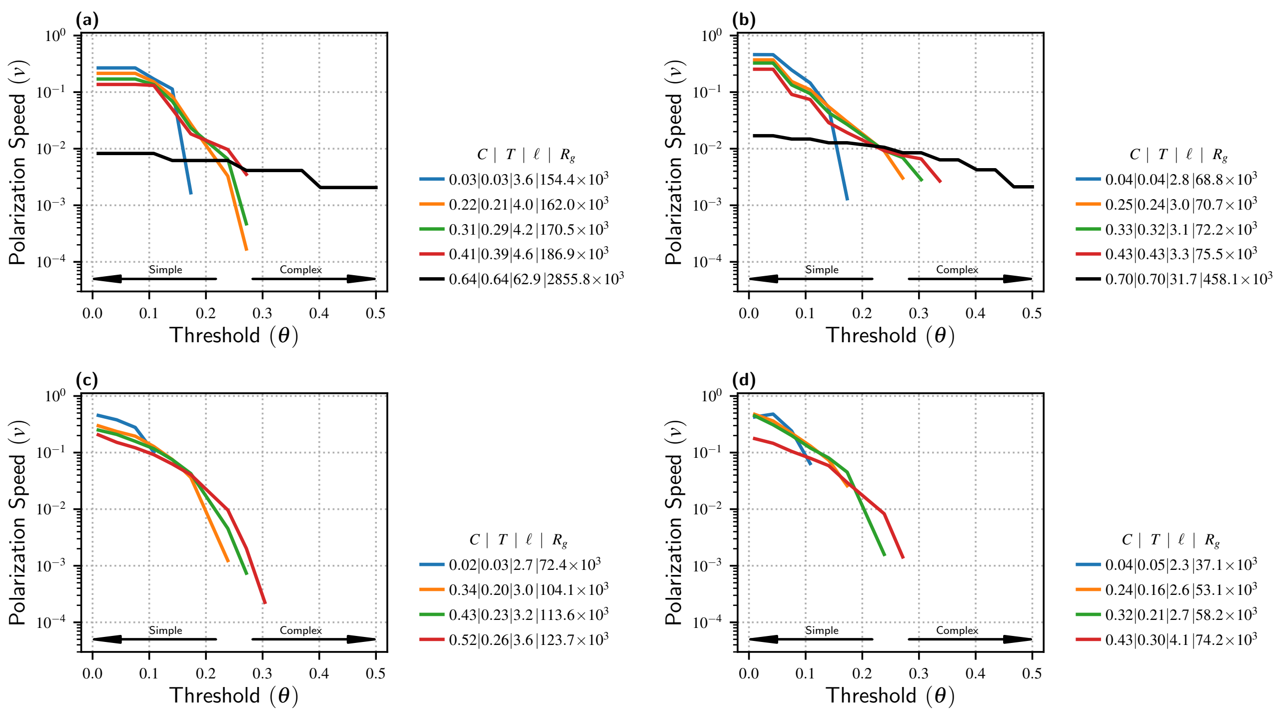

Figure S10. Linear threshold model for: (**a, b**) WS networks, and (**c,d**) MHK networks with $N = 1,000$ nodes, uniform threshold θ, and cascade size 0.3. (**a, c**) average degree $\langle k \rangle = 8$; (**b, d**) average degree $\langle k \rangle = 16$. Results are averaged over all seed nodes in the bottom 5% by degree that are initially and individually activated along with their neighbors. Selected networks have similar rewiring and triangular formation probabilities p.

