Electronic Supplementary Material

for the article

"*Natural ventilation of a room-atrium building with opposing wind: a deterministic and stochastic analysis*"

Environmental Fluid Mechanics

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Overshoot during the transient

Before reaching the steady state, the system may undergo a transient characterised by overshoots (previously identified for a single room by Kaye & Hunt [1] and Coomaraswamy & Caulfield [2]). The magnitude of the overshoots is affected by the presence of an atrium. In the following, the transient of a room-atrium system is displayed, focusing on an atrium geometry that enhances the ventilation (compared to the single room) for any value of W. The magnitude of the overshoots depends on the initial conditions and on the strength of the incoming opposing wind.

Since the overshoots occur when the system moves toward a steady state, the initial condition (at $\tau = 0$) differs from it. To this aim, the room and the atrium are assumed to be completely filled with ambient air (no buoyant fluid) for $\tau < 0$; then, at $\tau = 0$ the buoyancy source in the room is activated and a thin buoyant layer is formed instantaneously both in the room and in the atrium. Their density are slightly different from that of ambient air, so that the reduced gravities are small. In the no-wind case ($W = 0$), at $\tau = 0$ the system is governed by Regime $\mathcal{A}$ (the flow is forward as there is not an external wind, and there is a stratification). Conversely, if $W > 0$, at $\tau = 0$ the system is governed by Regime $\mathcal{B}$. Since in the atrium the buoyant layer is very thin (ζ_a is large) and its reduced gravity δ_a is small, the non dimensional pressure difference $\Delta\hat{P}_a = \delta_a(\hat{H}_a - \zeta_a)$ between the interior and the exterior

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of the atrium turns out to be very small, and so is the pressure difference $\Delta\hat{P}_r = \delta_r(1-\zeta_r)$. Therefore, any wind strength $W > \Delta\hat{P}_a + \Delta\hat{P}_r \simeq 0$ overcomes the stack effect and the flow is reverse (cf. Eq. (2.23) of the main text). For $\tau \geq 0$, the buoyancy source feeds and heats the buoyant layer of the room, so that its depth increases (ζ_r decreases) and the reduced gravity increases over time. Similarly, ζ_a decreases and δ_a increases. In this way, the pressure differences $\Delta\hat{P}_r$ and $\Delta\hat{P}_a$ increases over time.

Following Economidou [3], we use the experimental measures performed by Kaye & Hunt [1] to set the initial conditions at $\tau = 0$, i.e. when the source is activated and a thin buoyant layer is formed instantaneously. In dimensional form, the buoyant layer depth in the room is equal to the radius of the plume at the ceiling, i.e. $H_r 6\alpha/5$, and its reduced gravity is equal to that of the plume at the ceiling, i.e. $C^{-1}B^{2/3}H_r^{-5/3}$. In the atrium, the buoyant layer depth is equal to the radius of the plume a (cf. Fig. 1 in the main text) at the top of the atrium. The radius of the plume a in the atrium is assumed to be equal to the sum of the radius r_{H_r} of opening II (the opening connecting the room and the atrium, see Fig. 1 in the main text) and the so-called 'plume spread' $(H_a - H_r)6\alpha/5$ due to ambient air entrainment. The reduced gravity in the atrium is equal to the reduced gravity of the room buoyant layer. It follows that the initial conditions in non-dimensional form for the room read

$$\zeta_r(\tau = 0) = 1 - \frac{6\alpha}{5}, \quad \delta_r(\tau = 0) = 1, \tag{1}$$

while for the atrium

$$\zeta_a(\tau = 0) = \hat{H}_a - \left(\frac{r_{H_r}}{H_r} + \frac{6\alpha}{5}(\hat{H}_a - 1)\right), \quad \delta_a(\tau = 0) = 1. \tag{2}$$

We consider a room and an atrium of identical cross-sectional area (i.e., $\hat{S}_a = 1$) and identical effective opening area (i.e., $\hat{A}_a^* = 1$), and size $\mu = 5$. The non-dimensional atrium height is $\hat{H}_a = 2$. The ratio between the radius of opening II and the height of the room is $r_{H_r}/H_r = 0.08$.
In order to compare the transients dynamics and the overshoots magnitude in the two cases (room-atrium and single-room systems), Eqs. (2.22) and Eqs. (3.6) of the main text are solved using an algorithm based on Runge-Kutta formula. Some relevant examples of the time series of ζ_r, δ_r, q_r and ζ_s, δ_s, q_s are shown in Fig. S1, for different values of W.

