

Supplementary Information for: LARK: A Low-Cost, Accurate, Occlusion-Resilient, Kalman Filter-Assisted Tracking System for Image-Guided Surgery

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1 Supplementary Note 1: Landmark Registration

Landmark registration aligns a set of measured 3-D points with their corresponding ground-truth (GT) points to determine the rigid transformation between two coordinate systems. This method is central to IGS, where precise alignment ensures that surgical instruments and patient anatomy share a consistent spatial frame of reference.

We employ a point-based registration strategy using the Orthogonal Procrustes method [39]. The aim is to find a rotation matrix $S \in SO(3)$ and a translation vector $d \in \mathbb{R}^3$ that minimise the difference between corresponding points in the measured and GT sets. Suppose N_f fiducial points are available. The Fiducial Registration Error (FRE) is the root-mean-square distance between corresponding fiducials after alignment:

$$\text{FRE} = \sqrt{\frac{1}{N_f} \sum_{i=1}^{N_f} \|x_i^{\text{GT}} - (S x_i^{\text{meas}} + d)\|^2} \quad (1)$$

To determine S and d , the measured and GT point sets are first translated so that their centroids lie at the origin. Denote those centroids by $\bar{x}^{\text{meas}}$ and $\bar{x}^{\text{GT}}$. After centring, the covariance matrix is

$$G = \sum_{i=1}^{N_f} (x_i^{\text{meas}} - \bar{x}^{\text{meas}})(x_i^{\text{GT}} - \bar{x}^{\text{GT}})^{\top}. \quad (2)$$

Applying Singular Value Decomposition (SVD) to G ,

$$G = U \Sigma V^{\top},$$

the optimal rotation is

$$S = V U^{\top}.$$

If $\det(S) < 0$ the final column of V is negated to prevent a reflection. The translation that relates the two centroids is

$$d = \bar{x}^{\text{GT}} - S \bar{x}^{\text{meas}}.$$

With S and d fixed by the fiducials, we validate the transformation on a separate set of N_t target points. The Target Registration Error (TRE) is

$$\text{TRE} = \sqrt{\frac{1}{N_t} \sum_{j=1}^{N_t} \|y_j^{\text{GT}} - (S y_j^{\text{meas}} + d)\|^2}. \quad (3)$$

The same transformation can be applied to a measured trajectory $\{p_1^{\text{meas}}, \dots, p_{N_p}^{\text{meas}}\}$ to obtain

$$p_i^{\text{trans}} = S p_i^{\text{meas}} + d, \quad i = 1, \dots, N_p.$$

For each transformed point p_i^{trans} we locate its nearest neighbour q_i^{GT} in the GT trajectory T^{GT} :

$$q_i^{\text{GT}} = \underset{q \in T^{\text{GT}}}{\arg\min} \|p_i^{\text{trans}} - q\|.$$

The trajectory TRE is then

$$\text{TRE}_{\text{traj}} = \sqrt{\frac{1}{N_p} \sum_{i=1}^{N_p} \|p_i^{\text{trans}} - q_i^{\text{GT}}\|^2}.$$

Nearest-neighbour matching compensates for any temporal offset between the measured and GT trajectories, ensuring that spatial correspondence is preserved even when the two are sampled at different rates.

2 Supplementary Note 2: Noise Profiling

To compare the performance of our adaptive Kalman filter with a constant velocity Kalman filter using static covariance matrices, we first conduct a noise profiling procedure. The procedure allows us to estimate the static measurement noise covariance matrices R_i for each camera and the process noise covariance matrix Q for the non-adaptive filter.

The noise profiling involves collecting a series of pose measurements while the surgical instrument is held stationary at L distinct workspace locations indexed by $\ell = 1, \dots, L$. At every location we record $T = 30$ consecutive pose estimates from each active camera $i \in \{1, \dots, N\}$. The sample variance of those poses gives the location-specific measurement covariance

$$R_{\ell,i} = \text{Var}(y_{\tau,i} \mid \tau = 1, \dots, T), \quad (4)$$

where $y_{\tau,i}$ is the pose returned by camera i on frame τ while the instrument is fixed at location ℓ .

We then compute the average measurement noise covariance for each camera across all sampled positions:

$$\bar{R}_i = \frac{1}{L} \sum_{\ell=1}^L R_{\ell,i}, \quad (5)$$

which serves as the static measurement noise covariance matrix for camera i in the non-adaptive Kalman filter.

To formulate the static process noise covariance matrix Q , we calculate the average of the $\bar{R}_i$ matrices across all cameras:

$$Q = \frac{1}{N} \sum_{i=1}^N \bar{R}_i, \quad (6)$$

where N is the total number of cameras. This approach ensures that Q reflects the average noise characteristics across the entire system.

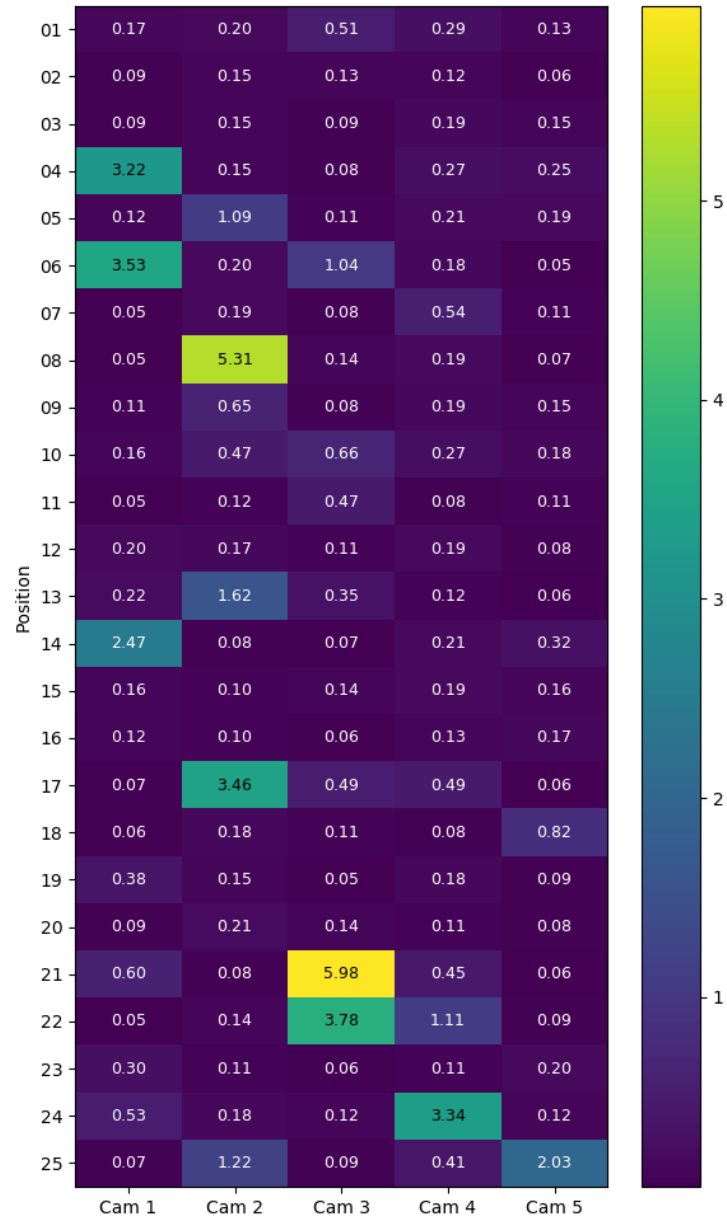
Beyond tuning the static covariance matrices for the non-adaptive Kalman filter, this procedure provides valuable insights into the spatial distribution of measurement noise within the workspace. By analyzing the norms of the covariance matrices $R_{\ell,i}$, we generate heatmaps illustrating the noise profile for each camera at different positions. The Frobenius norm of each $R_{\ell,i}$ matrix is computed as

$$\|R_{\ell,i}\|_F = \sqrt{\text{Tr}(R_{\ell,i} R_{\ell,i}^T)}. \quad (7)$$

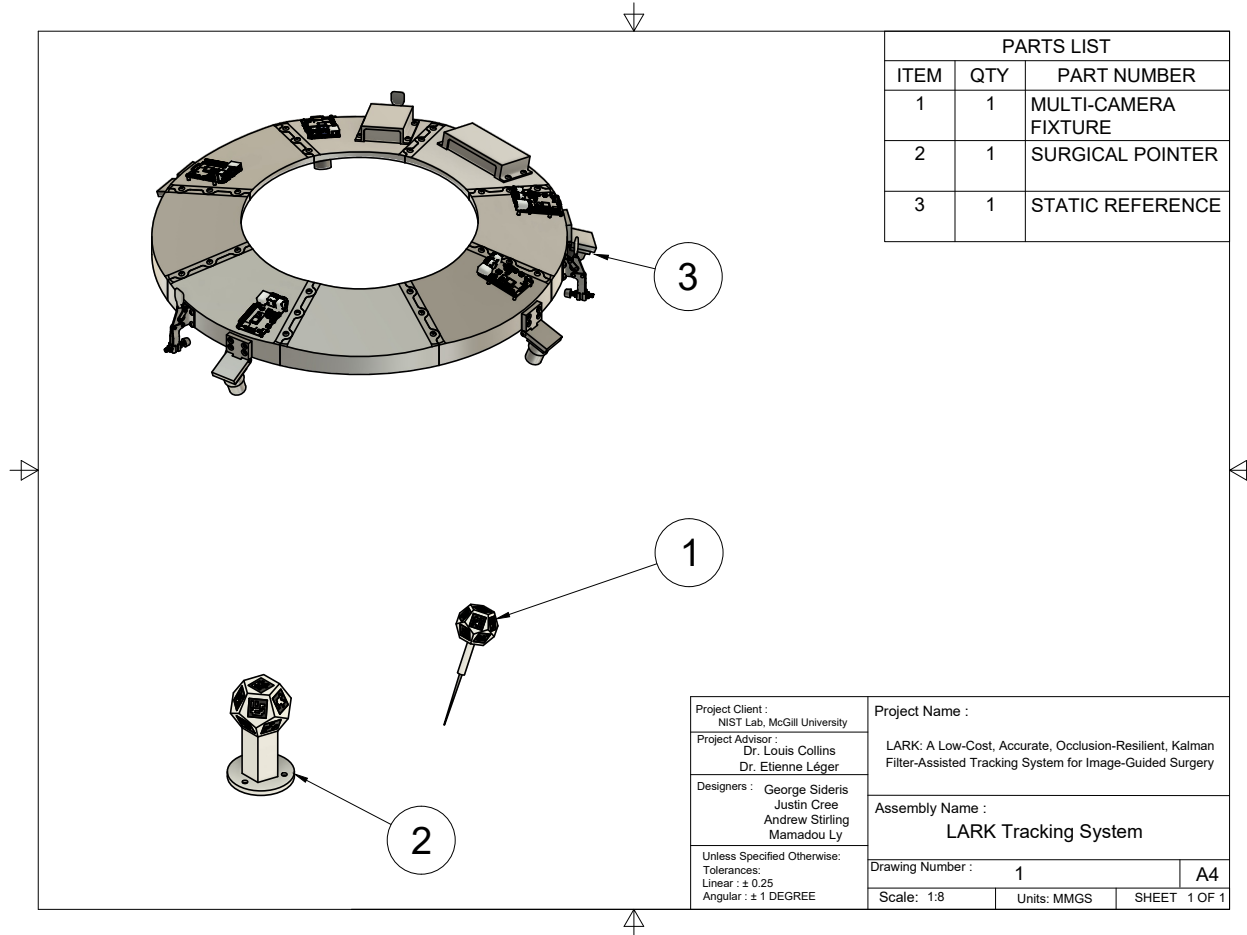
This norm provides a single scalar value to represent the overall noise level for each camera at each position. Supplementary Figure 1 shows the spatial distribution of noise across all measured positions for one of our experimental trials.

By employing this noise profiling procedure, we ensure that the static Kalman filter is optimally configured for our experimental conditions, providing a fair baseline for comparison with our adaptive Kalman filter. Moreover, the insights gained can guide adjustments to lighting, camera placement, and calibration to enhance measurement precision. In the OR, technicians can swiftly perform this procedure prior to surgery to ensure camera setups meet precision requirements, minimizing tracking variability.