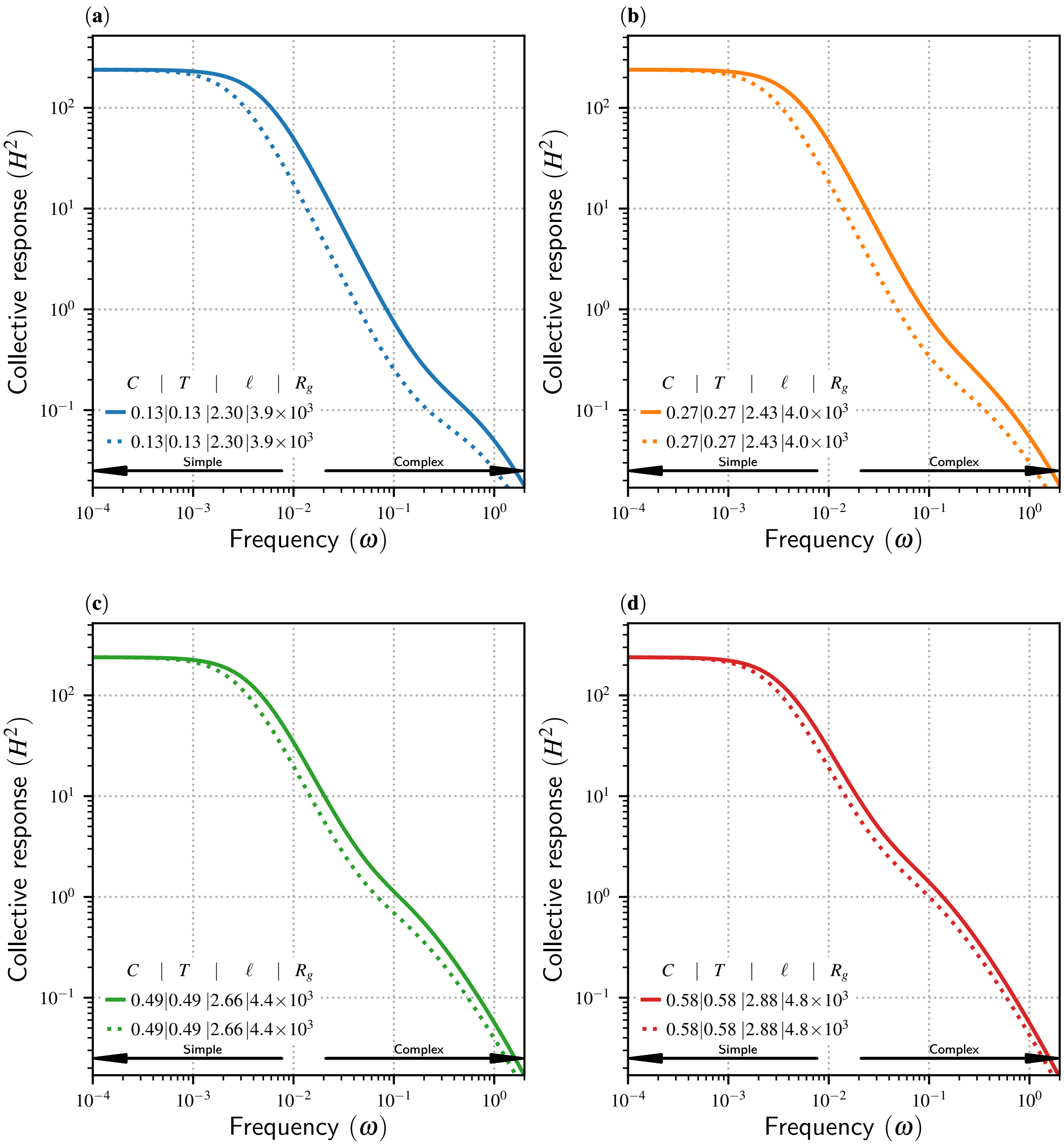

Figure S11. Collective response H^2 for WS networks with $N = 240$ nodes, average degree $\langle k \rangle = 16$. The results are averaged over all nodes serving as leaders: **solid lines** representing the top 5% by degree; **dashed lines** representing the bottom 5% by degree. The networks and the collective responses are the same as those in Fig.5 in the main text.