When $W = 0$ (red curves in Fig. S1), the system room-atrium remains in stratified forward flow regime (Regime $\mathcal{A}$) during the whole transient. The interface height ζ_r (see red continuous line in panel S1a) and the reduced gravity δ_r (see panel S1b) undergo a small overshoot before reaching the steady state. From the initial condition, ζ_r descends to a minimum value and then increases to attain the equilibrium. The reduced gravity (panel S1b) increases from the starting value, it reaches a maximum and then decreases to reach the steady state. The flow rate q_r (cf. panel S1c) reaches the equilibrium value without any overshoot occurring. In this first examined case ($W = 0$), the behaviour of the single room slightly differs from that of the room-atrium system. Also the single room remains in stratified forward flow (Regime $\mathcal{A}'$) during the whole transient. The interface height (see the red dashed line in panel S1a) is subjected to an overshoot (smaller than in case of room-atrium). Conversely to the case room-atrium, the reduced gravity and the flow rate reach directly the equilibrium.

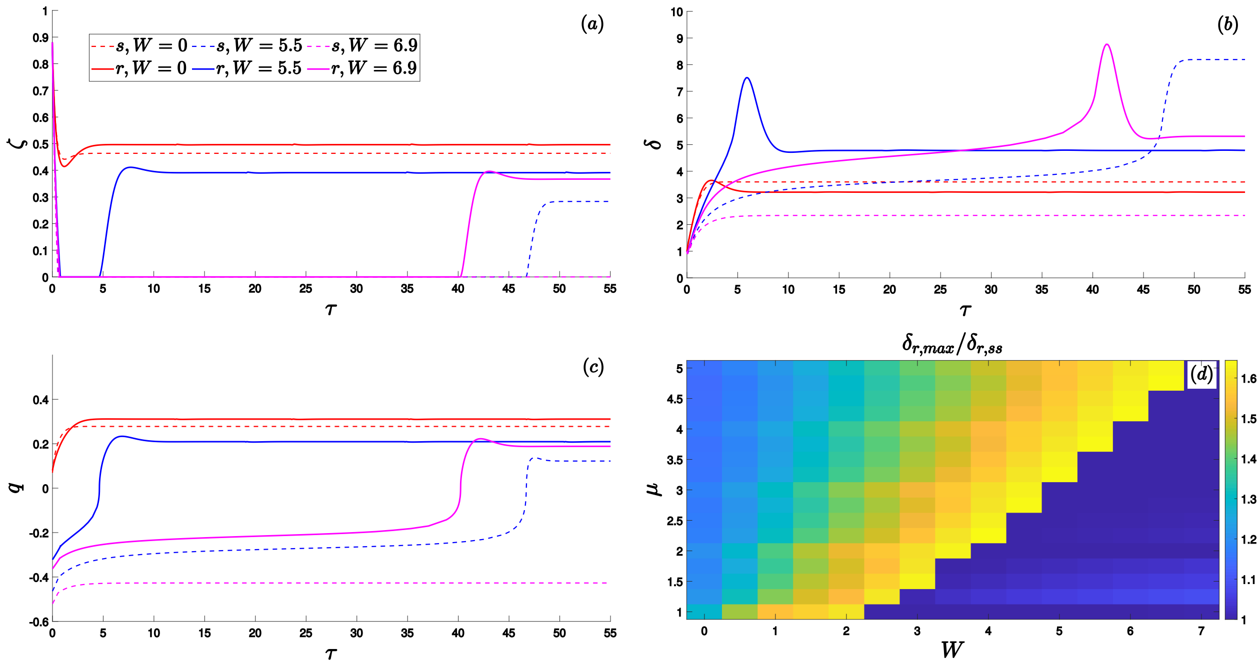

Figure S1: Time series of interface height (a), reduced gravity (b) and flow rate (c), for different values of W. In each panel, continuous lines correspond to the system room-atrium, dashed lines to a single room. Panel (d) shows the overshoot of the reduced gravity in the room linked to an atrium, namely $\delta_{r,max}/\delta_{r,0}$, where $\delta_{r,max}$ is the highest value reached while $\delta_{r,0}$ is the value reached at the equilibrium. When $\delta_{r,max}/\delta_{r,0} = 1$, no overshoot occurs. Overshoots are shown as a function of both the wind parameter W (on the x-axis) and the venting parameter μ (on the y-axis).

The room-atrium system reaches the equilibrium slightly later than the single room.