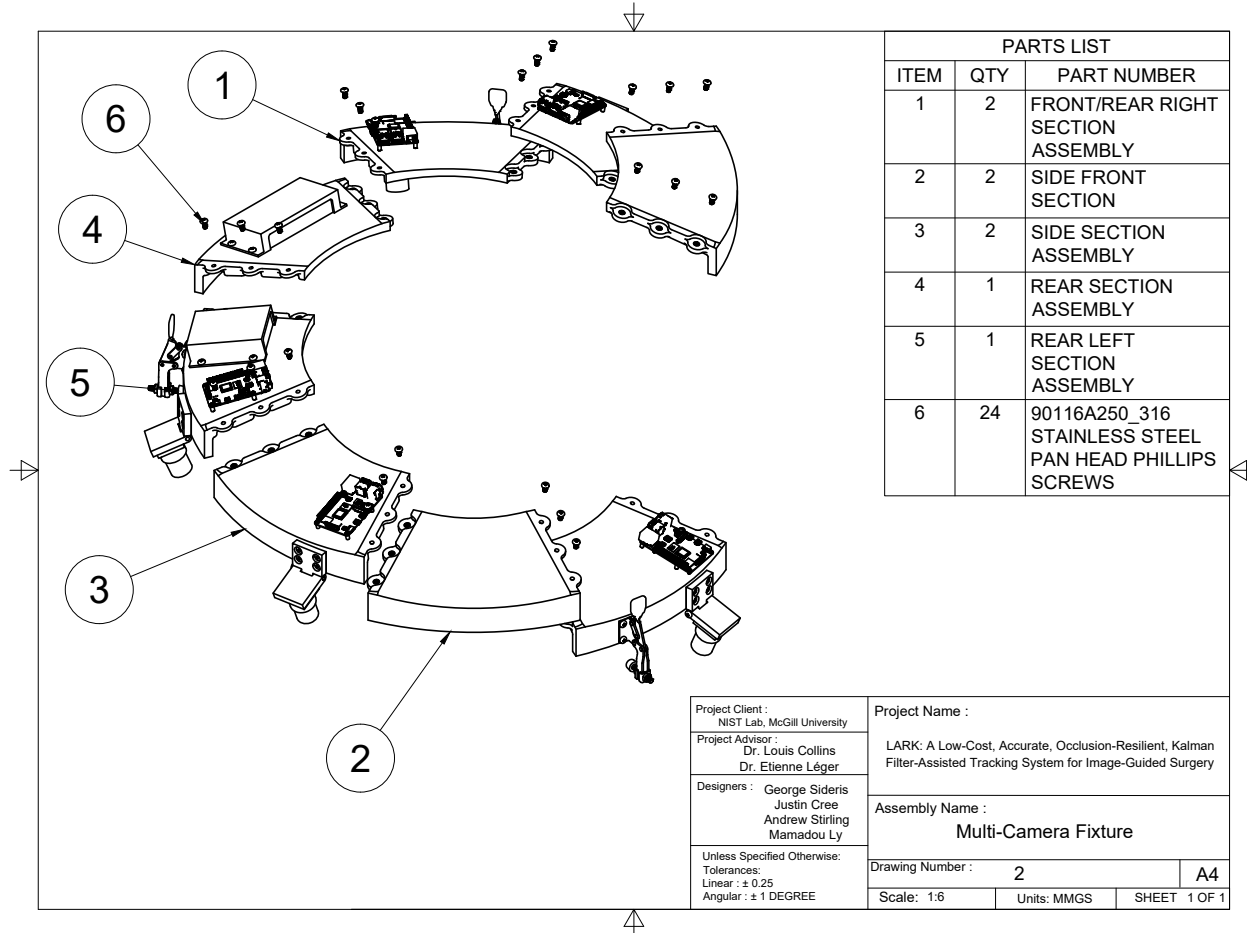
3 Supplementary Figures 1-26:



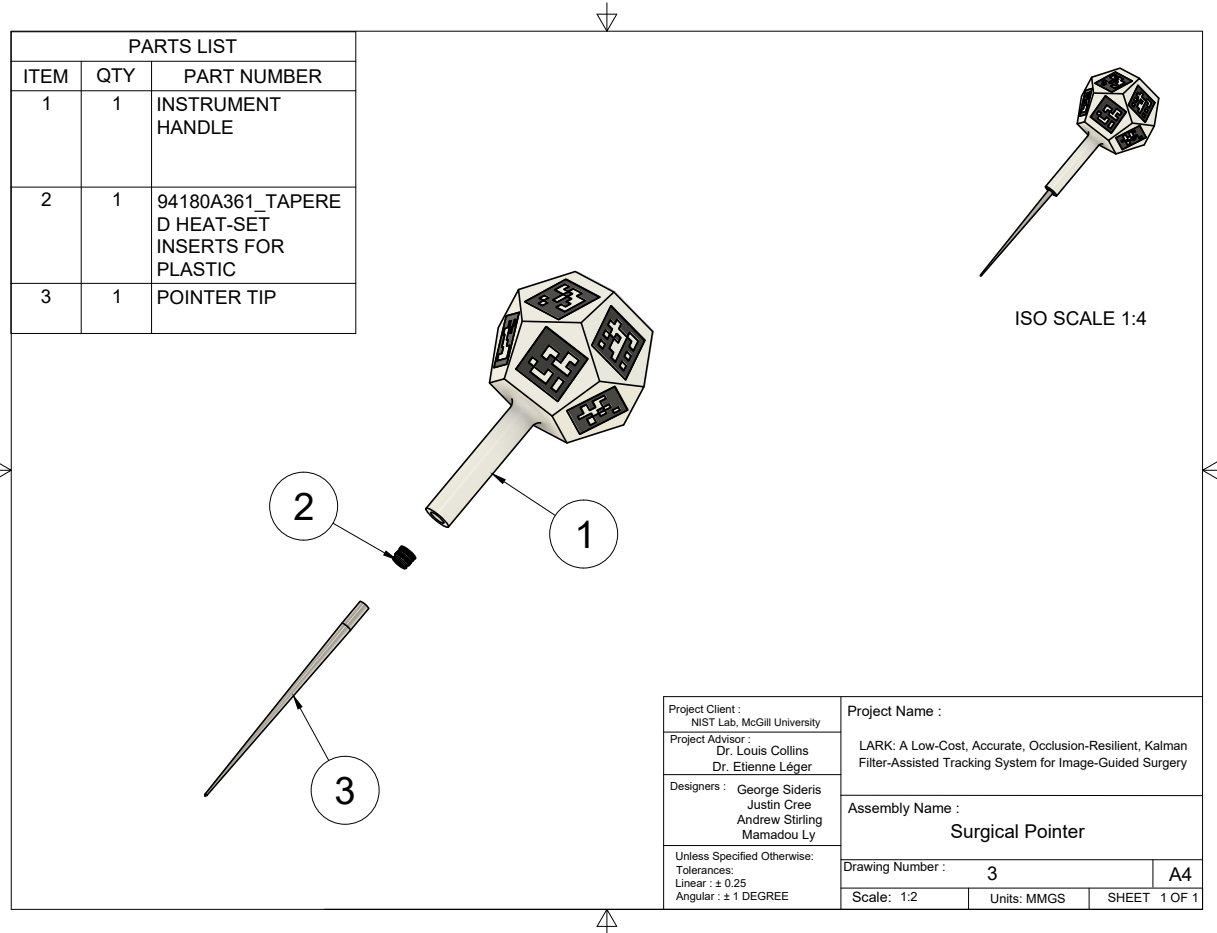
Supplementary Figure 1: Spatial distribution of the norm of the measurement noise covariance matrices $R_{\ell,i}$ for each camera across all measured positions.



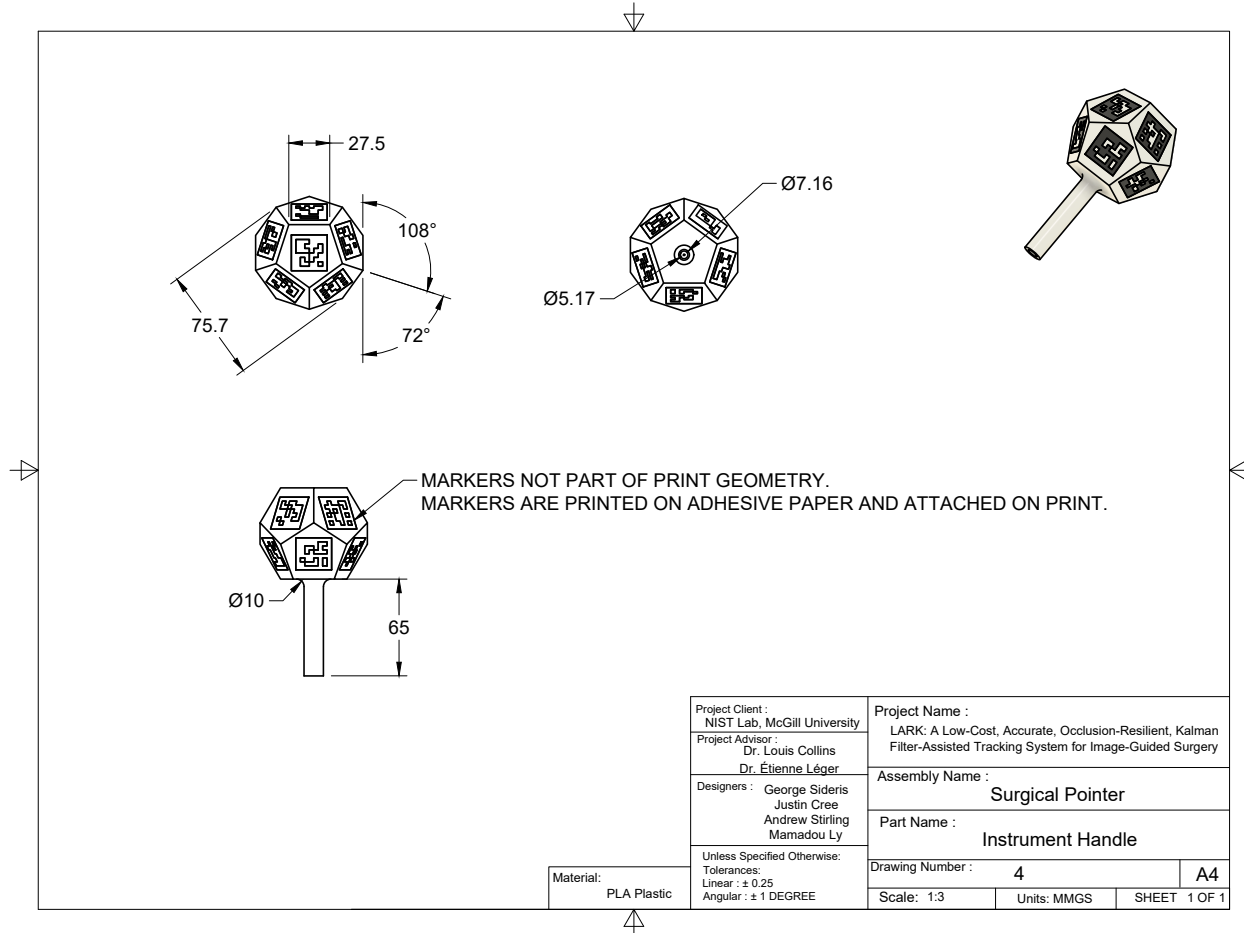
Supplementary Figure 3: LARK Tracking System Assembly Drawing.