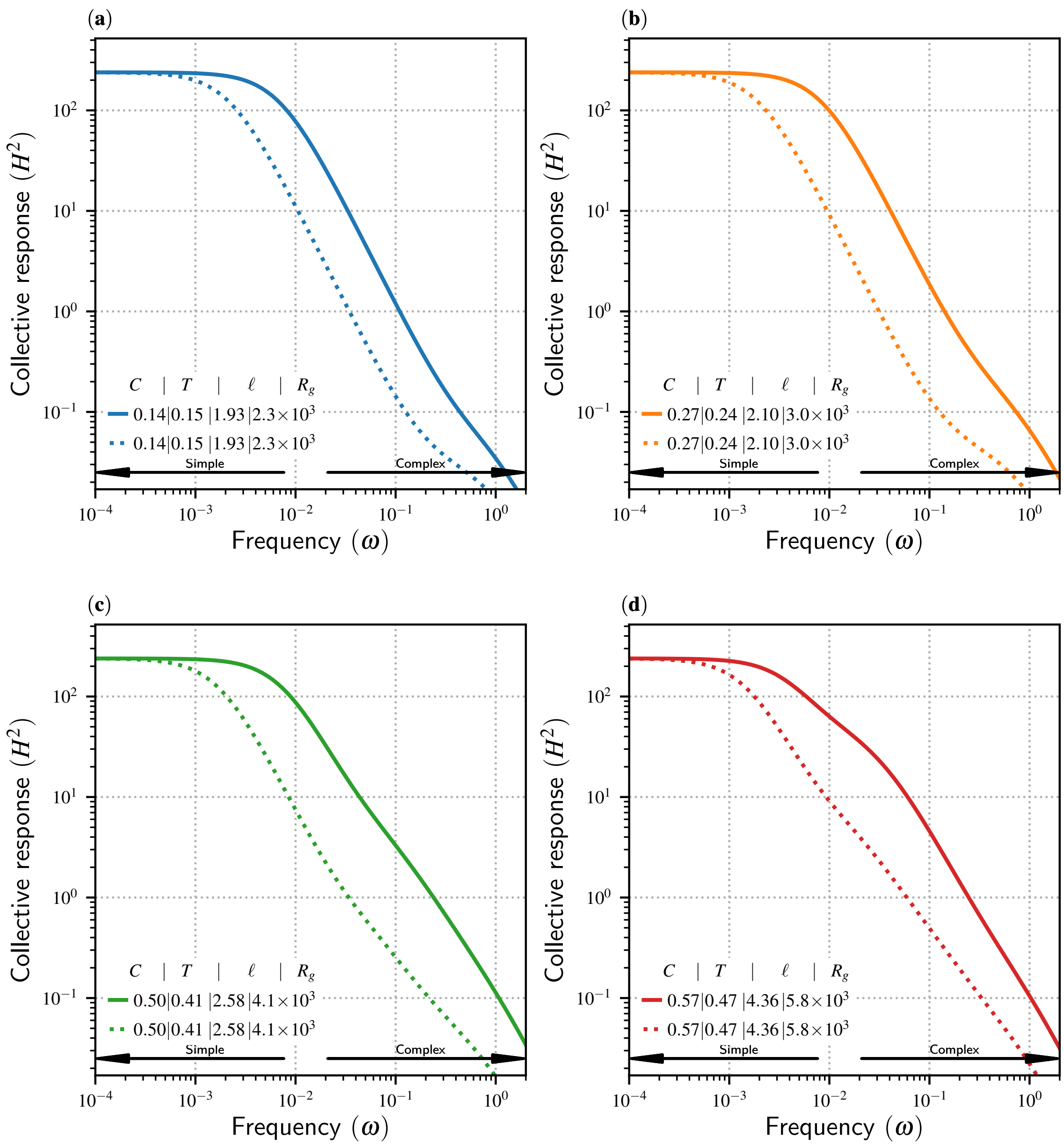

Figure S12. Collective response H^2 for MHK networks with $N = 240$ nodes, average degree $\langle k \rangle = 16$. The results are averaged over all nodes serving as leaders: **solid lines** representing the top 5% by degree; **dashed lines** representing the bottom 5% by degree. The networks and the collective responses are the same as those in Fig.6 in the main text.

Highly clustered scale-free model

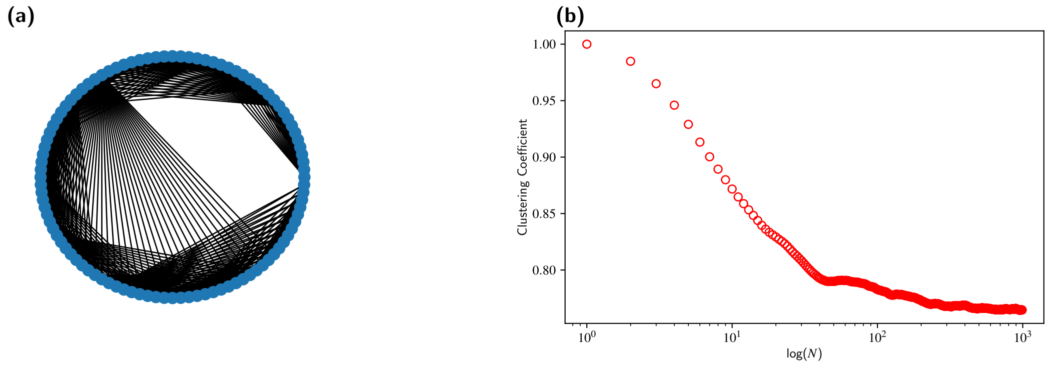

Figure S13. Highly clustered scale-free network. **(a)** Graph in a circular layout with $N = 100$ nodes and average degree $\langle k \rangle = 10$. **(b)** Variations of the clustering coefficient with N for $\langle k \rangle = 10$.