As W increases, at $\tau = 0$ the systems are governed by Regime $\mathcal{B}$ or $\mathcal{B}'$ and move immediately to the well-mixed reverse flow regime (Regime $\mathcal{C}$ or $\mathcal{C}'$). We consider $W = 5.5$ (slightly lower than $W_{s,crit} \simeq 5.53$) to investigate the dynamics of the single-room system when it is close to the bifurcation condition. As $W = 5.5$ is within the region in which only steady states governed by Regime $\mathcal{A}$ and $\mathcal{A}'$ exist, both the room-atrium and the single-room systems reach the buoyancy-driven equilibrium (see blue curves in Fig. S1). The interface height ζ_r in the room connected to an atrium (see blue continuous curves in panel S1a) undergoes a sudden increment during the transition from Regime $\mathcal{C}$ to $\mathcal{A}$; this increment leads ζ_r to its maximum value, and then to the equilibrium value that is slightly lower. It is noteworthy the behaviour of the reduced gravity: δ_r exhibits a great peak before reaching the steady state (cf. panel S1b). The peak occurs during the transition from Regime $\mathcal{C}$ to $\mathcal{A}$, thus when the flow rate q_r changes sign. After that, δ_r stabilises on the equilibrium value. The change of flow direction takes place in a very short time (see panel S1c), causing a small overshoot for q_r. The behaviour of a single room during the transient is different. The single room reaches the forward flow regime and the equilibrium later (cf. the blue continuous lines and the blue dashed lines in panels S1a, b, c). Moreover, the single-room variables undergo overshoots of smaller magnitude. As $\zeta_{s,0} < \zeta_{r,0}$, ζ_s undergoes a smaller increment during the transition from Regime $\mathcal{C}'$ to $\mathcal{A}'$; δ_s does not exhibit any peak, but it reaches the steady state almost immediately after the transition; q_s behaves similarly to q_r.

To investigate the dynamics of the room-atrium system close to the bifurcation condition, we analyse the case $W = 6.9$ (slightly lower than $W_{r,crit} \simeq 6.96$). Since $W < W_{r,crit}$, the system still can be attracted only by the equilibrium state of Regime $\mathcal{A}$. The variables of the room-atrium system (magenta continuous curves in S1) exhibit the same qualitative behaviour shown for $W = 5.5$, but the equilibrium state is reached later. The interface height ζ_r has a sudden increment when q_r changes sign (smaller than for $W = 5.5$ as the equilibrium value is lower, cf. continuous magenta line in panel S1a); the reduced gravity δ_r shows a great peak during the transition from Regime $\mathcal{C}$ to $\mathcal{A}$ (see panel S1b), bigger than in case of $W = 5.5$; the flow rate q_r shows a small overshoot (see panel S1c). Conversely, the single room is attracted by the equilibrium governed by Regime $\mathcal{C}'$, as $W > W_{s,crit}$. As soon as the opposing wind is activated, the ventilation of the single room moves immediately to the well-mixed reverse configuration (see dashed magenta lines in panels S1a, b, c).

From a physical point of view, overshoots are caused by a steep gradient of the flow rate that produces a sudden change of interface height and reduced gravity. For this reason, overshoots become much greater when the system undergoes a transition from reverse (Regimes $\mathcal{C}, \mathcal{C}'$) to forward (Regimes $\mathcal{A}, \mathcal{A}'$) flow. In this case, the flow rate switches sign with an almost vertical temporal gradient. The overshoots of ζ, δ and q in the room-atrium system are larger than for a single room. This means that during the transient, the natural ventilation of the system is worsened by the presence of the atrium; however, once reached the equilibrium, the atrium improves the ventilation. To understand the relation between the overshoots and the transition from reverse to forward flow, we analyse the peaks exhibited by the reduced gravity in the room connected to an atrium (see continuous blue and magenta lines in panel S1b). When the wind overcomes completely the stack effect at $\tau = 0$, the flow is reverse and the buoyancy source keeps heating the air in the room. After some time, the buoyancy force inside the room becomes strong enough to overcome the wind force. Thus, the flow rate changes direction and a stratification arises. Due to the stratification, an immediate density difference between the buoyant and the cold layer takes place, increasing instantaneously the reduced gravity of the room. At this stage, the buoyant air starts flowing out through opening II (the opening connecting the room to the atrium, see Fig. 1 in the main text) and accumulating in the atrium. This outflow of buoyancy causes a reduction of the reduced gravity in the room, which decreases until it attains the steady value. Differently, in case of a single room, the reduced gravity does not exhibit such a peak because once the system moves to regime $\mathcal{A}$, the buoyant fluid accumulates only in the room and cannot split (as it was the case in presence of the atrium). Thus, the reduced gravity does not decrease.

Panel S1d shows the magnitude of the peaks of δ_r, expressed as $\delta_{r,max}/\delta_{r,0}$, where $\delta_{r,max}$ is the highest value reached and $\delta_{r,0}$ is the value reached at the equilibrium. The larger $\delta_{r,max}/\delta_{r,0}$ is, the larger the peak is. If $\delta_{r,max}/\delta_{r,0} = 1$, no overshoot occurs; it is the case in which the system remains in the well-mixed state and attains the equilibrium of regime $\mathcal{C}$. The magnitude of the peaks depends on the value of W and the venting parameter μ. It increases as W increases, until W is great enough, so that wind-driven steady state is reached. Panel S1d shows that this threshold value of W increases fro growing μ.

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References

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