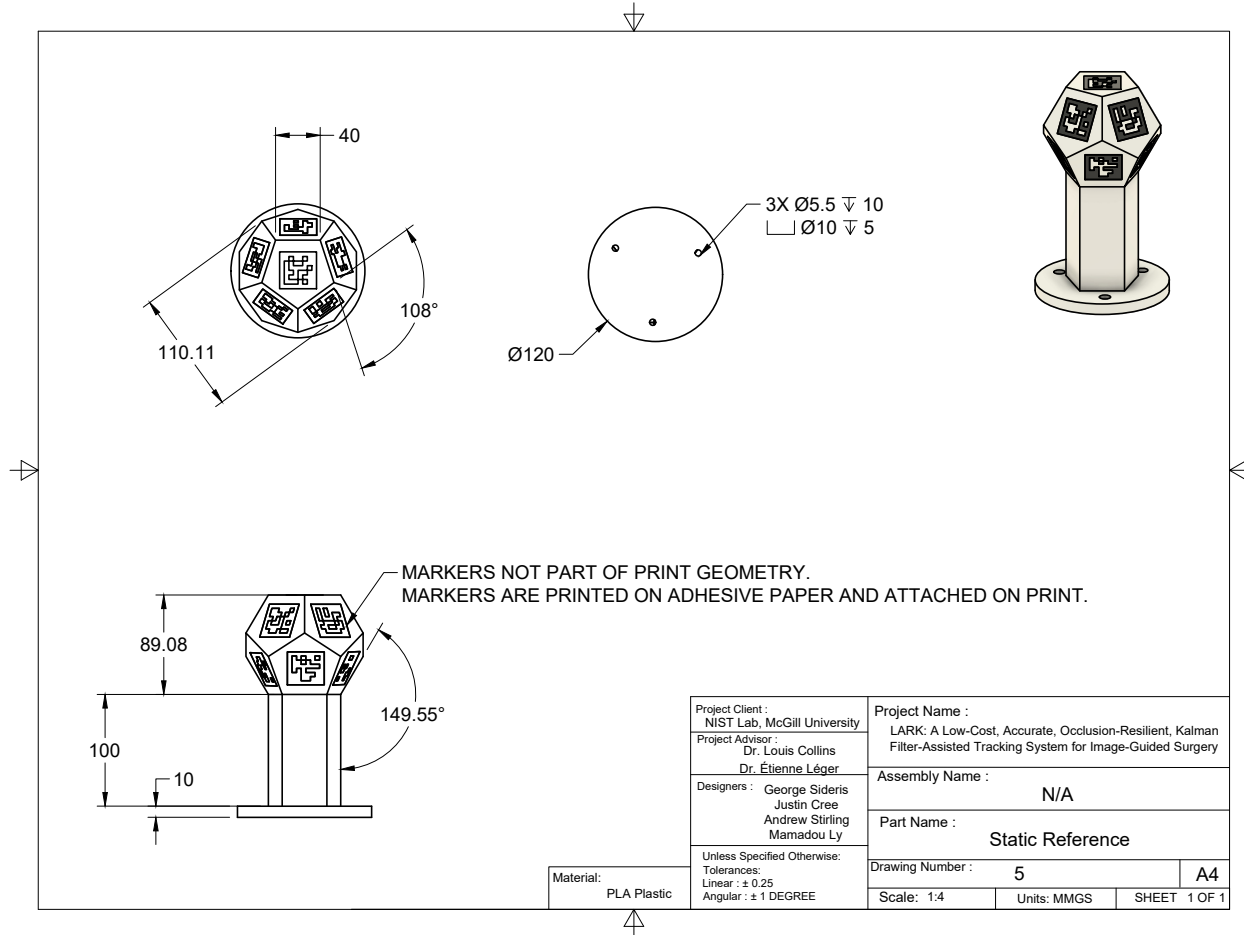
Supplementary Figure 4: Multi-Camera Fixture Assembly Drawing.



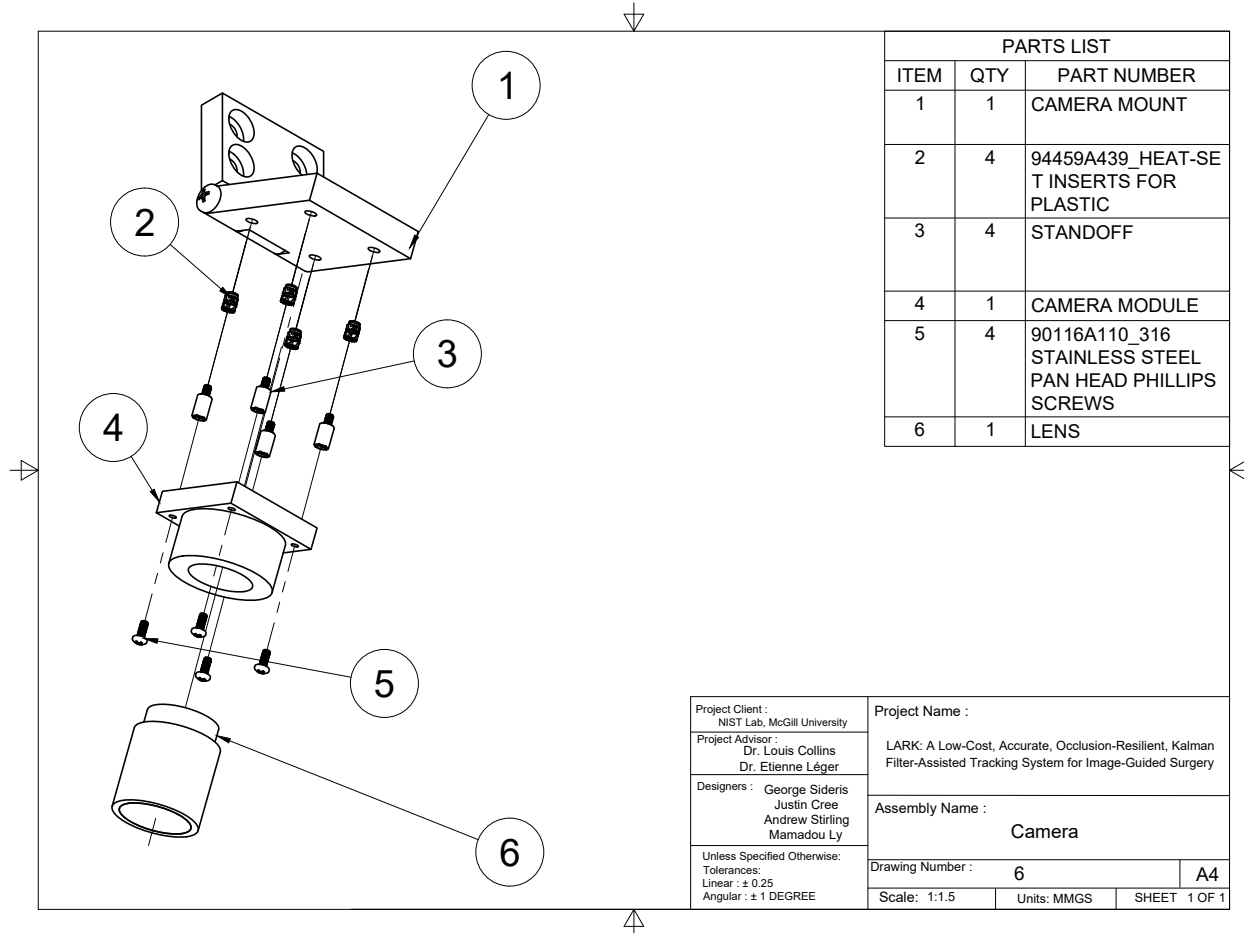
Supplementary Figure 5: Surgical Pointer Assembly Drawing.



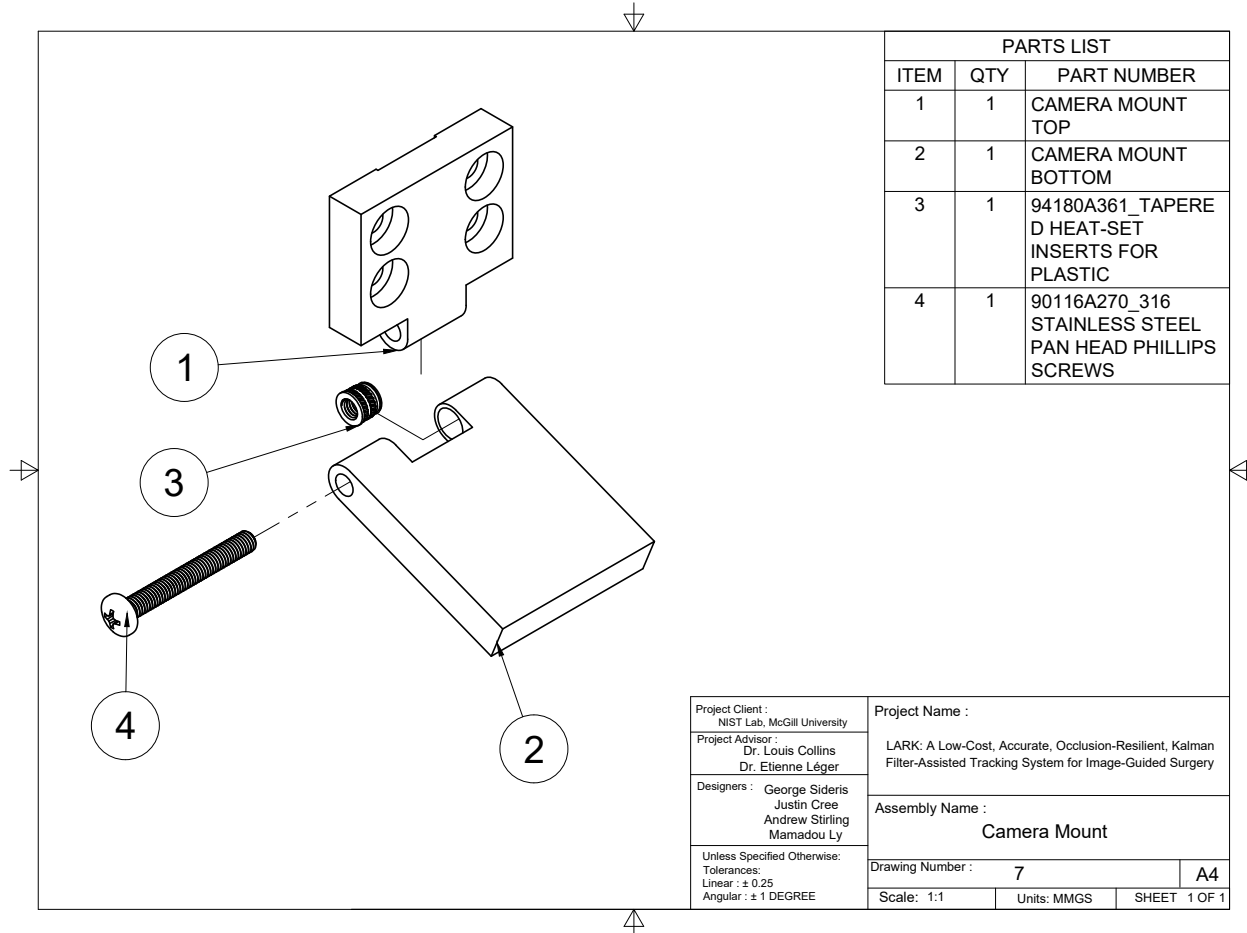
Supplementary Figure 6: Instrument Handle Technical Drawing.



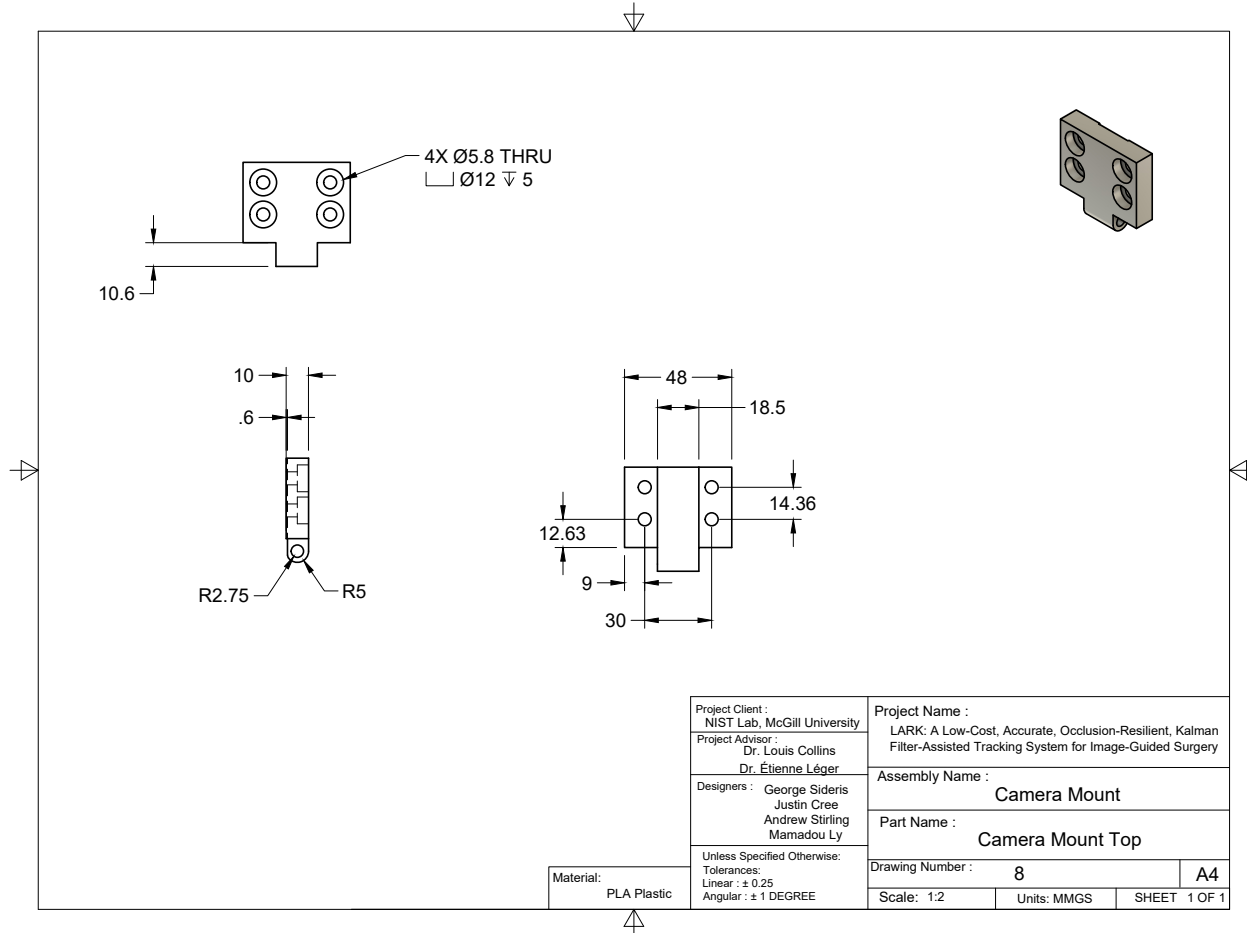
Supplementary Figure 7: Static Reference Technical Drawing.