Seeding effect on HCSF networks

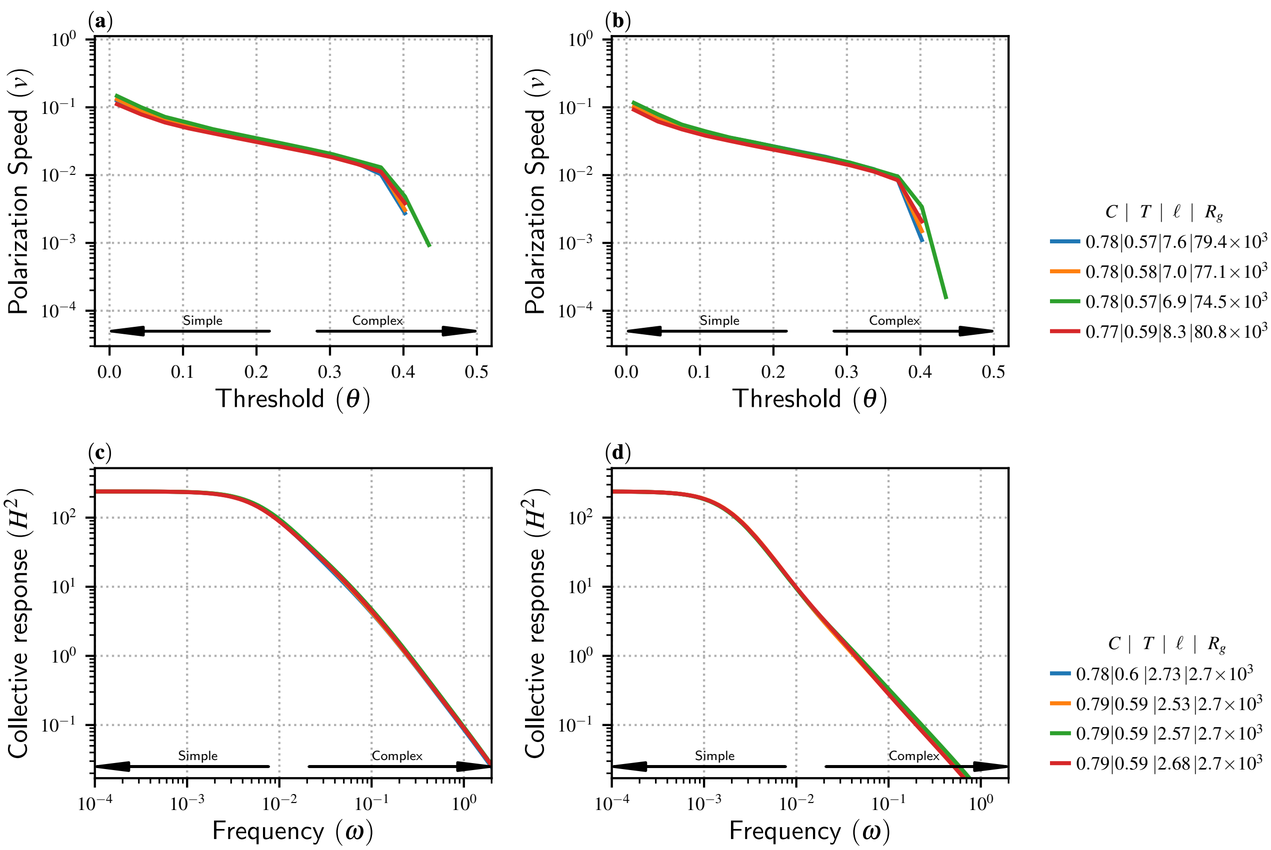

Figure S14. Linear threshold model for HCSF networks with $N = 1,000$ nodes, fixed average degree $\langle k \rangle = 16$, uniform threshold θ, with 30% cascade size. Results are averaged over all seed nodes in (**a**) the top 5% by degree; (**b**) the bottom 5% by degree, that are initially and individually activated along with their neighbors. Collective response H^2 for HCSF networks with $N = 240$ nodes, average degree $\langle k \rangle = 16$. The results are averaged over all nodes serving as leaders in (**c**) the top 5% by degree; (**d**) the bottom 5% by degree. The same sampled networks are used within each model type.