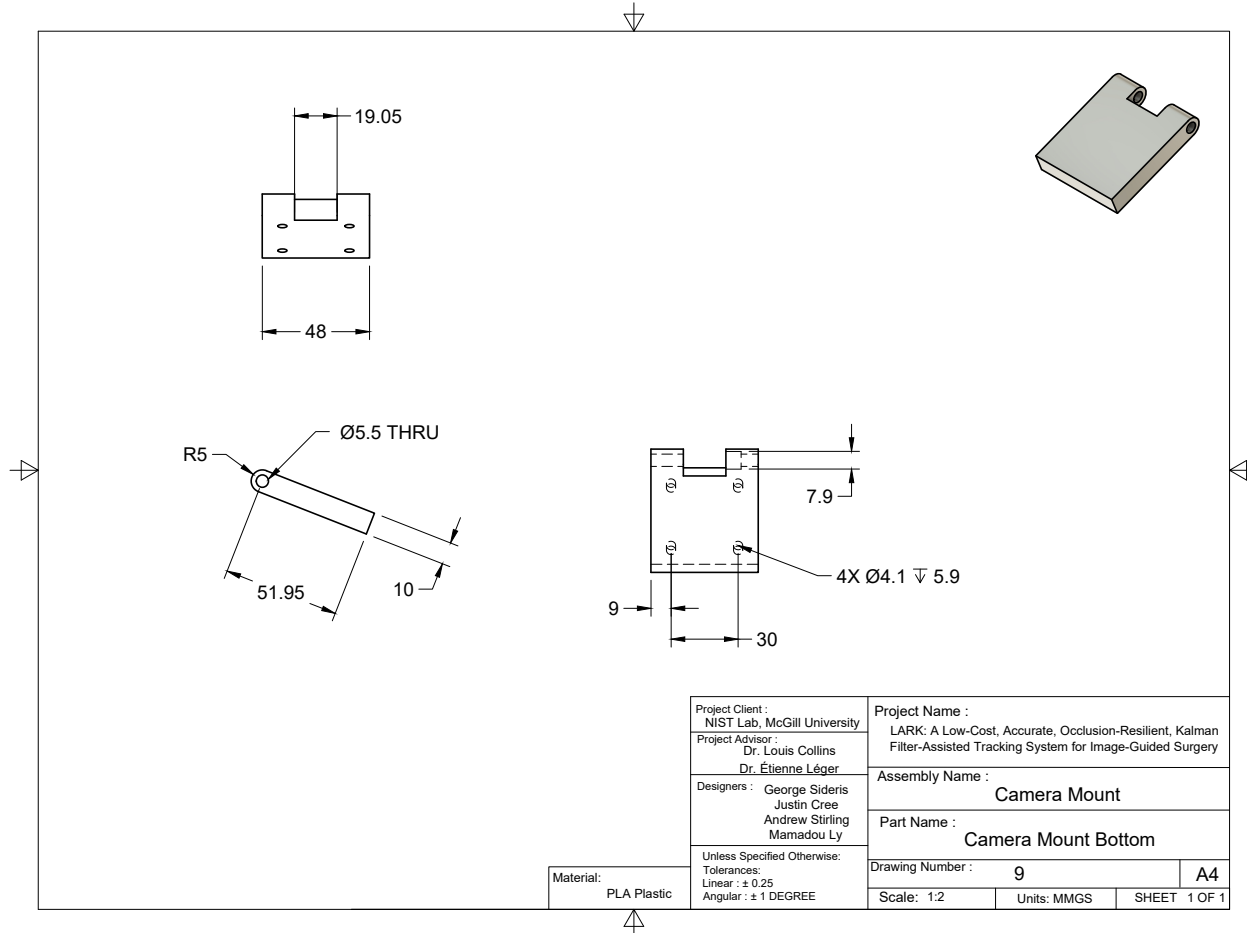
Supplementary Figure 8: Camera Assembly Drawing.



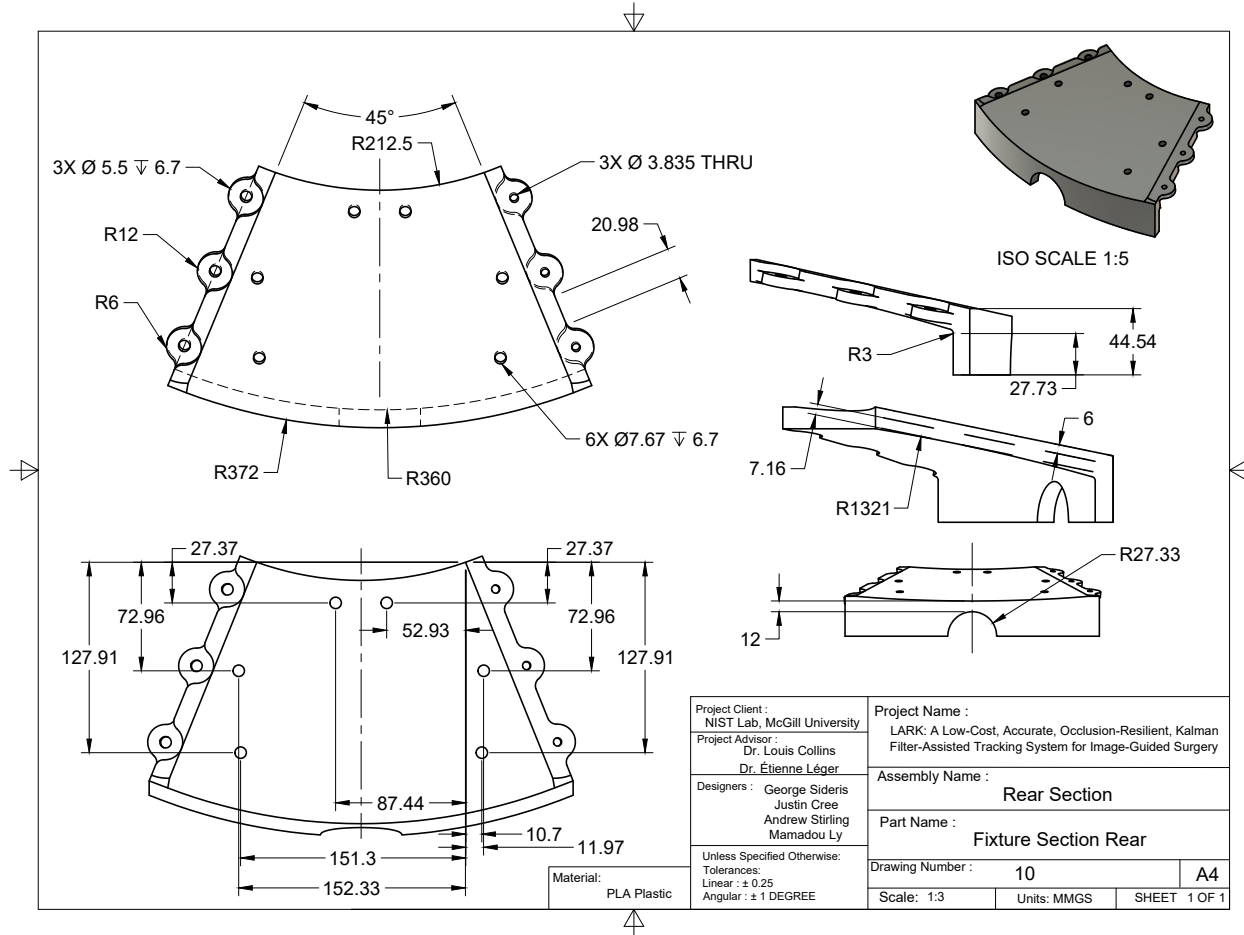
Supplementary Figure 9: Camera Mount Assembly Drawing.



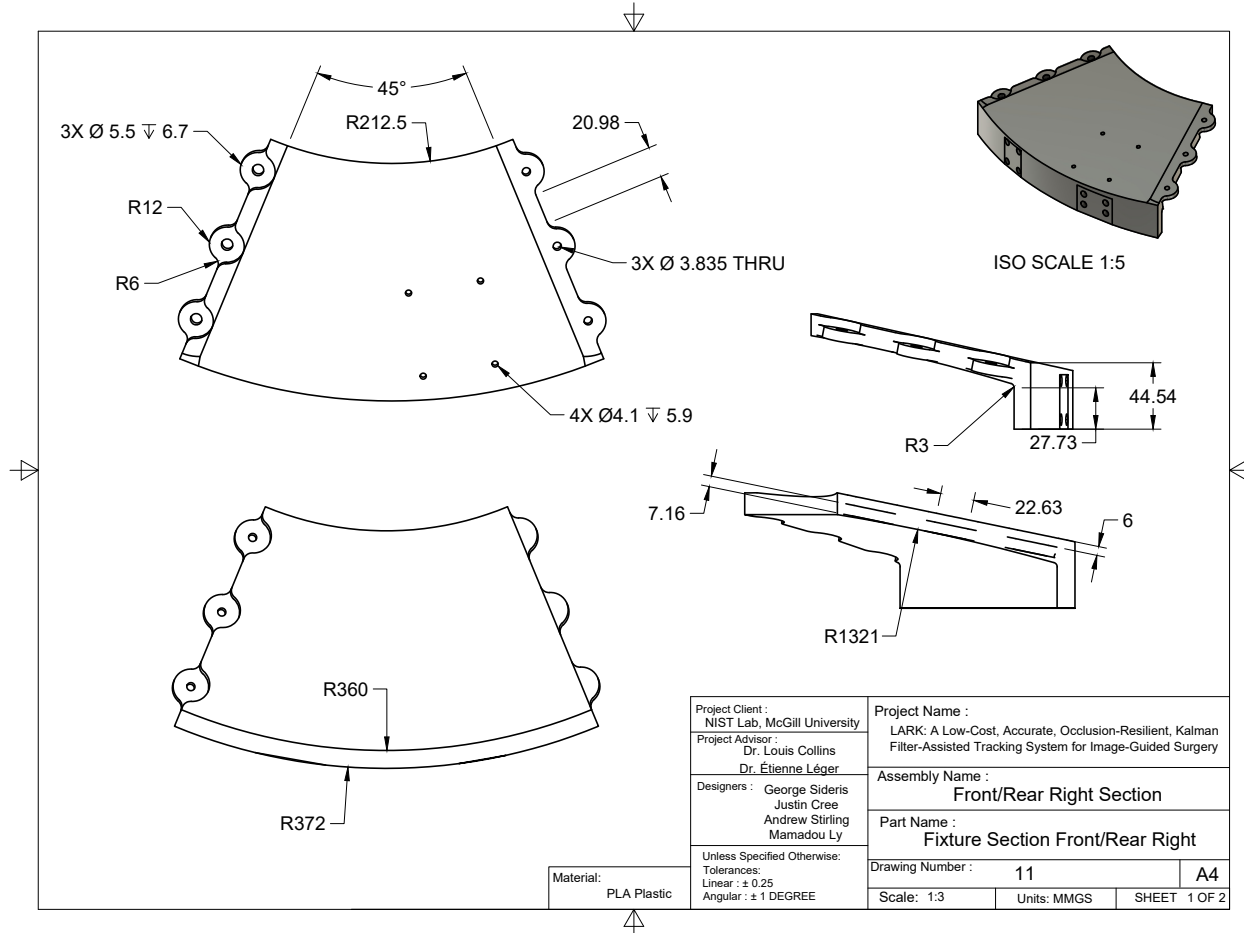
Supplementary Figure 10: Camera Mount Top Technical Drawing.



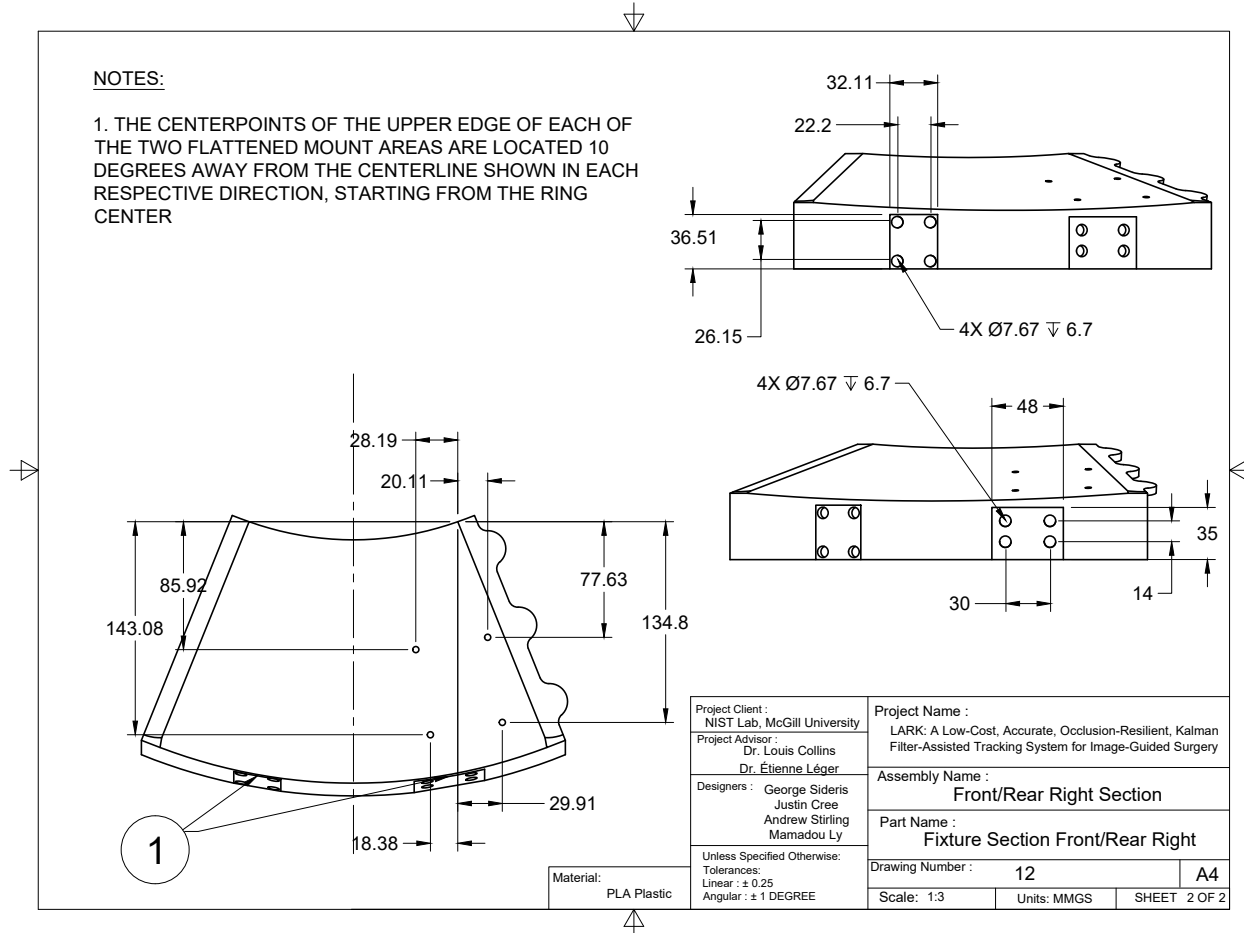
Supplementary Figure 11: Camera Mount Bottom Technical Drawing.



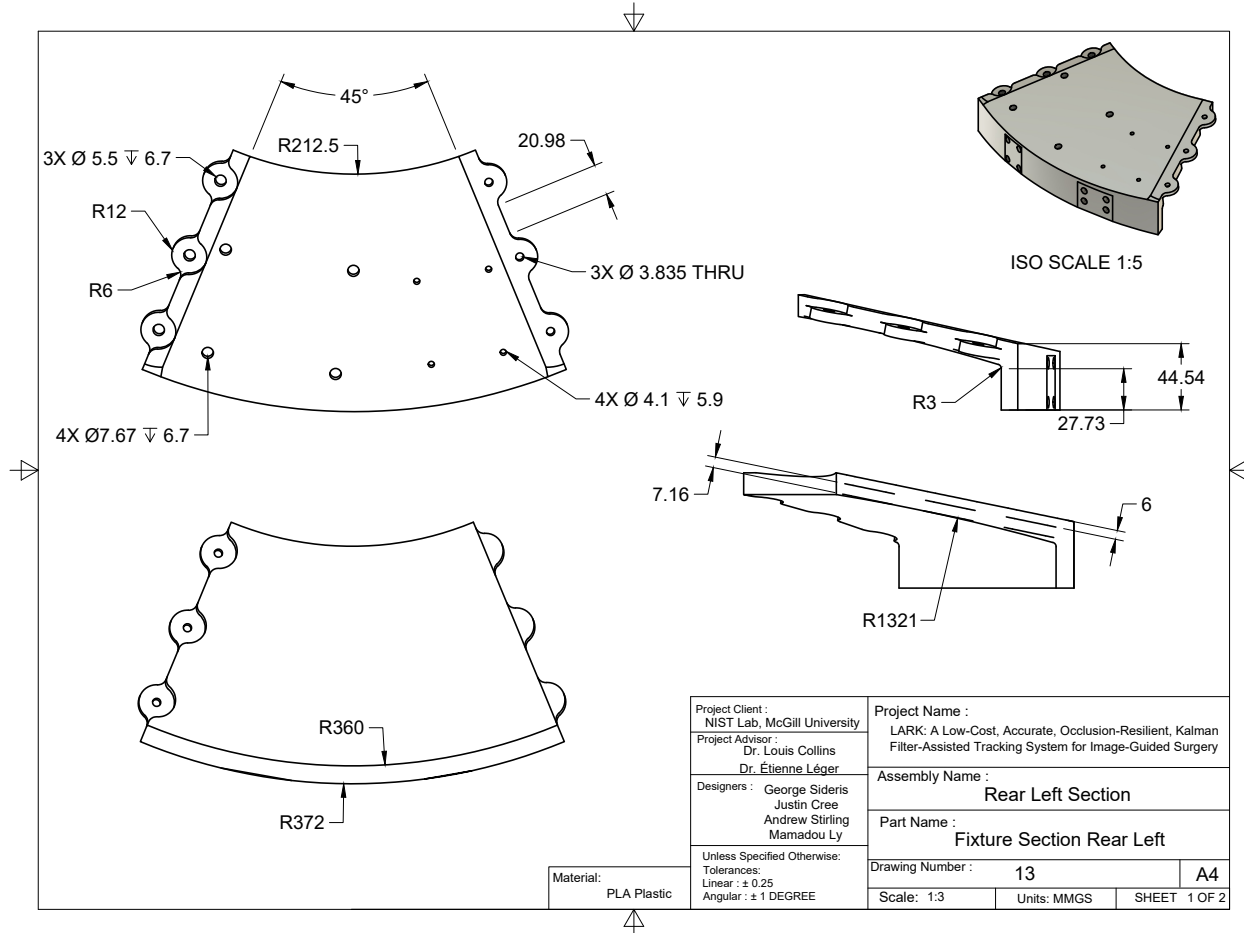
Supplementary Figure 12: Fixture Section Rear Technical Drawing.



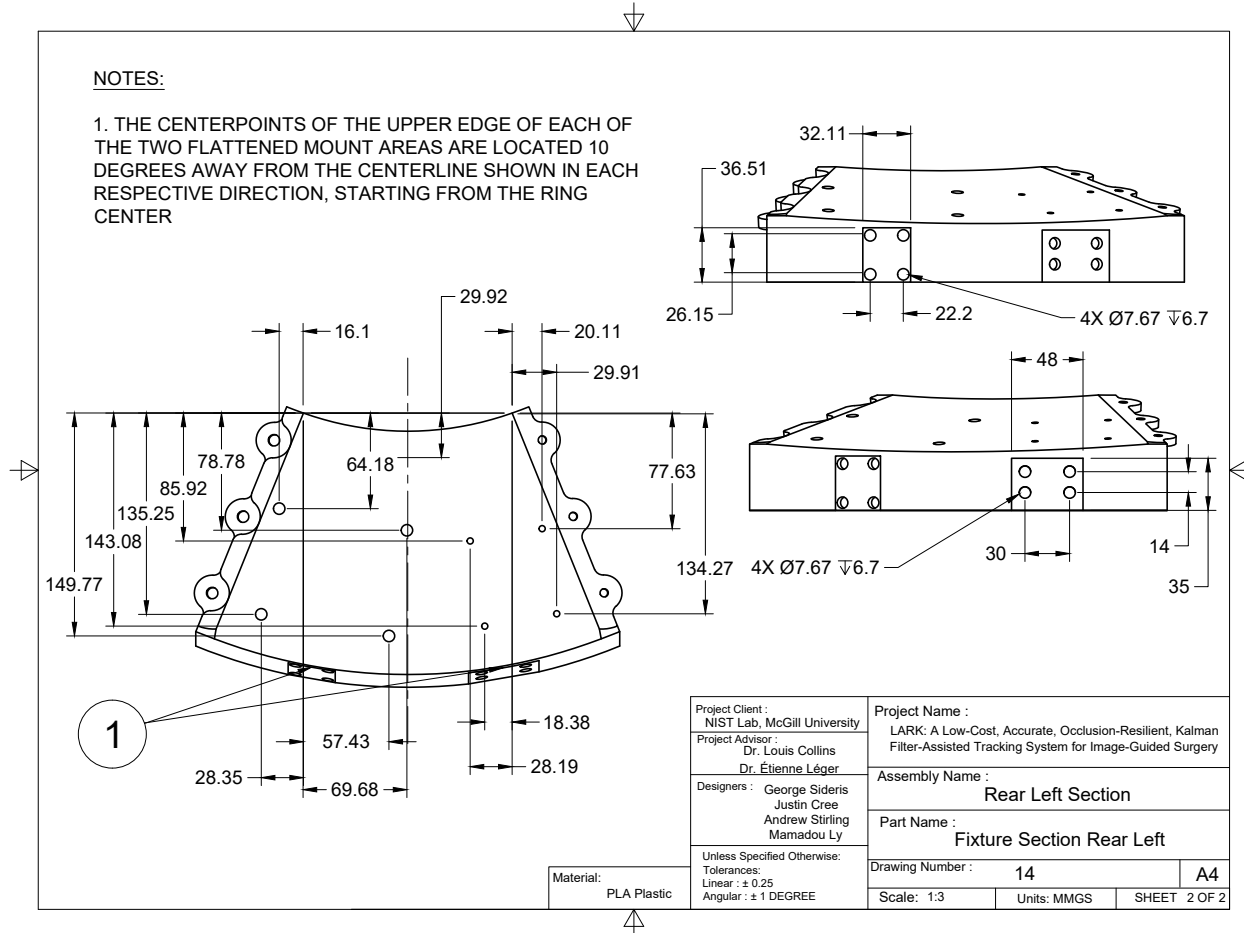
Supplementary Figure 13: Fixture Section Front/Rear Right (1/2) Technical Drawing.



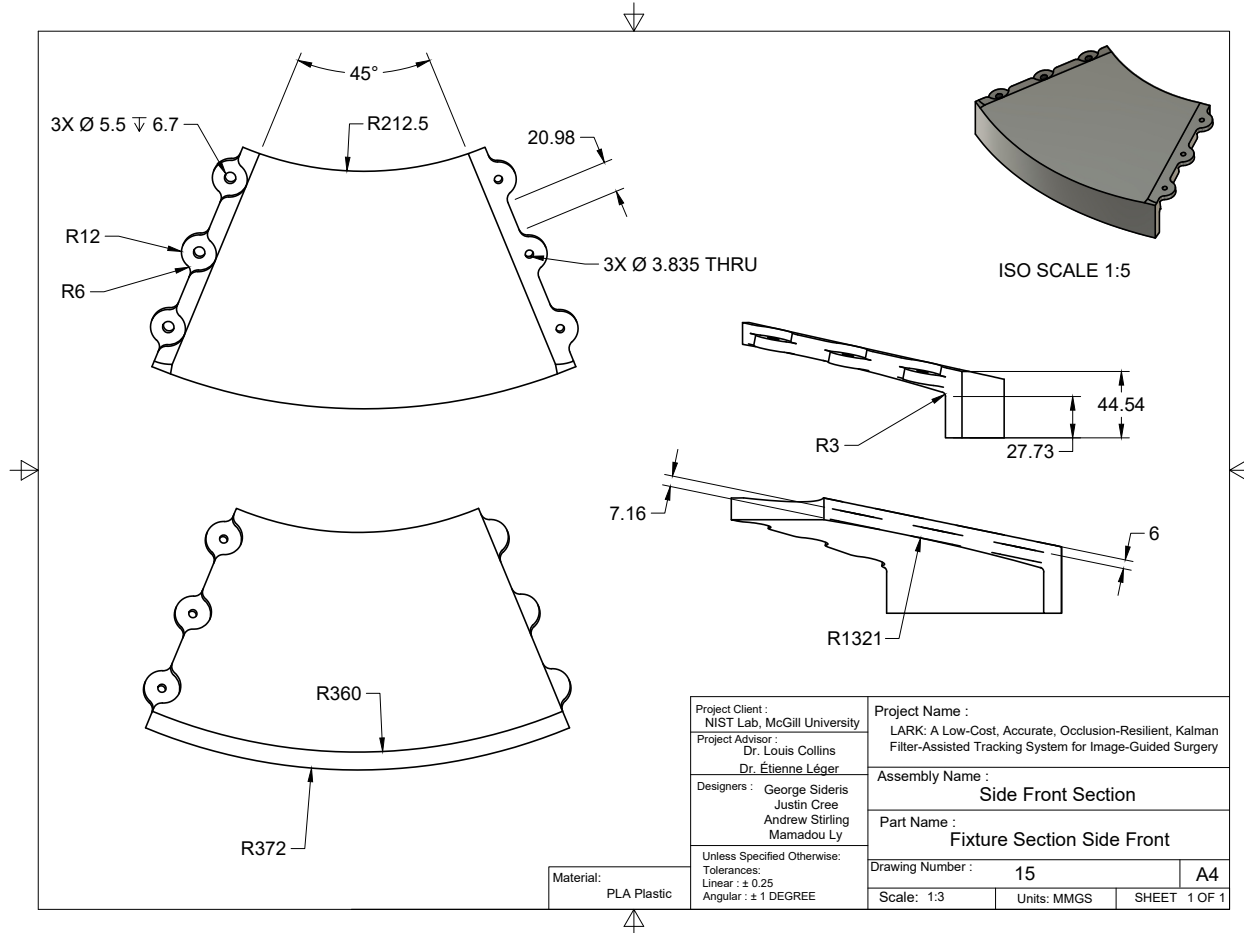
Supplementary Figure 14: Fixture Section Front/Rear Right (2/2) Technical Drawing.



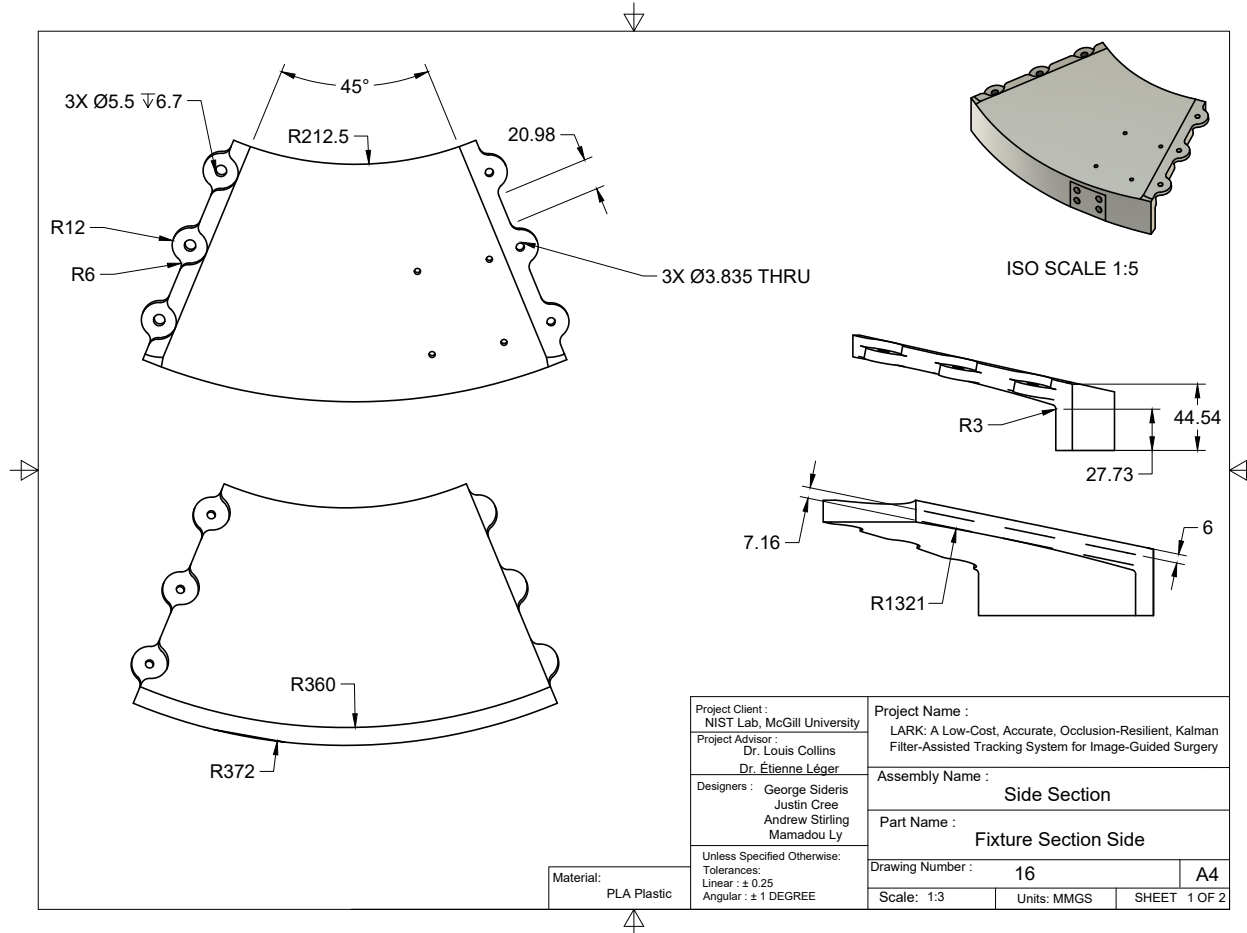
Supplementary Figure 15: Fixture Section Rear Left (1/2) Technical Drawing.



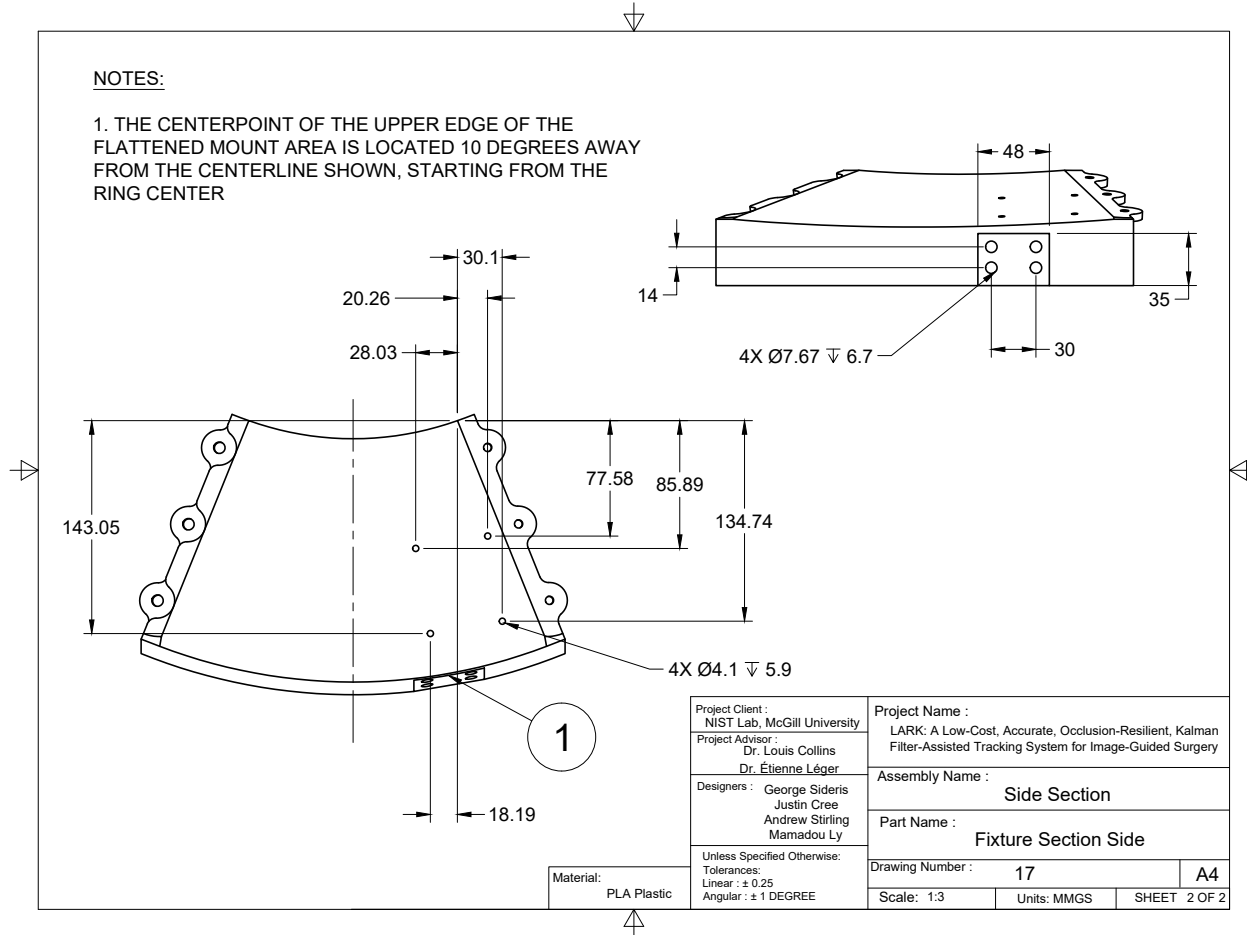
Supplementary Figure 16: Fixture Section Rear Left (2/2) Technical Drawing.



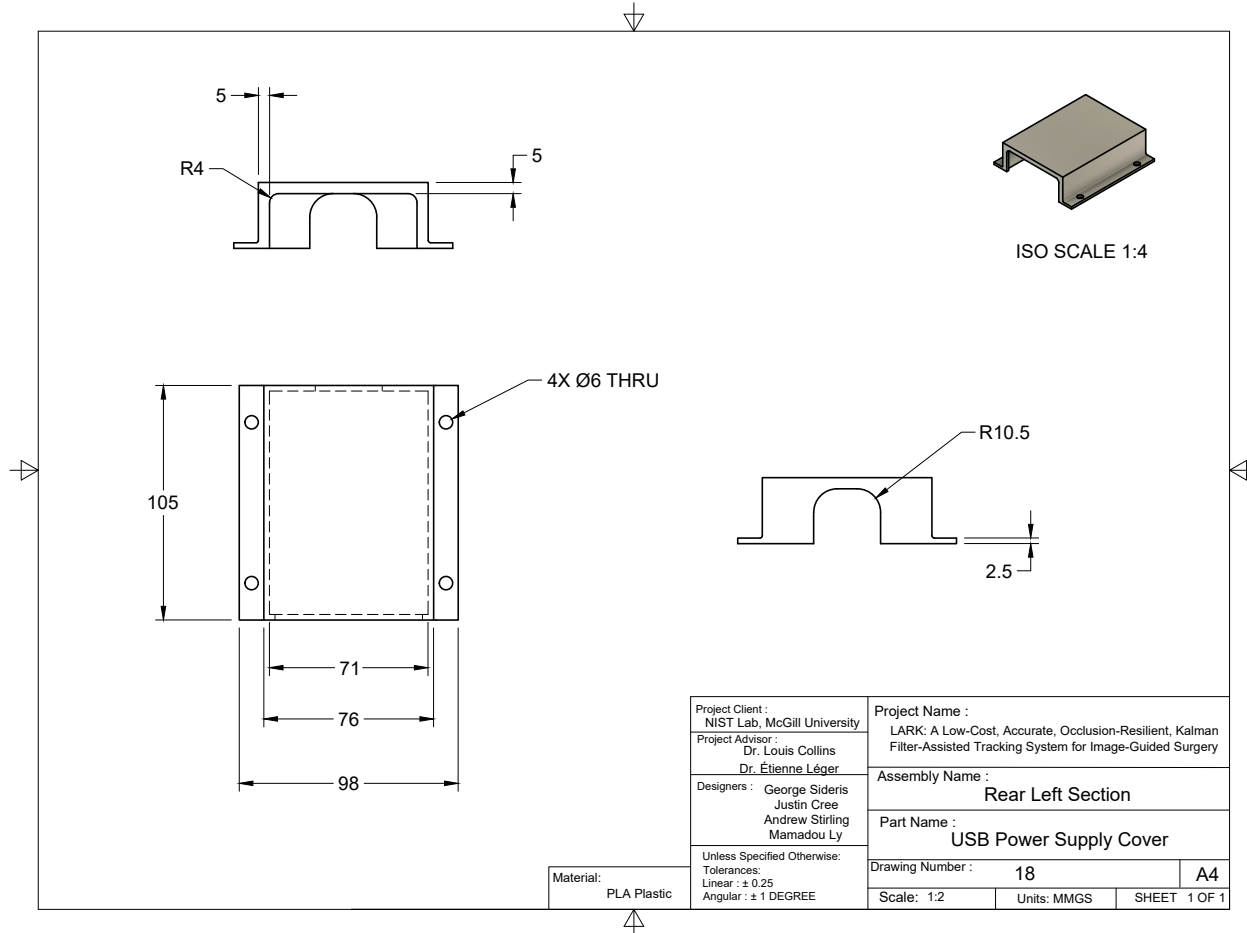
Supplementary Figure 17: Fixture Section Side Front Technical Drawing.



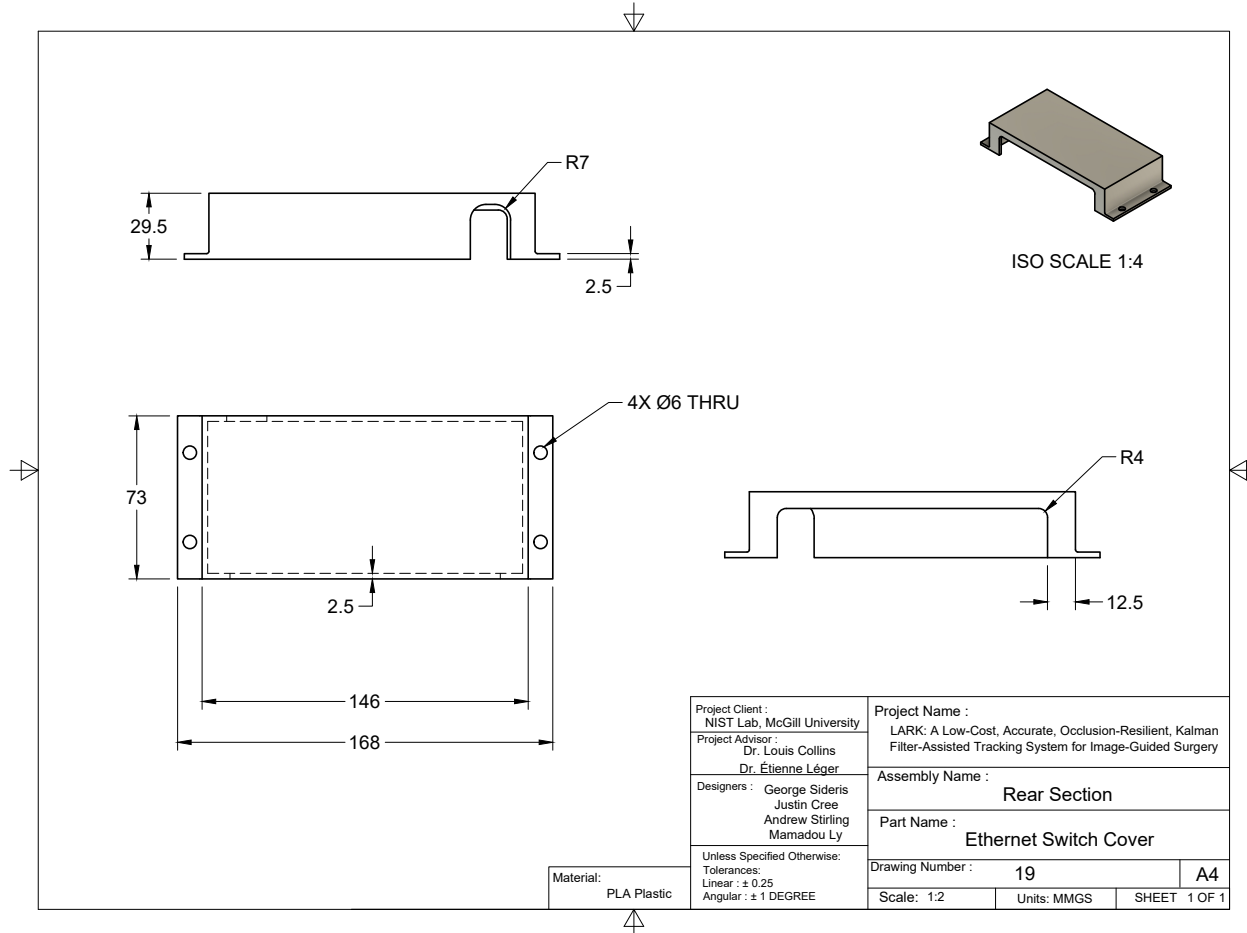
Supplementary Figure 18: Fixture Section Side (1/2) Technical Drawing.



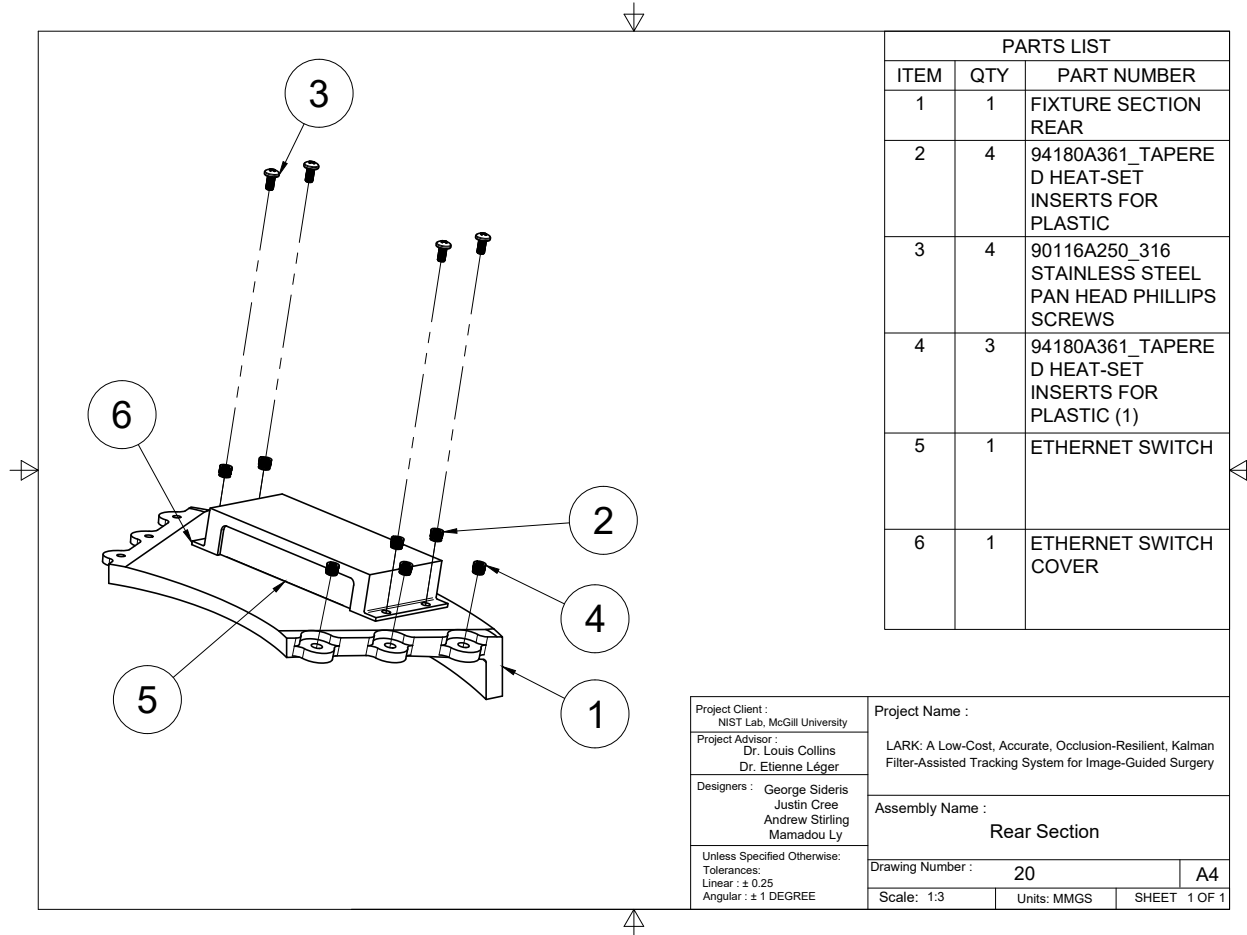
Supplementary Figure 19: Fixture Section Side (2/2) Technical Drawing.



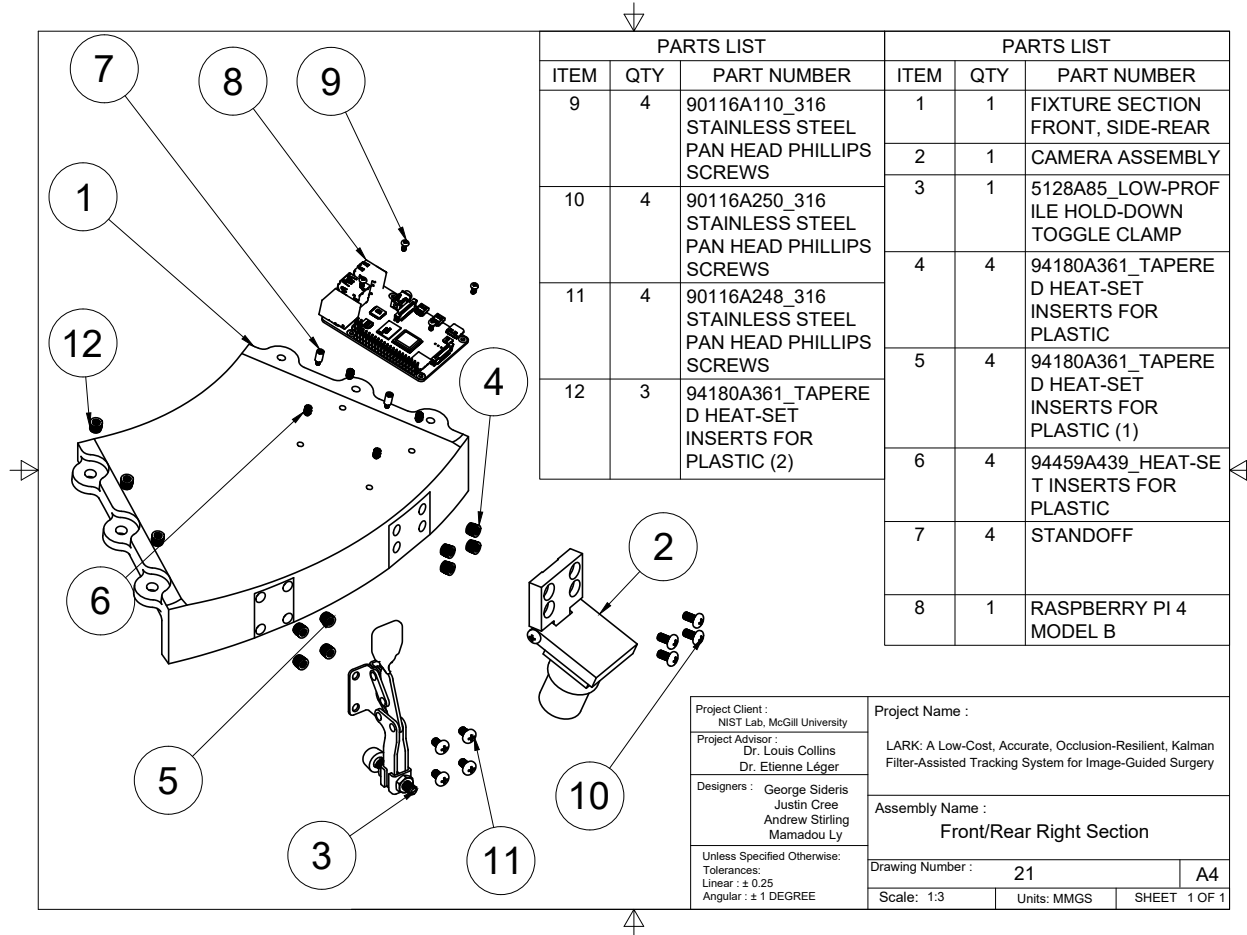
Supplementary Figure 20: USB Power Supply Cover Technical Drawing.



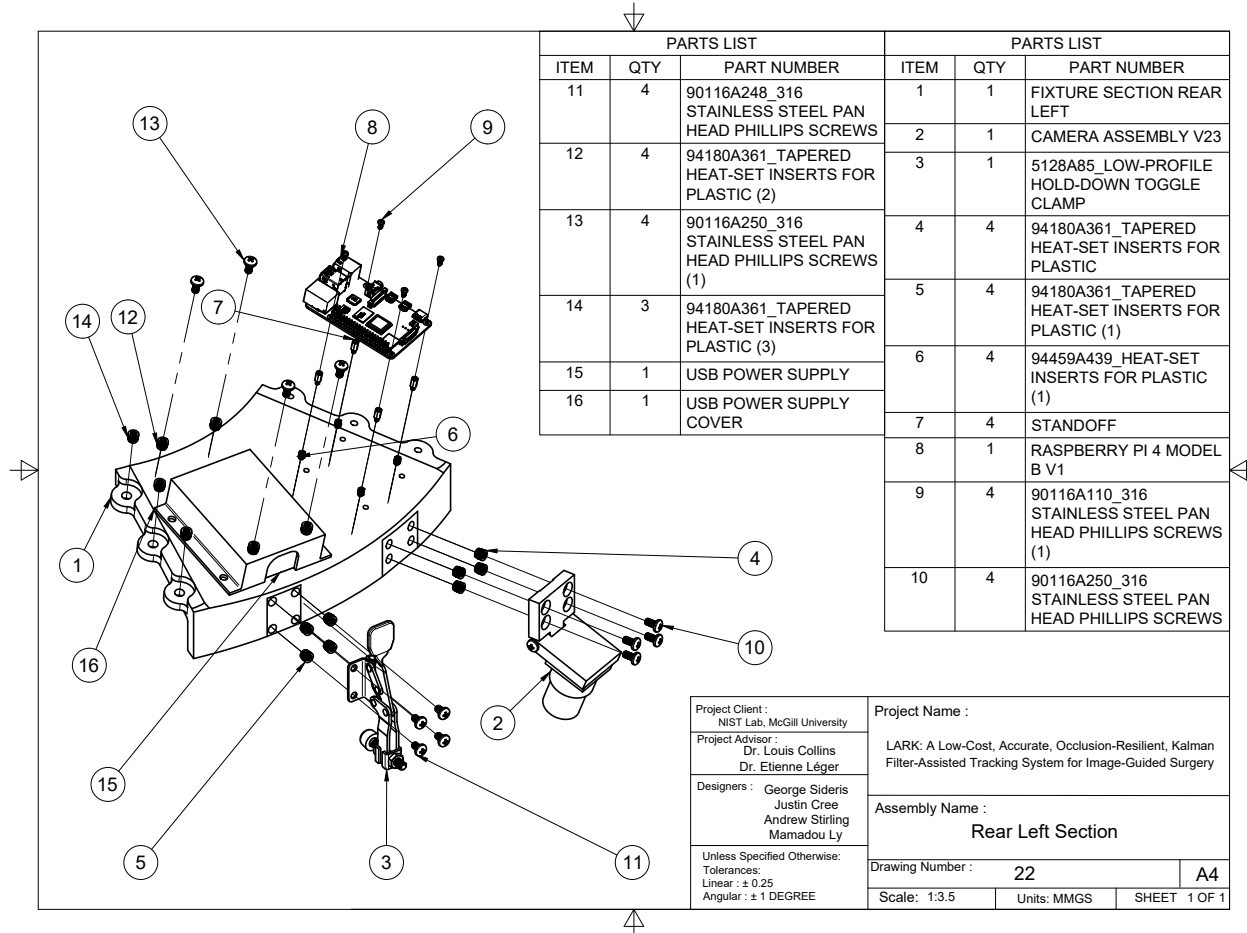
Supplementary Figure 21: Ethernet Switch Cover Technical Drawing.



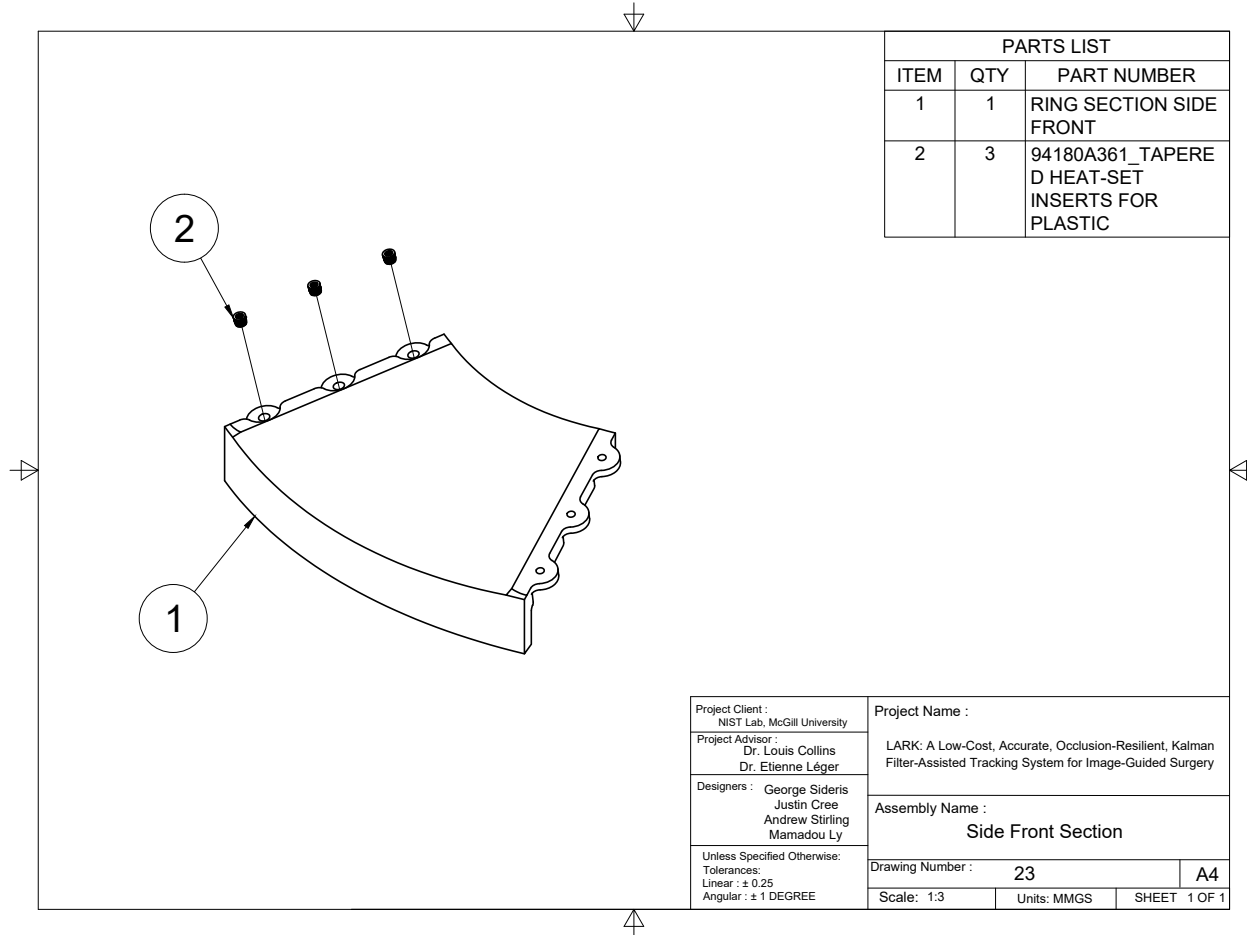
Supplementary Figure 22: Rear Section Assembly Drawing.



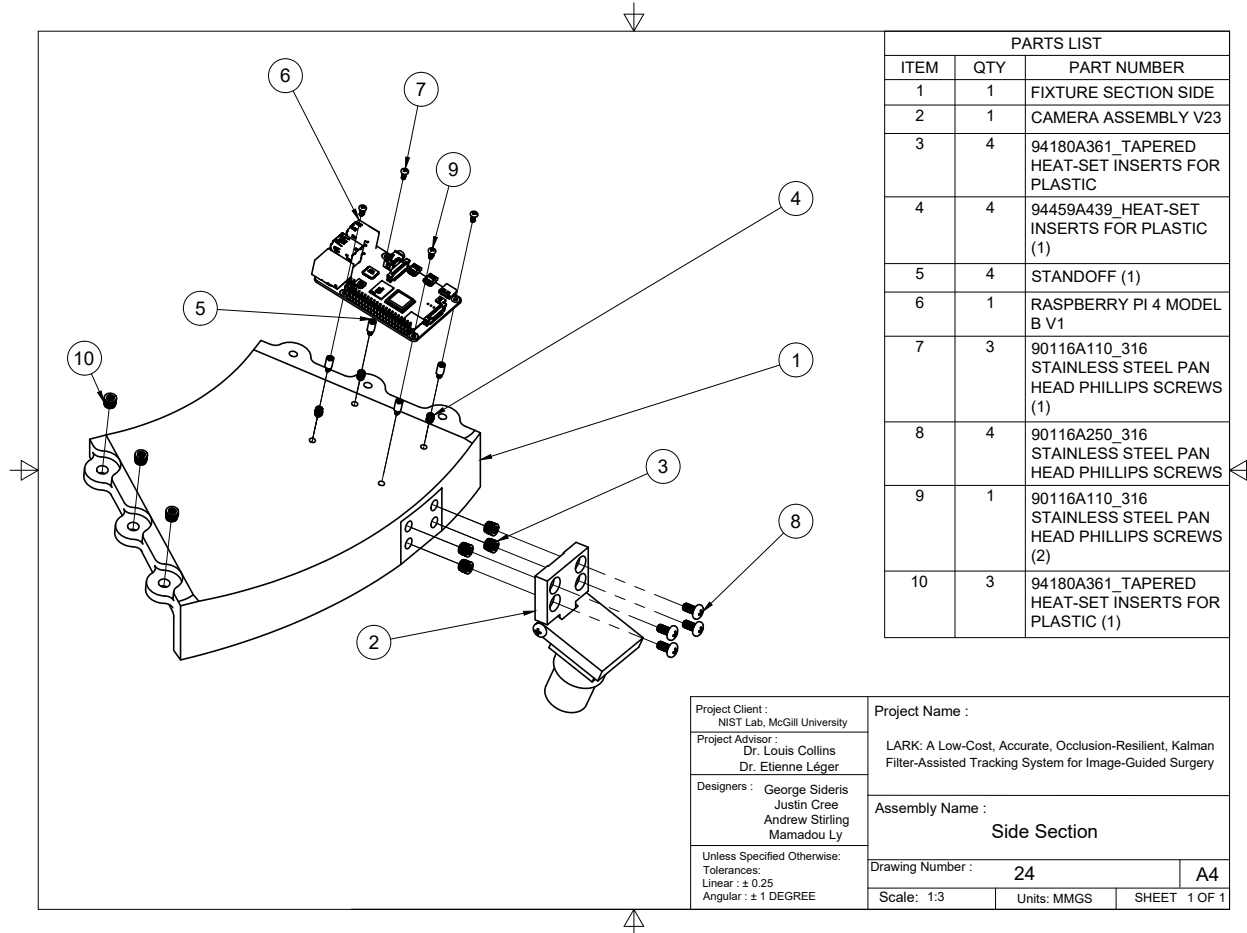
Supplementary Figure 23: Front/Rear Right Section Assembly Drawing.



Supplementary Figure 24: Rear Left Section Assembly Drawing.



Supplementary Figure 25: Side Front Section Assembly Drawing.



Supplementary Figure 26: Side Section Assembly Drawing.

References

- [39] Fitzpatrick, J.; West, J.; Maurer, C. Predicting error in rigid-body point-based registration. *IEEE Transactions on Medical Imaging* **1998**, *17*, 694–702. <https://doi.org/10.1109/42.736021>.